College Physics
Textbook Equity Edition

Volume 2 of 3: Chapters 13 - 24

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This is volume 2of 3, covering c**hapters 12 - 24**, plus the compete table of contents, appendices and index. Page numbers refer to the original 1269 page textbook pbuslished by OpenStax College as noted below. As a courtesy the original front matter, with some additional informate, and the preface are included in this edition.

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Orignial Front Matter

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6100 Main Street MS-380 Houston, Texas 77005

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College Physics Textbook Equity Editions Chapter Headings by Volume

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PREFACE

Textbook Equity* (textbookequity.org) provides high quality academic content and free access to any learner at any location, at the minim um possible cost. Education of everyone is the only true means to eliminate proverty and oppression. Creating barriers to education, whether intentio nal or not, by policy or price, leaves millions of geniuses undiscovered. We want them discovered. ***"Fearlessly copy, print, remix."tm**

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About This Book

Welcome to *College Physics*, an OpenStax College resource created with several goals in mind: accessibility, affordability, customization, and student engagement—all while encouraging learners toward high levels of learning. Instructors and students alike will find that this textbook offers a strong foundation in introductory physics, with algebra as a prerequisite. It is available for free online and in low-cost print and e-book editions.

To broaden access and encourage community curation, College Physics is “open source” licensed under a Creative Commons Attribution (CC-BY) license. Everyone is invited to submit examples, emerging research, and other feedback to enhance and strengthen the material and keep it current and relevant for today’s students. You can make suggestions by contacting us at info@openstaxcollege.org. You can find the status of the project, as well as alternate versions, corrections, etc., on the StaxDash at **http://openstaxcollege.org (http://openstaxcollege.org)** .

To the Student

This book is written for you. It is based on the teaching and research experience of numerous physicists and influenced by a strong recollection of their own struggles as students. After reading this book, we hope you see that physics is visible everywhere. Applications range from driving a car to launching a rocket, from a skater whirling on ice to a neutron star spinning in space, and from taking your temperature to taking a chest X-ray.

To the Instructor

This text is intended for one-year introductory courses requiring algebra and some trigonometry, but no calculus. OpenStax College provides the essential supplemental resources at http://openstaxcollege.org ; however, we have pared down the number of supplements to keep costs low. College Physics can be easily customized for your course using Connexions (http://cnx.org/content/col11406). Simply select the content most relevant to your curriculum and create a textbook that speaks directly to the needs of your class.

General Approach

College Physics is organized such that topics are introduced conceptually with a steady progression to precise definitions and analytical applications. The analytical aspect (problem solving) is tied back to the conceptual before moving on to another topic. Each introductory chapter, for example, opens with an engaging photograph relevant to the subject of the chapter and interesting applications that are easy for most students to visualize.

Organization, Level, and Content

There is considerable latitude on the part of the instructor regarding the use, organization, level, and content of this book. By choosing the types of problems assigned, the instructor can determine the level of sophistication required of the student.

Concepts and Calculations

The ability to calculate does not guarantee conceptual understanding. In order to unify conceptual, analytical, and calculation skills within the learning process, we have integrated Strategies and Discussions throughout the text.

Modern Perspective

The chapters on modern physics are more complete than many other texts on the market, with an entire chapter devoted to medical applications of nuclear physics and another to particle physics. The final chapter of the text, “Frontiers of Physics,” is devoted to the most exciting endeavors in physics. It ends with a module titled “Some Questions We Know to Ask.”

Supplements (Not necessarily endorsed by Textbook Equity)

Accompanying the main text are a **Student Solutions Manual and an Instructor Solutions Manual (http://openstaxcollege.org/textbooks/college-physics)** . The Student Solutions Manual provides worked-out solutions to select end-of-module Problems and Exercises. The Instructor Solutions Manual provides worked-out solutions to all Exercises.

Features of OpenStax *College Physics*

The following briefly describes the special features of this text.

Modularity

This textbook is organized on Connexions (http://cnx.org) as a collection of modules that can be rearranged and modified to suit the needs of a particular professor or class. That being said, modules often contain references to content in other modules, as most topics in physics cannot be discussed in isolation.

Learning Objectives

Every module begins with a set of learning objectives. These objectives are designed to guide the instructor in deciding what content to include or assign, and to guide the student with respect to what he or she can expect to learn. After completing the module and end-of-module exercises, students should be able to demonstrate mastery of the learning objectives.

Call-Outs

Key definitions, concepts, and equations are called out with a special design treatment. Call-outs are designed to catch readers' attention, to make it clear that a specific term, concept, or equation is particularly important, and to provide easy reference for a student reviewing content.

Key Terms

Key terms are in bold and are followed by a definition in context. Definitions of key terms are also listed in the Glossary, which appears at the end of the module.

Worked Examples

Worked examples have four distinct parts to promote both analytical and conceptual skills. Worked examples are introduced in words, always using some application that should be of interest. This is followed by a Strategy section that emphasizes the concepts involved and how solving the problem relates to those concepts. This is followed by the mathematical Solution and Discussion.

Many worked examples contain multiple-part problems to help the students learn how to approach normal situations, in which problems tend to have multiple parts. Finally, worked examples employ the techniques of the problem-solving strategies so that students can see how those strategies succeed in practice as well as in theory.

Problem-Solving Strategies

Problem-solving strategies are first presented in a special section and subsequently appear at crucial points in the text where students can benefit most from them. Problem-solving strategies have a logical structure that is reinforced in the worked examples and supported in certain places by line drawings that illustrate various steps.

Misconception Alerts

Students come to physics with preconceptions from everyday experiences and from previous courses. Some of these preconceptions are misconceptions, and many are very common among students and the general public. Some are inadvertently picked up through misunderstandings of lectures and texts. The Misconception Alerts feature is designed to point these out and correct them explicitly.

Take-Home Investigations

Take Home Investigations provide the opportunity for students to apply or explore what they have learned with a hands-on activity.

Things Great and Small

In these special topic essays, macroscopic phenomena (such as air pressure) are explained with submicroscopic phenomena (such as atoms bouncing off walls). These essays support the modern perspective by describing aspects of modern physics before they are formally treated in later chapters. Connections are also made between apparently disparate phenomena.

Simulations

Where applicable, students are directed to the interactive PHeT physics simulations developed by the University of Colorado (**http://phet.colorado.edu (http://phet.colorado.edu)**). There they can further explore the physics concepts they have learned about in the module.

Summary

Module summaries are thorough and functional and present all important definitions and equations. Students are able to find the definitions of all terms and symbols as well as their physical relationships. The structure of the summary makes plain the fundamental principles of the module or collection and serves as a useful study guide.

Glossary

At the end of every module or chapter is a glossary containing definitions of all of the key terms in the module or chapter.

End-of-Module Problems

At the end of every chapter is a set of Conceptual Questions and/or skills-based Problems & Exercises. Conceptual Questions challenge students' ability to explain what they have learned conceptually, independent of the mathematical details. Problems & Exercises challenge students to apply both concepts and skills to solve mathematical physics problems. Online, every other problem includes an answer that students can reveal immediately by clicking on a "Show Solution" button. Fully worked solutions to select problems are available in the Student Solutions Manual and the Teacher Solutions Manual.

In addition to traditional skills-based problems, there are three special types of end-of-module problems: Integrated Concept Problems, Unreasonable Results Problems, and Construct Your Own Problems. All of these problems are indicated with a subtitle preceding the problem.

Integrated Concept Problems

In Unreasonable Results Problems, students are challenged not only to apply concepts and skills to solve a problem, but also to analyze the answer with respect to how likely or realistic it really is. These problems contain a premise that produces an unreasonable answer and are designed to further emphasize that properly applied physics must describe nature accurately and is not simply the process of solving equations.

Unreasonable Results

In Unreasonable Results Problems, students are challenged to not only apply concepts and skills to solve a problem, but also to analyze the answer with respect to how likely or realistic it really is. These problems contain a premise that produces an unreasonable answer and are designed to further emphasize that properly applied physics must describe nature accurately and is not simply the process of solving equations.

Construct Your Own Problem

These problems require students to construct the details of a problem, justify their starting assumptions, show specific steps in the problem's solution, and finally discuss the meaning of the result. These types of problems relate well to both conceptual and analytical aspects of physics, emphasizing that physics must describe nature. Often they involve an integration of topics from more than one chapter. Unlike other problems, solutions are not provided since there is no single correct answer. Instructors should feel free to direct students regarding the level and scope of their considerations. Whether the problem is solved and described correctly will depend on initial assumptions.

Appendices

Appendix A: Atomic Masses
Appendix B: Selected Radioactive Isotopes
Appendix C: Useful Information
Appendix D: Glossary of Key Symbols and Notation

Acknowledgements

This text is based on the work completed by Dr. Paul Peter Urone in collaboration with Roger Hinrichs, Kim Dirks, and Manjula Sharma. We would like to thank the authors as well as the numerous professors (a partial list follows) who have contributed their time and energy to review and provide feedback on the manuscript. Their input has been critical in maintaining the pedagogical integrity and accuracy of the text.

Senior Contributing Authors

Dr. Paul Peter Urone
Dr. Roger Hinrichs, State University of New York, College at Oswego

Contributing Authors

Dr. Kim Dirks, University of Auckland, New Zealand
Dr. Manjula Sharma, University of Sydney, Australia

Expert Reviewers

Erik Christensen, P.E, South Florida Community College
Dr. Eric Kincanon, Gonzaga University
Dr. Douglas Ingram, Texas Christian University
Lee H. LaRue, Paris Junior College
Dr. Marc Sher, College of William and Mary
Dr. Ulrich Zurcher, Cleveland State University
Dr. Matthew Adams, Crafton Hills College, San Bernardino Community College District
Dr. Chuck Pearson, Virginia Intermont College

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Webassign is an independent online homework and assessment system that has been available commercially since 1998. WebAssign has recently begun to support the Open Education Resource community by creating a high quality online homework solution for selected open-source textbooks, available at an affordable price to students. These question collections include randomized values and variables, immediate feedback, links to the open-source textbook, and a variety of text-specific resources and tools; as well as the same level of rigorous coding and accuracy-checking as any commercially available online homework solution supporting traditionally available textbooks.

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13 TEMPERATURE, KINETIC THEORY, AND THE GAS LAWS

Figure 13.1 The welder's gloves and helmet protect him from the electric arc that transfers enough thermal energy to melt the rod, spray sparks, and burn the retina of an unprotected eye. The thermal energy can be felt on exposed skin a few meters away, and its light can be seen for kilometers. (credit: Kevin S. O'Brien/U.S. Navy)

Learning Objectives

13.1. Temperature
13.2. Thermal Expansion of Solids and Liquids
13.3. The Ideal Gas Law
13.4. Kinetic Theory: Atomic and Molecular Explanation of Pressure and Temperature
13.5. Phase Changes
13.6. Humidity, Evaporation, and Boiling

Introduction to Temperature, Kinetic Theory, and the Gas Laws

Heat is something familiar to each of us. We feel the warmth of the summer Sun, the chill of a clear summer night, the heat of coffee after a winter stroll, and the cooling effect of our sweat. Heat transfer is maintained by temperature differences. Manifestations of **heat transfer**—the movement of heat energy from one place or material to another—are apparent throughout the universe. Heat from beneath Earth's surface is brought to the surface in flows of incandescent lava. The Sun warms Earth's surface and is the source of much of the energy we find on it. Rising levels of atmospheric carbon dioxide threaten to trap more of the Sun's energy, perhaps fundamentally altering the ecosphere. In space, supernovas explode, briefly radiating more heat than an entire galaxy does.

What is heat? How do we define it? How is it related to temperature? What are heat's effects? How is it related to other forms of energy and to work? We will find that, in spite of the richness of the phenomena, there is a small set of underlying physical principles that unite the subjects and tie them to other fields.

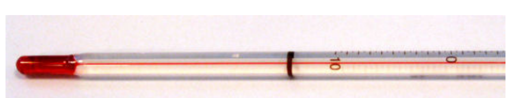


Figure 13.2 In a typical thermometer like this one, the alcohol, with a red dye, expands more rapidly than the glass containing it. When the thermometer's temperature increases, the liquid from the bulb is forced into the narrow tube, producing a large change in the length of the column for a small change in temperature. (credit: Chemical Engineer, Wikimedia Commons)

13.1 Temperature

The concept of temperature has evolved from the common concepts of hot and cold. Human perception of what feels hot or cold is a relative one. For example, if you place one hand in hot water and the other in cold water, and then place both hands in tepid water, the tepid water will feel cool to the hand that was in hot water, and warm to the one that was in cold water. The scientific definition of temperature is less ambiguous than your senses of hot and cold. **Temperature** is operationally defined to be what we measure with a thermometer. (Many physical quantities are defined solely in terms of how they are measured. We shall see later how temperature is related to the kinetic energies of atoms and molecules, a more physical explanation.) Two accurate thermometers, one placed in hot water and the other in cold water, will show the hot water to have a higher temperature. If they are then placed in the tepid water, both will give identical readings (within measurement uncertainties). In this section, we discuss temperature, its measurement by thermometers, and its relationship to thermal equilibrium. Again, temperature is the quantity measured by a thermometer.

Misconception Alert: Human Perception vs. Reality

On a cold winter morning, the wood on a porch feels warmer than the metal of your bike. The wood and bicycle are in thermal equilibrium with the outside air, and are thus the same temperature. They *feel* different because of the difference in the way that they conduct heat away from your skin. The metal conducts heat away from your body faster than the wood does (see more about conductivity in **Conduction**). This is just one example demonstrating that the human sense of hot and cold is not determined by temperature alone.

Another factor that affects our perception of temperature is humidity. Most people feel much hotter on hot, humid days than on hot, dry days. This is because on humid days, sweat does not evaporate from the skin as efficiently as it does on dry days. It is the evaporation of sweat (or water from a sprinkler or pool) that cools us off.

Any physical property that depends on temperature, and whose response to temperature is reproducible, can be used as the basis of a thermometer. Because many physical properties depend on temperature, the variety of thermometers is remarkable. For example, volume increases with temperature for most substances. This property is the basis for the common alcohol thermometer, the old mercury thermometer, and the bimetallic strip (**Figure 13.3**). Other properties used to measure temperature include electrical resistance and color, as shown in **Figure 13.4**, and the emission of infrared radiation, as shown in **Figure 13.5**.

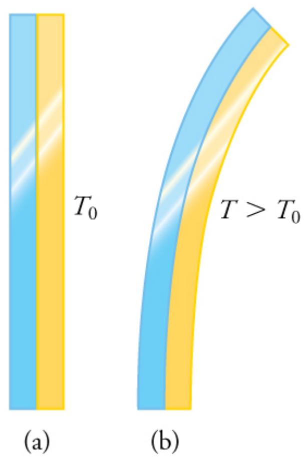


Figure 13.3 The curvature of a bimetallic strip depends on temperature. (a) The strip is straight at the starting temperature, where its two components have the same length. (b) At a higher temperature, this strip bends to the right, because the metal on the left has expanded more than the metal on the right.

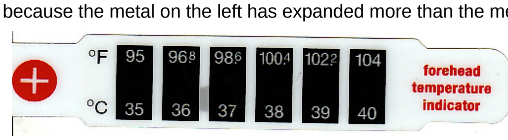


Figure 13.4 Each of the six squares on this plastic (liquid crystal) thermometer contains a film of a different heat-sensitive liquid crystal material. Below $95^\circ F$, all six squares are black. When the plastic thermometer is exposed to temperature that increases to $95^\circ F$, the first liquid crystal square changes color. When the temperature increases above $96.8^\circ F$ the second liquid crystal square also changes color, and so forth. (credit: Arkrishna, Wikimedia Commons)

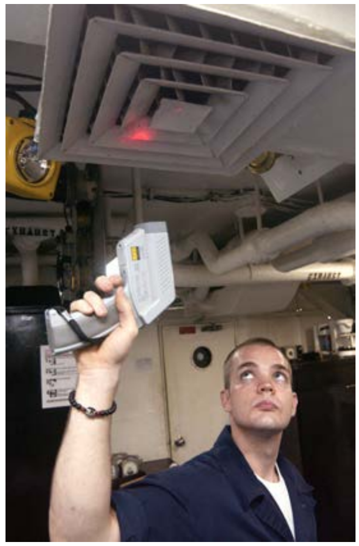


Figure 13.5 Fireman Jason Ormand uses a pyrometer to check the temperature of an aircraft carrier's ventilation system. Infrared radiation (whose emission varies with temperature) from the vent is measured and a temperature readout is quickly produced. Infrared measurements are also frequently used as a measure of body temperature. These modern thermometers, placed in the ear canal, are more accurate than alcohol thermometers placed under the tongue or in the armpit. (credit: Lamel J. Hinton/U.S. Navy)

Temperature Scales

Thermometers are used to measure temperature according to well-defined scales of measurement, which use pre-defined reference points to help compare quantities. The three most common temperature scales are the Fahrenheit, Celsius, and Kelvin scales. A temperature scale can be created by identifying two easily reproducible temperatures. The freezing and boiling temperatures of water at standard atmospheric pressure are commonly used.

The **Celsius** scale (which replaced the slightly different *centigrade* scale) has the freezing point of water at 0°C and the boiling point at 100°C. Its unit is the **degree Celsius** $(^\circ\text{C})$. On the **Fahrenheit** scale (still the most frequently used in the United States), the freezing point of water is at 32°F and the boiling point is at 212°F. The unit of temperature on this scale is the **degree Fahrenheit** $(^\circ\text{F})$. Note that a temperature difference of one degree Celsius is greater than a temperature difference of one degree Fahrenheit. Only 100 Celsius degrees span the same range as 180 Fahrenheit degrees, thus one degree on the Celsius scale is 1.8 times larger than one degree on the Fahrenheit scale $180 / 100 = 9 / 5.$

The **Kelvin** scale is the temperature scale that is commonly used in science. It is an *absolute temperature* scale defined to have 0 K at the lowest possible temperature, called **absolute zero**. The official temperature unit on this scale is the *kelvin*, which is abbreviated K, and is not accompanied by a degree sign. The freezing and boiling points of water are 273.15 K and 373.15 K, respectively. Thus, the magnitude of temperature differences is the same in units of kelvins and degrees Celsius. Unlike other temperature scales, the Kelvin scale is an absolute scale. It is used extensively in scientific work because a number of physical quantities, such as the volume of an ideal gas, are directly related to absolute temperature. The kelvin is the SI unit used in scientific work.

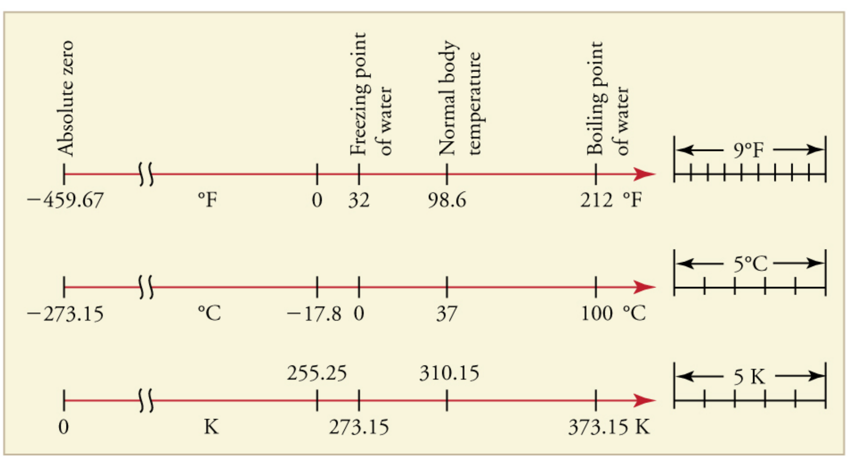


Figure 13.6 Relationships between the Fahrenheit, Celsius, and Kelvin temperature scales, rounded to the nearest degree. The relative sizes of the scales are also shown.

The relationships between the three common temperature scales is shown in **Figure 13.6**. Temperatures on these scales can be converted using the equations in **Table 13.1**.

Table 13.1 Temperature Conversions

To convert from . . .	Use this equation . . .	Also written as . . .
Celsius to Fahrenheit	$T(^\circ\text{F}) = \frac{9}{5}T(^\circ\text{C}) + 32$	$T_{^\circ\text{F}} = \frac{9}{5}T_{^\circ\text{C}} + 32$
Fahrenheit to Celsius	$T(^\circ\text{C}) = \frac{5}{9}(T(^\circ\text{F}) - 32)$	$T_{^\circ\text{C}} = \frac{5}{9}(T_{^\circ\text{F}} - 32)$
Celsius to Kelvin	$T(\text{K}) = T(^\circ\text{C}) + 273.15$	$T_\text{K} = T_{^\circ\text{C}} + 273.15$
Kelvin to Celsius	$T(^\circ\text{C}) = T(\text{K}) - 273.15$	$T_{^\circ\text{C}} = T_\text{K} - 273.15$
Fahrenheit to Kelvin	$T(\text{K}) = \frac{5}{9}(T(^\circ\text{F}) - 32) + 273.15$	$T_\text{K} = \frac{5}{9}(T_{^\circ\text{F}} - 32) + 273.15$
Kelvin to Fahrenheit	$T(^\circ\text{F}) = \frac{9}{5}(T(\text{K}) - 273.15) + 32$	$T_{^\circ\text{F}} = \frac{9}{5}(T_\text{K} - 273.15) + 32$

Notice that the conversions between Fahrenheit and Kelvin look quite complicated. In fact, they are simple combinations of the conversions between Fahrenheit and Celsius, and the conversions between Celsius and Kelvin.

Example 13.1 Converting between Temperature Scales: Room Temperature

"Room temperature" is generally defined to be 25°C. (a) What is room temperature in $^\circ\text{F}$? (b) What is it in K?

Strategy

To answer these questions, all we need to do is choose the correct conversion equations and plug in the known values.

Solution for (a)

1. Choose the right equation. To convert from $^\circ\text{C}$ to $^\circ\text{F}$, use the equation

$$T_{^\circ\text{F}} = \frac{9}{5}T_{^\circ\text{C}} + 32. \tag{13.1}$$

2. Plug the known value into the equation and solve:

$$T_{^\circ\text{F}} = \frac{9}{5}25^\circ\text{C} + 32 = 77^\circ\text{F}. \tag{13.2}$$

Solution for (b)

1. Choose the right equation. To convert from $^\circ\text{C}$ to K, use the equation

$$T_\text{K} = T_{^\circ\text{C}} + 273.15. \tag{13.3}$$

2. Plug the known value into the equation and solve:

$$T_\text{K} = 25^\circ\text{C} + 273.15 = 298\ \text{K}. \tag{13.4}$$

Example 13.2 Converting between Temperature Scales: the Reaumur Scale

The Reaumur scale is a temperature scale that was used widely in Europe in the 18th and 19th centuries. On the Reaumur temperature scale, the freezing point of water is 0°R and the boiling temperature is 80°R. If "room temperature" is 25°C on the Celsius scale, what is it on the Reaumur scale?

Strategy

To answer this question, we must compare the Reaumur scale to the Celsius scale. The difference between the freezing point and boiling point of water on the Reaumur scale is 80°R. On the Celsius scale it is 100°C. Therefore $100^\circ\text{C} = 80^\circ\text{R}$. Both scales start at 0° for freezing, so we can derive a simple formula to convert between temperatures on the two scales.

Solution

1. Derive a formula to convert from one scale to the other:

$$T_{^\circ\text{R}} = \frac{0.8^\circ\text{R}}{^\circ\text{C}} \times T_{^\circ\text{C}}. \tag{13.5}$$

2. Plug the known value into the equation and solve:

$$T_{^\circ\text{R}} = \frac{0.8^\circ\text{R}}{^\circ\text{C}} \times 25^\circ\text{C} = 20^\circ\text{R}. \tag{13.6}$$

Temperature Ranges in the Universe

Figure 13.8 shows the wide range of temperatures found in the universe. Human beings have been known to survive with body temperatures within a small range, from 24°C to 44°C $(75^\circ\text{F}$ to $111^\circ\text{F})$. The average normal body temperature is usually given as 37.0°C (98.6°F), and variations in this temperature can indicate a medical condition: a fever, an infection, a tumor, or circulatory problems (see **Figure 13.7**).

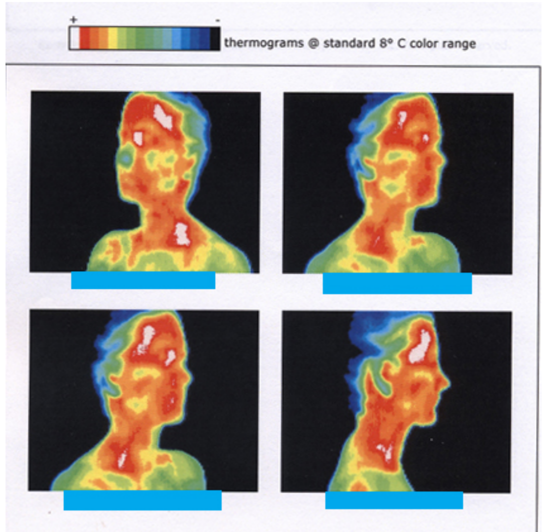


Figure 13.7 This image of radiation from a person's body (an infrared thermograph) shows the location of temperature abnormalities in the upper body. Dark blue corresponds to cold areas and red to white corresponds to hot areas. An elevated temperature might be an indication of malignant tissue (a cancerous tumor in the breast, for example), while a depressed temperature might be due to a decline in blood flow from a clot. In this case, the abnormalities are caused by a condition called hyperhidrosis. (credit: Porcelina81, Wikimedia Commons)

The lowest temperatures ever recorded have been measured during laboratory experiments: $4.5\times10^{-10}\text{ K}$ at the Massachusetts Institute of Technology (USA), and $1.0\times10^{-10}\text{ K}$ at Helsinki University of Technology (Finland). In comparison, the coldest recorded place on Earth's surface is Vostok, Antarctica at 183 K (-89°C), and the coldest place (outside the lab) known in the universe is the Boomerang Nebula, with a temperature of 1 K.

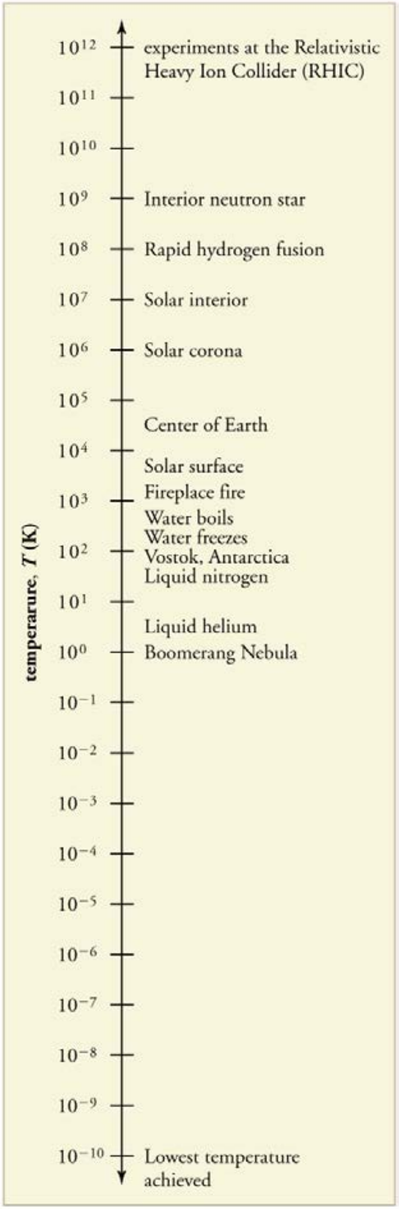


Figure 13.8 Each increment on this logarithmic scale indicates an increase by a factor of ten, and thus illustrates the tremendous range of temperatures in nature. Note that zero on a logarithmic scale would occur off the bottom of the page at infinity.

Making Connections: Absolute Zero

What is absolute zero? Absolute zero is the temperature at which all molecular motion has ceased. The concept of absolute zero arises from the behavior of gases. **Figure 13.9** shows how the pressure of gases at a constant volume decreases as temperature decreases. Various scientists have noted that the pressures of gases extrapolate to zero at the same temperature, -273.15ºC . This extrapolation implies that there is a lowest temperature. This temperature is called *absolute zero*. Today we know that most gases first liquefy and then freeze, and it is not actually possible to reach absolute zero. The numerical value of absolute zero temperature is -273.15ºC or 0 K.

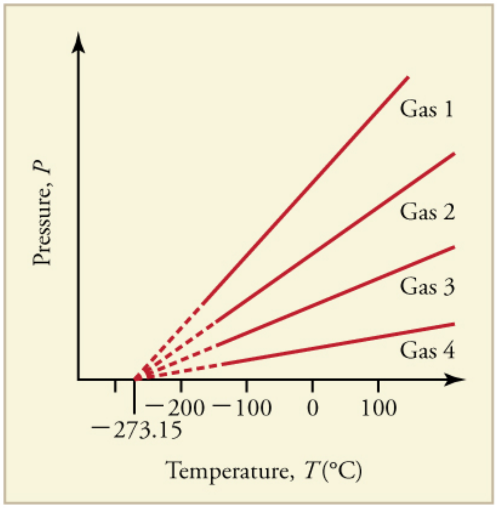


Figure 13.9 Graph of pressure versus temperature for various gases kept at a constant volume. Note that all of the graphs extrapolate to zero pressure at the same temperature.

Thermal Equilibrium and the Zeroth Law of Thermodynamics

Thermometers actually take their *own* temperature, not the temperature of the object they are measuring. This raises the question of how we can be certain that a thermometer measures the temperature of the object with which it is in contact. It is based on the fact that any two systems placed in *thermal contact* (meaning heat transfer can occur between them) will reach the same temperature. That is, heat will flow from the hotter object to the cooler one until they have exactly the same temperature. The objects are then in **thermal equilibrium**, and no further changes will occur. The systems interact and change because their temperatures differ, and the changes stop once their temperatures are the same. Thus, if enough time is allowed for this transfer of heat to run its course, the temperature a thermometer registers *does* represent the system with which it is in thermal equilibrium. Thermal equilibrium is established when two bodies are in contact with each other and can freely exchange energy.

Furthermore, experimentation has shown that if two systems, A and B, are in thermal equilibrium with each another, and B is in thermal equilibrium with a third system C, then A is also in thermal equilibrium with C. This conclusion may seem obvious, because all three have the same temperature, but it is basic to thermodynamics. It is called the **zeroth law of thermodynamics**.

The Zeroth Law of Thermodynamics

If two systems, A and B, are in thermal equilibrium with each other, and B is in thermal equilibrium with a third system, C, then A is also in thermal equilibrium with C.

This law was postulated in the 1930s, after the first and second laws of thermodynamics had been developed and named. It is called the *zeroth law* because it comes logically before the first and second laws (discussed in **Thermodynamics**). An example of this law in action is seen in babies in incubators: babies in incubators normally have very few clothes on, so to an observer they look as if they may not be warm enough. However, the temperature of the air, the cot, and the baby is the same, because they are in thermal equilibrium, which is accomplished by maintaining air temperature to keep the baby comfortable.

Check Your Understanding

Does the temperature of a body depend on its size?

Solution

No, the system can be divided into smaller parts each of which is at the same temperature. We say that the temperature is an *intensive* quantity. Intensive quantities are independent of size.

13.2 Thermal Expansion of Solids and Liquids

Figure 13.10 Thermal expansion joints like these in the Auckland Harbour Bridge in New Zealand allow bridges to change length without buckling. (credit: Ingolfson, Wikimedia Commons)

The expansion of alcohol in a thermometer is one of many commonly encountered examples of **thermal expansion**, the change in size or volume of a given mass with temperature. Hot air rises because its volume increases, which causes the hot air's density to be smaller than the density of surrounding air, causing a buoyant (upward) force on the hot air. The same happens in all liquids and gases, driving natural heat transfer upwards in homes, oceans, and weather systems. Solids also undergo thermal expansion. Railroad tracks and bridges, for example, have expansion joints to allow them to freely expand and contract with temperature changes.

What are the basic properties of thermal expansion? First, thermal expansion is clearly related to temperature change. The greater the temperature change, the more a bimetallic strip will bend. Second, it depends on the material. In a thermometer, for example, the expansion of alcohol is much greater than the expansion of the glass containing it.

What is the underlying cause of thermal expansion? As is discussed in **Kinetic Theory: Atomic and Molecular Explanation of Pressure and Temperature**, an increase in temperature implies an increase in the kinetic energy of the individual atoms. In a solid, unlike in a gas, the atoms or molecules are closely packed together, but their kinetic energy (in the form of small, rapid vibrations) pushes neighboring atoms or molecules apart from each other. This neighbor-to-neighbor pushing results in a slightly greater distance, on average, between neighbors, and adds up to a larger size for the whole body. For most substances under ordinary conditions, there is no preferred direction, and an increase in temperature will increase the solid's size by a certain fraction in each dimension.

Linear Thermal Expansion—Thermal Expansion in One Dimension

The change in length ΔL is proportional to length L. The dependence of thermal expansion on temperature, substance, and length is summarized in the equation

$$\Delta L = \alpha L \Delta T, \tag{13.7}$$

where ΔL is the change in length L, ΔT is the change in temperature, and α is the **coefficient of linear expansion**, which varies slightly with temperature.

Table 13.2 lists representative values of the coefficient of linear expansion, which may have units of $1/{}^\circ\text{C}$ or 1/K. Because the size of a kelvin and a degree Celsius are the same, both α and ΔT can be expressed in units of kelvins or degrees Celsius. The equation $\Delta L = \alpha L \Delta T$ is accurate for small changes in temperature and can be used for large changes in temperature if an average value of α is used.

Table 13.2 Thermal Expansion Coefficients at 20°C [1]

Material	Coefficient of linear expansion $\alpha(1/^{\circ}\text{C})$	Coefficient of volume expansion $\beta(1/^{\circ}\text{C})$
Solids		
Aluminum	25×10^{-6}	75×10^{-6}
Brass	19×10^{-6}	56×10^{-6}
Copper	17×10^{-6}	51×10^{-6}
Gold	14×10^{-6}	42×10^{-6}
Iron or Steel	12×10^{-6}	35×10^{-6}
Invar (Nickel-iron alloy)	0.9×10^{-6}	2.7×10^{-6}
Lead	29×10^{-6}	87×10^{-6}
Silver	18×10^{-6}	54×10^{-6}
Glass (ordinary)	9×10^{-6}	27×10^{-6}
Glass (Pyrex®)	3×10^{-6}	9×10^{-6}
Quartz	0.4×10^{-6}	1×10^{-6}
Concrete, Brick	$\sim12\times10^{-6}$	$\sim36\times10^{-6}$
Marble (average)	2.5×10^{-6}	7.5×10^{-6}
Liquids		
Ether		1650×10^{-6}
Ethyl alcohol		1100×10^{-6}
Petrol		950×10^{-6}
Glycerin		500×10^{-6}
Mercury		180×10^{-6}
Water		210×10^{-6}
Gases		
Air and most other gases at atmospheric pressure		3400×10^{-6}

Example 13.3 Calculating Linear Thermal Expansion: The Golden Gate Bridge

The main span of San Francisco's Golden Gate Bridge is 1275 m long at its coldest. The bridge is exposed to temperatures ranging from -15°C to 40°C. What is its change in length between these temperatures? Assume that the bridge is made entirely of steel.

Strategy

Use the equation for linear thermal expansion $\Delta L = \alpha L \Delta T$ to calculate the change in length, ΔL. Use the coefficient of linear expansion, α, for steel from **Table 13.2**, and note that the change in temperature, ΔT, is 55°C.

Solution

Plug all of the known values into the equation to solve for ΔL.

$$\Delta L = \alpha L \Delta T = \left(\frac{12\times10^{-6}}{^{\circ}\text{C}}\right)(1275\text{ m})(55^{\circ}\text{C}) = 0.84\text{ m}. \tag{13.8}$$

Discussion

1. Values for liquids and gases are approximate.

Although not large compared with the length of the bridge, this change in length is observable. It is generally spread over many expansion joints so that the expansion at each joint is small.

Thermal Expansion in Two and Three Dimensions

Objects expand in all dimensions, as illustrated in **Figure 13.11**. That is, their areas and volumes, as well as their lengths, increase with temperature. Holes also get larger with temperature. If you cut a hole in a metal plate, the remaining material will expand exactly as it would if the plug was still in place. The plug would get bigger, and so the hole must get bigger too. (Think of the ring of neighboring atoms or molecules on the wall of the hole as pushing each other farther apart as temperature increases. Obviously, the ring of neighbors must get slightly larger, so the hole gets slightly larger).

Thermal Expansion in Two Dimensions

For small temperature changes, the change in area ΔA is given by

$$\Delta A = 2\alpha A \Delta T, \tag{13.9}$$

where ΔA is the change in area A, ΔT is the change in temperature, and α is the coefficient of linear expansion, which varies slightly with temperature.

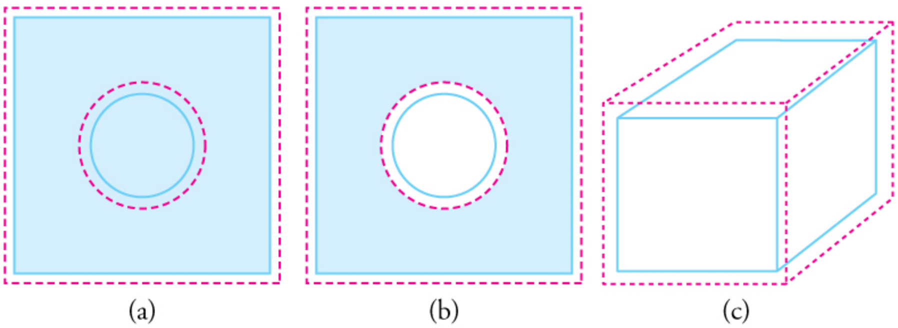


Figure 13.11 In general, objects expand in all directions as temperature increases. In these drawings, the original boundaries of the objects are shown with solid lines, and the expanded boundaries with dashed lines. (a) Area increases because both length and width increase. The area of a circular plug also increases. (b) If the plug is removed, the hole it leaves becomes larger with increasing temperature, just as if the expanding plug were still in place. (c) Volume also increases, because all three dimensions increase.

Thermal Expansion in Three Dimensions

The change in volume ΔV is very nearly $\Delta V = 3\alpha V \Delta T$. This equation is usually written as

$$\Delta V = \beta V \Delta T, \tag{13.10}$$

where β is the **coefficient of volume expansion** and $\beta \approx 3\alpha$. Note that the values of β in **Table 13.2** are almost exactly equal to 3α.

In general, objects will expand with increasing temperature. Water is the most important exception to this rule. Water expands with increasing temperature (its density *decreases*) when it is at temperatures greater than $4^\circ C (40^\circ F)$. However, it expands with *decreasing* temperature when it is between $+4^\circ C$ and $0^\circ C$ $(40^\circ F$ to $32^\circ F)$. Water is densest at $+4^\circ C$. (See **Figure 13.12**.) Perhaps the most striking effect of this phenomenon is the freezing of water in a pond. When water near the surface cools down to $4^\circ C$ it is denser than the remaining water and thus will sink to the bottom. This "turnover" results in a layer of warmer water near the surface, which is then cooled. Eventually the pond has a uniform temperature of $4^\circ C$. If the temperature in the surface layer drops below $4^\circ C$, the water is less dense than the water below, and thus stays near the top. As a result, the pond surface can completely freeze over. The ice on top of liquid water provides an insulating layer from winter's harsh exterior air temperatures. Fish and other aquatic life can survive in $4^\circ C$ water beneath ice, due to this unusual characteristic of water. It also produces circulation of water in the pond that is necessary for a healthy ecosystem of the body of water.

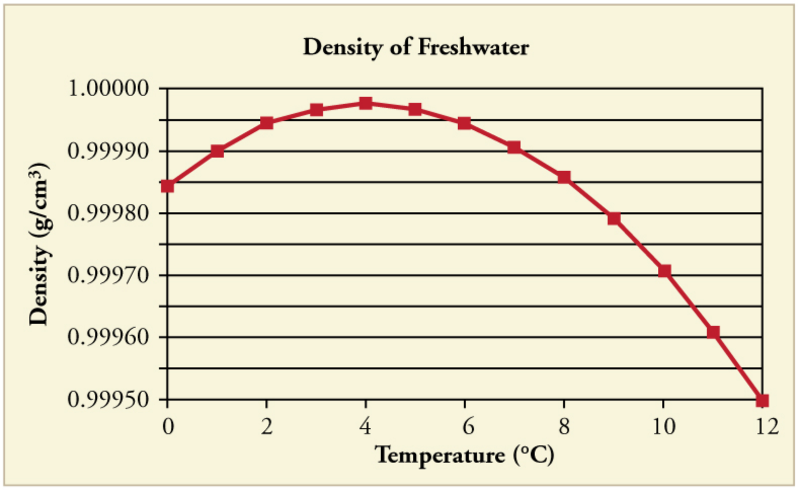


Figure 13.12 The density of water as a function of temperature. Note that the thermal expansion is actually very small. The maximum density at $+4^\circ\text{C}$ is only *0.0075%* greater than the density at 2°C , and *0.012%* greater than that at 0°C .

Making Connections: Real-World Connections—Filling the Tank

Differences in the thermal expansion of materials can lead to interesting effects at the gas station. One example is the dripping of gasoline from a freshly filled tank on a hot day. Gasoline starts out at the temperature of the ground under the gas station, which is cooler than the air temperature above. The gasoline cools the steel tank when it is filled. Both gasoline and steel tank expand as they warm to air temperature, but gasoline expands much more than steel, and so it may overflow.

This difference in expansion can also cause problems when interpreting the gasoline gauge. The actual amount (mass) of gasoline left in the tank when the gauge hits “empty” is a lot less in the summer than in the winter. The gasoline has the same volume as it does in the winter when the “add fuel” light goes on, but because the gasoline has expanded, there is less mass. If you are used to getting another 40 miles on “empty” in the winter, beware—you will probably run out much more quickly in the summer.

Figure 13.13 Because the gas expands more than the gas tank with increasing temperature, you can’t drive as many miles on “empty” in the summer as you can in the winter. (credit: Hector Alejandro, Flickr)

Example 13.4 Calculating Thermal Expansion: Gas vs. Gas Tank

Suppose your 60.0-L (15.9-gal) steel gasoline tank is full of gas, so both the tank and the gasoline have a temperature of 15.0°C . How much gasoline has spilled by the time they warm to 35.0°C ?

Strategy

The tank and gasoline increase in volume, but the gasoline increases more, so the amount spilled is the difference in their volume changes. (The gasoline tank can be treated as solid steel.) We can use the equation for volume expansion to calculate the change in volume of the gasoline and of the tank.

Solution

1. Use the equation for volume expansion to calculate the increase in volume of the steel tank:

$$\Delta V_{\text{s}} = \beta_{\text{s}} V_{\text{s}} \Delta T. \tag{13.11}$$

2. The increase in volume of the gasoline is given by this equation:

$$\Delta V_{\text{gas}} = \beta_{\text{gas}} V_{\text{gas}} \Delta T. \tag{13.12}$$

3. Find the difference in volume to determine the amount spilled as

$$V_{\text{spill}} = \Delta V_{\text{gas}} - \Delta V_{\text{s}}. \tag{13.13}$$

Alternatively, we can combine these three equations into a single equation. (Note that the original volumes are equal.)

$$\begin{aligned} V_{\text{spill}} &= \left(\beta_{\text{gas}} - \beta_{\text{s}}\right)V\Delta T \\ &= \left[(950 - 35)\times10^{-6}/\text{ºC}\right](60.0\ \text{L})(20.0\text{ºC}) \\ &= 1.10\ \text{L}. \end{aligned} \tag{13.14}$$

Discussion

This amount is significant, particularly for a 60.0-L tank. The effect is so striking because the gasoline and steel expand quickly. The rate of change in thermal properties is discussed in **Heat and Heat Transfer Methods**.

If you try to cap the tank tightly to prevent overflow, you will find that it leaks anyway, either around the cap or by bursting the tank. Tightly constricting the expanding gas is equivalent to compressing it, and both liquids and solids resist being compressed with extremely large forces. To avoid rupturing rigid containers, these containers have air gaps, which allow them to expand and contract without stressing them.

Thermal Stress

Thermal stress is created by thermal expansion or contraction (see **Elasticity: Stress and Strain** for a discussion of stress and strain). Thermal stress can be destructive, such as when expanding gasoline ruptures a tank. It can also be useful, for example, when two parts are joined together by heating one in manufacturing, then slipping it over the other and allowing the combination to cool. Thermal stress can explain many phenomena, such as the weathering of rocks and pavement by the expansion of ice when it freezes.

Example 13.5 Calculating Thermal Stress: Gas Pressure

What pressure would be created in the gasoline tank considered in **Example 13.4**, if the gasoline increases in temperature from 15.0ºC to 35.0ºC without being allowed to expand? Assume that the bulk modulus B for gasoline is $1.00\times10^{9}\ \text{N/m}^2$. (For more on bulk modulus, see **Elasticity: Stress and Strain**.)

Strategy

To solve this problem, we must use the following equation, which relates a change in volume ΔV to pressure:

$$\Delta V = \frac{1}{B}\frac{F}{A}V_0, \tag{13.15}$$

where F/A is pressure, V_0 is the original volume, and B is the bulk modulus of the material involved. We will use the amount spilled in **Example 13.4** as the change in volume, ΔV.

Solution

1. Rearrange the equation for calculating pressure:

$$P = \frac{F}{A} = \frac{\Delta V}{V_0}B. \tag{13.16}$$

2. Insert the known values. The bulk modulus for gasoline is $B = 1.00\times10^{9}\ \text{N/m}^2$. In the previous example, the change in volume $\Delta V = 1.10\ \text{L}$ is the amount that would spill. Here, $V_0 = 60.0\ \text{L}$ is the original volume of the gasoline. Substituting these values into the equation, we obtain

$$P = \frac{1.10\ \text{L}}{60.0\ \text{L}}\left(1.00\times10^{9}\ \text{Pa}\right) = 1.83\times10^{7}\ \text{Pa}. \tag{13.17}$$

Discussion

This pressure is about $2500\ \text{lb/in}^2$, *much* more than a gasoline tank can handle.

Forces and pressures created by thermal stress are typically as great as that in the example above. Railroad tracks and roadways can buckle on hot days if they lack sufficient expansion joints. (See **Figure 13.14**.) Power lines sag more in the summer than in the winter, and will snap in cold weather if there is insufficient slack. Cracks open and close in plaster walls as a house warms and cools. Glass cooking pans will crack if cooled rapidly or unevenly, because of differential contraction and the stresses it creates. (Pyrex® is less susceptible because of its small coefficient of thermal expansion.) Nuclear reactor pressure vessels are threatened by overly rapid cooling, and although none have failed, several have been cooled faster than considered desirable. Biological cells are ruptured when foods are frozen, detracting from their taste. Repeated thawing and freezing accentuate the damage. Even the oceans can be affected. A significant portion of the rise in sea level that is resulting from global warming is due to the thermal expansion of sea water.

Figure 13.14 Thermal stress contributes to the formation of potholes. (credit: Editor5807, Wikimedia Commons)

Metal is regularly used in the human body for hip and knee implants. Most implants need to be replaced over time because, among other things, metal does not bond with bone. Researchers are trying to find better metal coatings that would allow metal-to-bone bonding. One challenge is to find a coating that has an expansion coefficient similar to that of metal. If the expansion coefficients are too different, the thermal stresses during the manufacturing process lead to cracks at the coating-metal interface.

Another example of thermal stress is found in the mouth. Dental fillings can expand differently from tooth enamel. It can give pain when eating ice cream or having a hot drink. Cracks might occur in the filling. Metal fillings (gold, silver, etc.) are being replaced by composite fillings (porcelain), which have smaller coefficients of expansion, and are closer to those of teeth.

Check Your Understanding

Two blocks, A and B, are made of the same material. Block A has dimensions $l \times w \times h = L \times 2L \times L$ and Block B has dimensions $2L \times 2L \times 2L$. If the temperature changes, what is (a) the change in the volume of the two blocks, (b) the change in the cross-sectional area $l \times w$, and (c) the change in the height h of the two blocks?

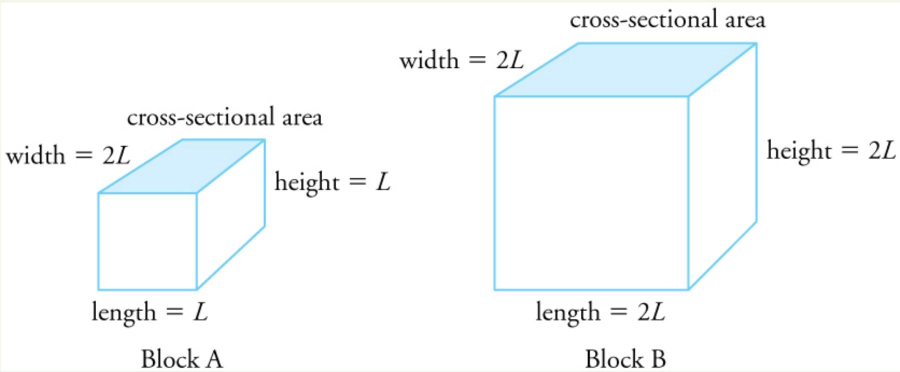


Figure 13.15

Solution

(a) The change in volume is proportional to the original volume. Block A has a volume of $L \times 2L \times L = 2L^3.$ Block B has a volume of $2L \times 2L \times 2L = 8L^3,$ which is 4 times that of Block A. Thus the change in volume of Block B should be 4 times the change in volume of Block A.

(b) The change in area is proportional to the area. The cross-sectional area of Block A is $L \times 2L = 2L^2,$ while that of Block B is $2L \times 2L = 4L^2.$ Because cross-sectional area of Block B is twice that of Block A, the change in the cross-sectional area of Block B is twice that of Block A.

(c) The change in height is proportional to the original height. Because the original height of Block B is twice that of A, the change in the height of Block B is twice that of Block A.

13.3 The Ideal Gas Law

Figure 13.16 The air inside this hot air balloon flying over Putrajaya, Malaysia, is hotter than the ambient air. As a result, the balloon experiences a buoyant force pushing it upward. (credit: Kevin Poh, Flickr)

In this section, we continue to explore the thermal behavior of gases. In particular, we examine the characteristics of atoms and molecules that compose gases. (Most gases, for example nitrogen, N_2, and oxygen, O_2, are composed of two or more atoms. We will primarily use the term "molecule" in discussing a gas because the term can also be applied to monatomic gases, such as helium.)

Gases are easily compressed. We can see evidence of this in **Table 13.2**, where you will note that gases have the *largest* coefficients of volume expansion. The large coefficients mean that gases expand and contract very rapidly with temperature changes. In addition, you will note that most gases expand at the *same* rate, or have the same β. This raises the question as to why gases should all act in nearly the same way, when liquids and solids have widely varying expansion rates.

The answer lies in the large separation of atoms and molecules in gases, compared to their sizes, as illustrated in **Figure 13.17**. Because atoms and molecules have large separations, forces between them can be ignored, except when they collide with each other during collisions. The motion of atoms and molecules (at temperatures well above the boiling temperature) is fast, such that the gas occupies all of the accessible volume and the expansion of gases is rapid. In contrast, in liquids and solids, atoms and molecules are closer together and are quite sensitive to the forces between them.

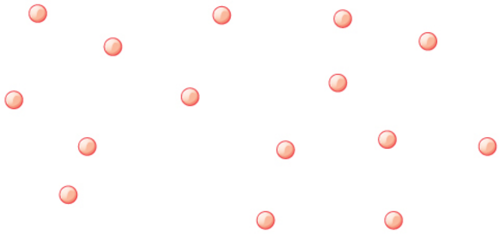


Figure 13.17 Atoms and molecules in a gas are typically widely separated, as shown. Because the forces between them are quite weak at these distances, the properties of a gas depend more on the number of atoms per unit volume and on temperature than on the type of atom.

To get some idea of how pressure, temperature, and volume of a gas are related to one another, consider what happens when you pump air into an initially deflated tire. The tire's volume first increases in direct proportion to the amount of air injected, without much increase in the tire pressure. Once the tire has expanded to nearly its full size, the walls limit volume expansion. If we continue to pump air into it, the pressure increases. The pressure will further increase when the car is driven and the tires move. Most manufacturers specify optimal tire pressure for cold tires. (See **Figure 13.18**.)

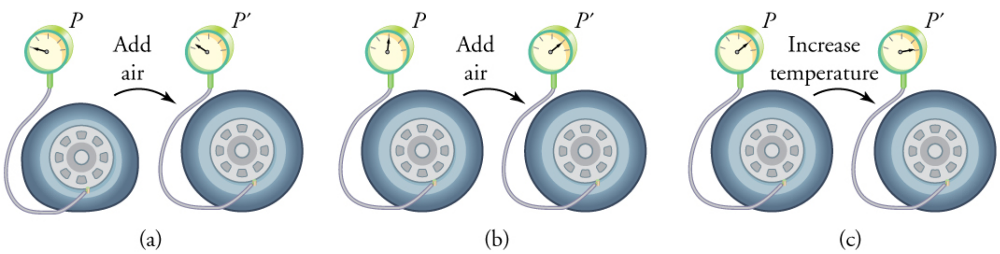


Figure 13.18 (a) When air is pumped into a deflated tire, its volume first increases without much increase in pressure. (b) When the tire is filled to a certain point, the tire walls resist further expansion and the pressure increases with more air. (c) Once the tire is inflated, its pressure increases with temperature.

At room temperatures, collisions between atoms and molecules can be ignored. In this case, the gas is called an ideal gas, in which case the relationship between the pressure, volume, and temperature is given by the equation of state called the ideal gas law.

Ideal Gas Law

The **ideal gas law** states that

$$PV = NkT, \tag{13.18}$$

where P is the absolute pressure of a gas, V is the volume it occupies, N is the number of atoms and molecules in the gas, and T is its absolute temperature. The constant k is called the **Boltzmann constant** in honor of Austrian physicist Ludwig Boltzmann (1844–1906) and has the value

$$k = 1.38\times10^{-23}\ \text{J/K}. \tag{13.19}$$

The ideal gas law can be derived from basic principles, but was originally deduced from experimental measurements of Charles' law (that volume occupied by a gas is proportional to temperature at a fixed pressure) and from Boyle's law (that for a fixed temperature, the product PV is a constant). In the ideal gas model, the volume occupied by its atoms and molecules is a negligible fraction of V. The ideal gas law describes the behavior of real gases under most conditions. (Note, for example, that N is the total number of atoms and molecules, independent of the type of gas.)

Let us see how the ideal gas law is consistent with the behavior of filling the tire when it is pumped slowly and the temperature is constant. At first, the pressure P is essentially equal to atmospheric pressure, and the volume V increases in direct proportion to the number of atoms and molecules N put into the tire. Once the volume of the tire is constant, the equation $PV = NkT$ predicts that the pressure should increase in proportion to *the number N of atoms and molecules*.

Example 13.6 Calculating Pressure Changes Due to Temperature Changes: Tire Pressure

Suppose your bicycle tire is fully inflated, with an absolute pressure of $7.00\times10^5\ \text{Pa}$ (a gauge pressure of just under $90.0\ \text{lb/in}^2$) at a temperature of 18.0°C. What is the pressure after its temperature has risen to 35.0°C? Assume that there are no appreciable leaks or changes in volume.

Strategy

The pressure in the tire is changing only because of changes in temperature. First we need to identify what we know and what we want to know, and then identify an equation to solve for the unknown.

We know the initial pressure $P_0 = 7.00\times10^5\ \text{Pa}$, the initial temperature $T_0 = 18.0^\circ\text{C}$, and the final temperature $T_\text{f} = 35.0^\circ\text{C}$. We must find the final pressure P_f. How can we use the equation $PV = NkT$? At first, it may seem that not enough information is given, because the volume V and number of atoms N are not specified. What we can do is use the equation twice: $P_0V_0 = NkT_0$ and $P_\text{f}V_\text{f} = NkT_\text{f}$. If we divide $P_\text{f}V_\text{f}$ by P_0V_0 we can come up with an equation that allows us to solve for P_f.

$$\frac{P_\text{f}V_\text{f}}{P_0V_0} = \frac{N_\text{f}kT_\text{f}}{N_0kT_0} \tag{13.20}$$

Since the volume is constant, V_f and V_0 are the same and they cancel out. The same is true for N_f and N_0, and k, which is a constant. Therefore,

$$\frac{P_\text{f}}{P_0} = \frac{T_\text{f}}{T_0}. \tag{13.21}$$

We can then rearrange this to solve for P_f:

$$P_\text{f} = P_0\frac{T_\text{f}}{T_0}, \tag{13.22}$$

where the temperature must be in units of kelvins, because T_0 and T_f are absolute temperatures.

Solution

1. Convert temperatures from Celsius to Kelvin.

$$\begin{aligned} T_0 &= (18.0 + 273)\text{K} = 291\ \text{K} \\ T_\text{f} &= (35.0 + 273)\text{K} = 308\ \text{K} \end{aligned} \tag{13.23}$$

2. Substitute the known values into the equation.

$$P_\text{f} = P_0\frac{T_\text{f}}{T_0} = 7.00\times10^5\ \text{Pa}\left(\frac{308\ \text{K}}{291\ \text{K}}\right) = 7.41\times10^5\ \text{Pa} \tag{13.24}$$

Discussion

The final temperature is about 6% greater than the original temperature, so the final pressure is about 6% greater as well. Note that *absolute* pressure and *absolute* temperature must be used in the ideal gas law.

Making Connections: Take-Home Experiment—Refrigerating a Balloon

Inflate a balloon at room temperature. Leave the inflated balloon in the refrigerator overnight. What happens to the balloon, and why?

Example 13.7 Calculating the Number of Molecules in a Cubic Meter of Gas

How many molecules are in a typical object, such as gas in a tire or water in a drink? We can use the ideal gas law to give us an idea of how large N typically is.

Calculate the number of molecules in a cubic meter of gas at standard temperature and pressure (STP), which is defined to be $0ºC$ and atmospheric pressure.

Strategy

Because pressure, volume, and temperature are all specified, we can use the ideal gas law $PV = NkT$, to find N .

Solution

1. Identify the knowns.

$$\begin{aligned} T &= 0ºC = 273\ \text{K} \\ P &= 1.01\times10^{5}\ \text{Pa} \\ V &= 1.00\ \text{m}^{3} \\ k &= 1.38\times10^{-23}\ \text{J/K} \end{aligned} \tag{13.25}$$

2. Identify the unknown: number of molecules, N .

3. Rearrange the ideal gas law to solve for N .

$$\begin{aligned} PV &= NkT \\ N &= \frac{PV}{kT} \end{aligned} \tag{13.26}$$

4. Substitute the known values into the equation and solve for N .

$$N = \frac{PV}{kT} = \frac{\left(1.01\times10^{5}\ \text{Pa}\right)\left(1.00\ \text{m}^{3}\right)}{\left(1.38\times10^{-23}\ \text{J/K}\right)(273\ \text{K})} = 2.68\times10^{25}\ \text{molecules} \tag{13.27}$$

Discussion

This number is undeniably large, considering that a gas is mostly empty space. N is huge, even in small volumes. For example, $1\ \text{cm}^3$ of a gas at STP has 2.68×10^{19} molecules in it. Once again, note that N is the same for all types or mixtures of gases.

Moles and Avogadro's Number

It is sometimes convenient to work with a unit other than molecules when measuring the amount of substance. A **mole** (abbreviated mol) is defined to be the amount of a substance that contains as many atoms or molecules as there are atoms in exactly 12 grams (0.012 kg) of carbon-12. The actual number of atoms or molecules in one mole is called **Avogadro's number** (N_A) , in recognition of Italian scientist Amedeo Avogadro (1776–1856).

He developed the concept of the mole, based on the hypothesis that equal volumes of gas, at the same pressure and temperature, contain equal numbers of molecules. That is, the number is independent of the type of gas. This hypothesis has been confirmed, and the value of Avogadro's number is

$$N_\text{A} = 6.02\times10^{23}\ \text{mol}^{-1}. \tag{13.28}$$

Avogadro's Number

One mole always contains 6.02×10^{23} particles (atoms or molecules), independent of the element or substance. A mole of any substance has a mass in grams equal to its molecular mass, which can be calculated from the atomic masses given in the periodic table of elements.

$$N_\text{A} = 6.02\times10^{23}\ \text{mol}^{-1} \tag{13.29}$$

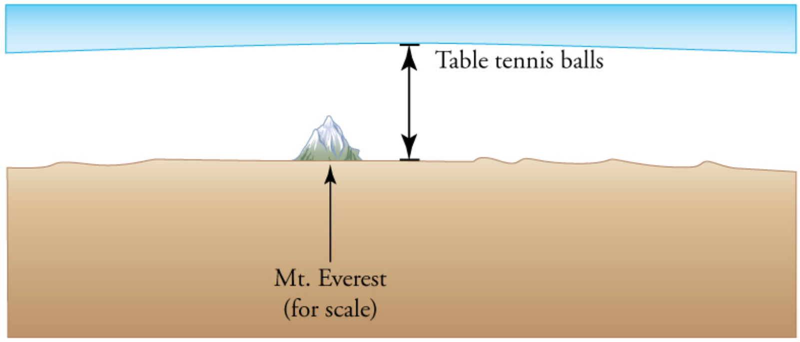


Figure 13.19 How big is a mole? On a macroscopic level, one mole of table tennis balls would cover the Earth to a depth of about 40 km.

Check Your Understanding

The active ingredient in a Tylenol pill is 325 mg of acetaminophen $(C_8H_9NO_2)$. Find the number of active molecules of acetaminophen in a single pill.

Solution

We first need to calculate the molar mass (the mass of one mole) of acetaminophen. To do this, we need to multiply the number of atoms of each element by the element's atomic mass.

$$\begin{aligned}&(8 \text{ moles of carbon})(12 \text{ grams/mole}) + (9 \text{ moles hydrogen})(1 \text{ gram/mole})\\&+(1 \text{ mole nitrogen})(14 \text{ grams/mole}) + (2 \text{ moles oxygen})(16 \text{ grams/mole}) = 151 \text{ g}\end{aligned} \tag{13.30}$$

Then we need to calculate the number of moles in 325 mg.

$$\left(\frac{325 \text{ mg}}{151 \text{ grams/mole}}\right)\left(\frac{1 \text{ gram}}{1000 \text{ mg}}\right) = 2.15\times10^{-3} \text{ moles} \tag{13.31}$$

Then use Avogadro's number to calculate the number of molecules.

$$N = \left(2.15\times10^{-3} \text{ moles}\right)\left(6.02\times10^{23} \text{ molecules/mole}\right) = 1.30\times10^{21} \text{ molecules} \tag{13.32}$$

Example 13.8 Calculating Moles per Cubic Meter and Liters per Mole

Calculate: (a) the number of moles in 1.00 m^3 of gas at STP, and (b) the number of liters of gas per mole.

Strategy and Solution

(a) We are asked to find the number of moles per cubic meter, and we know from **Example 13.7** that the number of molecules per cubic meter at STP is 2.68×10^{25}. The number of moles can be found by dividing the number of molecules by Avogadro's number. We let n stand for the number of moles,

$$n \text{ mol/m}^3 = \frac{N \text{ molecules/m}^3}{6.02\times10^{23} \text{ molecules/mol}} = \frac{2.68\times10^{25} \text{ molecules/m}^3}{6.02\times10^{23} \text{ molecules/mol}} = 44.5 \text{ mol/m}^3. \tag{13.33}$$

(b) Using the value obtained for the number of moles in a cubic meter, and converting cubic meters to liters, we obtain

$$\frac{\left(10^3 \text{ L/m}^3\right)}{44.5 \text{ mol/m}^3} = 22.5 \text{ L/mol.} \tag{13.34}$$

Discussion

This value is very close to the accepted value of 22.4 L/mol. The slight difference is due to rounding errors caused by using three-digit input. Again this number is the same for all gases. In other words, it is independent of the gas.

The (average) molar weight of air (approximately 80% N_2 and 20% O_2 is $M = 28.8 \text{ g}.$ Thus the mass of one cubic meter of air is 1.28 kg. If a living room has dimensions $5 \text{ m}\times5 \text{ m}\times3 \text{ m},$ the mass of air inside the room is 96 kg, which is the typical mass of a human.

Check Your Understanding

The density of air at standard conditions $(P = 1 \text{ atm}$ and $T = 20^\circ\text{C})$ is 1.28 kg/m^3. At what pressure is the density 0.64 kg/m^3 if the temperature and number of molecules are kept constant?

Solution

The best way to approach this question is to think about what is happening. If the density drops to half its original value and no molecules are lost, then the volume must double. If we look at the equation $PV = NkT$, we see that when the temperature is constant, the pressure is inversely proportional to volume. Therefore, if the volume doubles, the pressure must drop to half its original value, and $P_\text{f} = 0.50\text{ atm}$.

The Ideal Gas Law Restated Using Moles

A very common expression of the ideal gas law uses the number of moles, n, rather than the number of atoms and molecules, N. We start from the ideal gas law,

$$PV = NkT, \tag{13.35}$$

and multiply and divide the equation by Avogadro's number N_A. This gives

$$PV = \frac{N}{N_\text{A}} N_\text{A} kT. \tag{13.36}$$

Note that $n = N/N_\text{A}$ is the number of moles. We define the universal gas constant $R = N_\text{A}k$, and obtain the ideal gas law in terms of moles.

Ideal Gas Law (in terms of moles)

The ideal gas law (in terms of moles) is

$$PV = nRT. \tag{13.37}$$

The numerical value of R in SI units is

$$R = N_\text{A}k = \left(6.02\times10^{23}\text{ mol}^{-1}\right)\left(1.38\times10^{-23}\text{ J/K}\right) = 8.31\text{ J/mol}\cdot\text{K}. \tag{13.38}$$

In other units,

$$\begin{aligned} R &= 1.99\text{ cal/mol}\cdot\text{K} \\ R &= 0.0821\text{ L}\cdot\text{atm/mol}\cdot\text{K}. \end{aligned} \tag{13.39}$$

You can use whichever value of R is most convenient for a particular problem.

Example 13.9 Calculating Number of Moles: Gas in a Bike Tire

How many moles of gas are in a bike tire with a volume of $2.00\times10^{-3}\text{ m}^3(2.00\text{ L})$, a pressure of $7.00\times10^5\text{ Pa}$ (a gauge pressure of just under 90.0 lb/in^2), and at a temperature of 18.0°C?

Strategy

Identify the knowns and unknowns, and choose an equation to solve for the unknown. In this case, we solve the ideal gas law, $PV = nRT$, for the number of moles n.

Solution

1. Identify the knowns.

$$\begin{aligned} P &= 7.00\times10^5\text{ Pa} \\ V &= 2.00\times10^{-3}\text{ m}^3 \\ T &= 18.0^\circ\text{C} = 291\text{ K} \\ R &= 8.31\text{ J/mol}\cdot\text{K} \end{aligned} \tag{13.40}$$

2. Rearrange the equation to solve for n and substitute known values.

$$\begin{aligned} n &= \frac{PV}{RT} = \frac{\left(7.00\times10^5\text{ Pa}\right)\left(2.00\times10^{-3}\text{ m}^3\right)}{(8.31\text{ J/mol}\cdot\text{K})(291\text{ K})} \\ &= 0.579\text{ mol} \end{aligned} \tag{13.41}$$

Discussion

The most convenient choice for R in this case is $8.31\text{ J/mol}\cdot\text{K}$, because our known quantities are in SI units. The pressure and temperature are obtained from the initial conditions in **Example 13.6**, but we would get the same answer if we used the final values.

The ideal gas law can be considered to be another manifestation of the law of conservation of energy (see **Conservation of Energy**). Work done on a gas results in an increase in its energy, increasing pressure and/or temperature, or decreasing volume. This increased energy can also be viewed as increased internal kinetic energy, given the gas's atoms and molecules.

The Ideal Gas Law and Energy

Let us now examine the role of energy in the behavior of gases. When you inflate a bike tire by hand, you do work by repeatedly exerting a force through a distance. This energy goes into increasing the pressure of air inside the tire and increasing the temperature of the pump and the air.

The ideal gas law is closely related to energy: the units on both sides are joules. The right-hand side of the ideal gas law in $PV = NkT$ is NkT. This term is roughly the amount of translational kinetic energy of N atoms or molecules at an absolute temperature T, as we shall see formally in **Kinetic Theory: Atomic and Molecular Explanation of Pressure and Temperature**. The left-hand side of the ideal gas law is PV, which also has the units of joules. We know from our study of fluids that pressure is one type of potential energy per unit volume, so pressure multiplied by volume is energy. The important point is that there is energy in a gas related to both its pressure and its volume. The energy can be changed when the gas is doing work as it expands—something we explore in **Heat and Heat Transfer Methods**—similar to what occurs in gasoline or steam engines and turbines.

Problem-Solving Strategy: The Ideal Gas Law

Step 1 Examine the situation to determine that an ideal gas is involved. Most gases are nearly ideal.

Step 2 Make a list of what quantities are given, or can be inferred from the problem as stated (identify the known quantities). Convert known values into proper SI units (K for temperature, Pa for pressure, m^3 for volume, molecules for N, and moles for n).

Step 3 Identify exactly what needs to be determined in the problem (identify the unknown quantities). A written list is useful.

Step 4 Determine whether the number of molecules or the number of moles is known, in order to decide which form of the ideal gas law to use. The first form is $PV = NkT$ and involves N, the number of atoms or molecules. The second form is $PV = nRT$ and involves n, the number of moles.

Step 5 Solve the ideal gas law for the quantity to be determined (the unknown quantity). You may need to take a ratio of final states to initial states to eliminate the unknown quantities that are kept fixed.

Step 6 Substitute the known quantities, along with their units, into the appropriate equation, and obtain numerical solutions complete with units. Be certain to use absolute temperature and absolute pressure.

Step 7 Check the answer to see if it is reasonable: Does it make sense?

Check Your Understanding

Liquids and solids have densities about 1000 times greater than gases. Explain how this implies that the distances between atoms and molecules in gases are about 10 times greater than the size of their atoms and molecules.

Solution

Atoms and molecules are close together in solids and liquids. In gases they are separated by empty space. Thus gases have lower densities than liquids and solids. Density is mass per unit volume, and volume is related to the size of a body (such as a sphere) cubed. So if the distance between atoms and molecules increases by a factor of 10, then the volume occupied increases by a factor of 1000, and the density decreases by a factor of 1000.

13.4 Kinetic Theory: Atomic and Molecular Explanation of Pressure and Temperature

We have developed macroscopic definitions of pressure and temperature. Pressure is the force divided by the area on which the force is exerted, and temperature is measured with a thermometer. We gain a better understanding of pressure and temperature from the kinetic theory of gases, which assumes that atoms and molecules are in continuous random motion.

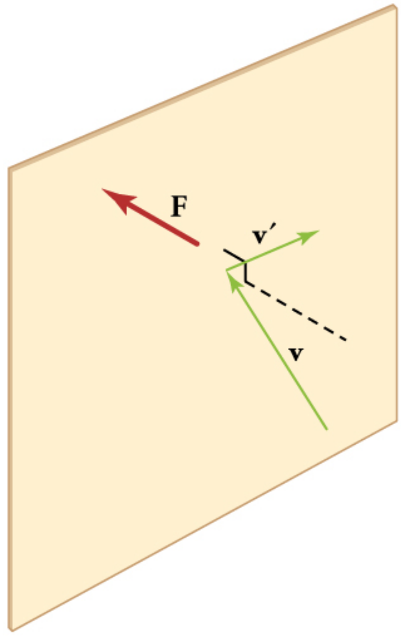


Figure 13.20 When a molecule collides with a rigid wall, the component of its momentum perpendicular to the wall is reversed. A force is thus exerted on the wall, creating pressure.

Figure 13.20 shows an elastic collision of a gas molecule with the wall of a container, so that it exerts a force on the wall (by Newton's third law). Because a huge number of molecules will collide with the wall in a short time, we observe an average force per unit area. These collisions are the source of pressure in a gas. As the number of molecules increases, the number of collisions and thus the pressure increase. Similarly, the gas pressure is higher if the average velocity of molecules is higher. The actual relationship is derived in the **Things Great and Small** feature below. The following relationship is found:

$$PV = \frac{1}{3}Nm\overline{v^2}, \tag{13.42}$$

where P is the pressure (average force per unit area), V is the volume of gas in the container, N is the number of molecules in the container, m is the mass of a molecule, and $\overline{v^2}$ is the average of the molecular speed squared.

What can we learn from this atomic and molecular version of the ideal gas law? We can derive a relationship between temperature and the average translational kinetic energy of molecules in a gas. Recall the previous expression of the ideal gas law:

$$PV = NkT. \tag{13.43}$$

Equating the right-hand side of this equation with the right-hand side of $PV = \frac{1}{3}Nm\overline{v^2}$ gives

$$\frac{1}{3}Nm\overline{v^2} = NkT. \tag{13.44}$$

Making Connections: Things Great and Small—Atomic and Molecular Origin of Pressure in a Gas

Figure 13.21 shows a box filled with a gas. We know from our previous discussions that putting more gas into the box produces greater pressure, and that increasing the temperature of the gas also produces a greater pressure. But why should increasing the temperature of the gas increase the pressure in the box? A look at the atomic and molecular scale gives us some answers, and an alternative expression for the ideal gas law.

The figure shows an expanded view of an elastic collision of a gas molecule with the wall of a container. Calculating the average force exerted by such molecules will lead us to the ideal gas law, and to the connection between temperature and molecular kinetic energy. We assume that a molecule is small compared with the separation of molecules in the gas, and that its interaction with other molecules can be ignored. We also assume the wall is rigid and that the molecule's direction changes, but that its speed remains constant (and hence its kinetic energy and the magnitude of its momentum remain constant as well). This assumption is not always valid, but the same result is obtained with a more detailed description of the molecule's exchange of energy and momentum with the wall.

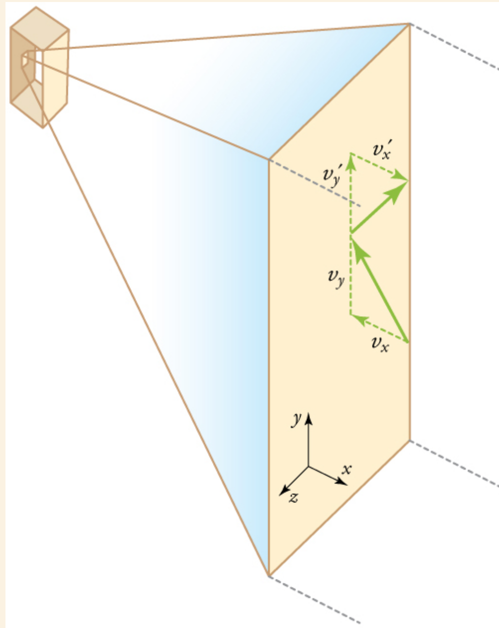


Figure 13.21 Gas in a box exerts an outward pressure on its walls. A molecule colliding with a rigid wall has the direction of its velocity and momentum in the x -direction reversed. This direction is perpendicular to the wall. The components of its velocity momentum in the y - and z -directions are not changed, which means there is no force parallel to the wall.

If the molecule's velocity changes in the x -direction, its momentum changes from $-mv_x$ to $+mv_x$. Thus, its change in momentum is $\Delta mv = +mv_x-(-mv_x) = 2mv_x$. The force exerted on the molecule is given by

$$F = \frac{\Delta p}{\Delta t} = \frac{2mv_x}{\Delta t}. \tag{13.45}$$

There is no force between the wall and the molecule until the molecule hits the wall. During the short time of the collision, the force between the molecule and wall is relatively large. We are looking for an average force; we take Δt to be the average time between collisions of the molecule with this wall. It is the time it would take the molecule to go across the box and back (a distance $2l)$ at a speed of v_x . Thus $\Delta t = 2l / v_x$, and the expression for the force becomes

$$F = \frac{2mv_x}{2l / v_x} = \frac{mv_x^2}{l}. \tag{13.46}$$

This force is due to *one* molecule. We multiply by the number of molecules N and use their average squared velocity to find the force

$$F = N\frac{m\overline{v_x^2}}{l}, \tag{13.47}$$

where the bar over a quantity means its average value. We would like to have the force in terms of the speed v , rather than the x -component of the velocity. We note that the total velocity squared is the sum of the squares of its components, so that

$$\overline{v^2} = \overline{v_x^2} + \overline{v_y^2} + \overline{v_z^2}. \tag{13.48}$$

Because the velocities are random, their average components in all directions are the same:

$$\overline{v_x^2} = \overline{v_y^2} = \overline{v_z^2}. \tag{13.49}$$

Thus,

$$\overline{v^2} = 3\overline{v_x^2}, \tag{13.50}$$

or

$$\overline{v_x^2} = \frac{1}{3}\overline{v^2}. \tag{13.51}$$

Substituting $\frac{1}{3}\overline{v^2}$ into the expression for F gives

$$F = N\frac{m\overline{v^2}}{3l}. \tag{13.52}$$

The pressure is $F / A,$ so that we obtain

$$P = \frac{F}{A} = N\frac{m\overline{v^2}}{3Al} = \frac{1}{3}\frac{Nm\overline{v^2}}{V}, \tag{13.53}$$

where we used $V = Al$ for the volume. This gives the important result.

$$PV = \frac{1}{3}Nm\overline{v^2} \tag{13.54}$$

This equation is another expression of the ideal gas law.

We can get the average kinetic energy of a molecule, $\frac{1}{2}mv^2$, from the left-hand side of the equation by canceling N and multiplying by 3/2. This calculation produces the result that the average kinetic energy of a molecule is directly related to absolute temperature.

$$\overline{\text{KE}} = \frac{1}{2}m\overline{v^2} = \frac{3}{2}kT \tag{13.55}$$

The average translational kinetic energy of a molecule, $\overline{\text{KE}}$, is called **thermal energy.** The equation $\overline{\text{KE}} = \frac{1}{2}m\overline{v^2} = \frac{3}{2}kT$ is a molecular interpretation of temperature, and it has been found to be valid for gases and reasonably accurate in liquids and solids. It is another definition of temperature based on an expression of the molecular energy.

It is sometimes useful to rearrange $\overline{\text{KE}} = \frac{1}{2}m\overline{v^2} = \frac{3}{2}kT$**,** and solve for the average speed of molecules in a gas in terms of temperature,

$$\sqrt{\overline{v^2}} = v_{\text{rms}} = \sqrt{\frac{3kT}{m}}, \tag{13.56}$$

where v_{rms} stands for root-mean-square (rms) speed.

Example 13.10 Calculating Kinetic Energy and Speed of a Gas Molecule

(a) What is the average kinetic energy of a gas molecule at 20.0ºC (room temperature)? (b) Find the rms speed of a nitrogen molecule (N_2) at this temperature.

Strategy for (a)

The known in the equation for the average kinetic energy is the temperature.

$$\overline{\text{KE}} = \frac{1}{2}m\overline{v^2} = \frac{3}{2}kT \tag{13.57}$$

Before substituting values into this equation, we must convert the given temperature to kelvins. This conversion gives $T = (20.0 + 273)\text{ K} = 293\text{ K}.$

Solution for (a)

The temperature alone is sufficient to find the average translational kinetic energy. Substituting the temperature into the translational kinetic energy equation gives

$$\overline{\text{KE}} = \frac{3}{2}kT = \frac{3}{2}\left(1.38\times10^{-23}\text{ J/K}\right)(293\text{ K}) = 6.07\times10^{-21}\text{ J}. \tag{13.58}$$

Strategy for (b)

Finding the rms speed of a nitrogen molecule involves a straightforward calculation using the equation

$$\sqrt{\overline{v^2}} = v_{\text{rms}} = \sqrt{\frac{3kT}{m}}, \tag{13.59}$$

but we must first find the mass of a nitrogen molecule. Using the molecular mass of nitrogen N_2 from the periodic table,

$$m = \frac{2(14.0067)\times10^{-3}\text{ kg/mol}}{6.02\times10^{23}\text{ mol}^{-1}} = 4.65\times10^{-26}\text{ kg}. \tag{13.60}$$

Solution for (b)

Substituting this mass and the value for k into the equation for v_{rms} yields

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3\left(1.38\times10^{-23}\ \text{J/K}\right)(293\ \text{K})}{4.65\times10^{-26}\ \text{kg}}} = 511\ \text{m/s}. \tag{13.61}$$

Discussion

Note that the average kinetic energy of the molecule is independent of the type of molecule. The average translational kinetic energy depends only on absolute temperature. The kinetic energy is very small compared to macroscopic energies, so that we do not feel when an air molecule is hitting our skin. The rms velocity of the nitrogen molecule is surprisingly large. These large molecular velocities do not yield macroscopic movement of air, since the molecules move in all directions with equal likelihood. The *mean free path* (the distance a molecule can move on average between collisions) of molecules in air is very small, and so the molecules move rapidly but do not get very far in a second. The high value for rms speed is reflected in the speed of sound, however, which is about 340 m/s at room temperature. The faster the rms speed of air molecules, the faster that sound vibrations can be transferred through the air. The speed of sound increases with temperature and is greater in gases with small molecular masses, such as helium. (See Figure 13.22.)

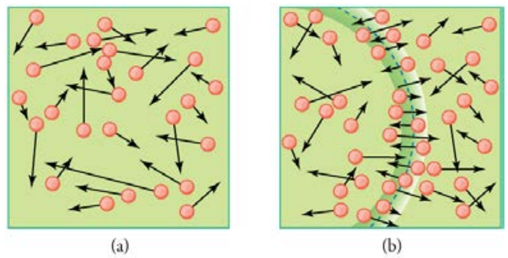


Figure 13.22 (a) There are many molecules moving so fast in an ordinary gas that they collide a billion times every second. (b) Individual molecules do not move very far in a small amount of time, but disturbances like sound waves are transmitted at speeds related to the molecular speeds.

Making Connections: Historical Note—Kinetic Theory of Gases

The kinetic theory of gases was developed by Daniel Bernoulli (1700–1782), who is best known in physics for his work on fluid flow (hydrodynamics). Bernoulli's work predates the atomistic view of matter established by Dalton.

Distribution of Molecular Speeds

The motion of molecules in a gas is random in magnitude and direction for individual molecules, but a gas of many molecules has a predictable distribution of molecular speeds. This distribution is called the *Maxwell-Boltzmann distribution*, after its originators, who calculated it based on kinetic theory, and has since been confirmed experimentally. (See Figure 13.23.) The distribution has a long tail, because a few molecules may go several times the rms speed. The most probable speed v_{p} is less than the rms speed v_{rms}. Figure 13.24 shows that the curve is shifted to higher speeds at higher temperatures, with a broader range of speeds.

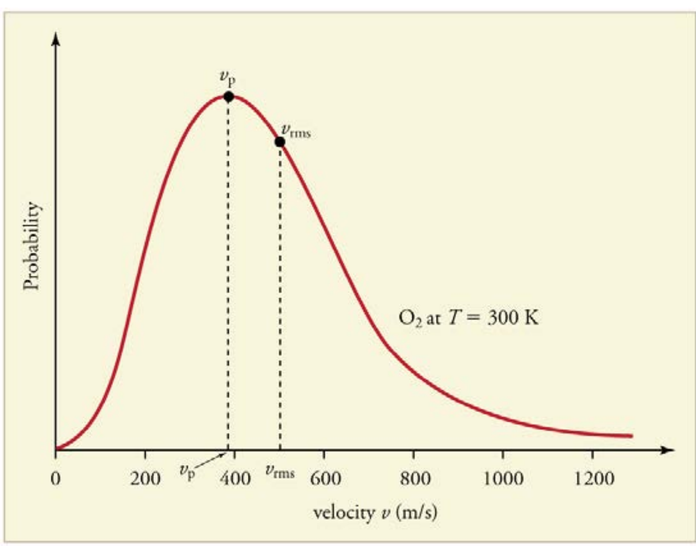


Figure 13.23 The Maxwell-Boltzmann distribution of molecular speeds in an ideal gas. The most likely speed v_{p} is less than the rms speed v_{rms}. Although very high speeds are possible, only a tiny fraction of the molecules have speeds that are an order of magnitude greater than v_{rms}.

The distribution of thermal speeds depends strongly on temperature. As temperature increases, the speeds are shifted to higher values and the distribution is broadened.

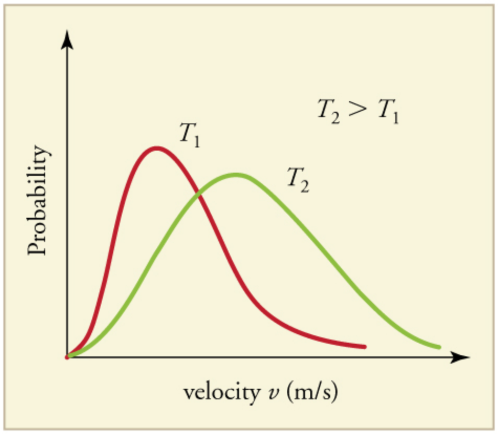


Figure 13.24 The Maxwell-Boltzmann distribution is shifted to higher speeds and is broadened at higher temperatures.

What is the implication of the change in distribution with temperature shown in **Figure 13.24** for humans? All other things being equal, if a person has a fever, he or she is likely to lose more water molecules, particularly from linings along moist cavities such as the lungs and mouth, creating a dry sensation in the mouth.

Example 13.11 Calculating Temperature: Escape Velocity of Helium Atoms

In order to escape Earth's gravity, an object near the top of the atmosphere (at an altitude of 100 km) must travel away from Earth at 11.1 km/s. This speed is called the *escape velocity*. At what temperature would helium atoms have an rms speed equal to the escape velocity?

Strategy

Identify the knowns and unknowns and determine which equations to use to solve the problem.

Solution

1. Identify the knowns: v is the escape velocity, 11.1 km/s.

2. Identify the unknowns: We need to solve for temperature, T. We also need to solve for the mass m of the helium atom.

3. Determine which equations are needed.

- To solve for mass m of the helium atom, we can use information from the periodic table:

$$m = \frac{\text{molar mass}}{\text{number of atoms per mole}}. \tag{13.62}$$

- To solve for temperature T, we can rearrange either

$$\overline{\text{KE}} = \frac{1}{2}m\overline{v^2} = \frac{3}{2}kT \tag{13.63}$$

or

$$\sqrt{\overline{v^2}} = v_{\text{rms}} = \sqrt{\frac{3kT}{m}} \tag{13.64}$$

to yield

$$T = \frac{m\overline{v^2}}{3k}, \tag{13.65}$$

where k is the Boltzmann constant and m is the mass of a helium atom.

4. Plug the known values into the equations and solve for the unknowns.

$$m = \frac{\text{molar mass}}{\text{number of atoms per mole}} = \frac{4.0026\times10^{-3}\text{ kg/mol}}{6.02\times10^{23}\text{ mol}} = 6.65\times10^{-27}\text{ kg} \tag{13.66}$$

$$T = \frac{\left(6.65\times10^{-27}\text{ kg}\right)\left(11.1\times10^{3}\text{ m/s}\right)^2}{3\left(1.38\times10^{-23}\text{ J/K}\right)} = 1.98\times10^{4}\text{ K} \tag{13.67}$$

Discussion

This temperature is much higher than atmospheric temperature, which is approximately 250 K $(-25^\circ\text{C}$ or $-10^\circ\text{F})$ at high altitude. Very few helium atoms are left in the atmosphere, but there were many when the atmosphere was formed. The reason for the loss of helium atoms is that there are a small number of helium atoms with speeds higher than Earth's escape velocity even at normal temperatures. The speed of a helium atom changes from one instant to the next, so that at any instant, there is a small, but nonzero chance that the speed is greater than the escape speed and the molecule escapes from Earth's gravitational pull. Heavier molecules, such as oxygen, nitrogen, and water (very little of which reach a very high altitude), have smaller rms speeds, and so it is much less likely that any of them will have speeds greater than the escape velocity. In fact, so few have speeds above the escape velocity that billions of years are required to lose significant amounts of the atmosphere.

Figure 13.25 shows the impact of a lack of an atmosphere on the Moon. Because the gravitational pull of the Moon is much weaker, it has lost almost its entire atmosphere. The comparison between Earth and the Moon is discussed in this chapter's Problems and Exercises.

Figure 13.25 This photograph of Apollo 17 Commander Eugene Cernan driving the lunar rover on the Moon in 1972 looks as though it was taken at night with a large spotlight. In fact, the light is coming from the Sun. Because the acceleration due to gravity on the Moon is so low (about 1/6 that of Earth), the Moon's escape velocity is much smaller. As a result, gas molecules escape very easily from the Moon, leaving it with virtually no atmosphere. Even during the daytime, the sky is black because there is no gas to scatter sunlight. (credit: Harrison H. Schmitt/NASA)

Check Your Understanding

If you consider a very small object such as a grain of pollen, in a gas, then the number of atoms and molecules striking its surface would also be relatively small. Would the grain of pollen experience any fluctuations in pressure due to statistical fluctuations in the number of gas atoms and molecules striking it in a given amount of time?

Solution

Yes. Such fluctuations actually occur for a body of any size in a gas, but since the numbers of atoms and molecules are immense for macroscopic bodies, the fluctuations are a tiny percentage of the number of collisions, and the averages spoken of in this section vary imperceptibly. Roughly speaking the fluctuations are proportional to the inverse square root of the number of collisions, so for small bodies they can become significant. This was actually observed in the 19th century for pollen grains in water, and is known as the Brownian effect.

PhET Explorations: Gas Properties

Pump gas molecules into a box and see what happens as you change the volume, add or remove heat, change gravity, and more. Measure the temperature and pressure, and discover how the properties of the gas vary in relation to each other.

PhET Interactive Simulation

Figure 13.26 Gas Properties (http://phet.colorado.edu/en/simulation/gas-properties)

13.5 Phase Changes

Up to now, we have considered the behavior of ideal gases. Real gases are like ideal gases at high temperatures. At lower temperatures, however, the interactions between the molecules and their volumes cannot be ignored. The molecules are very close (condensation occurs) and there is a dramatic decrease in volume, as seen in Figure 13.27. The substance changes from a gas to a liquid. When a liquid is cooled to even lower temperatures, it becomes a solid. The volume never reaches zero because of the finite volume of the molecules.

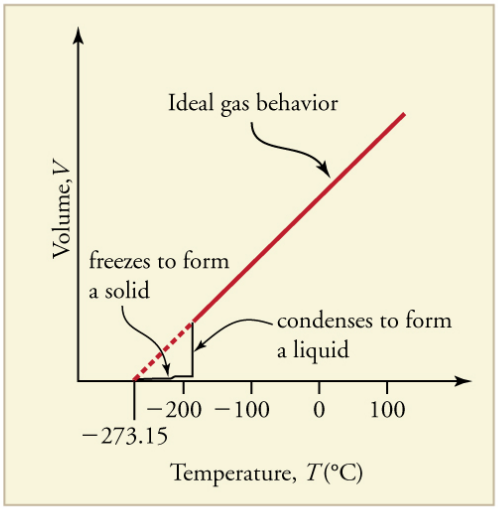


Figure 13.27 A sketch of volume versus temperature for a real gas at constant pressure. The linear (straight line) part of the graph represents ideal gas behavior—volume and temperature are directly and positively related and the line extrapolates to zero volume at $-273.15ºC$, or absolute zero. When the gas becomes a liquid, however, the volume actually decreases precipitously at the liquefaction point. The volume decreases slightly once the substance is solid, but it never becomes zero.

High pressure may also cause a gas to change phase to a liquid. Carbon dioxide, for example, is a gas at room temperature and atmospheric pressure, but becomes a liquid under sufficiently high pressure. If the pressure is reduced, the temperature drops and the liquid carbon dioxide solidifies into a snow-like substance at the temperature $-78ºC$. Solid CO_2 is called “dry ice.” Another example of a gas that can be in a liquid phase is liquid nitrogen (LN_2). LN_2 is made by liquefaction of atmospheric air (through compression and cooling). It boils at 77 K $(–196ºC)$ at atmospheric pressure. LN_2 is useful as a refrigerant and allows for the preservation of blood, sperm, and other biological materials. It is also used to reduce noise in electronic sensors and equipment, and to help cool down their current-carrying wires. In dermatology, LN_2 is used to freeze and painlessly remove warts and other growths from the skin.

PV Diagrams

We can examine aspects of the behavior of a substance by plotting a graph of pressure versus volume, called a ***PV* diagram**. When the substance behaves like an ideal gas, the ideal gas law describes the relationship between its pressure and volume. That is,

$$PV = NkT \text{ (ideal gas)}. \tag{13.68}$$

Now, assuming the number of molecules and the temperature are fixed,

$$PV = \text{constant (ideal gas, constant temperature)}. \tag{13.69}$$

For example, the volume of the gas will decrease as the pressure increases. If you plot the relationship $PV = \text{constant}$ on a PV diagram, you find a hyperbola. **Figure 13.28** shows a graph of pressure versus volume. The hyperbolas represent ideal-gas behavior at various fixed temperatures, and are called *isotherms*. At lower temperatures, the curves begin to look less like hyperbolas—the gas is not behaving ideally and may even contain liquid. There is a **critical point**—that is, a **critical temperature**—above which liquid cannot exist. At sufficiently high pressure above the critical point, the gas will have the density of a liquid but will not condense. Carbon dioxide, for example, cannot be liquefied at a temperature above $31.0ºC$. **Critical pressure** is the minimum pressure needed for liquid to exist at the critical temperature. **Table 13.3** lists representative critical temperatures and pressures.

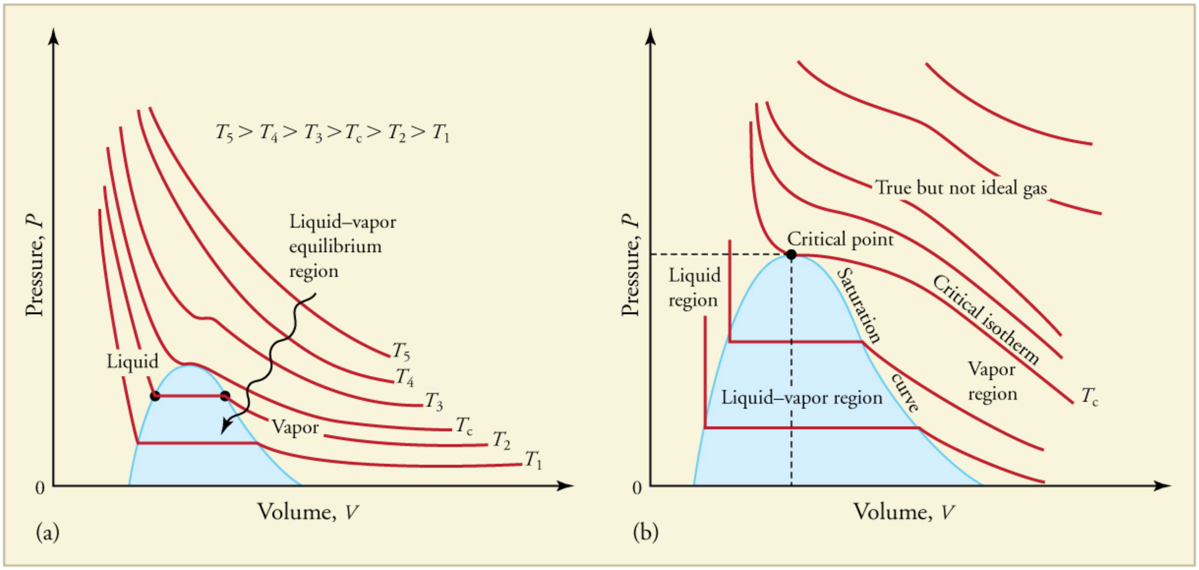


Figure 13.28 PV diagrams. (a) Each curve (isotherm) represents the relationship between P and V at a fixed temperature; the upper curves are at higher temperatures. The lower curves are not hyperbolas, because the gas is no longer an ideal gas. (b) An expanded portion of the PV diagram for low temperatures, where the phase can change from a gas to a liquid. The term "vapor" refers to the gas phase when it exists at a temperature below the boiling temperature.

Table 13.3 Critical Temperatures and Pressures

Substance	Critical temperature		Critical pressure	
	K	ºC	Pa	atm
Water	647.4	374.3	22.12×10^6	219.0
Sulfur dioxide	430.7	157.6	7.88×10^6	78.0
Ammonia	405.5	132.4	11.28×10^6	111.7
Carbon dioxide	304.2	31.1	7.39×10^6	73.2
Oxygen	154.8	−118.4	5.08×10^6	50.3
Nitrogen	126.2	−146.9	3.39×10^6	33.6
Hydrogen	33.3	−239.9	1.30×10^6	12.9
Helium	5.3	−267.9	0.229×10^6	2.27

Phase Diagrams

The plots of pressure versus temperatures provide considerable insight into thermal properties of substances. There are well-defined regions on these graphs that correspond to various phases of matter, so PT graphs are called **phase diagrams**. Figure 13.29 shows the phase diagram for water. Using the graph, if you know the pressure and temperature you can determine the phase of water. The solid lines—boundaries between phases—indicate temperatures and pressures at which the phases coexist (that is, they exist together in ratios, depending on pressure and temperature). For example, the boiling point of water is 100°C at 1.00 atm. As the pressure increases, the boiling temperature rises steadily to 374°C at a pressure of 218 atm. A pressure cooker (or even a covered pot) will cook food faster because the water can exist as a liquid at temperatures greater than 100°C without all boiling away. The curve ends at a point called the *critical point*, because at higher temperatures the liquid phase does not exist at any pressure. The critical point occurs at the critical temperature, as you can see for water from Table 13.3. The critical temperature for oxygen is -118°C, so oxygen cannot be liquefied above this temperature.

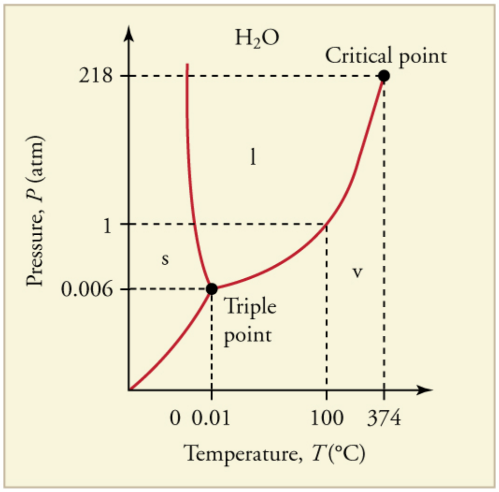


Figure 13.29 The phase diagram (PT graph) for water. Note that the axes are nonlinear and the graph is not to scale. This graph is simplified—there are several other exotic phases of ice at higher pressures.

Similarly, the curve between the solid and liquid regions in **Figure 13.29** gives the melting temperature at various pressures. For example, the melting point is 0°C at 1.00 atm, as expected. Note that, at a fixed temperature, you can change the phase from solid (ice) to liquid (water) by increasing the pressure. Ice melts from pressure in the hands of a snowball maker. From the phase diagram, we can also say that the melting temperature of ice rises with increased pressure. When a car is driven over snow, the increased pressure from the tires melts the snowflakes; afterwards the water refreezes and forms an ice layer.

At sufficiently low pressures there is no liquid phase, but the substance can exist as either gas or solid. For water, there is no liquid phase at pressures below 0.00600 atm. The phase change from solid to gas is called **sublimation**. It accounts for large losses of snow pack that never make it into a river, the routine automatic defrosting of a freezer, and the freeze-drying process applied to many foods. Carbon dioxide, on the other hand, sublimates at standard atmospheric pressure of 1 atm. (The solid form of CO_2 is known as dry ice because it does not melt. Instead, it moves directly from the solid to the gas state.)

All three curves on the phase diagram meet at a single point, the **triple point**, where all three phases exist in equilibrium. For water, the triple point occurs at 273.16 K (0.01°C), and is a more accurate calibration temperature than the melting point of water at 1.00 atm, or 273.15 K (0.0°C). See **Table 13.4** for the triple point values of other substances.

Equilibrium

Liquid and gas phases are in equilibrium at the boiling temperature. (See **Figure 13.30**.) If a substance is in a closed container at the boiling point, then the liquid is boiling and the gas is condensing at the same rate without net change in their relative amount. Molecules in the liquid escape as a gas at the same rate at which gas molecules stick to the liquid, or form droplets and become part of the liquid phase. The combination of temperature and pressure has to be "just right"; if the temperature and pressure are increased, equilibrium is maintained by the same increase of boiling and condensation rates.

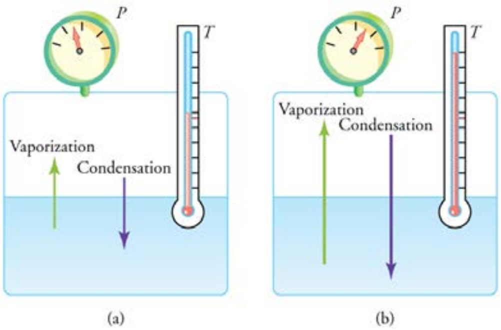


Figure 13.30 Equilibrium between liquid and gas at two different boiling points inside a closed container. (a) The rates of boiling and condensation are equal at this combination of temperature and pressure, so the liquid and gas phases are in equilibrium. (b) At a higher temperature, the boiling rate is faster and the rates at which molecules leave the liquid and enter the gas are also faster. Because there are more molecules in the gas, the gas pressure is higher and the rate at which gas molecules condense and enter the liquid is faster. As a result the gas and liquid are in equilibrium at this higher temperature.

Table 13.4 Triple Point Temperatures and Pressures

Substance	Temperature		Pressure	
	K	ºC	Pa	atm
Water	273.16	0.01	6.10×10^2	0.00600
Carbon dioxide	216.55	−56.60	5.16×10^5	5.11
Sulfur dioxide	197.68	−75.47	1.67×10^3	0.0167
Ammonia	195.40	−77.75	6.06×10^3	0.0600
Nitrogen	63.18	−210.0	1.25×10^4	0.124
Oxygen	54.36	−218.8	1.52×10^2	0.00151
Hydrogen	13.84	−259.3	7.04×10^3	0.0697

One example of equilibrium between liquid and gas is that of water and steam at 100°C and 1.00 atm. This temperature is the boiling point at that pressure, so they should exist in equilibrium. Why does an open pot of water at 100°C boil completely away? The gas surrounding an open pot is not pure water: it is mixed with air. If pure water and steam are in a closed container at 100°C and 1.00 atm, they would coexist—but with air over the pot, there are fewer water molecules to condense, and water boils. What about water at 20.0°C and 1.00 atm? This temperature and pressure correspond to the liquid region, yet an open glass of water at this temperature will completely evaporate. Again, the gas around it is air and not pure water vapor, so that the reduced evaporation rate is greater than the condensation rate of water from dry air. If the glass is sealed, then the liquid phase remains. We call the gas phase a **vapor** when it exists, as it does for water at 20.0°C , at a temperature below the boiling temperature.

Check Your Understanding

Explain why a cup of water (or soda) with ice cubes stays at 0°C , even on a hot summer day.

Solution

The ice and liquid water are in thermal equilibrium, so that the temperature stays at the freezing temperature as long as ice remains in the liquid. (Once all of the ice melts, the water temperature will start to rise.)

Vapor Pressure, Partial Pressure, and Dalton's Law

Vapor pressure is defined as the pressure at which a gas coexists with its solid or liquid phase. Vapor pressure is created by faster molecules that break away from the liquid or solid and enter the gas phase. The vapor pressure of a substance depends on both the substance and its temperature—an increase in temperature increases the vapor pressure.

Partial pressure is defined as the pressure a gas would create if it occupied the total volume available. In a mixture of gases, *the total pressure is the sum of partial pressures of the component gases*, assuming ideal gas behavior and no chemical reactions between the components. This law is known as **Dalton's law of partial pressures**, after the English scientist John Dalton (1766–1844), who proposed it. Dalton's law is based on kinetic theory, where each gas creates its pressure by molecular collisions, independent of other gases present. It is consistent with the fact that pressures add according to **Pascal's Principle**. Thus water evaporates and ice sublimates when their vapor pressures exceed the partial pressure of water vapor in the surrounding mixture of gases. If their vapor pressures are less than the partial pressure of water vapor in the surrounding gas, liquid droplets or ice crystals (frost) form.

Check Your Understanding

Is energy transfer involved in a phase change? If so, will energy have to be supplied to change phase from solid to liquid and liquid to gas? What about gas to liquid and liquid to solid? Why do they spray the orange trees with water in Florida when the temperatures are near or just below freezing?

Solution

Yes, energy transfer is involved in a phase change. We know that atoms and molecules in solids and liquids are bound to each other because we know that force is required to separate them. So in a phase change from solid to liquid and liquid to gas, a force must be exerted, perhaps by collision, to separate atoms and molecules. Force exerted through a distance is work, and energy is needed to do work to go from solid to liquid and liquid to gas. This is intuitively consistent with the need for energy to melt ice or boil water. The converse is also true. Going from gas to liquid or liquid to solid involves atoms and molecules pushing together, doing work and releasing energy.

PhET Explorations: States of Matter—Basics

Heat, cool, and compress atoms and molecules and watch as they change between solid, liquid, and gas phases.

PhET Interactive Simulation

Figure 13.31 States of Matter: Basics (http://phet.colorado.edu/en/simulation/states-of-matter-basics)

13.6 Humidity, Evaporation, and Boiling

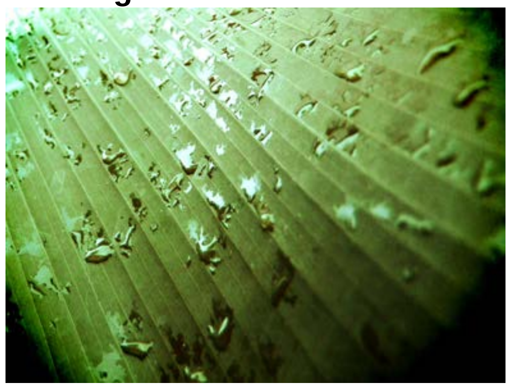


Figure 13.32 Dew drops like these, on a banana leaf photographed just after sunrise, form when the air temperature drops to or below the dew point. At the dew point, the air can no longer hold all of the water vapor it held at higher temperatures, and some of the water condenses to form droplets. (credit: Aaron Escobar, Flickr)

The expression "it's not the heat, it's the humidity" makes a valid point. We keep cool in hot weather by evaporating sweat from our skin and water from our breathing passages. Because evaporation is inhibited by high humidity, we feel hotter at a given temperature when the humidity is high. Low humidity, on the other hand, can cause discomfort from excessive drying of mucous membranes and can lead to an increased risk of respiratory infections.

When we say humidity, we really mean **relative humidity**. Relative humidity tells us how much water vapor is in the air compared with the maximum possible. At its maximum, denoted as **saturation**, the relative humidity is 100%, and evaporation is inhibited. The amount of water vapor the air can hold depends on its temperature. For example, relative humidity rises in the evening, as air temperature declines, sometimes reaching the **dew point**. At the dew point temperature, relative humidity is 100%, and fog may result from the condensation of water droplets if they are small enough to stay in suspension. Conversely, if you wish to dry something (perhaps your hair), it is more effective to blow hot air over it rather than cold air, because, among other things, hot air can hold more water vapor.

The capacity of air to hold water vapor is based on vapor pressure of water. The liquid and solid phases are continuously giving off vapor because some of the molecules have high enough speeds to enter the gas phase; see **Figure 13.33**(a). If a lid is placed over the container, as in **Figure 13.33**(b), evaporation continues, increasing the pressure, until sufficient vapor has built up for condensation to balance evaporation. Then equilibrium has been achieved, and the vapor pressure is equal to the partial pressure of water in the container. Vapor pressure increases with temperature because molecular speeds are higher as temperature increases. **Table 13.5** gives representative values of water vapor pressure over a range of temperatures.

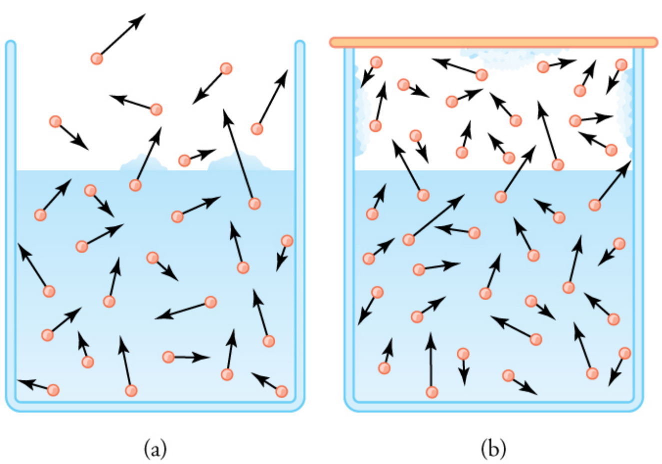


Figure 13.33 (a) Because of the distribution of speeds and kinetic energies, some water molecules can break away to the vapor phase even at temperatures below the ordinary boiling point. (b) If the container is sealed, evaporation will continue until there is enough vapor density for the condensation rate to equal the evaporation rate. This vapor density and the partial pressure it creates are the saturation values. They increase with temperature and are independent of the presence of other gases, such as air. They depend only on the vapor pressure of water.

Relative humidity is related to the partial pressure of water vapor in the air. At 100% humidity, the partial pressure is equal to the vapor pressure, and no more water can enter the vapor phase. If the partial pressure is less than the vapor pressure, then evaporation will take place, as humidity is less than 100%. If the partial pressure is greater than the vapor pressure, condensation takes place. The capacity of air to "hold" water vapor is determined by the vapor pressure of water and has nothing to do with the properties of air.

Table 13.5 Saturation Vapor Density of Water

Temperature $(^{\circ}\text{C})$	Vapor pressure (Pa)	Saturation vapor density (g/m^3)
−50	4.0	0.039
−20	1.04×10^{2}	0.89
−10	2.60×10^{2}	2.36
0	6.10×10^{2}	4.84
5	8.68×10^{2}	6.80
10	1.19×10^{3}	9.40
15	1.69×10^{3}	12.8
20	2.33×10^{3}	17.2
25	3.17×10^{3}	23.0
30	4.24×10^{3}	30.4
37	6.31×10^{3}	44.0
40	7.34×10^{3}	51.1
50	1.23×10^{4}	82.4
60	1.99×10^{4}	130
70	3.12×10^{4}	197
80	4.73×10^{4}	294
90	7.01×10^{4}	418
95	8.59×10^{4}	505
100	$\mathbf{1.01\times10^{5}}$	**598**
120	1.99×10^{5}	1095
150	4.76×10^{5}	2430
200	1.55×10^{6}	7090
220	2.32×10^{6}	10,200

Example 13.12 Calculating Density Using Vapor Pressure

Table 13.5 gives the vapor pressure of water at 20.0°C as $2.33\times10^{3}\text{ Pa}$. Use the ideal gas law to calculate the density of water vapor in g/m^3 that would create a partial pressure equal to this vapor pressure. Compare the result with the saturation vapor density given in the table.

Strategy

To solve this problem, we need to break it down into a two steps. The partial pressure follows the ideal gas law,

$$PV = nRT, \tag{13.70}$$

where n is the number of moles. If we solve this equation for n/V to calculate the number of moles per cubic meter, we can then convert this quantity to grams per cubic meter as requested. To do this, we need to use the molecular mass of water, which is given in the periodic table.

Solution

1. Identify the knowns and convert them to the proper units:

   a. temperature $T = 20^{\circ}\text{C}=293\text{ K}$

   b. vapor pressure P of water at 20°C is $2.33\times10^{3}\text{ Pa}$

c. molecular mass of water is 18.0 g/mol

2. Solve the ideal gas law for n / V.

$$\frac{n}{V} = \frac{P}{RT} \tag{13.71}$$

3. Substitute known values into the equation and solve for n / V.

$$\frac{n}{V} = \frac{P}{RT} = \frac{2.33 \times 10^3 \text{ Pa}}{(8.31 \text{ J/mol} \cdot \text{K})(293 \text{ K})} = 0.957 \text{ mol/m}^3 \tag{13.72}$$

4. Convert the density in moles per cubic meter to grams per cubic meter.

$$\rho = \left(0.957 \frac{\text{mol}}{\text{m}^3}\right)\left(\frac{18.0 \text{ g}}{\text{mol}}\right) = 17.2 \text{ g/m}^3 \tag{13.73}$$

Discussion

The density is obtained by assuming a pressure equal to the vapor pressure of water at 20.0ºC. The density found is identical to the value in **Table 13.5**, which means that a vapor density of 17.2 g/m^3 at 20.0ºC creates a partial pressure of $2.33 \times 10^3 \text{ Pa},$ equal to the vapor pressure of water at that temperature. If the partial pressure is equal to the vapor pressure, then the liquid and vapor phases are in equilibrium, and the relative humidity is 100%. Thus, there can be no more than 17.2 g of water vapor per m^3 at 20.0ºC, so that this value is the saturation vapor density at that temperature. This example illustrates how water vapor behaves like an ideal gas: the pressure and density are consistent with the ideal gas law (assuming the density in the table is correct). The saturation vapor densities listed in **Table 13.5** are the maximum amounts of water vapor that air can hold at various temperatures.

Percent Relative Humidity

We define **percent relative humidity** as the ratio of vapor density to saturation vapor density, or

$$\text{percent relative humidity} = \frac{\text{vapor density}}{\text{saturation vapor density}} \times 100 \tag{13.74}$$

We can use this and the data in **Table 13.5** to do a variety of interesting calculations, keeping in mind that relative humidity is based on the comparison of the partial pressure of water vapor in air and ice.

Example 13.13 Calculating Humidity and Dew Point

(a) Calculate the percent relative humidity on a day when the temperature is 25.0ºC and the air contains 9.40 g of water vapor per m^3. (b) At what temperature will this air reach 100% relative humidity (the saturation density)? This temperature is the dew point. (c) What is the humidity when the air temperature is 25.0ºC and the dew point is -10.0ºC?

Strategy and Solution

(a) Percent relative humidity is defined as the ratio of vapor density to saturation vapor density.

$$\text{percent relative humidity} = \frac{\text{vapor density}}{\text{saturation vapor density}} \times 100 \tag{13.75}$$

The first is given to be 9.40 g/m^3, and the second is found in **Table 13.5** to be 23.0 g/m^3. Thus,

$$\text{percent relative humidity} = \frac{9.40 \text{ g/m}^3}{23.0 \text{ g/m}^3} \times 100 = 40.9.\% \tag{13.76}$$

(b) The air contains 9.40 g/m^3 of water vapor. The relative humidity will be 100% at a temperature where 9.40 g/m^3 is the saturation density. Inspection of **Table 13.5** reveals this to be the case at 10.0ºC, where the relative humidity will be 100%. That temperature is called the dew point for air with this concentration of water vapor.

(c) Here, the dew point temperature is given to be -10.0ºC. Using **Table 13.5**, we see that the vapor density is 2.36 g/m^3, because this value is the saturation vapor density at -10.0ºC. The saturation vapor density at 25.0ºC is seen to be 23.0 g/m^3. Thus, the relative humidity at 25.0ºC is

$$\text{percent relative humidity} = \frac{2.36 \text{ g/m}^3}{23.0 \text{ g/m}^3} \times 100 = 10.3\%. \tag{13.77}$$

Discussion

The importance of dew point is that air temperature cannot drop below 10.0°C in part (b), or -10.0°C in part (c), without water vapor condensing out of the air. If condensation occurs, considerable transfer of heat occurs (discussed in **Heat and Heat Transfer Methods**), which prevents the temperature from further dropping. When dew points are below 0°C, freezing temperatures are a greater possibility, which explains why farmers keep track of the dew point. Low humidity in deserts means low dew-point temperatures. Thus condensation is unlikely. If the temperature drops, vapor does not condense in liquid drops. Because no heat is released into the air, the air temperature drops more rapidly compared to air with higher humidity. Likewise, at high temperatures, liquid droplets do not evaporate, so that no heat is removed from the gas to the liquid phase. This explains the large range of temperature in arid regions.

Why does water boil at 100°C? You will note from **Table 13.5** that the vapor pressure of water at 100°C is $1.01\times10^{5}\ \text{Pa}$, or 1.00 atm. Thus, it can evaporate without limit at this temperature and pressure. But why does it form bubbles when it boils? This is because water ordinarily contains significant amounts of dissolved air and other impurities, which are observed as small bubbles of air in a glass of water. If a bubble starts out at the bottom of the container at 20°C, it contains water vapor (about 2.30%). The pressure inside the bubble is fixed at 1.00 atm (we ignore the slight pressure exerted by the water around it). As the temperature rises, the amount of air in the bubble stays the same, but the water vapor increases; the bubble expands to keep the pressure at 1.00 atm. At 100°C, water vapor enters the bubble continuously since the partial pressure of water is equal to 1.00 atm in equilibrium. It cannot reach this pressure, however, since the bubble also contains air and total pressure is 1.00 atm. The bubble grows in size and thereby increases the buoyant force. The bubble breaks away and rises rapidly to the surface—we call this boiling! (See **Figure 13.34**.)

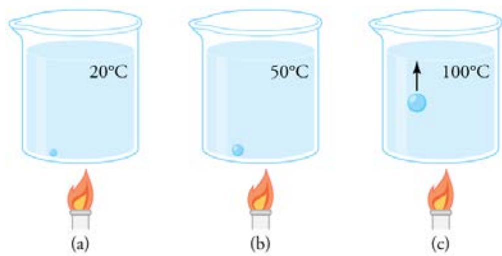


Figure 13.34 (a) An air bubble in water starts out saturated with water vapor at 20°C. (b) As the temperature rises, water vapor enters the bubble because its vapor pressure increases. The bubble expands to keep its pressure at 1.00 atm. (c) At 100°C, water vapor enters the bubble continuously because water's vapor pressure exceeds its partial pressure in the bubble, which must be less than 1.00 atm. The bubble grows and rises to the surface.

Check Your Understanding

Freeze drying is a process in which substances, such as foods, are dried by placing them in a vacuum chamber and lowering the atmospheric pressure around them. How does the lowered atmospheric pressure speed the drying process, and why does it cause the temperature of the food to drop?

Solution

Decreased the atmospheric pressure results in decreased partial pressure of water, hence a lower humidity. So evaporation of water from food, for example, will be enhanced. The molecules of water most likely to break away from the food will be those with the greatest velocities. Those remaining thus have a lower average velocity and a lower temperature. This can (and does) result in the freezing and drying of the food; hence the process is aptly named freeze drying.

PhET Explorations: States of Matter

Watch different types of molecules form a solid, liquid, or gas. Add or remove heat and watch the phase change. Change the temperature or volume of a container and see a pressure-temperature diagram respond in real time. Relate the interaction potential to the forces between molecules.

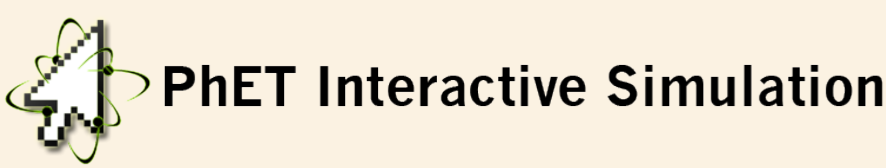


Figure 13.35 States of Matter: Basics (http://phet.colorado.edu/en/simulation/states-of-matter)

Glossary

Avogadro's number: N_{A}, the number of molecules or atoms in one mole of a substance; $N_{\text{A}} = 6.02\times10^{23}$ particles/mole

absolute zero: the lowest possible temperature; the temperature at which all molecular motion ceases

Boltzmann constant: k, a physical constant that relates energy to temperature; $k = 1.38\times10^{-23}\ \text{J/K}$

Celsius scale: temperature scale in which the freezing point of water is 0°C and the boiling point of water is 100°C

coefficient of linear expansion: α, the change in length, per unit length, per 1ºC change in temperature; a constant used in the calculation of linear expansion; the coefficient of linear expansion depends on the material and to some degree on the temperature of the material

coefficient of volume expansion: β, the change in volume, per unit volume, per 1ºC change in temperature

critical point: the temperature above which a liquid cannot exist

critical pressure: the minimum pressure needed for a liquid to exist at the critical temperature

critical temperature: the temperature above which a liquid cannot exist

Dalton's law of partial pressures: the physical law that states that the total pressure of a gas is the sum of partial pressures of the component gases

degree Celsius: unit on the Celsius temperature scale

degree Fahrenheit: unit on the Fahrenheit temperature scale

dew point: the temperature at which relative humidity is 100%; the temperature at which water starts to condense out of the air

Fahrenheit scale: temperature scale in which the freezing point of water is 32ºF and the boiling point of water is 212ºF

ideal gas law: the physical law that relates the pressure and volume of a gas to the number of gas molecules or number of moles of gas and the temperature of the gas

Kelvin scale: temperature scale in which 0 K is the lowest possible temperature, representing absolute zero

mole: the quantity of a substance whose mass (in grams) is equal to its molecular mass

PV diagram: a graph of pressure vs. volume

partial pressure: the pressure a gas would create if it occupied the total volume of space available

percent relative humidity: the ratio of vapor density to saturation vapor density

phase diagram: a graph of pressure vs. temperature of a particular substance, showing at which pressures and temperatures the three phases of the substance occur

relative humidity: the amount of water in the air relative to the maximum amount the air can hold

saturation: the condition of 100% relative humidity

sublimation: the phase change from solid to gas

temperature: the quantity measured by a thermometer

thermal energy: $\overline{\text{KE}}$, the average translational kinetic energy of a molecule

thermal equilibrium: the condition in which heat no longer flows between two objects that are in contact; the two objects have the same temperature

thermal expansion: the change in size or volume of an object with change in temperature

thermal stress: stress caused by thermal expansion or contraction

triple point: the pressure and temperature at which a substance exists in equilibrium as a solid, liquid, and gas

vapor pressure: the pressure at which a gas coexists with its solid or liquid phase

vapor: a gas at a temperature below the boiling temperature

zeroth law of thermodynamics: law that states that if two objects are in thermal equilibrium, and a third object is in thermal equilibrium with one of those objects, it is also in thermal equilibrium with the other object

Section Summary

13.1 Temperature

- Temperature is the quantity measured by a thermometer.
- Temperature is related to the average kinetic energy of atoms and molecules in a system.
- Absolute zero is the temperature at which there is no molecular motion.
- There are three main temperature scales: Celsius, Fahrenheit, and Kelvin.
- Temperatures on one scale can be converted to temperatures on another scale using the following equations:

$$T_{\text{ºF}} = \frac{9}{5}T_{\text{ºC}} + 32$$

$$T_{°C} = \frac{5}{9}(T_{°F} - 32)$$

$$T_K = T_{°C} + 273.15$$

$$T_{°C} = T_K - 273.15$$

- Systems are in thermal equilibrium when they have the same temperature.
- Thermal equilibrium occurs when two bodies are in contact with each other and can freely exchange energy.
- The zeroth law of thermodynamics states that when two systems, A and B, are in thermal equilibrium with each other, and B is in thermal equilibrium with a third system, C, then A is also in thermal equilibrium with C.

13.2 Thermal Expansion of Solids and Liquids

- Thermal expansion is the increase, or decrease, of the size (length, area, or volume) of a body due to a change in temperature.
- Thermal expansion is large for gases, and relatively small, but not negligible, for liquids and solids.
- Linear thermal expansion is
$$\Delta L = \alpha L \Delta T,$$
where ΔL is the change in length L, ΔT is the change in temperature, and α is the coefficient of linear expansion, which varies slightly with temperature.
- The change in area due to thermal expansion is
$$\Delta A = 2\alpha A \Delta T,$$
where ΔA is the change in area.
- The change in volume due to thermal expansion is
$$\Delta V = \beta V \Delta T,$$
where β is the coefficient of volume expansion and $\beta \approx 3\alpha$. Thermal stress is created when thermal expansion is constrained.

13.3 The Ideal Gas Law

- The ideal gas law relates the pressure and volume of a gas to the number of gas molecules and the temperature of the gas.
- The ideal gas law can be written in terms of the number of molecules of gas:
$$PV = NkT,$$
where P is pressure, V is volume, T is temperature, N is number of molecules, and k is the Boltzmann constant
$$k = 1.38\times10^{-23}\ \text{J/K}.$$
- A mole is the number of atoms in a 12-g sample of carbon-12.
- The number of molecules in a mole is called Avogadro's number N_A,
$$N_A = 6.02\times10^{23}\ \text{mol}^{-1}.$$
- A mole of any substance has a mass in grams equal to its molecular weight, which can be determined from the periodic table of elements.
- The ideal gas law can also be written and solved in terms of the number of moles of gas:
$$PV = nRT,$$
where n is number of moles and R is the universal gas constant,
$$R = 8.31\ \text{J/mol} \cdot \text{K}.$$
- The ideal gas law is generally valid at temperatures well above the boiling temperature.

13.4 Kinetic Theory: Atomic and Molecular Explanation of Pressure and Temperature

- Kinetic theory is the atomistic description of gases as well as liquids and solids.
- Kinetic theory models the properties of matter in terms of continuous random motion of atoms and molecules.
- The ideal gas law can also be expressed as
$$PV = \frac{1}{3}Nm\overline{v^2},$$
where P is the pressure (average force per unit area), V is the volume of gas in the container, N is the number of molecules in the container, m is the mass of a molecule, and $\overline{v^2}$ is the average of the molecular speed squared.
- Thermal energy is defined to be the average translational kinetic energy $\overline{\text{KE}}$ of an atom or molecule.
- The temperature of gases is proportional to the average translational kinetic energy of atoms and molecules.
$$\overline{\text{KE}} = \frac{1}{2}m\overline{v^2} = \frac{3}{2}kT$$
or
$$\sqrt{\overline{v^2}} = v_{\text{rms}} = \sqrt{\frac{3kT}{m}}.$$
- The motion of individual molecules in a gas is random in magnitude and direction. However, a gas of many molecules has a predictable distribution of molecular speeds, known as the *Maxwell-Boltzmann distribution*.

13.5 Phase Changes

- Most substances have three distinct phases: gas, liquid, and solid.
- Phase changes among the various phases of matter depend on temperature and pressure.

- The existence of the three phases with respect to pressure and temperature can be described in a phase diagram.
- Two phases coexist (i.e., they are in thermal equilibrium) at a set of pressures and temperatures. These are described as a line on a phase diagram.
- The three phases coexist at a single pressure and temperature. This is known as the triple point and is described by a single point on a phase diagram.
- A gas at a temperature below its boiling point is called a vapor.
- Vapor pressure is the pressure at which a gas coexists with its solid or liquid phase.
- Partial pressure is the pressure a gas would create if it existed alone.
- Dalton's law states that the total pressure is the sum of the partial pressures of all of the gases present.

13.6 Humidity, Evaporation, and Boiling

- Relative humidity is the fraction of water vapor in a gas compared to the saturation value.
- The saturation vapor density can be determined from the vapor pressure for a given temperature.
- Percent relative humidity is defined to be

$$\text{percent relative humidity} = \frac{\text{vapor density}}{\text{saturation vapor density}} \times 100.$$

- The dew point is the temperature at which air reaches 100% relative humidity.

Conceptual Questions

13.1 Temperature

1. What does it mean to say that two systems are in thermal equilibrium?

2. Give an example of a physical property that varies with temperature and describe how it is used to measure temperature.

3. When a cold alcohol thermometer is placed in a hot liquid, the column of alcohol goes *down* slightly before going up. Explain why.

4. If you add boiling water to a cup at room temperature, what would you expect the final equilibrium temperature of the unit to be? You will need to include the surroundings as part of the system. Consider the zeroth law of thermodynamics.

13.2 Thermal Expansion of Solids and Liquids

5. Thermal stresses caused by uneven cooling can easily break glass cookware. Explain why Pyrex®, a glass with a small coefficient of linear expansion, is less susceptible.

6. Water expands significantly when it freezes: a volume increase of about 9% occurs. As a result of this expansion and because of the formation and growth of crystals as water freezes, anywhere from 10% to 30% of biological cells are burst when animal or plant material is frozen. Discuss the implications of this cell damage for the prospect of preserving human bodies by freezing so that they can be thawed at some future date when it is hoped that all diseases are curable.

7. One method of getting a tight fit, say of a metal peg in a hole in a metal block, is to manufacture the peg slightly larger than the hole. The peg is then inserted when at a different temperature than the block. Should the block be hotter or colder than the peg during insertion? Explain your answer.

8. Does it really help to run hot water over a tight metal lid on a glass jar before trying to open it? Explain your answer.

9. Liquids and solids expand with increasing temperature, because the kinetic energy of a body's atoms and molecules increases. Explain why some materials *shrink* with increasing temperature.

13.3 The Ideal Gas Law

10. Find out the human population of Earth. Is there a mole of people inhabiting Earth? If the average mass of a person is 60 kg, calculate the mass of a mole of people. How does the mass of a mole of people compare with the mass of Earth?

11. Under what circumstances would you expect a gas to behave significantly differently than predicted by the ideal gas law?

12. A constant-volume gas thermometer contains a fixed amount of gas. What property of the gas is measured to indicate its temperature?

13.4 Kinetic Theory: Atomic and Molecular Explanation of Pressure and Temperature

13. How is momentum related to the pressure exerted by a gas? Explain on the atomic and molecular level, considering the behavior of atoms and molecules.

13.5 Phase Changes

14. A pressure cooker contains water and steam in equilibrium at a pressure greater than atmospheric pressure. How does this greater pressure increase cooking speed?

15. Why does condensation form most rapidly on the coldest object in a room—for example, on a glass of ice water?

16. What is the vapor pressure of solid carbon dioxide (dry ice) at -78.5°C?

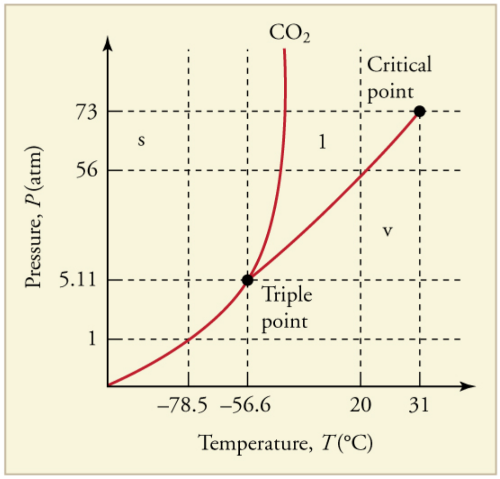


Figure 13.36 The phase diagram for carbon dioxide. The axes are nonlinear, and the graph is not to scale. Dry ice is solid carbon dioxide and has a sublimation temperature of -78.5°C.

17. Can carbon dioxide be liquefied at room temperature (20°C)? If so, how? If not, why not? (See **Figure 13.36**.)

18. Oxygen cannot be liquefied at room temperature by placing it under a large enough pressure to force its molecules together. Explain why this is.

19. What is the distinction between gas and vapor?

13.6 Humidity, Evaporation, and Boiling

20. Because humidity depends only on water's vapor pressure and temperature, are the saturation vapor densities listed in **Table 13.5** valid in an atmosphere of helium at a pressure of $1.01\times10^5\ \text{N/m}^2$, rather than air? Are those values affected by altitude on Earth?

21. Why does a beaker of 40.0°C water placed in a vacuum chamber start to boil as the chamber is evacuated (air is pumped out of the chamber)? At what pressure does the boiling begin? Would food cook any faster in such a beaker?

22. Why does rubbing alcohol evaporate much more rapidly than water at STP (standard temperature and pressure)?

Problems & Exercises

13.1 Temperature

23. What is the Fahrenheit temperature of a person with a $39.0^{\circ}C$ fever?

24. Frost damage to most plants occurs at temperatures of $28.0^{\circ}F$ or lower. What is this temperature on the Kelvin scale?

25. To conserve energy, room temperatures are kept at $68.0^{\circ}F$ in the winter and $78.0^{\circ}F$ in the summer. What are these temperatures on the Celsius scale?

26. A tungsten light bulb filament may operate at 2900 K. What is its Fahrenheit temperature? What is this on the Celsius scale?

27. The surface temperature of the Sun is about 5750 K. What is this temperature on the Fahrenheit scale?

28. One of the hottest temperatures ever recorded on the surface of Earth was $134^{\circ}F$ in Death Valley, CA. What is this temperature in Celsius degrees? What is this temperature in Kelvin?

29. (a) Suppose a cold front blows into your locale and drops the temperature by 40.0 Fahrenheit degrees. How many degrees Celsius does the temperature decrease when there is a $40.0^{\circ}F$ decrease in temperature? (b) Show that any change in temperature in Fahrenheit degrees is nine-fifths the change in Celsius degrees.

30. (a) At what temperature do the Fahrenheit and Celsius scales have the same numerical value? (b) At what temperature do the Fahrenheit and Kelvin scales have the same numerical value?

13.2 Thermal Expansion of Solids and Liquids

31. The height of the Washington Monument is measured to be 170 m on a day when the temperature is $35.0^{\circ}C$. What will its height be on a day when the temperature falls to $-10.0^{\circ}C$? Although the monument is made of limestone, assume that its thermal coefficient of expansion is the same as marble's.

32. How much taller does the Eiffel Tower become at the end of a day when the temperature has increased by $15^{\circ}C$? Its original height is 321 m and you can assume it is made of steel.

33. What is the change in length of a 3.00-cm-long column of mercury if its temperature changes from $37.0^{\circ}C$ to $40.0^{\circ}C$, assuming the mercury is unconstrained?

34. How large an expansion gap should be left between steel railroad rails if they may reach a maximum temperature $35.0^{\circ}C$ greater than when they were laid? Their original length is 10.0 m.

35. You are looking to purchase a small piece of land in Hong Kong. The price is "only" $60,000 per square meter! The land title says the dimensions are $20\text{ m}\times 30\text{ m}.$ By how much would the total price change if you measured the parcel with a steel tape measure on a day when the temperature was $20^{\circ}C$ above normal?

36. Global warming will produce rising sea levels partly due to melting ice caps but also due to the expansion of water as average ocean temperatures rise. To get some idea of the size of this effect, calculate the change in length of a column of water 1.00 km high for a temperature increase of $1.00^{\circ}C.$ Note that this calculation is only approximate because ocean warming is not uniform with depth.

37. Show that 60.0 L of gasoline originally at $15.0^{\circ}C$ will expand to 61.1 L when it warms to $35.0^{\circ}C,$ as claimed in **Example 13.4**.

38. (a) Suppose a meter stick made of steel and one made of invar (an alloy of iron and nickel) are the same length at $0^{\circ}C$. What is their difference in length at $22.0^{\circ}C$? (b) Repeat the calculation for two 30.0-m-long surveyor's tapes.

39. (a) If a 500-mL glass beaker is filled to the brim with ethyl alcohol at a temperature of $5.00^{\circ}C,$ how much will overflow when its temperature reaches $22.0^{\circ}C$? (b) How much less water would overflow under the same conditions?

40. Most automobiles have a coolant reservoir to catch radiator fluid that may overflow when the engine is hot. A radiator is made of copper and is filled to its 16.0-L capacity when at $10.0^{\circ}C.$ What volume of radiator fluid will overflow when the radiator and fluid reach their $95.0^{\circ}C$ operating temperature, given that the fluid's volume coefficient of expansion is $\beta = 400\times 10^{-6}/^{\circ}C$? Note that this coefficient is approximate, because most car radiators have operating temperatures of greater than $95.0^{\circ}C.$

41. A physicist makes a cup of instant coffee and notices that, as the coffee cools, its level drops 3.00 mm in the glass cup. Show that this decrease cannot be due to thermal contraction by calculating the decrease in level if the 350 cm^3 of coffee is in a 7.00-cm-diameter cup and decreases in temperature from $95.0^{\circ}C$ to $45.0^{\circ}C.$ (Most of the drop in level is actually due to escaping bubbles of air.)

42. (a) The density of water at $0^{\circ}C$ is very nearly 1000 kg/m^3 (it is actually 999.84 kg/m^3), whereas the density of ice at $0^{\circ}C$ is 917 kg/m^3. Calculate the pressure necessary to keep ice from expanding when it freezes, neglecting the effect such a large pressure would have on the freezing temperature. (This problem gives you only an indication of how large the forces associated with freezing water might be.) (b) What are the implications of this result for biological cells that are frozen?

43. Show that $\beta \approx 3\alpha,$ by calculating the change in volume ΔV of a cube with sides of length $L.$

13.3 The Ideal Gas Law

44. The gauge pressure in your car tires is $2.50\times 10^5\text{ N/m}^2$ at a temperature of $35.0^{\circ}C$ when you drive it onto a ferry boat to Alaska. What is their gauge pressure later, when their temperature has dropped to $-40.0^{\circ}C$?

45. Convert an absolute pressure of $7.00\times 10^5\text{ N/m}^2$ to gauge pressure in $\text{lb/in}^2.$ (This value was stated to be just less than 90.0 lb/in^2 in **Example 13.9**. Is it?)

46. Suppose a gas-filled incandescent light bulb is manufactured so that the gas inside the bulb is at atmospheric pressure when the bulb has a temperature of $20.0^{\circ}C$. (a) Find the gauge pressure inside such a bulb when it is hot, assuming its average temperature is $60.0^{\circ}C$ (an approximation) and neglecting any change in volume due to thermal expansion or gas leaks. (b) The actual final pressure for the light bulb will be less than calculated in part (a) because the glass bulb will expand. What will the actual final pressure be, taking this into account? Is this a negligible difference?

47. Large helium-filled balloons are used to lift scientific equipment to high altitudes. (a) What is the pressure inside such a balloon if it starts out at sea level with a temperature of $10.0^{\circ}C$ and rises to an altitude where its volume is twenty times the original volume and its temperature is $-50.0^{\circ}C$? (b) What is the gauge pressure? (Assume atmospheric pressure is constant.)

48. Confirm that the units of nRT are those of energy for each value of R: (a) $8.31\text{ J/mol}\cdot\text{K}$, (b) $1.99\text{ cal/mol}\cdot\text{K}$, and (c) $0.0821\text{ L}\cdot\text{atm/mol}\cdot\text{K}$.

49. In the text, it was shown that $N/V = 2.68\times10^{25}\ \text{m}^{-3}$ for gas at STP. (a) Show that this quantity is equivalent to $N/V = 2.68\times10^{19}\ \text{cm}^{-3}$, as stated. (b) About how many atoms are there in one μm^3 (a cubic micrometer) at STP? (c) What does your answer to part (b) imply about the separation of atoms and molecules?

50. Calculate the number of moles in the 2.00-L volume of air in the lungs of the average person. Note that the air is at 37.0°C (body temperature).

51. An airplane passenger has $100\ \text{cm}^3$ of air in his stomach just before the plane takes off from a sea-level airport. What volume will the air have at cruising altitude if cabin pressure drops to $7.50\times10^4\ \text{N/m}^2$?

52. (a) What is the volume (in km^3) of Avogadro's number of sand grains if each grain is a cube and has sides that are 1.0 mm long? (b) How many kilometers of beaches in length would this cover if the beach averages 100 m in width and 10.0 m in depth? Neglect air spaces between grains.

53. An expensive vacuum system can achieve a pressure as low as $1.00\times10^{-7}\ \text{N/m}^2$ at 20°C. How many atoms are there in a cubic centimeter at this pressure and temperature?

54. The number density of gas atoms at a certain location in the space above our planet is about $1.00\times10^{11}\ \text{m}^{-3}$, and the pressure is $2.75\times10^{-10}\ \text{N/m}^2$ in this space. What is the temperature there?

55. A bicycle tire has a pressure of $7.00\times10^5\ \text{N/m}^2$ at a temperature of 18.0°C and contains 2.00 L of gas. What will its pressure be if you let out an amount of air that has a volume of $100\ \text{cm}^3$ at atmospheric pressure? Assume tire temperature and volume remain constant.

56. A high-pressure gas cylinder contains 50.0 L of toxic gas at a pressure of $1.40\times10^7\ \text{N/m}^2$ and a temperature of 25.0°C. Its valve leaks after the cylinder is dropped. The cylinder is cooled to dry ice temperature (-78.5°C) to reduce the leak rate and pressure so that it can be safely repaired. (a) What is the final pressure in the tank, assuming a negligible amount of gas leaks while being cooled and that there is no phase change? (b) What is the final pressure if one-tenth of the gas escapes? (c) To what temperature must the tank be cooled to reduce the pressure to 1.00 atm (assuming the gas does not change phase and that there is no leakage during cooling)? (d) Does cooling the tank appear to be a practical solution?

57. Find the number of moles in 2.00 L of gas at 35.0°C and under $7.41\times10^7\ \text{N/m}^2$ of pressure.

58. Calculate the depth to which Avogadro's number of table tennis balls would cover Earth. Each ball has a diameter of 3.75 cm. Assume the space between balls adds an extra 25.0% to their volume and assume they are not crushed by their own weight.

59. (a) What is the gauge pressure in a 25.0°C car tire containing 3.60 mol of gas in a 30.0 L volume? (b) What will its gauge pressure be if you add 1.00 L of gas originally at atmospheric pressure and 25.0°C? Assume the temperature returns to 25.0°C and the volume remains constant.

60. (a) In the deep space between galaxies, the density of atoms is as low as $10^6\ \text{atoms/m}^3$, and the temperature is a frigid 2.7 K. What is the pressure? (b) What volume (in m^3) is occupied by 1 mol of gas? (c) If this volume is a cube, what is the length of its sides in kilometers?

13.4 Kinetic Theory: Atomic and Molecular Explanation of Pressure and Temperature

61. Some incandescent light bulbs are filled with argon gas. What is v_{rms} for argon atoms near the filament, assuming their temperature is 2500 K?

62. Average atomic and molecular speeds (v_{rms}) are large, even at low temperatures. What is v_{rms} for helium atoms at 5.00 K, just one degree above helium's liquefaction temperature?

63. (a) What is the average kinetic energy in joules of hydrogen atoms on the 5500°C surface of the Sun? (b) What is the average kinetic energy of helium atoms in a region of the solar corona where the temperature is $6.00\times10^5\ \text{K}$?

64. The escape velocity of any object from Earth is 11.2 km/s. (a) Express this speed in m/s and km/h. (b) At what temperature would oxygen molecules (molecular mass is equal to 32.0 g/mol) have an average velocity v_{rms} equal to Earth's escape velocity of 11.1 km/s?

65. The escape velocity from the Moon is much smaller than from Earth and is only 2.38 km/s. At what temperature would hydrogen molecules (molecular mass is equal to 2.016 g/mol) have an average velocity v_{rms} equal to the Moon's escape velocity?

66. Nuclear fusion, the energy source of the Sun, hydrogen bombs, and fusion reactors, occurs much more readily when the average kinetic energy of the atoms is high—that is, at high temperatures. Suppose you want the atoms in your fusion experiment to have average kinetic energies of $6.40\times10^{-14}\ \text{J}$. What temperature is needed?

67. Suppose that the average velocity (v_{rms}) of carbon dioxide molecules (molecular mass is equal to 44.0 g/mol) in a flame is found to be $1.05\times10^5\ \text{m/s}$. What temperature does this represent?

68. Hydrogen molecules (molecular mass is equal to 2.016 g/mol) have an average velocity v_{rms} equal to 193 m/s. What is the temperature?

69. Much of the gas near the Sun is atomic hydrogen. Its temperature would have to be $1.5\times10^7\ \text{K}$ for the average velocity v_{rms} to equal the escape velocity from the Sun. What is that velocity?

70. There are two important isotopes of uranium— ^{235}U and ^{238}U; these isotopes are nearly identical chemically but have different atomic masses. Only ^{235}U is very useful in nuclear reactors. One of the techniques for separating them (gas diffusion) is based on the different average velocities v_{rms} of uranium hexafluoride gas, UF_6. (a) The molecular masses for $^{235}\text{U}\ \text{UF}_6$ and $^{238}\text{U}\ \text{UF}_6$ are 349.0 g/mol and 352.0 g/mol, respectively. What is the ratio of their average velocities? (b) At what temperature would their average velocities differ by 1.00 m/s? (c) Do your answers in this problem imply that this technique may be difficult?

13.6 Humidity, Evaporation, and Boiling

71. Dry air is 78.1% nitrogen. What is the partial pressure of nitrogen when the atmospheric pressure is $1.01\times10^5\ \text{N/m}^2$?

72. (a) What is the vapor pressure of water at 20.0°C? (b) What percentage of atmospheric pressure does this correspond to? (c) What percent of 20.0°C air is water vapor if it has 100% relative humidity? (The density of dry air at 20.0°C is $1.20\ \text{kg/m}^3$.)

73. Pressure cookers increase cooking speed by raising the boiling temperature of water above its value at atmospheric pressure. (a) What pressure is necessary to raise the boiling point to 120.0°C? (b) What gauge pressure does this correspond to?

74. (a) At what temperature does water boil at an altitude of 1500 m (about 5000 ft) on a day when atmospheric pressure is

$8.59\times10^4\ \text{N/m}^2$? (b) What about at an altitude of 3000 m (about 10,000 ft) when atmospheric pressure is $7.00\times10^4\ \text{N/m}^2$?

75. What is the atmospheric pressure on top of Mt. Everest on a day when water boils there at a temperature of 70.0°C?

76. At a spot in the high Andes, water boils at 80.0°C, greatly reducing the cooking speed of potatoes, for example. What is atmospheric pressure at this location?

77. What is the relative humidity on a 25.0°C day when the air contains $18.0\ \text{g/m}^3$ of water vapor?

78. What is the density of water vapor in g/m^3 on a hot dry day in the desert when the temperature is 40.0°C and the relative humidity is 6.00%?

79. A deep-sea diver should breathe a gas mixture that has the same oxygen partial pressure as at sea level, where dry air contains 20.9% oxygen and has a total pressure of $1.01\times10^5\ \text{N/m}^2$. (a) What is the partial pressure of oxygen at sea level? (b) If the diver breathes a gas mixture at a pressure of $2.00\times10^6\ \text{N/m}^2$, what percent oxygen should it be to have the same oxygen partial pressure as at sea level?

80. The vapor pressure of water at 40.0°C is $7.34\times10^3\ \text{N/m}^2$. Using the ideal gas law, calculate the density of water vapor in g/m^3 that creates a partial pressure equal to this vapor pressure. The result should be the same as the saturation vapor density at that temperature $(51.1\ \text{g/m}^3)$.

81. Air in human lungs has a temperature of 37.0°C and a saturation vapor density of $44.0\ \text{g/m}^3$. (a) If 2.00 L of air is exhaled and very dry air inhaled, what is the maximum loss of water vapor by the person? (b) Calculate the partial pressure of water vapor having this density, and compare it with the vapor pressure of $6.31\times10^3\ \text{N/m}^2$.

82. If the relative humidity is 90.0% on a muggy summer morning when the temperature is 20.0°C, what will it be later in the day when the temperature is 30.0°C, assuming the water vapor density remains constant?

83. Late on an autumn day, the relative humidity is 45.0% and the temperature is 20.0°C. What will the relative humidity be that evening when the temperature has dropped to 10.0°C, assuming constant water vapor density?

84. Atmospheric pressure atop Mt. Everest is $3.30\times10^4\ \text{N/m}^2$. (a) What is the partial pressure of oxygen there if it is 20.9% of the air? (b) What percent oxygen should a mountain climber breathe so that its partial pressure is the same as at sea level, where atmospheric pressure is $1.01\times10^5\ \text{N/m}^2$? (c) One of the most severe problems for those climbing very high mountains is the extreme drying of breathing passages. Why does this drying occur?

85. What is the dew point (the temperature at which 100% relative humidity would occur) on a day when relative humidity is 39.0% at a temperature of 20.0°C?

86. On a certain day, the temperature is 25.0°C and the relative humidity is 90.0%. How many grams of water must condense out of each cubic meter of air if the temperature falls to 15.0°C? Such a drop in temperature can, thus, produce heavy dew or fog.

87. Integrated Concepts

The boiling point of water increases with depth because pressure increases with depth. At what depth will fresh water have a boiling point of 150°C, if the surface of the water is at sea level?

88. Integrated Concepts

(a) At what depth in fresh water is the critical pressure of water reached, given that the surface is at sea level? (b) At what temperature will this water boil? (c) Is a significantly higher temperature needed to boil water at a greater depth?

89. Integrated Concepts

To get an idea of the small effect that temperature has on Archimedes' principle, calculate the fraction of a copper block's weight that is supported by the buoyant force in 0°C water and compare this fraction with the fraction supported in 95.0°C water.

90. Integrated Concepts

If you want to cook in water at 150°C, you need a pressure cooker that can withstand the necessary pressure. (a) What pressure is required for the boiling point of water to be this high? (b) If the lid of the pressure cooker is a disk 25.0 cm in diameter, what force must it be able to withstand at this pressure?

91. Unreasonable Results

(a) How many moles per cubic meter of an ideal gas are there at a pressure of $1.00\times10^{14}\ \text{N/m}^2$ and at 0°C? (b) What is unreasonable about this result? (c) Which premise or assumption is responsible?

92. Unreasonable Results

(a) An automobile mechanic claims that an aluminum rod fits loosely into its hole on an aluminum engine block because the engine is hot and the rod is cold. If the hole is 10.0% bigger in diameter than the 22.0°C rod, at what temperature will the rod be the same size as the hole? (b) What is unreasonable about this temperature? (c) Which premise is responsible?

93. Unreasonable Results

The temperature inside a supernova explosion is said to be $2.00\times10^{13}\ \text{K}$. (a) What would the average velocity v_{rms} of hydrogen atoms be? (b) What is unreasonable about this velocity? (c) Which premise or assumption is responsible?

94. Unreasonable Results

Suppose the relative humidity is 80% on a day when the temperature is 30.0°C. (a) What will the relative humidity be if the air cools to 25.0°C and the vapor density remains constant? (b) What is unreasonable about this result? (c) Which premise is responsible?

14 HEAT AND HEAT TRANSFER METHODS

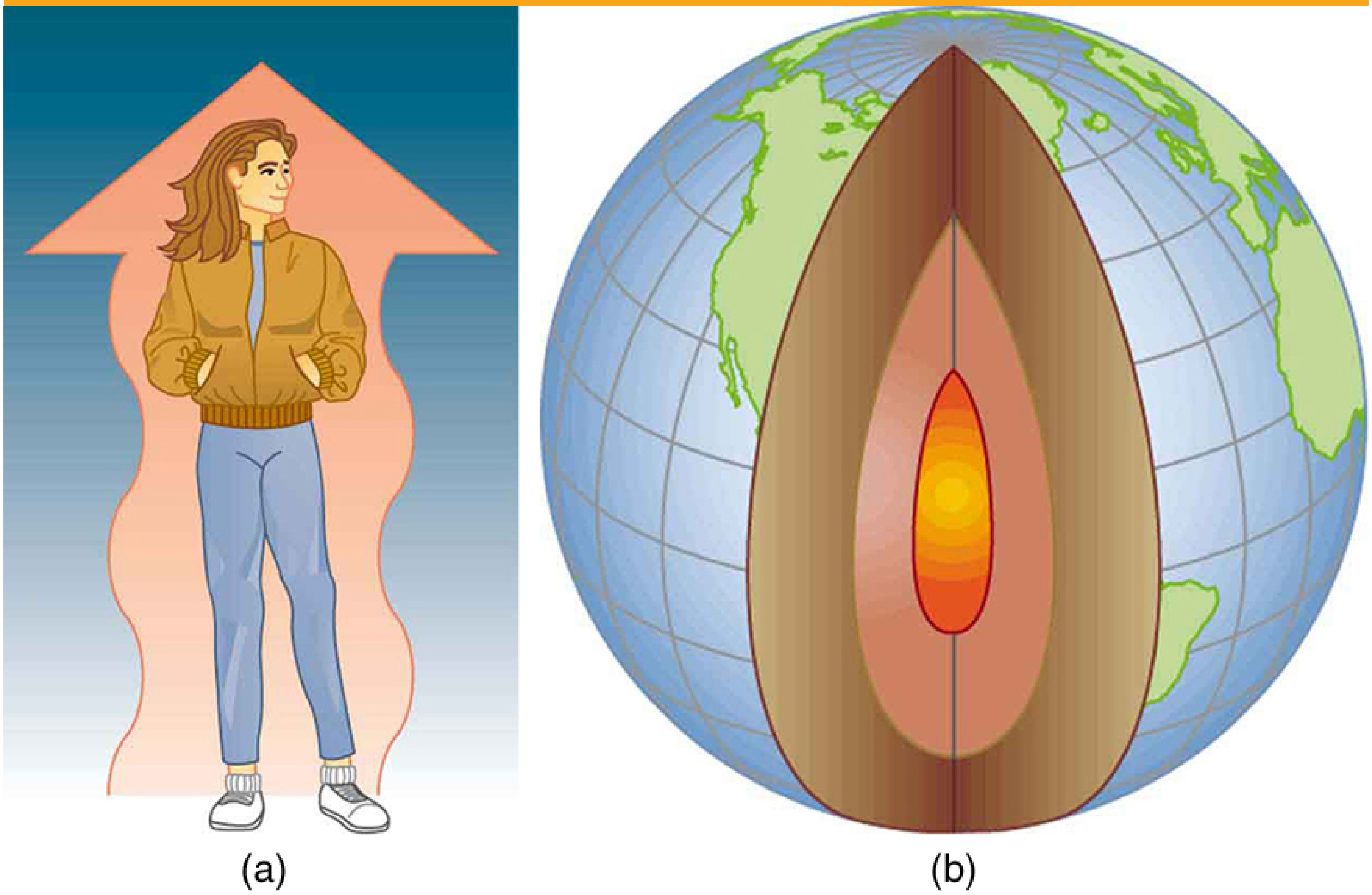


Figure 14.1 (a) The chilling effect of a clear breezy night is produced by the wind and by radiative heat transfer to cold outer space. (b) There was once great controversy about the Earth's age, but it is now generally accepted to be about 4.5 billion years old. Much of the debate is centered on the Earth's molten interior. According to our understanding of heat transfer, if the Earth is really that old, its center should have cooled off long ago. The discovery of radioactivity in rocks revealed the source of energy that keeps the Earth's interior molten, despite heat transfer to the surface, and from there to cold outer space.

Learning Objectives

Introduction to Heat and Heat Transfer Methods

Energy can exist in many forms and heat is one of the most intriguing. Heat is often hidden, as it only exists when in transit, and is transferred by a number of distinctly different methods. Heat transfer touches every aspect of our lives and helps us understand how the universe functions. It explains the chill we feel on a clear breezy night, or why Earth's core has yet to cool. This chapter defines and explores heat transfer, its effects, and the methods by which heat is transferred. These topics are fundamental, as well as practical, and will often be referred to in the chapters ahead.

14.1 Heat

In **Work, Energy, and Energy Resources**, we defined work as force times distance and learned that work done on an object changes its kinetic energy. We also saw in **Temperature, Kinetic Theory, and the Gas Laws** that temperature is proportional to the (average) kinetic energy of atoms and molecules. We say that a thermal system has a certain internal energy: its internal energy is higher if the temperature is higher. If two objects at different temperatures are brought in contact with each other, energy is transferred from the hotter to the colder object until equilibrium is reached and the bodies reach thermal equilibrium (i.e., they are at the same temperature). No work is done by either object, because no force acts through a distance. The transfer of energy is caused by the temperature difference, and ceases once the temperatures are equal. These observations lead to the following definition of **heat**: Heat is the spontaneous transfer of energy due to a temperature difference.

As noted in **Temperature, Kinetic Theory, and the Gas Laws**, heat is often confused with temperature. For example, we may say the heat was unbearable, when we actually mean that the temperature was high. Heat is a form of energy, whereas temperature is not. The misconception arises because we are sensitive to the flow of heat, rather than the temperature.

Owing to the fact that heat is a form of energy, it has the SI unit of *joule* (J). The *calorie* (cal) is a common unit of energy, defined as the energy needed to change the temperature of 1.00 g of water by 1.00ºC —specifically, between 14.5ºC and 15.5ºC, since there is a slight temperature dependence. Perhaps the most common unit of heat is the **kilocalorie** (kcal), which is the energy needed to change the temperature of 1.00 kg of water by 1.00ºC. Since mass is most often specified in kilograms, kilocalorie is commonly used. Food calories (given the notation Cal, and sometimes called "big calorie") are actually kilocalories ($1\text{ kilocalorie} = 1000\text{ calories}$), a fact not easily determined from package labeling.

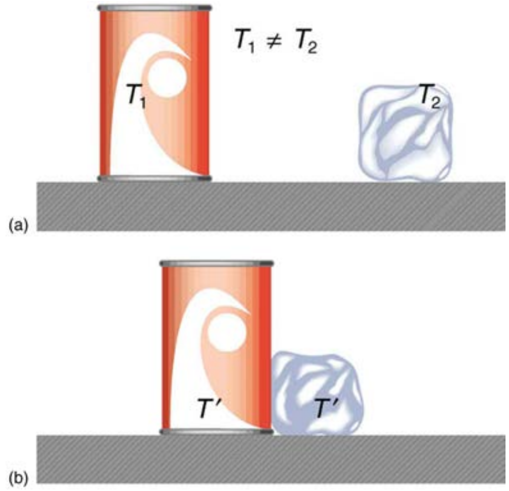


Figure 14.2 In figure (a) the soft drink and the ice have different temperatures, T_1 and T_2, and are not in thermal equilibrium. In figure (b), when the soft drink and ice are allowed to interact, energy is transferred until they reach the same temperature T', achieving equilibrium. Heat transfer occurs due to the difference in temperatures. In fact, since the soft drink and ice are both in contact with the surrounding air and bench, the equilibrium temperature will be the same for both.

Mechanical Equivalent of Heat

It is also possible to change the temperature of a substance by doing work. Work can transfer energy into or out of a system. This realization helped establish the fact that heat is a form of energy. James Prescott Joule (1818–1889) performed many experiments to establish the **mechanical equivalent of heat**—*the work needed to produce the same effects as heat transfer*. In terms of the units used for these two terms, the best modern value for this equivalence is

$$1.000\text{ kcal} = 4186\text{ J}. \tag{14.1}$$

We consider this equation as the conversion between two different units of energy.

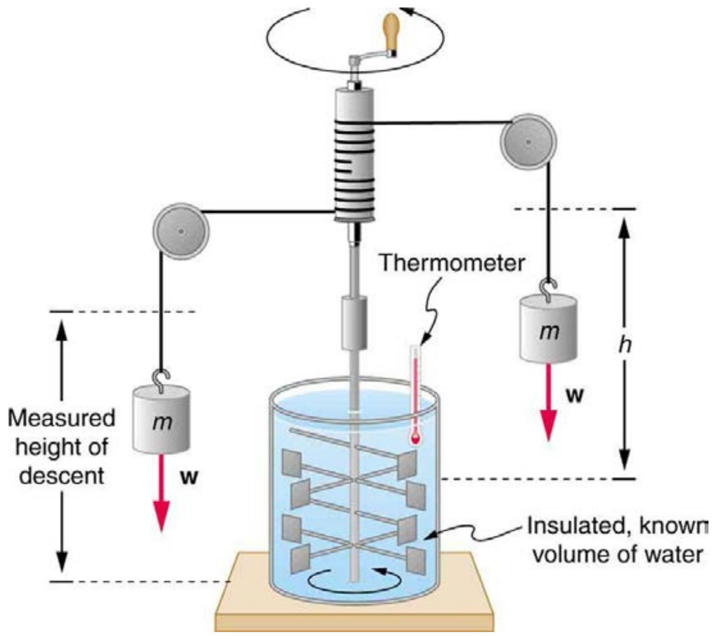


Figure 14.3 Schematic depiction of Joule's experiment that established the equivalence of heat and work.

The figure above shows one of Joule's most famous experimental setups for demonstrating the mechanical equivalent of heat. It demonstrated that work and heat can produce the same effects, and helped establish the principle of conservation of energy. Gravitational potential energy (PE) (work done by the gravitational force) is converted into kinetic energy (KE), and then randomized by viscosity and turbulence into increased average kinetic energy of atoms and molecules in the system, producing a temperature increase. His contributions to the field of thermodynamics were so significant that the SI unit of energy was named after him.

Heat added or removed from a system changes its internal energy and thus its temperature. Such a temperature increase is observed while cooking. However, adding heat does not necessarily increase the temperature. An example is melting of ice; that is, when a substance changes from one phase to another. Work done on the system or by the system can also change the internal energy of the system. Joule demonstrated that the temperature of a system can be increased by stirring. If an ice cube is rubbed against a rough surface, work is done by the frictional force. A system has a well-defined internal energy, but we cannot say that it has a certain “heat content” or “work content”. We use the phrase “heat transfer” to emphasize its nature.

Check Your Understanding

Two samples (A and B) of the same substance are kept in a lab. Someone adds 10 kilojoules (kJ) of heat to one sample, while 10 kJ of work is done on the other sample. How can you tell to which sample the heat was added?

Solution

Heat and work both change the internal energy of the substance. However, the properties of the sample only depend on the internal energy so that it is impossible to tell whether heat was added to sample A or B.

14.2 Temperature Change and Heat Capacity

One of the major effects of heat transfer is temperature change: heating increases the temperature while cooling decreases it. We assume that there is no phase change and that no work is done on or by the system. Experiments show that the transferred heat depends on three factors—the change in temperature, the mass of the system, and the substance and phase of the substance.

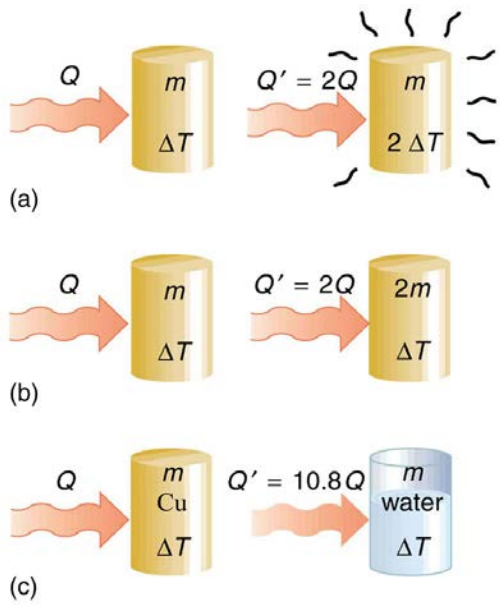


Figure 14.4 The heat Q transferred to cause a temperature change depends on the magnitude of the temperature change, the mass of the system, and the substance and phase involved. (a) The amount of heat transferred is directly proportional to the temperature change. To double the temperature change of a mass m, you need to add twice the heat. (b) The amount of heat transferred is also directly proportional to the mass. To cause an equivalent temperature change in a doubled mass, you need to add twice the heat. (c) The amount of heat transferred depends on the substance and its phase. If it takes an amount Q of heat to cause a temperature change ΔT in a given mass of copper, it will take 10.8 times that amount of heat to cause the equivalent temperature change in the same mass of water assuming no phase change in either substance.

The dependence on temperature change and mass are easily understood. Owing to the fact that the (average) kinetic energy of an atom or molecule is proportional to the absolute temperature, the internal energy of a system is proportional to the absolute temperature and the number of atoms or molecules. Owing to the fact that the transferred heat is equal to the change in the internal energy, the heat is proportional to the mass of the substance and the temperature change. The transferred heat also depends on the substance so that, for example, the heat necessary to raise the temperature is less for alcohol than for water. For the same substance, the transferred heat also depends on the phase (gas, liquid, or solid).

Heat Transfer and Temperature Change

The quantitative relationship between heat transfer and temperature change contains all three factors:

$$Q = mc\Delta T, \tag{14.2}$$

where Q is the symbol for heat transfer, m is the mass of the substance, and ΔT is the change in temperature. The symbol c stands for **specific heat** and depends on the material and phase. The specific heat is the amount of heat necessary to change the temperature of 1.00 kg of mass by 1.00°C. The specific heat c is a property of the substance; its SI unit is $\text{J/(kg} \cdot \text{K)}$ or $\text{J/(kg}\cdot{}^{\circ}\text{C)}$. Recall that the temperature change (ΔT) is the same in units of kelvin and degrees Celsius. If heat transfer is measured in kilocalories, then *the unit of specific heat* is $\text{kcal/(kg}\cdot{}^{\circ}\text{C)}$.

Values of specific heat must generally be looked up in tables, because there is no simple way to calculate them. In general, the specific heat also depends on the temperature. **Table 14.1** lists representative values of specific heat for various substances. Except for gases, the temperature and volume dependence of the specific heat of most substances is weak. We see from this table that the specific heat of water is five times that of glass and ten times that of iron, which means that it takes five times as much heat to raise the temperature of water the same amount as for glass and ten times as much heat to raise the temperature of water as for iron. In fact, water has one of the largest specific heats of any material, which is important for sustaining life on Earth.

Example 14.1 Calculating the Required Heat: Heating Water in an Aluminum Pan

A 0.500 kg aluminum pan on a stove is used to heat 0.250 liters of water from 20.0ºC to 80.0ºC. (a) How much heat is required? What percentage of the heat is used to raise the temperature of (b) the pan and (c) the water?

Strategy

The pan and the water are always at the same temperature. When you put the pan on the stove, the temperature of the water and the pan is increased by the same amount. We use the equation for the heat transfer for the given temperature change and mass of water and aluminum. The specific heat values for water and aluminum are given in **Table 14.1**.

Solution

Because water is in thermal contact with the aluminum, the pan and the water are at the same temperature.

1. Calculate the temperature difference:

$$\Delta T = T_\text{f} - T_\text{i} = 60.0\text{ºC}. \tag{14.3}$$

2. Calculate the mass of water. Because the density of water is $1000\ \text{kg/m}^3$, one liter of water has a mass of 1 kg, and the mass of 0.250 liters of water is $m_\text{w} = 0.250\ \text{kg}$.
3. Calculate the heat transferred to the water. Use the specific heat of water in **Table 14.1**:

$$Q_\text{w} = m_\text{w}c_\text{w}\Delta T = (0.250\ \text{kg})(4186\ \text{J/kgºC})(60.0\text{ºC}) = 62.8\ \text{kJ}. \tag{14.4}$$

4. Calculate the heat transferred to the aluminum. Use the specific heat for aluminum in **Table 14.1**:

$$Q_\text{Al} = m_\text{Al}c_\text{Al}\Delta T = (0.500\ \text{kg})(900\ \text{J/kgºC})(60.0\text{ºC}) = 27.0 \times 10^4\ \text{J} = 27.0\ \text{kJ}. \tag{14.5}$$

5. Compare the percentage of heat going into the pan versus that going into the water. First, find the total transferred heat:

$$Q_\text{Total} = Q_\text{W} + Q_\text{Al} = 62.8\ \text{kJ} + 27.0\ \text{kJ} = 89.8\ \text{kJ}. \tag{14.6}$$

Thus, the amount of heat going into heating the pan is

$$\frac{27.0\ \text{kJ}}{89.8\ \text{kJ}} \times 100\% = 30.1\%, \tag{14.7}$$

and the amount going into heating the water is

$$\frac{62.8\ \text{kJ}}{89.8\ \text{kJ}} \times 100\% = 69.9\%. \tag{14.8}$$

Discussion

In this example, the heat transferred to the container is a significant fraction of the total transferred heat. Although the mass of the pan is twice that of the water, the specific heat of water is over four times greater than that of aluminum. Therefore, it takes a bit more than twice the heat to achieve the given temperature change for the water as compared to the aluminum pan.

Figure 14.5 The smoking brakes on this truck are a visible evidence of the mechanical equivalent of heat.

Example 14.2 Calculating the Temperature Increase from the Work Done on a Substance: Truck Brakes Overheat on Downhill Runs

Truck brakes used to control speed on a downhill run do work, converting gravitational potential energy into increased internal energy (higher temperature) of the brake material. This conversion prevents the gravitational potential energy from being converted into kinetic energy of the truck. The problem is that the mass of the truck is large compared with that of the brake material absorbing the energy, and the temperature increase may occur too fast for sufficient heat to transfer from the brakes to the environment.

Calculate the temperature increase of 100 kg of brake material with an average specific heat of $800\text{ J/kg}\cdot\text{ºC}$ if the material retains 10% of the energy from a 10,000-kg truck descending 75.0 m (in vertical displacement) at a constant speed.

Strategy

If the brakes are not applied, gravitational potential energy is converted into kinetic energy. When brakes are applied, gravitational potential energy is converted into internal energy of the brake material. We first calculate the gravitational potential energy (Mgh) that the entire truck loses in its descent and then find the temperature increase produced in the brake material alone.

Solution

1. Calculate the change in gravitational potential energy as the truck goes downhill

$$Mgh = (10{,}000\text{ kg})\left(9.80\text{ m/s}^2\right)(75.0\text{ m}) = 7.35\times10^6\text{ J}. \tag{14.9}$$

2. Calculate the temperature from the heat transferred using $Q{=}Mgh$ and

$$\Delta T = \frac{Q}{mc}, \tag{14.10}$$

where m is the mass of the brake material. Insert the values $m = 100\text{ kg}$ and $c = 800\text{ J/kg}\cdot\text{ºC}$ to find

$$\Delta T = \frac{\left(7.35\times10^6\text{ J}\right)}{(100\text{ kg})(800\text{ J/kgºC})} = 92\text{ºC}. \tag{14.11}$$

Discussion

This temperature is close to the boiling point of water. If the truck had been traveling for some time, then just before the descent, the brake temperature would likely be higher than the ambient temperature. The temperature increase in the descent would likely raise the temperature of the brake material above the boiling point of water, so this technique is not practical. However, the same idea underlies the recent hybrid technology of cars, where mechanical energy (gravitational potential energy) is converted by the brakes into electrical energy (battery).

Table 14.1 Specific Heats[1] of Various Substances

Substances	Specific heat (*c*)	
Solids	J/kg·ºC	kcal/kg·ºC[2]
Aluminum	900	0.215
Asbestos	800	0.19
Concrete, granite (average)	840	0.20
Copper	387	0.0924
Glass	840	0.20
Gold	129	0.0308
Human body (average at 37 °C)	3500	0.83
Ice (average, -50°C to 0°C)	2090	0.50
Iron, steel	452	0.108
Lead	128	0.0305
Silver	235	0.0562
Wood	1700	0.4
Liquids		
Benzene	1740	0.415
Ethanol	2450	0.586
Glycerin	2410	0.576
Mercury	139	0.0333
Water (15.0 °C)	4186	1.000
Gases [3]		
Air (dry)	721 (1015)	0.172 (0.242)
Ammonia	1670 (2190)	0.399 (0.523)
Carbon dioxide	638 (833)	0.152 (0.199)
Nitrogen	739 (1040)	0.177 (0.248)
Oxygen	651 (913)	0.156 (0.218)
Steam (100°C)	1520 (2020)	0.363 (0.482)

Note that **Example 14.2** is an illustration of the mechanical equivalent of heat. Alternatively, the temperature increase could be produced by a blow torch instead of mechanically.

Example 14.3 Calculating the Final Temperature When Heat Is Transferred Between Two Bodies: Pouring Cold Water in a Hot Pan

Suppose you pour 0.250 kg of 20.0ºC water (about a cup) into a 0.500-kg aluminum pan off the stove with a temperature of 150ºC. Assume that the pan is placed on an insulated pad and that a negligible amount of water boils off. What is the temperature when the water and pan reach thermal equilibrium a short time later?

Strategy

The pan is placed on an insulated pad so that little heat transfer occurs with the surroundings. Originally the pan and water are not in thermal equilibrium: the pan is at a higher temperature than the water. Heat transfer then restores thermal equilibrium once the water and pan are in contact. Because heat transfer between the pan and water takes place rapidly, the mass of evaporated water is negligible and the magnitude of the heat lost by the pan is equal to the heat gained by the water. The exchange of heat stops once a thermal equilibrium between the pan and the water is achieved. The heat exchange can be written as $\mid Q_{\text{hot}} \mid = Q_{\text{cold}}$.

Solution

1. Use the equation for heat transfer $Q = mc\Delta T$ to express the heat lost by the aluminum pan in terms of the mass of the pan, the specific heat of aluminum, the initial temperature of the pan, and the final temperature:

$$Q_{\text{hot}} = m_{\text{Al}}c_{\text{Al}}(T_{\text{f}} - 150\text{ºC}). \tag{14.12}$$

1. The values for solids and liquids are at constant volume and at 25ºC, except as noted.
2. These values are identical in units of $\text{cal/g}\cdot\text{ºC}$.
3. c_{v} at constant volume and at 20.0ºC, except as noted, and at 1.00 atm average pressure. Values in parentheses are c_{p} at a constant pressure of 1.00 atm.

2. Express the heat gained by the water in terms of the mass of the water, the specific heat of water, the initial temperature of the water and the final temperature:

$$Q_{\text{cold}} = m_{\text{W}} c_{\text{W}} (T_{\text{f}} - 20.0^{\circ}\text{C}). \tag{14.13}$$

3. Note that $Q_{\text{hot}} < 0$ and $Q_{\text{cold}} > 0$ and that they must sum to zero because the heat lost by the hot pan must be the same as the heat gained by the cold water:

$$\begin{aligned} Q_{\text{cold}} + Q_{\text{hot}} &= 0, \\ Q_{\text{cold}} &= -Q_{\text{hot}}, \\ m_{\text{W}} c_{\text{W}} (T_{\text{f}} - 20.0^{\circ}\text{C}) &= -m_{\text{Al}} c_{\text{Al}} (T_{\text{f}} - 150^{\circ}\text{C}.) \end{aligned} \tag{14.14}$$

4. This an equation for the unknown final temperature, T_{f}
5. Bring all terms involving T_{f} on the left hand side and all other terms on the right hand side. Solve for T_{f},

$$T_{\text{f}} = \frac{m_{\text{Al}} c_{\text{Al}} (150^{\circ}\text{C}) + m_{\text{W}} c_{\text{W}} (20.0^{\circ}\text{C})}{m_{\text{Al}} c_{\text{Al}} + m_{\text{W}} c_{\text{W}}}, \tag{14.15}$$

and insert the numerical values:

$$\begin{aligned} T_{\text{f}} &= \frac{(0.500 \text{ kg})(900 \text{ J/kg}^{\circ}\text{C})(150^{\circ}\text{C}) + (0.250 \text{ kg})(4186 \text{ J/kg}^{\circ}\text{C})(20.0^{\circ}\text{C})}{(0.500 \text{ kg})(900 \text{ J/kg}^{\circ}\text{C}) + (0.250 \text{ kg})(4186 \text{ J/kg}^{\circ}\text{C})} \\ &= \frac{88430 \text{ J}}{1496.5 \text{ J/}^{\circ}\text{C}} \\ &= 59.1^{\circ}\text{C}. \end{aligned} \tag{14.16}$$

Discussion

This is a typical *calorimetry* problem—two bodies at different temperatures are brought in contact with each other and exchange heat until a common temperature is reached. Why is the final temperature so much closer to 20.0°C than 150°C ? The reason is that water has a greater specific heat than most common substances and thus undergoes a small temperature change for a given heat transfer. A large body of water, such as a lake, requires a large amount of heat to increase its temperature appreciably. This explains why the temperature of a lake stays relatively constant during a day even when the temperature change of the air is large. However, the water temperature does change over longer times (e.g., summer to winter).

Take-Home Experiment: Temperature Change of Land and Water

What heats faster, land or water?

To study differences in heat capacity:

- Place equal masses of dry sand (or soil) and water at the same temperature into two small jars. (The average density of soil or sand is about 1.6 times that of water, so you can achieve approximately equal masses by using 50% more water by volume.)
- Heat both (using an oven or a heat lamp) for the same amount of time.
- Record the final temperature of the two masses.
- Now bring both jars to the same temperature by heating for a longer period of time.
- Remove the jars from the heat source and measure their temperature every 5 minutes for about 30 minutes.

Which sample cools off the fastest? This activity replicates the phenomena responsible for land breezes and sea breezes.

Check Your Understanding

If 25 kJ is necessary to raise the temperature of a block from 25°C to 30°C, how much heat is necessary to heat the block from 45°C to 50°C ?

Solution

The heat transfer depends only on the temperature difference. Since the temperature differences are the same in both cases, the same 25 kJ is necessary in the second case.

14.3 Phase Change and Latent Heat

So far we have discussed temperature change due to heat transfer. No temperature change occurs from heat transfer if ice melts and becomes liquid water (i.e., during a phase change). For example, consider water dripping from icicles melting on a roof warmed by the Sun. Conversely, water freezes in an ice tray cooled by lower-temperature surroundings.

Figure 14.6 Heat from the air transfers to the ice causing it to melt. (credit: Mike Brand)

Energy is required to melt a solid because the cohesive bonds between the molecules in the solid must be broken apart such that, in the liquid, the molecules can move around at comparable kinetic energies; thus, there is no rise in temperature. Similarly, energy is needed to vaporize a liquid, because molecules in a liquid interact with each other via attractive forces. There is no temperature change until a phase change is complete. The temperature of a cup of soda initially at 0°C stays at 0°C until all the ice has melted. Conversely, energy is released during freezing and condensation, usually in the form of thermal energy. Work is done by cohesive forces when molecules are brought together. The corresponding energy must be given off (dissipated) to allow them to stay together Figure 14.7.

The energy involved in a phase change depends on two major factors: the number and strength of bonds or force pairs. The number of bonds is proportional to the number of molecules and thus to the mass of the sample. The strength of forces depends on the type of molecules. The heat Q required to change the phase of a sample of mass m is given by

$$Q = mL_{\text{f}}\ \text{(melting/freezing)}, \tag{14.17}$$

$$Q = mL_{\text{v}}\ \text{(vaporization/condensation)}, \tag{14.18}$$

where the latent heat of fusion, L_{f}, and latent heat of vaporization, L_{v}, are material constants that are determined experimentally. See (Table 14.2).

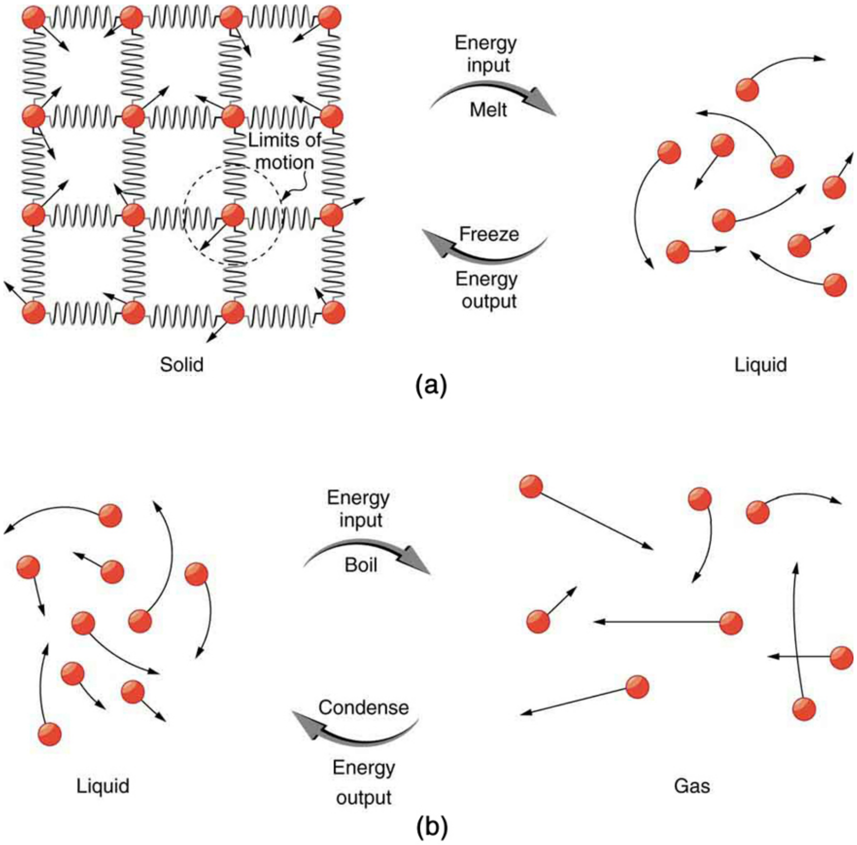


Figure 14.7 (a) Energy is required to partially overcome the attractive forces between molecules in a solid to form a liquid. That same energy must be removed for freezing to take place. (b) Molecules are separated by large distances when going from liquid to vapor, requiring significant energy to overcome molecular attraction. The same energy must be removed for condensation to take place. There is no temperature change until a phase change is complete.

Latent heat is measured in units of J/kg. Both L_f and L_v depend on the substance, particularly on the strength of its molecular forces as noted earlier. L_f and L_v are collectively called **latent heat coefficients**. They are *latent*, or hidden, because in phase changes, energy enters or leaves a system without causing a temperature change in the system; so, in effect, the energy is hidden. **Table 14.2** lists representative values of L_f and L_v, together with melting and boiling points.

The table shows that significant amounts of energy are involved in phase changes. Let us look, for example, at how much energy is needed to melt a kilogram of ice at $0^\circ\mathrm{C}$ to produce a kilogram of water at $0^\circ\mathrm{C}$. Using the equation for a change in temperature and the value for water from **Table 14.2**, we find that $Q = mL_\mathrm{f} = (1.0\ \mathrm{kg})(334\ \mathrm{kJ/kg}) = 334\ \mathrm{kJ}$ is the energy to melt a kilogram of ice. This is a lot of energy as it represents the same amount of energy needed to raise the temperature of 1 kg of liquid water from $0^\circ\mathrm{C}$ to $79.8^\circ\mathrm{C}$. Even more energy is required to vaporize water; it would take 2256 kJ to change 1 kg of liquid water at the normal boiling point ($100^\circ\mathrm{C}$ at atmospheric pressure) to steam (water vapor). This example shows that the energy for a phase change is enormous compared to energy associated with temperature changes without a phase change.

Table 14.2 Heats of Fusion and Vaporization [4]

		L_f			L_v	
Substance	**Melting point (ºC)**	**kJ/kg**	**kcal/kg**	**Boiling point (°C)**	**kJ/kg**	**kcal/kg**
Helium	−269.7	5.23	1.25	−268.9	20.9	4.99
Hydrogen	−259.3	58.6	14.0	−252.9	452	108
Nitrogen	−210.0	25.5	6.09	−195.8	201	48.0
Oxygen	−218.8	13.8	3.30	−183.0	213	50.9
Ethanol	−114	104	24.9	78.3	854	204
Ammonia	−75		108	−33.4	1370	327
Mercury	−38.9	11.8	2.82	357	272	65.0
Water	0.00	334	79.8	100.0	2256[5]	539[6]
Sulfur	119	38.1	9.10	444.6	326	77.9
Lead	327	24.5	5.85	1750	871	208
Antimony	631	165	39.4	1440	561	134
Aluminum	660	380	90	2450	11400	2720
Silver	961	88.3	21.1	2193	2336	558
Gold	1063	64.5	15.4	2660	1578	377
Copper	1083	134	32.0	2595	5069	1211
Uranium	1133	84	20	3900	1900	454
Tungsten	3410	184	44	5900	4810	1150

Phase changes can have a tremendous stabilizing effect even on temperatures that are not near the melting and boiling points, because evaporation and condensation (conversion of a gas into a liquid state) occur even at temperatures below the boiling point. Take, for example, the fact that air temperatures in humid climates rarely go above $35.0^\circ\mathrm{C}$, which is because most heat transfer goes into evaporating water into the air. Similarly, temperatures in humid weather rarely fall below the dew point because enormous heat is released when water vapor condenses.

We examine the effects of phase change more precisely by considering adding heat into a sample of ice at $-20^\circ\mathrm{C}$ (**Figure 14.8**). The temperature of the ice rises linearly, absorbing heat at a constant rate of $0.50\ \mathrm{cal/g{\cdot}^\circ C}$ until it reaches $0^\circ\mathrm{C}$. Once at this temperature, the ice begins to melt until all the ice has melted, absorbing 79.8 cal/g of heat. The temperature remains constant at $0^\circ\mathrm{C}$ during this phase change. Once all the ice has melted, the temperature of the liquid water rises, absorbing heat at a new constant rate of $1.00\ \mathrm{cal/g{\cdot}^\circ C}$. At $100^\circ\mathrm{C}$, the water begins to boil and the temperature again remains constant while the water absorbs 539 cal/g of heat during this phase change. When all the liquid has become steam vapor, the temperature rises again, absorbing heat at a rate of $0.482\ \mathrm{cal/g{\cdot}^\circ C}$.

4. Values quoted at the normal melting and boiling temperatures at standard atmospheric pressure (1 atm).
5. At $37.0^\circ\mathrm{C}$ (body temperature), the heat of vaporization L_v for water is 2430 kJ/kg or 580 kcal/kg
6. At $37.0^\circ\mathrm{C}$ (body temperature), the heat of vaporization L_v for water is 2430 kJ/kg or 580 kcal/kg

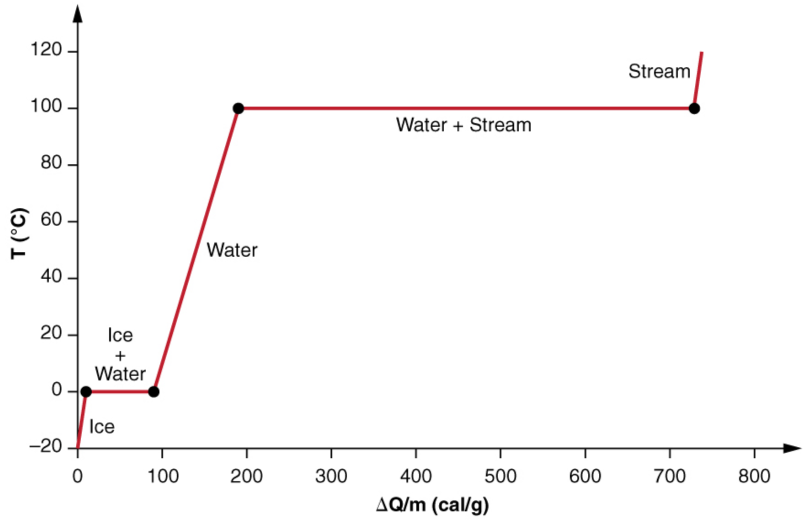


Figure 14.8 A graph of temperature versus energy added. The system is constructed so that no vapor evaporates while ice warms to become liquid water, and so that, when vaporization occurs, the vapor remains in of the system. The long stretches of constant temperature values at $0ºC$ and $100ºC$ reflect the large latent heat of melting and vaporization, respectively.

Water can evaporate at temperatures below the boiling point. More energy is required than at the boiling point, because the kinetic energy of water molecules at temperatures below $100ºC$ is less than that at $100ºC$, hence less energy is available from random thermal motions. Take, for example, the fact that, at body temperature, perspiration from the skin requires a heat input of 2428 kJ/kg, which is about 10 percent higher than the latent heat of vaporization at $100ºC$. This heat comes from the skin, and thus provides an effective cooling mechanism in hot weather. High humidity inhibits evaporation, so that body temperature might rise, leaving unevaporated sweat on your brow.

Example 14.4 Calculate Final Temperature from Phase Change: Cooling Soda with Ice Cubes

Three ice cubes are used to chill a soda at $20ºC$ with mass $m_{\text{soda}} = 0.25\text{ kg}$. The ice is at $0ºC$ and each ice cube has a mass of 6.0 g. Assume that the soda is kept in a foam container so that heat loss can be ignored. Assume the soda has the same heat capacity as water. Find the final temperature when all ice has melted.

Strategy

The ice cubes are at the melting temperature of $0ºC$. Heat is transferred from the soda to the ice for melting. Melting of ice occurs in two steps: first the phase change occurs and solid (ice) transforms into liquid water at the melting temperature, then the temperature of this water rises. Melting yields water at $0ºC$, so more heat is transferred from the soda to this water until the water plus soda system reaches thermal equilibrium,

$$Q_{\text{ice}} = -Q_{\text{soda}}. \tag{14.19}$$

The heat transferred to the ice is $Q_{\text{ice}} = m_{\text{ice}}L_{\text{f}} + m_{\text{ice}}c_{\text{W}}(T_{\text{f}} - 0ºC)$. The heat given off by the soda is $Q_{\text{soda}} = m_{\text{soda}}c_{\text{W}}(T_{\text{f}} - 20ºC)$. Since no heat is lost, $Q_{\text{ice}} = -Q_{\text{soda}}$, so that

$$m_{\text{ice}}L_{\text{f}} + m_{\text{ice}}c_{\text{W}}(T_{\text{f}} - 0ºC) = -m_{\text{soda}}c_{\text{W}}(T_{\text{f}} - 20ºC). \tag{14.20}$$

Bring all terms involving T_{f} on the left-hand-side and all other terms on the right-hand-side. Solve for the unknown quantity T_{f}:

$$T_{\text{f}} = \frac{m_{\text{soda}}c_{\text{W}}(20ºC) - m_{\text{ice}}L_{\text{f}}}{(m_{\text{soda}} + m_{\text{ice}})c_{\text{W}}}. \tag{14.21}$$

Solution

1. Identify the known quantities. The mass of ice is $m_{\text{ice}} = 3\times6.0\text{ g} = 0.018\text{ kg}$ and the mass of soda is $m_{\text{soda}} = 0.25\text{ kg}$.
2. Calculate the terms in the numerator:
$$m_{\text{soda}}c_{\text{W}}(20ºC) = (0.25\text{ kg})(4186\text{ J/kg}\cdot\text{ºC})(20ºC) = 20{,}930\text{ J} \tag{14.22}$$
and
$$m_{\text{ice}}L_{\text{f}} = (0.018\text{ kg})(334{,}000\text{ J/kg}) = 6012\text{ J}. \tag{14.23}$$
3. Calculate the denominator:
$$(m_{\text{soda}} + m_{\text{ice}})c_{\text{W}} = (0.25\text{ kg} + 0.018\text{ kg})(4186\text{ K/(kg}\cdot\text{ºC}) = 1122\text{ J/ºC}. \tag{14.24}$$
4. Calculate the final temperature:
$$T_{\text{f}} = \frac{20{,}930\text{ J} - 6012\text{ J}}{1122\text{ J/ºC}} = 13ºC. \tag{14.25}$$

Discussion

This example illustrates the enormous energies involved during a phase change. The mass of ice is about 7 percent the mass of water but leads to a noticeable change in the temperature of soda. Although we assumed that the ice was at the freezing temperature, this is incorrect: the typical temperature is -6°C . However, this correction gives a final temperature that is essentially identical to the result we found. Can you explain why?

We have seen that vaporization requires heat transfer to a liquid from the surroundings, so that energy is released by the surroundings. Condensation is the reverse process, increasing the temperature of the surroundings. This increase may seem surprising, since we associate condensation with cold objects—the glass in the figure, for example. However, energy must be removed from the condensing molecules to make a vapor condense. The energy is exactly the same as that required to make the phase change in the other direction, from liquid to vapor, and so it can be calculated from $Q = mL_\text{v}$.

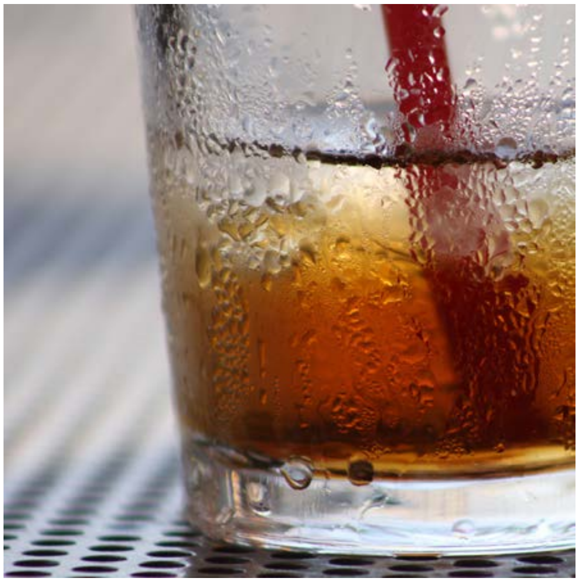


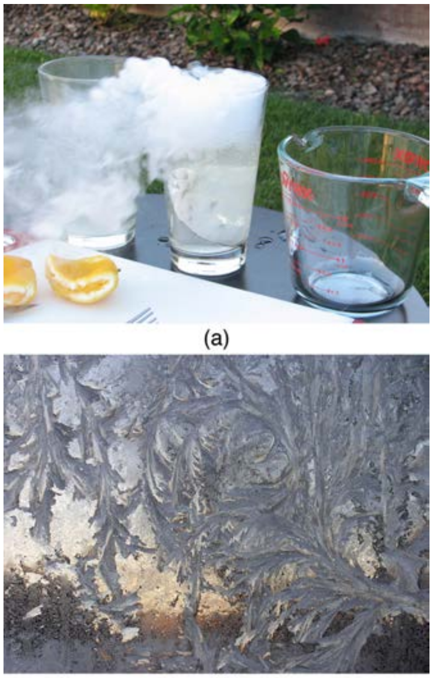
Figure 14.9 Condensation forms on this glass of iced tea because the temperature of the nearby air is reduced to below the dew point. The air cannot hold as much water as it did at room temperature, and so water condenses. Energy is released when the water condenses, speeding the melting of the ice in the glass. (credit: Jenny Downing)

Real-World Application

Energy is also released when a liquid freezes. This phenomenon is used by fruit growers in Florida to protect oranges when the temperature is close to the freezing point (0°C) . Growers spray water on the plants in orchards so that the water freezes and heat is released to the growing oranges on the trees. This prevents the temperature inside the orange from dropping below freezing, which would damage the fruit.

Figure 14.10 The ice on these trees released large amounts of energy when it froze, helping to prevent the temperature of the trees from dropping below 0°C . Water is intentionally sprayed on orchards to help prevent hard frosts. (credit: Hermann Hammer)

Sublimation is the transition from solid to vapor phase. You may have noticed that snow can disappear into thin air without a trace of liquid water, or the disappearance of ice cubes in a freezer. The reverse is also true: Frost can form on very cold windows without going through the liquid stage. A popular effect is the making of “smoke” from dry ice, which is solid carbon dioxide. Sublimation occurs because the equilibrium vapor pressure of solids is not zero. Certain air fresheners use the sublimation of a solid to inject a perfume into the room. Moth balls are a slightly toxic example of a phenol (an organic compound) that sublimates, while some solids, such as osmium tetroxide, are so toxic that they must be kept in sealed containers to prevent human exposure to their sublimation-produced vapors.



(a)

(b)

Figure 14.11 Direct transitions between solid and vapor are common, sometimes useful, and even beautiful. (a) Dry ice sublimates directly to carbon dioxide gas. The visible vapor is made of water droplets. (credit: Windell Oskay) (b) Frost forms patterns on a very cold window, an example of a solid formed directly from a vapor. (credit: Liz West)

All phase transitions involve heat. In the case of direct solid-vapor transitions, the energy required is given by the equation $Q = mL_{\text{s}}$, where L_{s} is the **heat of sublimation**, which is the energy required to change 1.00 kg of a substance from the solid phase to the vapor phase. L_{s} is analogous to L_{f} and L_{v}, and its value depends on the substance. Sublimation requires energy input, so that dry ice is an effective coolant, whereas the reverse process (i.e., frosting) releases energy. The amount of energy required for sublimation is of the same order of magnitude as that for other phase transitions.

The material presented in this section and the preceding section allows us to calculate any number of effects related to temperature and phase change. In each case, it is necessary to identify which temperature and phase changes are taking place and then to apply the appropriate equation. Keep in mind that heat transfer and work can cause both temperature and phase changes.

Problem-Solving Strategies for the Effects of Heat Transfer

1. *Examine the situation to determine that there is a change in the temperature or phase. Is there heat transfer into or out of the system?* When the presence or absence of a phase change is not obvious, you may wish to first solve the problem as if there were no phase changes, and examine the temperature change obtained. If it is sufficient to take you past a boiling or melting point, you should then go back and do the problem in steps—temperature change, phase change, subsequent temperature change, and so on.
2. *Identify and list all objects that change temperature and phase.*
3. *Identify exactly what needs to be determined in the problem (identify the unknowns).* A written list is useful.
4. *Make a list of what is given or what can be inferred from the problem as stated (identify the knowns).*
5. *Solve the appropriate equation for the quantity to be determined (the unknown).* If there is a temperature change, the transferred heat depends on the specific heat (see **Table 14.1**) whereas, for a phase change, the transferred heat depends on the latent heat. See **Table 14.2**.
6. *Substitute the knowns along with their units into the appropriate equation and obtain numerical solutions complete with units.* You will need to do this in steps if there is more than one stage to the process (such as a temperature change followed by a phase change).
7. *Check the answer to see if it is reasonable: Does it make sense?* As an example, be certain that the temperature change does not also cause a phase change that you have not taken into account.

Check Your Understanding

Why does snow remain on mountain slopes even when daytime temperatures are higher than the freezing temperature?

Solution

Snow is formed from ice crystals and thus is the solid phase of water. Because enormous heat is necessary for phase changes, it takes a certain amount of time for this heat to be accumulated from the air, even if the air is above 0^{o}C. The warmer the air is, the faster this heat exchange occurs and the faster the snow melts.

14.4 Heat Transfer Methods

Equally as interesting as the effects of heat transfer on a system are the methods by which this occurs. Whenever there is a temperature difference, heat transfer occurs. Heat transfer may occur rapidly, such as through a cooking pan, or slowly, such as through the walls of a picnic ice chest. We can control rates of heat transfer by choosing materials (such as thick wool clothing for the winter), controlling air movement (such as the use of weather stripping around doors), or by choice of color (such as a white roof to reflect summer sunlight). So many processes involve heat transfer, so that it is hard to imagine a situation where no heat transfer occurs. Yet every process involving heat transfer takes place by only three methods:

1. **Conduction** is heat transfer through stationary matter by physical contact. (The matter is stationary on a macroscopic scale—we know there is thermal motion of the atoms and molecules at any temperature above absolute zero.) Heat transferred between the electric burner of a stove and the bottom of a pan is transferred by conduction.
2. **Convection** is the heat transfer by the macroscopic movement of a fluid. This type of transfer takes place in a forced-air furnace and in weather systems, for example.
3. Heat transfer by **radiation** occurs when microwaves, infrared radiation, visible light, or another form of electromagnetic radiation is emitted or absorbed. An obvious example is the warming of the Earth by the Sun. A less obvious example is thermal radiation from the human body.

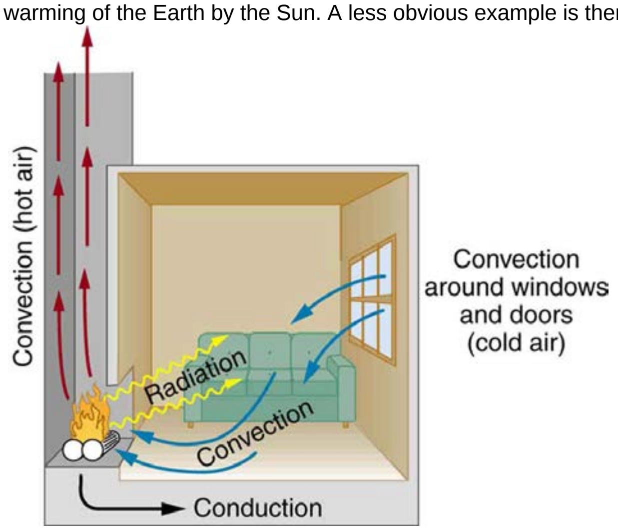


Figure 14.12 In a fireplace, heat transfer occurs by all three methods: conduction, convection, and radiation. Radiation is responsible for most of the heat transferred into the room. Heat transfer also occurs through conduction into the room, but at a much slower rate. Heat transfer by convection also occurs through cold air entering the room around windows and hot air leaving the room by rising up the chimney.

We examine these methods in some detail in the three following modules. Each method has unique and interesting characteristics, but all three do have one thing in common: they transfer heat solely because of a temperature difference Figure 14.12.

Check Your Understanding

Name an example from daily life (different from the text) for each mechanism of heat transfer.

Solution

Conduction: Heat transfers into your hands as you hold a hot cup of coffee.

Convection: Heat transfers as the barista “steams” cold milk to make hot *cocoa*.

Radiation: Reheating a cold cup of coffee in a microwave oven.

14.5 Conduction

Figure 14.13 Insulation is used to limit the conduction of heat from the inside to the outside (in winters) and from the outside to the inside (in summers). (credit: Giles Douglas)

Your feet feel cold as you walk barefoot across the living room carpet in your cold house and then step onto the kitchen tile floor. This result is intriguing, since the carpet and tile floor are both at the same temperature. The different sensation you feel is explained by the different rates of heat transfer: the heat loss during the same time interval is greater for skin in contact with the tiles than with the carpet, so the temperature drop is greater on the tiles.

Some materials conduct thermal energy faster than others. In general, good conductors of electricity (metals like copper, aluminum, gold, and silver) are also good heat conductors, whereas insulators of electricity (wood, plastic, and rubber) are poor heat conductors. Figure 14.14 shows molecules in two bodies at different temperatures. The (average) kinetic energy of a molecule in the hot body is higher than in the colder body. If two molecules collide, an energy transfer from the hot to the cold molecule occurs. The cumulative effect from all collisions results in a net flux of heat from the hot body to the colder body. The heat flux thus depends on the temperature difference $\Delta T = T_{\text{hot}} - T_{\text{cold}}$. Therefore, you will get a more severe burn from boiling water than from hot tap water. Conversely, if the temperatures are the same, the net heat transfer rate falls to zero, and equilibrium is achieved. Owing to the fact that the number of collisions increases with increasing area, heat conduction depends on the cross-sectional area. If you touch a cold wall with your palm, your hand cools faster than if you just touch it with your fingertip.

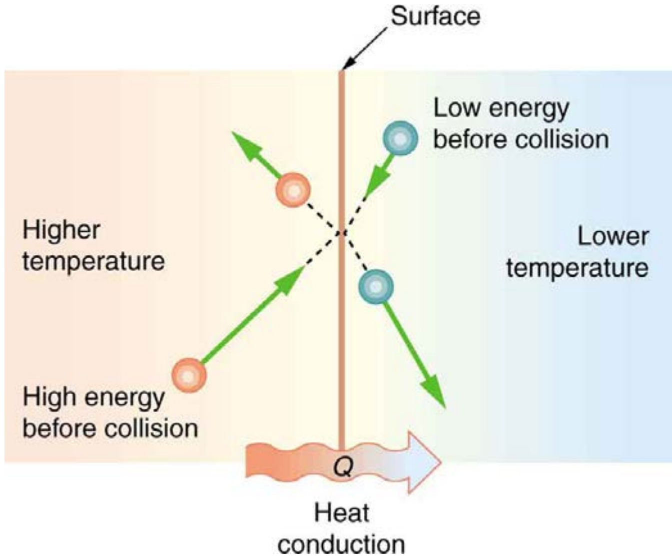


Figure 14.14 The molecules in two bodies at different temperatures have different average kinetic energies. Collisions occurring at the contact surface tend to transfer energy from high-temperature regions to low-temperature regions. In this illustration, a molecule in the lower temperature region (right side) has low energy before collision, but its energy increases after colliding with the contact surface. In contrast, a molecule in the higher temperature region (left side) has high energy before collision, but its energy decreases after colliding with the contact surface.

A third factor in the mechanism of conduction is the thickness of the material through which heat transfers. The figure below shows a slab of material with different temperatures on either side. Suppose that T_2 is greater than T_1, so that heat is transferred from left to right. Heat transfer from the left side to the right side is accomplished by a series of molecular collisions. The thicker the material, the more time it takes to transfer the same amount of heat. This model explains why thick clothing is warmer than thin clothing in winters, and why Arctic mammals protect themselves with thick blubber.

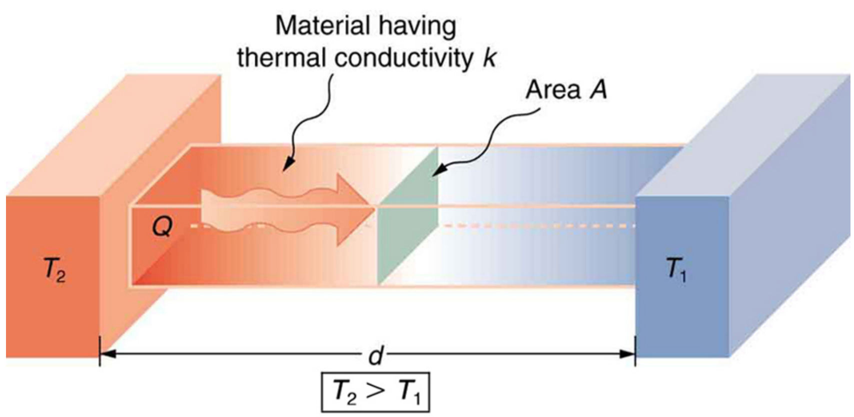


Figure 14.15 Heat conduction occurs through any material, represented here by a rectangular bar, whether window glass or walrus blubber. The temperature of the material is T_2 on the left and T_1 on the right, where T_2 is greater than T_1. The rate of heat transfer by conduction is directly proportional to the surface area A, the temperature difference $T_2 - T_1$, and the substance's conductivity k. The rate of heat transfer is inversely proportional to the thickness d.

Lastly, the heat transfer rate depends on the material properties described by the coefficient of thermal conductivity. All four factors are included in a simple equation that was deduced from and is confirmed by experiments. The **rate of conductive heat transfer** through a slab of material, such as the one in **Figure 14.15**, is given by

$$\frac{Q}{t} = \frac{kA(T_2 - T_1)}{d}, \tag{14.26}$$

where Q/t is the rate of heat transfer in watts or kilocalories per second, k is the **thermal conductivity** of the material, A and d are its surface area and thickness, as shown in **Figure 14.15**, and $(T_2 - T_1)$ is the temperature difference across the slab. **Table 14.3** gives representative values of thermal conductivity.

Example 14.5 Calculating Heat Transfer Through Conduction: Conduction Rate Through an Ice Box

A Styrofoam ice box has a total area of $0.950\ \text{m}^2$ and walls with an average thickness of 2.50 cm. The box contains ice, water, and canned beverages at 0°C. The inside of the box is kept cold by melting ice. How much ice melts in one day if the ice box is kept in the trunk of a car at 35.0°C?

Strategy

This question involves both heat for a phase change (melting of ice) and the transfer of heat by conduction. To find the amount of ice melted, we must find the net heat transferred. This value can be obtained by calculating the rate of heat transfer by conduction and multiplying by time.

Solution

1. Identify the knowns.

$$A = 0.950\ \text{m}^2;\ d = 2.50\ \text{cm} = 0.0250\ \text{m};\ T_1 = 0^\circ\text{C};\ T_2 = 35.0^\circ\text{C},\ t = 1\ \text{day} = 24\ \text{hours} = 86{,}400\ \text{s}. \tag{14.27}$$

2. Identify the unknowns. We need to solve for the mass of the ice, m. We will also need to solve for the net heat transferred to melt the ice, Q.

3. Determine which equations to use. The rate of heat transfer by conduction is given by

$$\frac{Q}{t} = \frac{kA(T_2 - T_1)}{d}. \tag{14.28}$$

4. The heat is used to melt the ice: $Q = mL_\text{f}$.

5. Insert the known values:

$$\frac{Q}{t} = \frac{(0.010\ \text{J/s}\cdot\text{m}\cdot^\circ\text{C})\left(0.950\ \ \text{m}^2\right)(35.0^\circ\text{C} - 0^\circ\text{C})}{0.0250\ \text{m}} = 13.3\ \text{J/s}. \tag{14.29}$$

6. Multiply the rate of heat transfer by the time ($1\ \text{day} = 86{,}400\ \text{s}$):

$$Q = (Q\,/\,t)t = (13.3\ \text{J/s})(86{,}400\ \text{s}) = 1.15\times10^6\ \text{J}. \tag{14.30}$$

7. Set this equal to the heat transferred to melt the ice: $Q = mL_\text{f}$. Solve for the mass m:

$$m = \frac{Q}{L_\text{f}} = \frac{1.15\times10^6\ \text{J}}{334\times10^3\ \text{J/kg}} = 3.44\text{kg}. \tag{14.31}$$

Discussion

The result of 3.44 kg, or about 7.6 lbs, seems about right, based on experience. You might expect to use about a 4 kg (7–10 lb) bag of ice per day. A little extra ice is required if you add any warm food or beverages.

Inspecting the conductivities in **Table 14.3** shows that Styrofoam is a very poor conductor and thus a good insulator. Other good insulators include fiberglass, wool, and goose-down feathers. Like Styrofoam, these all incorporate many small pockets of air, taking advantage of air's poor thermal conductivity.

Table 14.3 Thermal Conductivities of Common Substances[7]

Substance	Thermal conductivity k (J/s⋅m⋅ºC)
Silver	420
Copper	390
Gold	318
Aluminum	220
Steel iron	80
Steel (stainless)	14
Ice	2.2
Glass (average)	0.84
Concrete brick	0.84
Water	0.6
Fatty tissue (without blood)	0.2
Asbestos	0.16
Plasterboard	0.16
Wood	0.08–0.16
Snow (dry)	0.10
Cork	0.042
Glass wool	0.042
Wool	0.04
Down feathers	0.025
Air	0.023
Styrofoam	0.010

A combination of material and thickness is often manipulated to develop good insulators—the smaller the conductivity k and the larger the thickness d, the better. The ratio of d/k will thus be large for a good insulator. The ratio d/k is called the $\boldsymbol{R}$ **factor**. The rate of conductive heat transfer is inversely proportional to R. The larger the value of R, the better the insulation. R factors are most commonly quoted for household insulation, refrigerators, and the like—unfortunately, it is still in non-metric units of ft^2·°F·h/Btu, although the unit usually goes unstated (1 British thermal unit [Btu] is the amount of energy needed to change the temperature of 1.0 lb of water by 1.0 °F). A couple of representative values are an R factor of 11 for 3.5-in-thick fiberglass batts (pieces) of insulation and an R factor of 19 for 6.5-in-thick fiberglass batts. Walls are usually insulated with 3.5-in batts, while ceilings are usually insulated with 6.5-in batts. In cold climates, thicker batts may be used in ceilings and walls.

Figure 14.16 The fiberglass batt is used for insulation of walls and ceilings to prevent heat transfer between the inside of the building and the outside environment.

Note that in **Table 14.3**, the best thermal conductors—silver, copper, gold, and aluminum—are also the best electrical conductors, again related to the density of free electrons in them. Cooking utensils are typically made from good conductors.

Example 14.6 Calculating the Temperature Difference Maintained by a Heat Transfer: Conduction Through an Aluminum Pan

Water is boiling in an aluminum pan placed on an electrical element on a stovetop. The sauce pan has a bottom that is 0.800 cm thick and 14.0 cm in diameter. The boiling water is evaporating at the rate of 1.00 g/s. What is the temperature difference across (through) the bottom of the pan?

Strategy

Conduction through the aluminum is the primary method of heat transfer here, and so we use the equation for the rate of heat transfer and solve for the temperature difference.

7. At temperatures near 0ºC.

$$T_2 - T_1 = \frac{Q}{t}\left(\frac{d}{kA}\right). \tag{14.32}$$

Solution

1. Identify the knowns and convert them to the SI units.

   The thickness of the pan, $d = 0.800 \text{ cm} = 8.0 \times 10^{-3} \text{ m}$, the area of the pan, $A = \pi(0.14/2)^2 \text{ m}^2 = 1.54 \times 10^{-2} \text{ m}^2$, and the thermal conductivity, $k = 220 \text{ J/s} \cdot \text{m} \cdot {}^\circ\text{C}$.
2. Calculate the necessary heat of vaporization of 1 g of water:

$$Q = mL_\text{v} = \left(1.00 \times 10^{-3} \text{ kg}\right)\left(2256 \times 10^3 \text{ J/kg}\right) = 2256 \text{ J}. \tag{14.33}$$

3. Calculate the rate of heat transfer given that 1 g of water melts in one second:

$$Q/t = 2256 \text{ J/s or } 2.26 \text{ kW}. \tag{14.34}$$

4. Insert the knowns into the equation and solve for the temperature difference:

$$T_2 - T_1 = \frac{Q}{t}\left(\frac{d}{kA}\right) = (2256 \text{ J/s})\frac{8.00 \times 10^{-3}\text{m}}{(220 \text{ J/s} \cdot \text{m} \cdot {}^\circ\text{C})\left(1.54 \times 10^{-2} \text{ m}^2\right)} = 5.33^\circ\text{C}. \tag{14.35}$$

Discussion

The value for the heat transfer $Q/t = 2.26\text{kW or } 2256 \text{ J/s}$ is typical for an electric stove. This value gives a remarkably small temperature difference between the stove and the pan. Consider that the stove burner is red hot while the inside of the pan is nearly 100°C because of its contact with boiling water. This contact effectively cools the bottom of the pan in spite of its proximity to the very hot stove burner. Aluminum is such a good conductor that it only takes this small temperature difference to produce a heat transfer of 2.26 kW into the pan.

Conduction is caused by the random motion of atoms and molecules. As such, it is an ineffective mechanism for heat transport over macroscopic distances and short time distances. Take, for example, the temperature on the Earth, which would be unbearably cold during the night and extremely hot during the day if heat transport in the atmosphere was to be only through conduction. In another example, car engines would overheat unless there was a more efficient way to remove excess heat from the pistons.

Check Your Understanding

How does the rate of heat transfer by conduction change when all spatial dimensions are doubled?

Solution

Because area is the product of two spatial dimensions, it increases by a factor of four when each dimension is doubled $\left(A_\text{final} = (2d)^2 = 4d^2 = 4A_\text{initial}\right)$. The distance, however, simply doubles. Because the temperature difference and the coefficient of thermal conductivity are independent of the spatial dimensions, the rate of heat transfer by conduction increases by a factor of four divided by two, or two:

$$\left(\frac{Q}{t}\right)_\text{final} = \frac{kA_\text{final}(T_2 - T_1)}{d_\text{final}} = \frac{k(4\text{A}_\text{initial})(T_2 - T_1)}{2\text{d}_\text{initial}} = 2\frac{kA_\text{initial}(T_2 - T_1)}{d_\text{initial}} = 2\left(\frac{Q}{t}\right)_\text{initial}. \tag{14.36}$$

14.6 Convection

Convection is driven by large-scale flow of matter. In the case of Earth, the atmospheric circulation is caused by the flow of hot air from the tropics to the poles, and the flow of cold air from the poles toward the tropics. (Note that Earth's rotation causes the observed easterly flow of air in the northern hemisphere). Car engines are kept cool by the flow of water in the cooling system, with the water pump maintaining a flow of cool water to the pistons. The circulatory system is used the body: when the body overheats, the blood vessels in the skin expand (dilate), which increases the blood flow to the skin where it can be cooled by sweating. These vessels become smaller when it is cold outside and larger when it is hot (so more fluid flows, and more energy is transferred).

The body also loses a significant fraction of its heat through the breathing process.

While convection is usually more complicated than conduction, we can describe convection and do some straightforward, realistic calculations of its effects. Natural convection is driven by buoyant forces: hot air rises because density decreases as temperature increases. The house in Figure 14.17 is kept warm in this manner, as is the pot of water on the stove in Figure 14.18. Ocean currents and large-scale atmospheric circulation transfer energy from one part of the globe to another. Both are examples of natural convection.

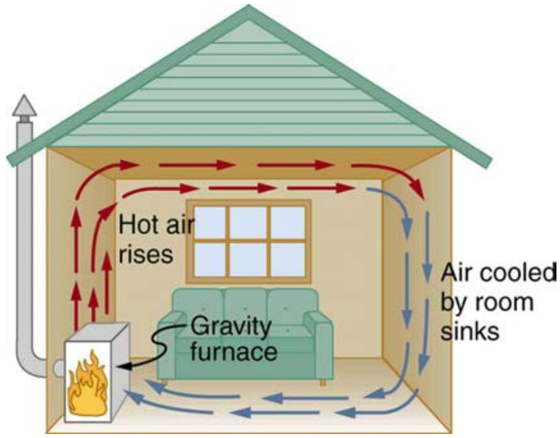


Figure 14.17 Air heated by the so-called gravity furnace expands and rises, forming a convective loop that transfers energy to other parts of the room. As the air is cooled at the ceiling and outside walls, it contracts, eventually becoming denser than room air and sinking to the floor. A properly designed heating system using natural convection, like this one, can be quite efficient in uniformly heating a home.

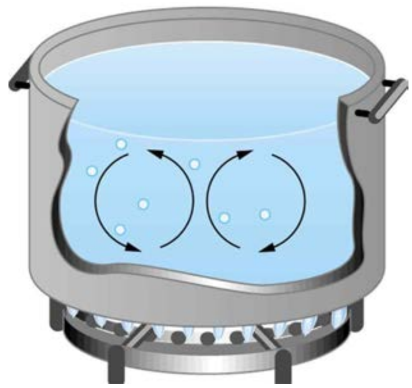


Figure 14.18 Convection plays an important role in heat transfer inside this pot of water. Once conducted to the inside, heat transfer to other parts of the pot is mostly by convection. The hotter water expands, decreases in density, and rises to transfer heat to other regions of the water, while colder water sinks to the bottom. This process keeps repeating.

Take-Home Experiment: Convection Rolls in a Heated Pan

Take two small pots of water and use an eye dropper to place a drop of food coloring near the bottom of each. Leave one on a bench top and heat the other over a stovetop. Watch how the color spreads and how long it takes the color to reach the top. Watch how convective loops form.

Example 14.7 Calculating Heat Transfer by Convection: Convection of Air Through the Walls of a House

Most houses are not airtight: air goes in and out around doors and windows, through cracks and crevices, following wiring to switches and outlets, and so on. The air in a typical house is completely replaced in less than an hour. Suppose that a moderately-sized house has inside dimensions $12.0\text{m}\times18.0\text{m}\times3.00\text{m}$ high, and that all air is replaced in 30.0 min. Calculate the heat transfer per unit time in watts needed to warm the incoming cold air by 10.0ºC, thus replacing the heat transferred by convection alone.

Strategy

Heat is used to raise the temperature of air so that $Q = mc\Delta T$. The rate of heat transfer is then Q/t, where t is the time for air turnover. We are given that ΔT is 10.0ºC, but we must still find values for the mass of air and its specific heat before we can calculate Q. The specific heat of air is a weighted average of the specific heats of nitrogen and oxygen, which gives $c = c_\text{p} \cong 1000\text{ J/kg}\cdot\text{ºC}$ from **Table 14.4** (note that the specific heat at constant pressure must be used for this process).

Solution

1. Determine the mass of air from its density and the given volume of the house. The density is given from the density ρ and the volume

$$m = \rho V = \left(1.29\text{ kg/m}^3\right)(12.0\text{ m}\times18.0\text{ m}\times3.00\text{ m}) = 836\text{ kg}. \tag{14.37}$$

2. Calculate the heat transferred from the change in air temperature: $Q = mc\Delta T$ so that

$$Q = (836\text{ kg})(1000\text{ J/kg}\cdot\text{ºC})(10.0\text{ºC}) = 8.36\times10^6\text{ J}. \tag{14.38}$$

3. Calculate the heat transfer from the heat Q and the turnover time t. Since air is turned over in $t = 0.500\text{ h} = 1800\text{ s}$, the heat transferred per unit time is

$$\frac{Q}{t} = \frac{8.36\times10^6\text{ J}}{1800\text{ s}} = 4.64\text{ kW}. \tag{14.39}$$

Discussion

This rate of heat transfer is equal to the power consumed by about forty-six 100-W light bulbs. Newly constructed homes are designed for a turnover time of 2 hours or more, rather than 30 minutes for the house of this example. Weather stripping, caulking, and improved window seals are commonly employed. More extreme measures are sometimes taken in very cold (or hot) climates to achieve a tight standard of more than 6 hours for one air turnover. Still longer turnover times are unhealthy, because a minimum amount of fresh air is necessary to supply oxygen for breathing and to dilute household pollutants. The term used for the process by which outside air leaks into the house from cracks around windows, doors, and the foundation is called "air infiltration."

A cold wind is much more chilling than still cold air, because convection combines with conduction in the body to increase the rate at which energy is transferred away from the body. The table below gives approximate wind-chill factors, which are the temperatures of still air that produce the same rate of cooling as air of a given temperature and speed. Wind-chill factors are a dramatic reminder of convection's ability to transfer heat faster than conduction. For example, a 15.0 m/s wind at 0ºC has the chilling equivalent of still air at about -18ºC .

Table 14.4 Wind-Chill Factors

Moving air temperature	Wind speed (m/s)				
(ºC)	2	5	10	15	20
5	3	−1	−8	−10	−12
2	0	−7	−12	−16	−18
0	−2	−9	−15	−18	−20
−5	−7	−15	−22	−26	−29
−10	−12	−21	−29	−34	−36
−20	−23	−34	−44	−50	−52
−10	−12	−21	−29	−34	−36
−20	−23	−34	−44	−50	−52
−40	−44	−59	−73	−82	−84

Although air can transfer heat rapidly by convection, it is a poor conductor and thus a good insulator. The amount of available space for airflow determines whether air acts as an insulator or conductor. The space between the inside and outside walls of a house, for example, is about 9 cm (3.5 in) —large enough for convection to work effectively. The addition of wall insulation prevents airflow, so heat loss (or gain) is decreased. Similarly, the gap between the two panes of a double-paned window is about 1 cm, which prevents convection and takes advantage of air's low conductivity to prevent greater loss. Fur, fiber, and fiberglass also take advantage of the low conductivity of air by trapping it in spaces too small to support convection, as shown in the figure. Fur and feathers are lightweight and thus ideal for the protection of animals.

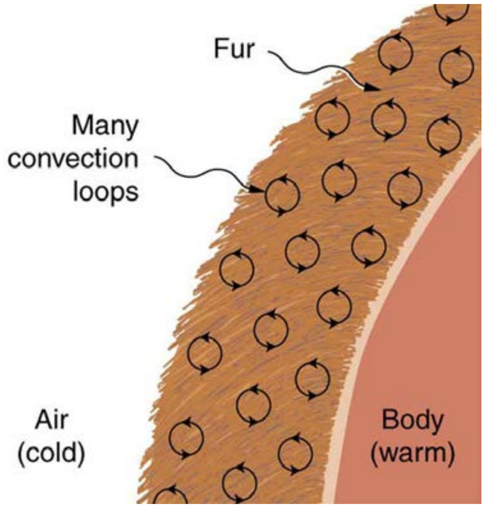


Figure 14.19 Fur is filled with air, breaking it up into many small pockets. Convection is very slow here, because the loops are so small. The low conductivity of air makes fur a very good lightweight insulator.

Some interesting phenomena happen *when convection is accompanied by a phase change*. It allows us to cool off by sweating, even if the temperature of the surrounding air exceeds body temperature. Heat from the skin is required for sweat to evaporate from the skin, but without air flow, the air becomes saturated and evaporation stops. Air flow caused by convection replaces the saturated air by dry air and evaporation continues.

Example 14.8 Calculate the Flow of Mass during Convection: Sweat-Heat Transfer away from the Body

The average person produces heat at the rate of about 120 W when at rest. At what rate must water evaporate from the body to get rid of all this energy? (This evaporation might occur when a person is sitting in the shade and surrounding temperatures are the same as skin temperature, eliminating heat transfer by other methods.)

Strategy

Energy is needed for a phase change ($Q = mL_\mathrm{v}$). Thus, the energy loss per unit time is

$$\frac{Q}{t} = \frac{mL_\mathrm{v}}{t} = 120 \text{ W} = 120 \text{ J/s.} \tag{14.40}$$

We divide both sides of the equation by L_v to find that the mass evaporated per unit time is

$$\frac{m}{t} = \frac{120 \text{ J/s}}{L_\mathrm{v}}. \tag{14.41}$$

Solution

(1) Insert the value of the latent heat from **Table 14.2**, $L_\mathrm{v} = 2430 \text{ kJ/kg} = 2430 \text{ J/g}$. This yields

$$\frac{m}{t} = \frac{120 \text{ J/s}}{2430 \text{ J/g}} = 0.0494 \text{ g/s} = 2.96 \text{ g/min.} \tag{14.42}$$

Discussion

Evaporating about 3 g/min seems reasonable. This would be about 180 g (about 7 oz) per hour. If the air is very dry, the sweat may evaporate without even being noticed. A significant amount of evaporation also takes place in the lungs and breathing passages.

Another important example of the combination of phase change and convection occurs when water evaporates from the oceans. Heat is removed from the ocean when water evaporates. If the water vapor condenses in liquid droplets as clouds form, heat is released in the atmosphere. Thus, there is an overall transfer of heat from the ocean to the atmosphere. This process is the driving power behind thunderheads, those great cumulus clouds that rise as much as 20.0 km into the stratosphere. Water vapor carried in by convection condenses, releasing tremendous amounts of energy. This energy causes the air to expand and rise, where it is colder. More condensation occurs in these colder regions, which in turn drives the cloud even higher. Such a mechanism is called positive feedback, since the process reinforces and accelerates itself. These systems sometimes produce violent storms, with lightning and hail, and constitute the mechanism driving hurricanes.

Figure 14.20 Cumulus clouds are caused by water vapor that rises because of convection. The rise of clouds is driven by a positive feedback mechanism. (credit: Mike Love)

Figure 14.21 Convection accompanied by a phase change releases the energy needed to drive this thunderhead into the stratosphere. (credit: Gerardo García Moretti)

Figure 14.22 The phase change that occurs when this iceberg melts involves tremendous heat transfer. (credit: Dominic Alves)

The movement of icebergs is another example of convection accompanied by a phase change. Suppose an iceberg drifts from Greenland into warmer Atlantic waters. Heat is removed from the warm ocean water when the ice melts and heat is released to the land mass when the iceberg forms on Greenland.

Check Your Understanding

Explain why using a fan in the summer feels refreshing!

Solution

Using a fan increases the flow of air: warm air near your body is replaced by cooler air from elsewhere. Convection increases the rate of heat transfer so that moving air “feels” cooler than still air.

14.7 Radiation

You can feel the heat transfer from a fire and from the Sun. Similarly, you can sometimes tell that the oven is hot without touching its door or looking inside—it may just warm you as you walk by. The space between the Earth and the Sun is largely empty, without any possibility of heat transfer by convection or conduction. In these examples, heat is transferred by radiation. That is, the hot body emits electromagnetic waves that are absorbed by our skin: no medium is required for electromagnetic waves to propagate. Different names are used for electromagnetic waves of different wavelengths: radio waves, microwaves, infrared **radiation**, visible light, ultraviolet radiation, X-rays, and gamma rays.

Figure 14.23 Most of the heat transfer from this fire to the observers is through infrared radiation. The visible light, although dramatic, transfers relatively little thermal energy. Convection transfers energy away from the observers as hot air rises, while conduction is negligibly slow here. Skin is very sensitive to infrared radiation, so that you can sense the presence of a fire without looking at it directly. (credit: Daniel X. O'Neil)

The energy of electromagnetic radiation depends on the wavelength (color) and varies over a wide range: a smaller wavelength (or higher frequency) corresponds to a higher energy. Because more heat is radiated at higher temperatures, a temperature change is accompanied by a color change. Take, for example, an electrical element on a stove, which glows from red to orange, while the higher-temperature steel in a blast furnace glows from yellow to white. The radiation you feel is mostly infrared, which corresponds to a lower temperature than that of the electrical element and the steel. The radiated energy depends on its intensity, which is represented in the figure below by the height of the distribution.

Electromagnetic Waves explains more about the electromagnetic spectrum and **Introduction to Quantum Physics** discusses how the decrease in wavelength corresponds to an increase in energy.

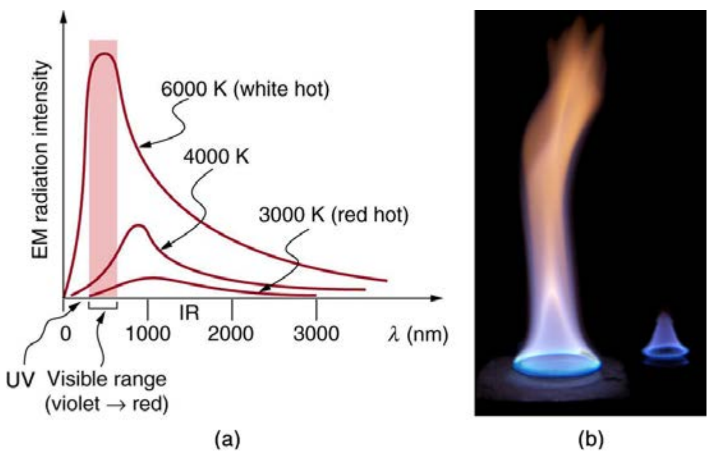


Figure 14.24 (a) A graph of the spectra of electromagnetic waves emitted from an ideal radiator at three different temperatures. The intensity or rate of radiation emission increases dramatically with temperature, and the spectrum shifts toward the visible and ultraviolet parts of the spectrum. The shaded portion denotes the visible part of the spectrum. It is apparent that the shift toward the ultraviolet with temperature makes the visible appearance shift from red to white to blue as temperature increases. (b) Note the variations in color corresponding to variations in flame temperature. (credit: Tuohirulla)

All objects absorb and emit electromagnetic radiation. The rate of heat transfer by radiation is largely determined by the color of the object. Black is the most effective, and white is the least effective. People living in hot climates generally avoid wearing black clothing, for instance (see **Take-Home Experiment: Temperature in the Sun**). Similarly, black asphalt in a parking lot will be hotter than adjacent gray sidewalk on a summer day, because black absorbs better than gray. The reverse is also true—black radiates better than gray. Thus, on a clear summer night, the asphalt will be colder than the gray sidewalk, because black radiates the energy more rapidly than gray. An *ideal radiator* is the same color as an *ideal absorber*, and captures all the radiation that falls on it. In contrast, white is a poor absorber and is also a poor radiator. A white object reflects all radiation, like a mirror. (A perfect, polished white surface is mirror-like in appearance, and a crushed mirror looks white.)

Figure 14.25 This illustration shows that the darker pavement is hotter than the lighter pavement (much more of the ice on the right has melted), although both have been in the sunlight for the same time. The thermal conductivities of the pavements are the same.

Gray objects have a uniform ability to absorb all parts of the electromagnetic spectrum. Colored objects behave in similar but more complex ways, which gives them a particular color in the visible range and may make them special in other ranges of the nonvisible spectrum. Take, for example, the strong absorption of infrared radiation by the skin, which allows us to be very sensitive to it.

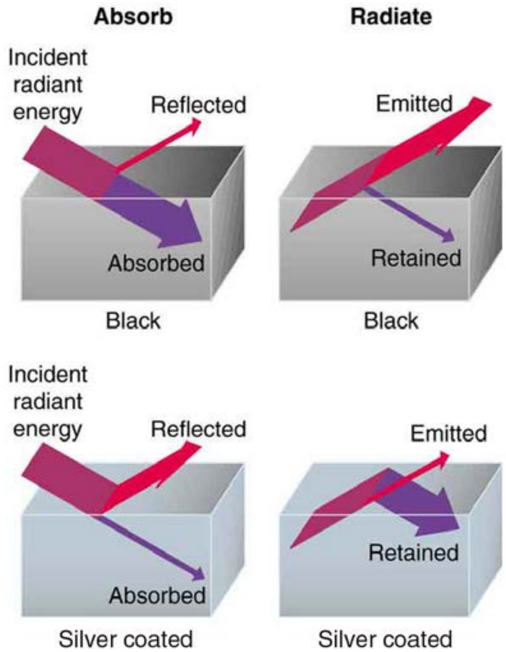


Figure 14.26 A black object is a good absorber and a good radiator, while a white (or silver) object is a poor absorber and a poor radiator. It is as if radiation from the inside is reflected back into the silver object, whereas radiation from the inside of the black object is "absorbed" when it hits the surface and finds itself on the outside and is strongly emitted.

The rate of heat transfer by emitted radiation is determined by the **Stefan-Boltzmann law of radiation**:

$$\frac{Q}{t} = \sigma e A T^4, \tag{14.43}$$

where $\sigma = 5.67 \times 10^{-8}\ \text{J/s} \cdot \text{m}^2 \cdot \text{K}^4$ is the Stefan-Boltzmann constant, A is the surface area of the object, and T is its absolute temperature in kelvin. The symbol e stands for the **emissivity** of the object, which is a measure of how well it radiates. An ideal jet-black (or black body) radiator has $e = 1$, whereas a perfect reflector has $e = 0$. Real objects fall between these two values. Take, for example, tungsten light bulb filaments which have an e of about 0.5, and carbon black (a material used in printer toner), which has the (greatest known) emissivity of about 0.99.

The radiation rate is directly proportional to the *fourth power* of the absolute temperature—a remarkably strong temperature dependence. Furthermore, the radiated heat is proportional to the surface area of the object. If you knock apart the coals of a fire, there is a noticeable increase in radiation due to an increase in radiating surface area.

Figure 14.27 A thermograph of part of a building shows temperature variations, indicating where heat transfer to the outside is most severe. Windows are a major region of heat transfer to the outside of homes. (credit: U.S. Army)

Skin is a remarkably good absorber and emitter of infrared radiation, having an emissivity of 0.97 in the infrared spectrum. Thus, we are all nearly (jet) black in the infrared, in spite of the obvious variations in skin color. This high infrared emissivity is why we can so easily feel radiation on our skin. It is also the basis for the use of night scopes used by law enforcement and the military to detect human beings. Even small temperature variations can be detected because of the T^4 dependence. Images, called *thermographs*, can be used medically to detect regions of abnormally high temperature in the body, perhaps indicative of disease. Similar techniques can be used to detect heat leaks in homes **Figure 14.27**, optimize performance of blast furnaces, improve comfort levels in work environments, and even remotely map the Earth's temperature profile.

All objects emit and absorb radiation. The *net* rate of heat transfer by radiation (absorption minus emission) is related to both the temperature of the object and the temperature of its surroundings. Assuming that an object with a temperature T_1 is surrounded by an environment with uniform temperature T_2, the **net rate of heat transfer by radiation** is

$$\frac{Q_{\text{net}}}{t} = \sigma e A\left(T_2^4 - T_1^4\right), \tag{14.44}$$

where e is the emissivity of the object alone. In other words, it does not matter whether the surroundings are white, gray, or black; the balance of radiation into and out of the object depends on how well it emits and absorbs radiation. When $T_2 > T_1$, the quantity Q_{net} / t is positive; that is, the net heat transfer is from hot to cold.

Take-Home Experiment: Temperature in the Sun

Place a thermometer out in the sunshine and shield it from direct sunlight using an aluminum foil. What is the reading? Now remove the shield, and note what the thermometer reads. Take a handkerchief soaked in nail polish remover, wrap it around the thermometer and place it in the sunshine. What does the thermometer read?

Example 14.9 Calculate the Net Heat Transfer of a Person: Heat Transfer by Radiation

What is the rate of heat transfer by radiation, with an unclothed person standing in a dark room whose ambient temperature is 22.0°C. The person has a normal skin temperature of 33.0°C and a surface area of 1.50 m^2. The emissivity of skin is 0.97 in the infrared, where the radiation takes place.

Strategy

We can solve this by using the equation for the rate of radiative heat transfer.

Solution

Insert the temperatures values $T_2 = 295\text{ K}$ and $T_1 = 306\text{ K}$, so that

$$\frac{Q}{t} = \sigma e A\left(T_2^4 - T_1^4\right) \tag{14.45}$$

$$= \left(5.67\times10^{-8}\text{ J/s}\cdot\text{ m}^2\cdot\text{ K}^4\right)(0.97)\left(1.50\text{ m}^2\right)\left[(295\text{ K})^4 - (306\text{ K})^4\right] \tag{14.46}$$

$$= -99\text{ J/s} = -99\text{ W}. \tag{14.47}$$

Discussion

This value is a significant rate of heat transfer to the environment (note the minus sign), considering that a person at rest may produce energy at the rate of 125 W and that conduction and convection will also be transferring energy to the environment. Indeed, we would probably expect this person to feel cold. Clothing significantly reduces heat transfer to the environment by many methods, because clothing slows down both conduction and convection, and has a lower emissivity (especially if it is white) than skin.

The Earth receives almost all its energy from radiation of the Sun and reflects some of it back into outer space. Because the Sun is hotter than the Earth, the net energy flux is from the Sun to the Earth. However, the rate of energy transfer is less than the equation for the radiative heat transfer would predict because the Sun does not fill the sky. The average emissivity (e) of the Earth is about 0.65, but the calculation of this value is complicated by the fact that the highly reflective cloud coverage varies greatly from day to day. There is a negative feedback (one in which a change produces an effect that opposes that change) between clouds and heat transfer; greater temperatures evaporate more water to form more clouds, which reflect more radiation back into space, reducing the temperature. The often mentioned **greenhouse effect** is directly related to the variation of the Earth's emissivity with radiation type (see the figure given below). The greenhouse effect is a natural phenomenon responsible for providing temperatures suitable for life on Earth. The Earth's relatively constant temperature is a result of the energy balance between the incoming solar radiation and the energy radiated from the Earth. Most of the infrared radiation emitted from the Earth is absorbed by carbon dioxide (CO_2) and water (H_2O) in the atmosphere and then re-radiated back to the Earth or into outer space. Re-radiation back to the Earth maintains its surface temperature about 40°C higher than it would be if there was no atmosphere, similar to the way glass increases temperatures in a greenhouse.

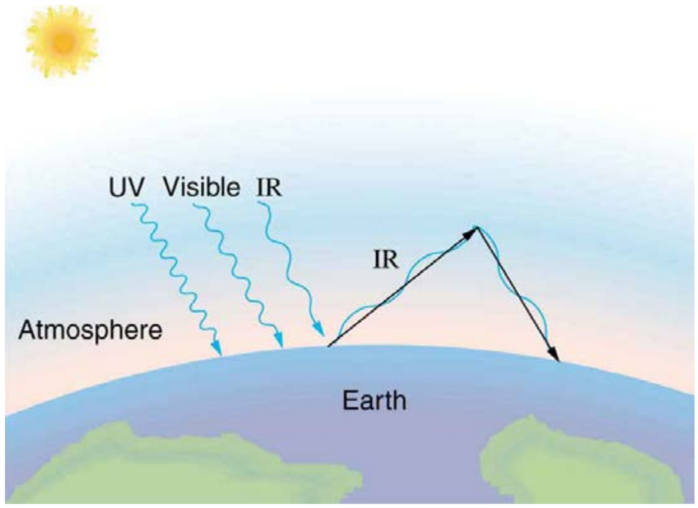


Figure 14.28 The greenhouse effect is a name given to the trapping of energy in the Earth's atmosphere by a process similar to that used in greenhouses. The atmosphere, like window glass, is transparent to incoming visible radiation and most of the Sun's infrared. These wavelengths are absorbed by the Earth and re-emitted as infrared. Since Earth's temperature is much lower than that of the Sun, the infrared radiated by the Earth has a much longer wavelength. The atmosphere, like glass, traps these longer infrared rays, keeping the Earth warmer than it would otherwise be. The amount of trapping depends on concentrations of trace gases like carbon dioxide, and a change in the concentration of these gases is believed to affect the Earth's surface temperature.

The greenhouse effect is also central to the discussion of global warming due to emission of carbon dioxide and methane (and other so-called greenhouse gases) into the Earth's atmosphere from industrial production and farming. Changes in global climate could lead to more intense storms, precipitation changes (affecting agriculture), reduction in rain forest biodiversity, and rising sea levels.

Heating and cooling are often significant contributors to energy use in individual homes. Current research efforts into developing environmentally friendly homes quite often focus on reducing conventional heating and cooling through better building materials, strategically positioning windows to optimize radiation gain from the Sun, and opening spaces to allow convection. It is possible to build a zero-energy house that allows for comfortable living in most parts of the United States with hot and humid summers and cold winters.

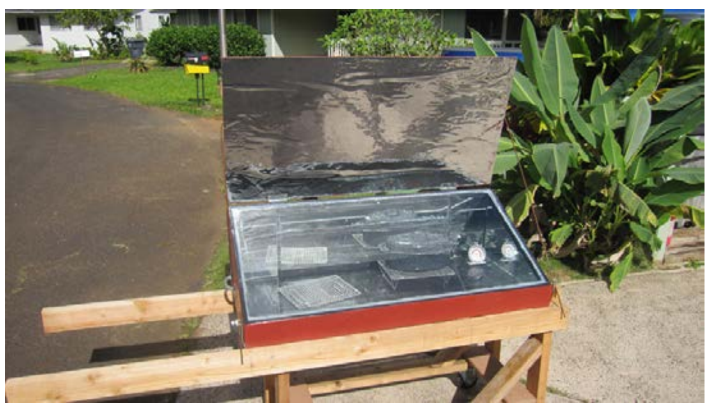


Figure 14.29 This simple but effective solar cooker uses the greenhouse effect and reflective material to trap and retain solar energy. Made of inexpensive, durable materials, it saves money and labor, and is of particular economic value in energy-poor developing countries. (credit: E.B. Kauai)

Conversely, dark space is very cold, about $3\text{K}(-454\text{ºF})$, so that the Earth radiates energy into the dark sky. Owing to the fact that clouds have lower emissivity than either oceans or land masses, they reflect some of the radiation back to the surface, greatly reducing heat transfer into dark space, just as they greatly reduce heat transfer into the atmosphere during the day. The rate of heat transfer from soil and grasses can be so rapid that frost may occur on clear summer evenings, even in warm latitudes.

Check Your Understanding

What is the change in the rate of the radiated heat by a body at the temperature $T_1 = 20\text{ºC}$ compared to when the body is at the temperature $T_2 = 40\text{ºC}$?

Solution

The radiated heat is proportional to the fourth power of the *absolute temperature*. Because $T_1 = 293\text{ K}$ and $T_2 = 313\text{ K}$, the rate of heat transfer increases by about 30 percent of the original rate.

Career Connection: Energy Conservation Consultation

The cost of energy is generally believed to remain very high for the foreseeable future. Thus, passive control of heat loss in both commercial and domestic housing will become increasingly important. Energy consultants measure and analyze the flow of energy into and out of houses and ensure that a healthy exchange of air is maintained inside the house. The job prospects for an energy consultant are strong.

Problem-Solving Strategies for the Methods of Heat Transfer

1. *Examine the situation to determine what type of heat transfer is involved.*
2. *Identify the type(s) of heat transfer—conduction, convection, or radiation.*
3. *Identify exactly what needs to be determined in the problem (identify the unknowns). A written list is very useful.*
4. *Make a list of what is given or can be inferred from the problem as stated (identify the knowns).*
5. *Solve the appropriate equation for the quantity to be determined (the unknown).*
6. For conduction, equation $\frac{Q}{t} = \frac{kA(T_2 - T_1)}{d}$ is appropriate. Table 14.3 lists thermal conductivities. For convection, determine the amount of matter moved and use equation $Q = mc\Delta T$, to calculate the heat transfer involved in the temperature change of the fluid. If a phase change accompanies convection, equation $Q = mL_\text{f}$ or $Q = mL_\text{v}$ is appropriate to find the heat transfer involved in the phase change. Table 14.2 lists information relevant to phase change. For radiation, equation $\frac{Q_\text{net}}{t} = \sigma eA\left(T_2^4 - T_1^4\right)$ gives the net heat transfer rate.
7. *Insert the knowns along with their units into the appropriate equation and obtain numerical solutions complete with units.*
8. *Check the answer to see if it is reasonable. Does it make sense?*

Glossary

conduction: heat transfer through stationary matter by physical contact

convection: heat transfer by the macroscopic movement of fluid

emissivity: measure of how well an object radiates

greenhouse effect: warming of the Earth that is due to gases such as carbon dioxide and methane that absorb infrared radiation from the Earth's surface and reradiate it in all directions, thus sending a fraction of it back toward the surface of the Earth

heat of sublimation: the energy required to change a substance from the solid phase to the vapor phase

heat: the spontaneous transfer of energy due to a temperature difference

kilocalorie: $1 \text{ kilocalorie} = 1000 \text{ calories}$

latent heat coefficient: a physical constant equal to the amount of heat transferred for every 1 kg of a substance during the change in phase of the substance

mechanical equivalent of heat: the work needed to produce the same effects as heat transfer

net rate of heat transfer by radiation: is $\frac{Q_\text{net}}{t} = \sigma eA\left(T_2^4 - T_1^4\right)$

radiation: heat transfer which occurs when microwaves, infrared radiation, visible light, or other electromagnetic radiation is emitted or absorbed

radiation: energy transferred by electromagnetic waves directly as a result of a temperature difference

rate of conductive heat transfer: rate of heat transfer from one material to another

Stefan-Boltzmann law of radiation: $\frac{Q}{t} = \sigma eAT^4,$ where σ is the Stefan-Boltzmann constant, A is the surface area of the object, T is the absolute temperature, and e is the emissivity

specific heat: the amount of heat necessary to change the temperature of 1.00 kg of a substance by 1.00 ºC

sublimation: the transition from the solid phase to the vapor phase

thermal conductivity: the property of a material's ability to conduct heat

Section Summary

14.1 Heat

- Heat and work are the two distinct methods of energy transfer.
- Heat is energy transferred solely due to a temperature difference.
- Any energy unit can be used for heat transfer, and the most common are kilocalorie (kcal) and joule (J).
- Kilocalorie is defined to be the energy needed to change the temperature of 1.00 kg of water between 14.5°C and 15.5°C.
- The mechanical equivalent of this heat transfer is $1.00 \text{ kcal} = 4186 \text{ J}.$

14.2 Temperature Change and Heat Capacity

- The transfer of heat Q that leads to a change ΔT in the temperature of a body with mass m is $Q = mc\Delta T$, where c is the specific heat of the material. This relationship can also be considered as the definition of specific heat.

14.3 Phase Change and Latent Heat

- Most substances can exist either in solid, liquid, and gas forms, which are referred to as "phases."
- Phase changes occur at fixed temperatures for a given substance at a given pressure, and these temperatures are called boiling and freezing (or melting) points.
- During phase changes, heat absorbed or released is given by:

$$Q = mL,$$

where L is the latent heat coefficient.

14.4 Heat Transfer Methods

- Heat is transferred by three different methods: conduction, convection, and radiation.

14.5 Conduction

- Heat conduction is the transfer of heat between two objects in direct contact with each other.
- The rate of heat transfer Q/t (energy per unit time) is proportional to the temperature difference $T_2 - T_1$ and the contact area A and inversely proportional to the distance d between the objects:

$$\frac{Q}{t} = \frac{kA(T_2 - T_1)}{d}.$$

14.6 Convection

- Convection is heat transfer by the macroscopic movement of mass. Convection can be natural or forced and generally transfers thermal energy faster than conduction. Table 14.4 gives wind-chill factors, indicating that moving air has the same chilling effect of much colder stationary air. *Convection that occurs along with a phase change* can transfer energy from cold regions to warm ones.

14.7 Radiation

- Radiation is the rate of heat transfer through the emission or absorption of electromagnetic waves.
- The rate of heat transfer depends on the surface area and the fourth power of the absolute temperature:

$$\frac{Q}{t} = \sigma e A T^4,$$

where $\sigma = 5.67\times10^{-8}\ \text{J/s} \cdot \text{m}^2 \cdot \text{K}^4$ is the Stefan-Boltzmann constant and e is the emissivity of the body. For a black body, $e = 1$ whereas a shiny white or perfect reflector has $e = 0$, with real objects having values of e between 1 and 0. The net rate of heat transfer by radiation is

$$\frac{Q_{\text{net}}}{t} = \sigma e A\left(T_2^4 - T_1^4\right)$$

where T_1 is the temperature of an object surrounded by an environment with uniform temperature T_2 and e is the emissivity of the *object*.

Conceptual Questions

14.1 Heat

1. How is heat transfer related to temperature?

2. Describe a situation in which heat transfer occurs. What are the resulting forms of energy?

3. When heat transfers into a system, is the energy stored as heat? Explain briefly.

14.2 Temperature Change and Heat Capacity

4. What three factors affect the heat transfer that is necessary to change an object's temperature?

5. The brakes in a car increase in temperature by ΔT when bringing the car to rest from a speed v. How much greater would ΔT be if the car initially had twice the speed? You may assume the car to stop sufficiently fast so that no heat transfers out of the brakes.

14.3 Phase Change and Latent Heat

6. Heat transfer can cause temperature and phase changes. What else can cause these changes?

7. How does the latent heat of fusion of water help slow the decrease of air temperatures, perhaps preventing temperatures from falling significantly below 0°C, in the vicinity of large bodies of water?

8. What is the temperature of ice right after it is formed by freezing water?

9. If you place 0°C ice into 0°C water in an insulated container, what will happen? Will some ice melt, will more water freeze, or will neither take place?

10. What effect does condensation on a glass of ice water have on the rate at which the ice melts? Will the condensation speed up the melting process or slow it down?

11. In very humid climates where there are numerous bodies of water, such as in Florida, it is unusual for temperatures to rise above about $35^\circ\text{C}(95^\circ\text{F})$. In deserts, however, temperatures can rise far above this. Explain how the evaporation of water helps limit high temperatures in humid climates.

12. In winters, it is often warmer in San Francisco than in nearby Sacramento, 150 km inland. In summers, it is nearly always hotter in Sacramento. Explain how the bodies of water surrounding San Francisco moderate its extreme temperatures.

13. Putting a lid on a boiling pot greatly reduces the heat transfer necessary to keep it boiling. Explain why.

14. Freeze-dried foods have been dehydrated in a vacuum. During the process, the food freezes and must be heated to facilitate dehydration. Explain both how the vacuum speeds up dehydration and why the food freezes as a result.

15. When still air cools by radiating at night, it is unusual for temperatures to fall below the dew point. Explain why.

16. In a physics classroom demonstration, an instructor inflates a balloon by mouth and then cools it in liquid nitrogen. When cold, the shrunken balloon has a small amount of light blue liquid in it, as well as some snow-like crystals. As it warms up, the liquid boils, and part of the crystals sublimate, with some crystals lingering for awhile and then producing a liquid. Identify the blue liquid and the two solids in the cold balloon. Justify your identifications using data from **Table 14.2**.

14.4 Heat Transfer Methods

17. What are the main methods of heat transfer from the hot core of Earth to its surface? From Earth's surface to outer space?

When our bodies get too warm, they respond by sweating and increasing blood circulation to the surface to transfer thermal energy away from the core. What effect will this have on a person in a 40.0°C hot tub?

Figure 14.30 shows a cut-away drawing of a thermos bottle (also known as a Dewar flask), which is a device designed specifically to slow down all forms of heat transfer. Explain the functions of the various parts, such as the vacuum, the silvering of the walls, the thin-walled long glass neck, the rubber support, the air layer, and the stopper.

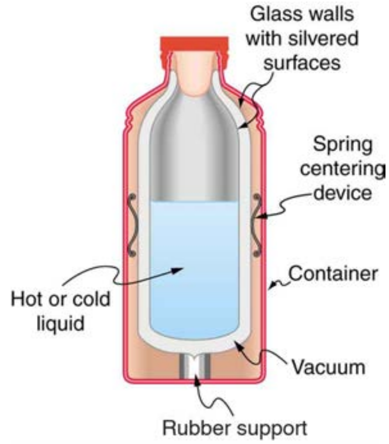


Figure 14.30 The construction of a thermos bottle is designed to inhibit all methods of heat transfer.

14.5 Conduction

18. Some electric stoves have a flat ceramic surface with heating elements hidden beneath. A pot placed over a heating element will be heated, while it is safe to touch the surface only a few centimeters away. Why is ceramic, with a conductivity less than that of a metal but greater than that of a good insulator, an ideal choice for the stove top?

19. Loose-fitting white clothing covering most of the body is ideal for desert dwellers, both in the hot Sun and during cold evenings. Explain how such clothing is advantageous during both day and night.

Figure 14.31 A jellabiya is worn by many men in Egypt. (credit: Zerida)

14.6 Convection

20. One way to make a fireplace more energy efficient is to have an external air supply for the combustion of its fuel. Another is to have room air circulate around the outside of the fire box and back into the room. Detail the methods of heat transfer involved in each.

21. On cold, clear nights horses will sleep under the cover of large trees. How does this help them keep warm?

14.7 Radiation

22. When watching a daytime circus in a large, dark-colored tent, you sense significant heat transfer from the tent. Explain why this occurs.

23. Satellites designed to observe the radiation from cold (3 K) dark space have sensors that are shaded from the Sun, Earth, and Moon and that are cooled to very low temperatures. Why must the sensors be at low temperature?

24. Why are cloudy nights generally warmer than clear ones?

25. Why are thermometers that are used in weather stations shielded from the sunshine? What does a thermometer measure if it is shielded from the sunshine and also if it is not?

26. On average, would Earth be warmer or cooler without the atmosphere? Explain your answer.

Problems & Exercises

14.2 Temperature Change and Heat Capacity

27. On a hot day, the temperature of an 80,000-L swimming pool increases by 1.50°C. What is the net heat transfer during this heating? Ignore any complications, such as loss of water by evaporation.

28. Show that $1\ \text{cal/g} \cdot {}^\circ\text{C} = 1\ \text{kcal/kg} \cdot {}^\circ\text{C}$.

29. To sterilize a 50.0-g glass baby bottle, we must raise its temperature from 22.0°C to 95.0°C. How much heat transfer is required?

30. The same heat transfer into identical masses of different substances produces different temperature changes. Calculate the final temperature when 1.00 kcal of heat transfers into 1.00 kg of the following, originally at 20.0°C: (a) water; (b) concrete; (c) steel; and (d) mercury.

31. Rubbing your hands together warms them by converting work into thermal energy. If a woman rubs her hands back and forth for a total of 20 rubs, at a distance of 7.50 cm per rub, and with an average frictional force of 40.0 N, what is the temperature increase? The mass of tissues warmed is only 0.100 kg, mostly in the palms and fingers.

32. A 0.250-kg block of a pure material is heated from 20.0°C to 65.0°C by the addition of 4.35 kJ of energy. Calculate its specific heat and identify the substance of which it is most likely composed.

33. Suppose identical amounts of heat transfer into different masses of copper and water, causing identical changes in temperature. What is the ratio of the mass of copper to water?

34. (a) The number of kilocalories in food is determined by calorimetry techniques in which the food is burned and the amount of heat transfer is measured. How many kilocalories per gram are there in a 5.00-g peanut if the energy from burning it is transferred to 0.500 kg of water held in a 0.100-kg aluminum cup, causing a 54.9°C temperature increase? (b) Compare your answer to labeling information found on a package of peanuts and comment on whether the values are consistent.

35. Following vigorous exercise, the body temperature of an 80.0-kg person is 40.0°C. At what rate in watts must the person transfer thermal energy to reduce the the body temperature to 37.0°C in 30.0 min, assuming the body continues to produce energy at the rate of 150 W? $(1\ \text{watt} = 1\ \text{joule/second or } 1\ \text{W} = 1\ \text{J/s})$.

36. Even when shut down after a period of normal use, a large commercial nuclear reactor transfers thermal energy at the rate of 150 MW by the radioactive decay of fission products. This heat transfer causes a rapid increase in temperature if the cooling system fails $(1\ \text{watt} = 1\ \text{joule/second or } 1\ \text{W} = 1\ \text{J/s and } 1\ \text{MW} = 1\ \text{megawatt})$. (a) Calculate the rate of temperature increase in degrees Celsius per second $({}^\circ\text{C/s})$ if the mass of the reactor core is $1.60\times10^5\ \text{kg}$ and it has an average specific heat of $0.3349\ \text{kJ/kg}^\circ \cdot \text{C}$. (b) How long would it take to obtain a temperature increase of 2000°C, which could cause some metals holding the radioactive materials to melt? (The initial rate of temperature increase would be greater than that calculated here because the heat transfer is concentrated in a smaller mass. Later, however, the temperature increase would slow down because the $5\times10^5\text{-kg}$ steel containment vessel would also begin to heat up.)

Figure 14.32 Radioactive spent-fuel pool at a nuclear power plant. Spent fuel stays hot for a long time. (credit: U.S. Department of Energy)

14.3 Phase Change and Latent Heat

37. How much heat transfer (in kilocalories) is required to thaw a 0.450-kg package of frozen vegetables originally at 0°C if their heat of fusion is the same as that of water?

38. A bag containing 0°C ice is much more effective in absorbing energy than one containing the same amount of 0°C water.

a. How much heat transfer is necessary to raise the temperature of 0.800 kg of water from 0°C to 30.0°C?
b. How much heat transfer is required to first melt 0.800 kg of 0°C ice and then raise its temperature?
c. Explain how your answer supports the contention that the ice is more effective.

39. (a) How much heat transfer is required to raise the temperature of a 0.750-kg aluminum pot containing 2.50 kg of water from 30.0°C to the boiling point and then boil away 0.750 kg of water? (b) How long does this take if the rate of heat transfer is 500 W $1\ \text{watt} = 1\ \text{joule/second}\ (1\ \text{W} = 1\ \text{J/s})$?

40. The formation of condensation on a glass of ice water causes the ice to melt faster than it would otherwise. If 8.00 g of condensation forms on a glass containing both water and 200 g of ice, how many grams of the ice will melt as a result? Assume no other heat transfer occurs.

41. On a trip, you notice that a 3.50-kg bag of ice lasts an average of one day in your cooler. What is the average power in watts entering the ice if it starts at 0°C and completely melts to 0°C water in exactly one day $1\ \text{watt} = 1\ \text{joule/second}\ \ (1\ \text{W} = 1\ \text{J/s})$?

42. On a certain dry sunny day, a swimming pool's temperature would rise by 1.50°C if not for evaporation. What fraction of the water must evaporate to carry away precisely enough energy to keep the temperature constant?

43. (a) How much heat transfer is necessary to raise the temperature of a 0.200-kg piece of ice from -20.0°C to 130°C, including the energy needed for phase changes?
(b) How much time is required for each stage, assuming a constant 20.0 kJ/s rate of heat transfer?
(c) Make a graph of temperature versus time for this process.

44. In 1986, a gargantuan iceberg broke away from the Ross Ice Shelf in Antarctica. It was approximately a rectangle 160 km long, 40.0 km wide, and 250 m thick.

(a) What is the mass of this iceberg, given that the density of ice is 917 kg/m^3 ?

(b) How much heat transfer (in joules) is needed to melt it?

(c) How many years would it take sunlight alone to melt ice this thick, if the ice absorbs an average of 100 W/m^2 , 12.00 h per day?

45. How many grams of coffee must evaporate from 350 g of coffee in a 100-g glass cup to cool the coffee from 95.0°C to 45.0°C ? You may assume the coffee has the same thermal properties as water and that the average heat of vaporization is 2340 kJ/kg (560 cal/g). (You may neglect the change in mass of the coffee as it cools, which will give you an answer that is slightly larger than correct.)

46. (a) It is difficult to extinguish a fire on a crude oil tanker, because each liter of crude oil releases $2.80\times10^7 \text{ J}$ of energy when burned. To illustrate this difficulty, calculate the number of liters of water that must be expended to absorb the energy released by burning 1.00 L of crude oil, if the water has its temperature raised from 20.0°C to 100°C , it boils, and the resulting steam is raised to 300°C . (b) Discuss additional complications caused by the fact that crude oil has a smaller density than water.

47. The energy released from condensation in thunderstorms can be very large. Calculate the energy released into the atmosphere for a small storm of radius 1 km, assuming that 1.0 cm of rain is precipitated uniformly over this area.

48. To help prevent frost damage, 4.00 kg of 0°C water is sprayed onto a fruit tree.

(a) How much heat transfer occurs as the water freezes?

(b) How much would the temperature of the 200-kg tree decrease if this amount of heat transferred from the tree? Take the specific heat to be $3.35 \text{ kJ/kg}\cdot^\circ\text{C}$, and assume that no phase change occurs.

49. A 0.250-kg aluminum bowl holding 0.800 kg of soup at 25.0°C is placed in a freezer. What is the final temperature if 377 kJ of energy is transferred from the bowl and soup, assuming the soup's thermal properties are the same as that of water? Explicitly show how you follow the steps in **Problem-Solving Strategies for the Effects of Heat Transfer**.

50. A 0.0500-kg ice cube at -30.0°C is placed in 0.400 kg of 35.0°C water in a very well-insulated container. What is the final temperature?

51. If you pour 0.0100 kg of 20.0°C water onto a 1.20-kg block of ice (which is initially at -15.0°C), what is the final temperature? You may assume that the water cools so rapidly that effects of the surroundings are negligible.

52. Indigenous people sometimes cook in watertight baskets by placing hot rocks into water to bring it to a boil. What mass of 500°C rock must be placed in 4.00 kg of 15.0°C water to bring its temperature to 100°C , if 0.0250 kg of water escapes as vapor from the initial sizzle? You may neglect the effects of the surroundings and take the average specific heat of the rocks to be that of granite.

53. What would be the final temperature of the pan and water in **Calculating the Final Temperature When Heat Is Transferred Between Two Bodies: Pouring Cold Water in a Hot Pan** if 0.260 kg of water was placed in the pan and 0.0100 kg of the water evaporated immediately, leaving the remainder to come to a common temperature with the pan?

54. In some countries, liquid nitrogen is used on dairy trucks instead of mechanical refrigerators. A 3.00-hour delivery trip requires 200 L of liquid nitrogen, which has a density of 808 kg/m^3 .

(a) Calculate the heat transfer necessary to evaporate this amount of liquid nitrogen and raise its temperature to 3.00°C . (Use c_p and assume it is constant over the temperature range.) This value is the amount of cooling the liquid nitrogen supplies.

(b) What is this heat transfer rate in kilowatt-hours?

(c) Compare the amount of cooling obtained from melting an identical mass of 0°C ice with that from evaporating the liquid nitrogen.

55. Some gun fanciers make their own bullets, which involves melting and casting the lead slugs. How much heat transfer is needed to raise the temperature and melt 0.500 kg of lead, starting from 25.0°C ?

14.5 Conduction

56. (a) Calculate the rate of heat conduction through house walls that are 13.0 cm thick and that have an average thermal conductivity twice that of glass wool. Assume there are no windows or doors. The surface area of the walls is 120 m^2 and their inside surface is at 18.0°C , while their outside surface is at 5.00°C . (b) How many 1-kW room heaters would be needed to balance the heat transfer due to conduction?

57. The rate of heat conduction out of a window on a winter day is rapid enough to chill the air next to it. To see just how rapidly the windows transfer heat by conduction, calculate the rate of conduction in watts through a 3.00-m^2 window that is 0.635 cm thick (1/4 in) if the temperatures of the inner and outer surfaces are 5.00°C and -10.0°C , respectively. This rapid rate will not be maintained—the inner surface will cool, and even result in frost formation.

58. Calculate the rate of heat conduction out of the human body, assuming that the core internal temperature is 37.0°C , the skin temperature is 34.0°C , the thickness of the tissues between averages 1.00 cm , and the surface area is 1.40 m^2 .

59. Suppose you stand with one foot on ceramic flooring and one foot on a wool carpet, making contact over an area of 80.0 cm^2 with each foot. Both the ceramic and the carpet are 2.00 cm thick and are 10.0°C on their bottom sides. At what rate must heat transfer occur from each foot to keep the top of the ceramic and carpet at 33.0°C ?

60. A man consumes 3000 kcal of food in one day, converting most of it to maintain body temperature. If he loses half this energy by evaporating water (through breathing and sweating), how many kilograms of water evaporate?

61. (a) A firewalker runs across a bed of hot coals without sustaining burns. Calculate the heat transferred by conduction into the sole of one foot of a firewalker given that the bottom of the foot is a 3.00-mm-thick callus with a conductivity at the low end of the range for wood and its density is 300 kg/m^3 . The area of contact is 25.0 cm^2 , the temperature of the coals is 700°C , and the time in contact is 1.00 s.

(b) What temperature increase is produced in the 25.0 cm^3 of tissue affected?

(c) What effect do you think this will have on the tissue, keeping in mind that a callus is made of dead cells?

62. (a) What is the rate of heat conduction through the 3.00-cm-thick fur of a large animal having a 1.40-m^2 surface area? Assume that the animal's skin temperature is 32.0°C , that the air temperature is -5.00°C , and that fur has the same thermal conductivity as air. (b) What food intake will the animal need in one day to replace this heat transfer?

63. A walrus transfers energy by conduction through its blubber at the rate of 150 W when immersed in -1.00°C water. The walrus's internal core temperature is 37.0°C , and it has a surface area of 2.00 m^2 . What is the average thickness of its blubber, which has the conductivity of fatty tissues without blood?

Figure 14.33 Walrus on ice. (credit: Captain Budd Christman, NOAA Corps)

64. Compare the rate of heat conduction through a 13.0-cm-thick wall that has an area of $10.0\ \text{m}^2$ and a thermal conductivity twice that of glass wool with the rate of heat conduction through a window that is 0.750 cm thick and that has an area of $2.00\ \text{m}^2$, assuming the same temperature difference across each.

65. Suppose a person is covered head to foot by wool clothing with average thickness of 2.00 cm and is transferring energy by conduction through the clothing at the rate of 50.0 W. What is the temperature difference across the clothing, given the surface area is $1.40\ \text{m}^2$?

66. Some stove tops are smooth ceramic for easy cleaning. If the ceramic is 0.600 cm thick and heat conduction occurs through the same area and at the same rate as computed in **Example 14.6**, what is the temperature difference across it? Ceramic has the same thermal conductivity as glass and brick.

67. One easy way to reduce heating (and cooling) costs is to add extra insulation in the attic of a house. Suppose the house already had 15 cm of fiberglass insulation in the attic and in all the exterior surfaces. If you added an extra 8.0 cm of fiberglass to the attic, then by what percentage would the heating cost of the house drop? Take the single story house to be of dimensions 10 m by 15 m by 3.0 m. Ignore air infiltration and heat loss through windows and doors.

68. (a) Calculate the rate of heat conduction through a double-paned window that has a 1.50-m^2 area and is made of two panes of 0.800-cm-thick glass separated by a 1.00-cm air gap. The inside surface temperature is 15.0°C, while that on the outside is -10.0°C. (Hint: There are identical temperature drops across the two glass panes. First find these and then the temperature drop across the air gap. This problem ignores the increased heat transfer in the air gap due to convection.)

(b) Calculate the rate of heat conduction through a 1.60-cm-thick window of the same area and with the same temperatures. Compare your answer with that for part (a).

69. Many decisions are made on the basis of the payback period: the time it will take through savings to equal the capital cost of an investment. Acceptable payback times depend upon the business or philosophy one has. (For some industries, a payback period is as small as two years.) Suppose you wish to install the extra insulation in **Exercise 14.67**. If energy cost \$1.00 per million joules and the insulation was \$4.00 per square meter, then calculate the simple payback time. Take the average ΔT for the 120 day heating season to be 15.0°C.

70. For the human body, what is the rate of heat transfer by conduction through the body's tissue with the following conditions: the tissue thickness is 3.00 cm, the change in temperature is 2.00°C, and the skin area is $1.50\ \text{m}^2$. How does this compare with the average heat transfer rate to the body resulting from an energy intake of about 2400 kcal per day? (No exercise is included.)

14.6 Convection

71. At what wind speed does -10°C air cause the same chill factor as still air at -29°C?

72. At what temperature does still air cause the same chill factor as -5°C air moving at 15 m/s?

73. The "steam" above a freshly made cup of instant coffee is really water vapor droplets condensing after evaporating from the hot coffee. What is the final temperature of 250 g of hot coffee initially at 90.0°C if 2.00 g evaporates from it? The coffee is in a Styrofoam cup, so other methods of heat transfer can be neglected.

74. (a) How many kilograms of water must evaporate from a 60.0-kg woman to lower her body temperature by 0.750°C?

(b) Is this a reasonable amount of water to evaporate in the form of perspiration, assuming the relative humidity of the surrounding air is low?

75. On a hot dry day, evaporation from a lake has just enough heat transfer to balance the $1.00\ \text{kW/m}^2$ of incoming heat from the Sun. What mass of water evaporates in 1.00 h from each square meter? Explicitly show how you follow the steps in the **Problem-Solving Strategies for the Effects of Heat Transfer**.

76. One winter day, the climate control system of a large university classroom building malfunctions. As a result, $500\ \text{m}^3$ of excess cold air is brought in each minute. At what rate in kilowatts must heat transfer occur to warm this air by 10.0°C (that is, to bring the air to room temperature)?

77. The Kilauea volcano in Hawaii is the world's most active, disgorging about $5\times10^5\ \text{m}^3$ of 1200°C lava per day. What is the rate of heat transfer out of Earth by convection if this lava has a density of $2700\ \text{kg/m}^3$ and eventually cools to 30°C? Assume that the specific heat of lava is the same as that of granite.

Figure 14.34 Lava flow on Kilauea volcano in Hawaii. (credit: J. P. Eaton, U.S. Geological Survey)

78. During heavy exercise, the body pumps 2.00 L of blood per minute to the surface, where it is cooled by 2.00°C. What is the rate of heat transfer from this forced convection alone, assuming blood has the same specific heat as water and its density is $1050\ \text{kg/m}^3$?

79. A person inhales and exhales 2.00 L of 37.0°C air, evaporating $4.00\times10^{-2}\ \text{g}$ of water from the lungs and breathing passages with each breath.

(a) How much heat transfer occurs due to evaporation in each breath?

(b) What is the rate of heat transfer in watts if the person is breathing at a moderate rate of 18.0 breaths per minute?

(c) If the inhaled air had a temperature of 20.0°C, what is the rate of heat transfer for warming the air?

(d) Discuss the total rate of heat transfer as it relates to typical metabolic rates. Will this breathing be a major form of heat transfer for this person?

80. A glass coffee pot has a circular bottom with a 9.00-cm diameter in contact with a heating element that keeps the coffee warm with a continuous heat transfer rate of 50.0 W

(a) What is the temperature of the bottom of the pot, if it is 3.00 mm thick and the inside temperature is 60.0°C ?

(b) If the temperature of the coffee remains constant and all of the heat transfer is removed by evaporation, how many grams per minute evaporate? Take the heat of vaporization to be 2340 kJ/kg.

14.7 Radiation

81. At what net rate does heat radiate from a 275-m^2 black roof on a night when the roof's temperature is 30.0°C and the surrounding temperature is 15.0°C ? The emissivity of the roof is 0.900.

82. (a) Cherry-red embers in a fireplace are at 850°C and have an exposed area of $0.200\ \text{m}^2$ and an emissivity of 0.980. The surrounding room has a temperature of 18.0°C . If 50% of the radiant energy enters the room, what is the net rate of radiant heat transfer in kilowatts? (b) Does your answer support the contention that most of the heat transfer into a room by a fireplace comes from infrared radiation?

83. Radiation makes it impossible to stand close to a hot lava flow. Calculate the rate of heat transfer by radiation from $1.00\ \text{m}^2$ of 1200°C fresh lava into 30.0°C surroundings, assuming lava's emissivity is 1.00.

84. (a) Calculate the rate of heat transfer by radiation from a car radiator at 110°C into a 50.0°C environment, if the radiator has an emissivity of 0.750 and a 1.20-m^2 surface area. (b) Is this a significant fraction of the heat transfer by an automobile engine? To answer this, assume a horsepower of $200\ \text{hp}\ (1.5\ \text{kW})$ and the efficiency of automobile engines as 25%.

85. Find the net rate of heat transfer by radiation from a skier standing in the shade, given the following. She is completely clothed in white (head to foot, including a ski mask), the clothes have an emissivity of 0.200 and a surface temperature of 10.0°C , the surroundings are at -15.0°C , and her surface area is $1.60\ \text{m}^2$.

86. Suppose you walk into a sauna that has an ambient temperature of 50.0°C . (a) Calculate the rate of heat transfer to you by radiation given your skin temperature is 37.0°C , the emissivity of skin is 0.98, and the surface area of your body is $1.50\ \text{m}^2$. (b) If all other forms of heat transfer are balanced (the net heat transfer is zero), at what rate will your body temperature increase if your mass is 75.0 kg?

87. Thermography is a technique for measuring radiant heat and detecting variations in surface temperatures that may be medically, environmentally, or militarily meaningful.(a) What is the percent increase in the rate of heat transfer by radiation from a given area at a temperature of 34.0°C compared with that at 33.0°C , such as on a person's skin? (b) What is the percent increase in the rate of heat transfer by radiation from a given area at a temperature of 34.0°C compared with that at 20.0°C , such as for warm and cool automobile hoods?

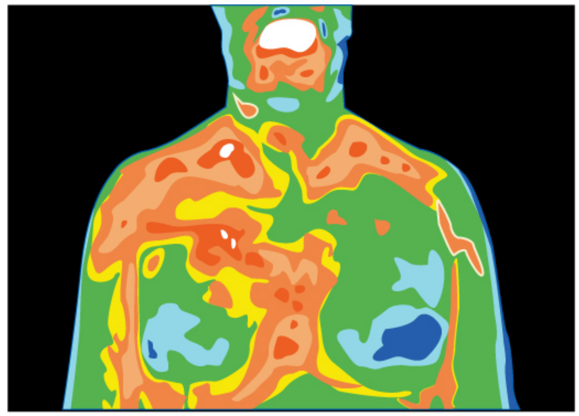


Figure 14.35 Artist's rendition of a thermograph of a patient's upper body, showing the distribution of heat represented by different colors.

88. The Sun radiates like a perfect black body with an emissivity of exactly 1. (a) Calculate the surface temperature of the Sun, given that it is a sphere with a $7.00\times10^{8}\text{-m}$ radius that radiates $3.80\times10^{26}\ \text{W}$ into 3-K space. (b) How much power does the Sun radiate per square meter of its surface? (c) How much power in watts per square meter is that value at the distance of Earth, $1.50\times10^{11}\ \text{m}$ away? (This number is called the solar constant.)

89. A large body of lava from a volcano has stopped flowing and is slowly cooling. The interior of the lava is at 1200°C , its surface is at 450°C , and the surroundings are at 27.0°C . (a) Calculate the rate at which energy is transferred by radiation from $1.00\ \text{m}^2$ of surface lava into the surroundings, assuming the emissivity is 1.00. (b) Suppose heat conduction to the surface occurs at the same rate. What is the thickness of the lava between the 450°C surface and the 1200°C interior, assuming that the lava's conductivity is the same as that of brick?

90. Calculate the temperature the entire sky would have to be in order to transfer energy by radiation at $1000\ \text{W/m}^2$ —about the rate at which the Sun radiates when it is directly overhead on a clear day. This value is the effective temperature of the sky, a kind of average that takes account of the fact that the Sun occupies only a small part of the sky but is much hotter than the rest. Assume that the body receiving the energy has a temperature of 27.0°C .

91. (a) A shirtless rider under a circus tent feels the heat radiating from the sunlit portion of the tent. Calculate the temperature of the tent canvas based on the following information: The shirtless rider's skin temperature is 34.0°C and has an emissivity of 0.970. The exposed area of skin is $0.400\ \text{m}^2$. He receives radiation at the rate of 20.0 W—half what you would calculate if the entire region behind him was hot. The rest of the surroundings are at 34.0°C . (b) Discuss how this situation would change if the sunlit side of the tent was nearly pure white and if the rider was covered by a white tunic.

92. Integrated Concepts

One 30.0°C day the relative humidity is 75.0% , and that evening the temperature drops to 20.0°C , well below the dew point. (a) How many grams of water condense from each cubic meter of air? (b) How much heat transfer occurs by this condensation? (c) What temperature increase could this cause in dry air?

93. Integrated Concepts

Large meteors sometimes strike the Earth, converting most of their kinetic energy into thermal energy. (a) What is the kinetic energy of a $10^9\ \text{kg}$ meteor moving at 25.0 km/s? (b) If this meteor lands in a deep ocean and 80% of its kinetic energy goes into heating water, how many kilograms of water could it raise by 5.0°C? (c) Discuss how the energy of the meteor is more likely to be deposited in the ocean and the likely effects of that energy.

94. Integrated Concepts

Frozen waste from airplane toilets has sometimes been accidentally ejected at high altitude. Ordinarily it breaks up and disperses over a large area, but sometimes it holds together and strikes the ground. Calculate the mass of 0ºC ice that can be melted by the conversion of kinetic and gravitational potential energy when a 20.0 kg piece of frozen waste is released at 12.0 km altitude while moving at 250 m/s and strikes the ground at 100 m/s (since less than 20.0 kg melts, a significant mess results).

95. Integrated Concepts

(a) A large electrical power facility produces 1600 MW of "waste heat," which is dissipated to the environment in cooling towers by warming air flowing through the towers by 5.00ºC . What is the necessary flow rate of air in m^3/s ? (b) Is your result consistent with the large cooling towers used by many large electrical power plants?

96. Integrated Concepts

(a) Suppose you start a workout on a Stairmaster, producing power at the same rate as climbing 116 stairs per minute. Assuming your mass is 76.0 kg and your efficiency is 20.0% , how long will it take for your body temperature to rise 1.00ºC if all other forms of heat transfer in and out of your body are balanced? (b) Is this consistent with your experience in getting warm while exercising?

97. Integrated Concepts

A 76.0-kg person suffering from hypothermia comes indoors and shivers vigorously. How long does it take the heat transfer to increase the person's body temperature by 2.00ºC if all other forms of heat transfer are balanced?

98. Integrated Concepts

In certain large geographic regions, the underlying rock is hot. Wells can be drilled and water circulated through the rock for heat transfer for the generation of electricity. (a) Calculate the heat transfer that can be extracted by cooling 1.00 km^3 of granite by 100ºC . (b) How long will it take for heat transfer at the rate of 300 MW, assuming no heat transfers back into the 1.00 km^3 of rock by its surroundings?

99. Integrated Concepts

Heat transfers from your lungs and breathing passages by evaporating water. (a) Calculate the maximum number of grams of water that can be evaporated when you inhale 1.50 L of 37ºC air with an original relative humidity of 40.0%. (Assume that body temperature is also 37ºC .) (b) How many joules of energy are required to evaporate this amount? (c) What is the rate of heat transfer in watts from this method, if you breathe at a normal resting rate of 10.0 breaths per minute?

100. Integrated Concepts

(a) What is the temperature increase of water falling 55.0 m over Niagara Falls? (b) What fraction must evaporate to keep the temperature constant?

101. Integrated Concepts

Hot air rises because it has expanded. It then displaces a greater volume of cold air, which increases the buoyant force on it. (a) Calculate the ratio of the buoyant force to the weight of 50.0ºC air surrounded by 20.0ºC air. (b) What energy is needed to cause 1.00 m^3 of air to go from 20.0ºC to 50.0ºC ? (c) What gravitational potential energy is gained by this volume of air if it rises 1.00 m? Will this cause a significant cooling of the air?

102. Unreasonable Results

(a) What is the temperature increase of an 80.0 kg person who consumes 2500 kcal of food in one day with 95.0% of the energy transferred as heat to the body? (b) What is unreasonable about this result? (c) Which premise or assumption is responsible?

103. Unreasonable Results

A slightly deranged Arctic inventor surrounded by ice thinks it would be much less mechanically complex to cool a car engine by melting ice on it than by having a water-cooled system with a radiator, water pump, antifreeze, and so on. (a) If 80.0% of the energy in 1.00 gal of gasoline is converted into "waste heat" in a car engine, how many kilograms of 0ºC ice could it melt? (b) Is this a reasonable amount of ice to carry around to cool the engine for 1.00 gal of gasoline consumption? (c) What premises or assumptions are unreasonable?

104. Unreasonable Results

(a) Calculate the rate of heat transfer by conduction through a window with an area of 1.00 m^2 that is 0.750 cm thick, if its inner surface is at 22.0ºC and its outer surface is at 35.0ºC . (b) What is unreasonable about this result? (c) Which premise or assumption is responsible?

105. Unreasonable Results

A meteorite 1.20 cm in diameter is so hot immediately after penetrating the atmosphere that it radiates 20.0 kW of power. (a) What is its temperature, if the surroundings are at 20.0ºC and it has an emissivity of 0.800? (b) What is unreasonable about this result? (c) Which premise or assumption is responsible?

106. Construct Your Own Problem

Consider a new model of commercial airplane having its brakes tested as a part of the initial flight permission procedure. The airplane is brought to takeoff speed and then stopped with the brakes alone. Construct a problem in which you calculate the temperature increase of the brakes during this process. You may assume most of the kinetic energy of the airplane is converted to thermal energy in the brakes and surrounding materials, and that little escapes. Note that the brakes are expected to become so hot in this procedure that they ignite and, in order to pass the test, the airplane must be able to withstand the fire for some time without a general conflagration.

107. Construct Your Own Problem

Consider a person outdoors on a cold night. Construct a problem in which you calculate the rate of heat transfer from the person by all three heat transfer methods. Make the initial circumstances such that at rest the person will have a net heat transfer and then decide how much physical activity of a chosen type is necessary to balance the rate of heat transfer. Among the things to consider are the size of the person, type of clothing, initial metabolic rate, sky conditions, amount of water evaporated, and volume of air breathed. Of course, there are many other factors to consider and your instructor may wish to guide you in the assumptions made as well as the detail of analysis and method of presenting your results.

15 THERMODYNAMICS

Figure 15.1 A steam engine uses heat transfer to do work. Tourists regularly ride this narrow-gauge steam engine train near the San Juan Skyway in Durango, Colorado, part of the National Scenic Byways Program. (credit: Dennis Adams)

Learning Objectives

Introduction to Thermodynamics

Heat transfer is energy in transit, and it can be used to do work. It can also be converted to any other form of energy. A car engine, for example, burns fuel for heat transfer into a gas. Work is done by the gas as it exerts a force through a distance, converting its energy into a variety of other forms—into the car's kinetic or gravitational potential energy; into electrical energy to run the spark plugs, radio, and lights; and back into stored energy in the car's battery. But most of the heat transfer produced from burning fuel in the engine does not do work on the gas. Rather, the energy is released into the environment, implying that the engine is quite inefficient.

It is often said that modern gasoline engines cannot be made to be significantly more efficient. We hear the same about heat transfer to electrical energy in large power stations, whether they are coal, oil, natural gas, or nuclear powered. Why is that the case? Is the inefficiency caused by design problems that could be solved with better engineering and superior materials? Is it part of some money-making conspiracy by those who sell energy? Actually, the truth is more interesting, and reveals much about the nature of heat transfer.

Basic physical laws govern how heat transfer for doing work takes place and place insurmountable limits onto its efficiency. This chapter will explore these laws as well as many applications and concepts associated with them. These topics are part of *thermodynamics*—the study of heat transfer and its relationship to doing work.

15.1 The First Law of Thermodynamics

Figure 15.2 This boiling tea kettle represents energy in motion. The water in the kettle is turning to water vapor because heat is being transferred from the stove to the kettle. As the entire system gets hotter, work is done—from the evaporation of the water to the whistling of the kettle. (credit: Gina Hamilton)

If we are interested in how heat transfer is converted into doing work, then the conservation of energy principle is important. The first law of thermodynamics applies the conservation of energy principle to systems where heat transfer and doing work are the methods of transferring energy into and out of the system. The **first law of thermodynamics** states that the change in internal energy of a system equals the net heat transfer *into* the system minus the net work done *by* the system. In equation form, the first law of thermodynamics is

$$\Delta U = Q - W. \tag{15.1}$$

Here ΔU is the *change in internal energy* U of the system. Q is the *net heat transferred into the system*—that is, Q is the sum of all heat transfer into and out of the system. W is the *net work done by the system*—that is, W is the sum of all work done on or by the system. We use the following sign conventions: if Q is positive, then there is a net heat transfer into the system; if W is positive, then there is net work done by the system. So positive Q adds energy to the system and positive W takes energy from the system. Thus $\Delta U = Q - W$. Note also that if more heat transfer into the system occurs than work done, the difference is stored as internal energy. Heat engines are a good example of this—heat transfer into them takes place so that they can do work. (See **Figure 15.3**.) We will now examine Q , W , and ΔU further.

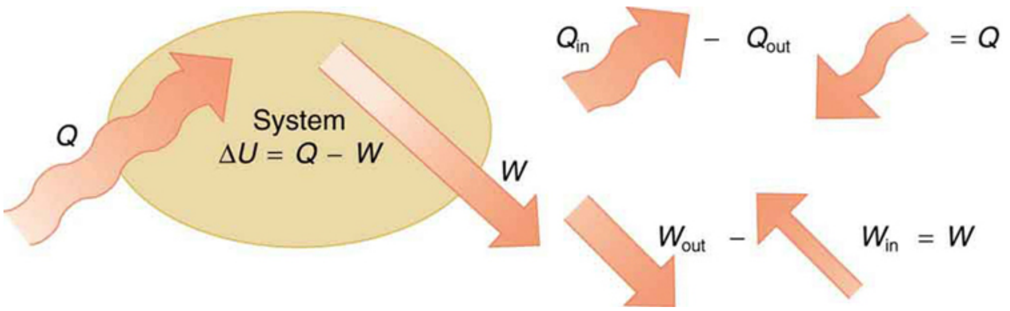


Figure 15.3 The first law of thermodynamics is the conservation-of-energy principle stated for a system where heat and work are the methods of transferring energy for a system in thermal equilibrium. Q represents the net heat transfer—it is the sum of all heat transfers into and out of the system. Q is positive for net heat transfer *into* the system. W is the total work done on and by the system. W is positive when more work is done *by* the system than on it. The change in the internal energy of the system, ΔU , is related to heat and work by the first law of thermodynamics, $\Delta U = Q - W$.

Making Connections: Law of Thermodynamics and Law of Conservation of Energy

The first law of thermodynamics is actually the law of conservation of energy stated in a form most useful in thermodynamics. The first law gives the relationship between heat transfer, work done, and the change in internal energy of a system.

Heat *Q* and Work *W*

Heat transfer (Q) and doing work (W) are the two everyday means of bringing energy into or taking energy out of a system. The processes are quite different. Heat transfer, a less organized process, is driven by temperature differences. Work, a quite organized process, involves a macroscopic force exerted through a distance. Nevertheless, heat and work can produce identical results.For example, both can cause a temperature increase. Heat transfer into a system, such as when the Sun warms the air in a bicycle tire, can increase its temperature, and so can work done on the system, as when the bicyclist pumps air into the tire. Once the temperature increase has occurred, it is impossible to tell whether it was caused by heat transfer or by doing work. This uncertainty is an important point. Heat transfer and work are both energy in transit—neither is stored as such in a system. However, both can change the internal energy U of a system. Internal energy is a form of energy completely different from either heat or work.

Internal Energy *U*

We can think about the internal energy of a system in two different but consistent ways. The first is the atomic and molecular view, which examines the system on the atomic and molecular scale. The **internal energy** U of a system is the sum of the kinetic and potential energies of its atoms and molecules. Recall that kinetic plus potential energy is called mechanical energy. Thus internal energy is the sum of atomic and molecular mechanical energy. Because it is impossible to keep track of all individual atoms and molecules, we must deal with averages and distributions. A second way to view the internal energy of a system is in terms of its macroscopic characteristics, which are very similar to atomic and molecular average values.

Macroscopically, we define the change in internal energy ΔU to be that given by the first law of thermodynamics:

$$\Delta U = Q - W. \tag{15.2}$$

Many detailed experiments have verified that $\Delta U = Q - W$, where ΔU is the change in total kinetic and potential energy of all atoms and molecules in a system. It has also been determined experimentally that the internal energy U of a system depends only on the state of the system and *not how it reached that state.* More specifically, U is found to be a function of a few macroscopic quantities (pressure, volume, and temperature, for example), independent of past history such as whether there has been heat transfer or work done. This independence means that if we know the state of a system, we can calculate changes in its internal energy U from a few macroscopic variables.

Making Connections: Macroscopic and Microscopic

In thermodynamics, we often use the macroscopic picture when making calculations of how a system behaves, while the atomic and molecular picture gives underlying explanations in terms of averages and distributions. We shall see this again in later sections of this chapter. For example, in the topic of entropy, calculations will be made using the atomic and molecular view.

To get a better idea of how to think about the internal energy of a system, let us examine a system going from State 1 to State 2. The system has internal energy U_1 in State 1, and it has internal energy U_2 in State 2, no matter how it got to either state. So the change in internal energy $\Delta U = U_2 - U_1$ is independent of what caused the change. In other words, ΔU *is independent of path*. By path, we mean the method of getting from the starting point to the ending point. Why is this independence important? Note that $\Delta U = Q - W$. Both Q and W *depend on path*, but ΔU does not. This path independence means that internal energy U is easier to consider than either heat transfer or work done.

Example 15.1 Calculating Change in Internal Energy: The Same Change in U is Produced by Two Different Processes

(a) Suppose there is heat transfer of 40.00 J to a system, while the system does 10.00 J of work. Later, there is heat transfer of 25.00 J out of the system while 4.00 J of work is done on the system. What is the net change in internal energy of the system?

(b) What is the change in internal energy of a system when a total of 150.00 J of heat transfer occurs out of (from) the system and 159.00 J of work is done on the system? (See **Figure 15.4**).

Strategy

In part (a), we must first find the net heat transfer and net work done from the given information. Then the first law of thermodynamics $(\Delta U = Q - W)$ can be used to find the change in internal energy. In part (b), the net heat transfer and work done are given, so the equation can be used directly.

Solution for (a)

The net heat transfer is the heat transfer into the system minus the heat transfer out of the system, or

$$Q = 40.00 \text{ J} - 25.00 \text{ J} = 15.00 \text{ J}. \tag{15.3}$$

Similarly, the total work is the work done by the system minus the work done on the system, or

$$W = 10.00 \text{ J} - 4.00 \text{ J} = 6.00 \text{ J}. \tag{15.4}$$

Thus the change in internal energy is given by the first law of thermodynamics:

$$\Delta U = Q - W = 15.00 \text{ J} - 6.00 \text{ J} = 9.00 \text{ J}. \tag{15.5}$$

We can also find the change in internal energy for each of the two steps. First, consider 40.00 J of heat transfer in and 10.00 J of work out, or

$$\Delta U_1 = Q_1 - W_1 = 40.00 \text{ J} - 10.00 \text{ J} = 30.00 \text{ J}. \tag{15.6}$$

Now consider 25.00 J of heat transfer out and 4.00 J of work in, or

$$\Delta U_2 = Q_2 - W_2 = -25.00 \text{ J} - (-4.00 \text{ J}) = -21.00 \text{ J}. \tag{15.7}$$

The total change is the sum of these two steps, or

$$\Delta U = \Delta U_1 + \Delta U_2 = 30.00 \text{ J} + (-21.00 \text{ J}) = 9.00 \text{ J}. \tag{15.8}$$

Discussion on (a)

No matter whether you look at the overall process or break it into steps, the change in internal energy is the same.

Solution for (b)

Here the net heat transfer and total work are given directly to be $Q = -150.00\text{ J}$ and $W = -159.00\text{ J}$, so that

$$\Delta U = Q - W = -150.00\text{ J} - (-159.00\text{ J}) = 9.00\text{ J}. \tag{15.9}$$

Discussion on (b)

A very different process in part (b) produces the same 9.00-J change in internal energy as in part (a). Note that the change in the system in both parts is related to ΔU and not to the individual Q s or W s involved. The system ends up in the *same* state in both (a) and (b). Parts (a) and (b) present two different paths for the system to follow between the same starting and ending points, and the change in internal energy for each is the same—it is independent of path.

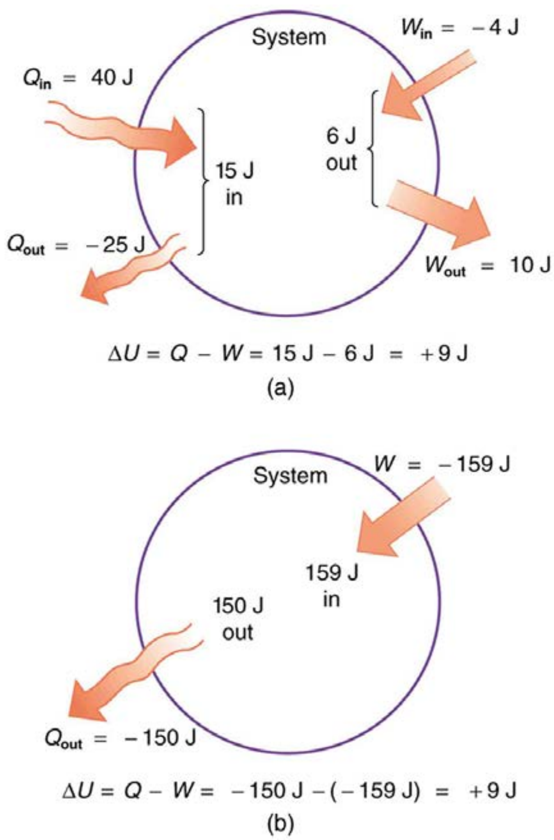


Figure 15.4 Two different processes produce the same change in a system. (a) A total of 15.00 J of heat transfer occurs into the system, while work takes out a total of 6.00 J. The change in internal energy is $\Delta U = Q - W = 9.00\text{ J}$. (b) Heat transfer removes 150.00 J from the system while work puts 159.00 J into it, producing an increase of 9.00 J in internal energy. If the system starts out in the same state in (a) and (b), it will end up in the same final state in either case—its final state is related to internal energy, not how that energy was acquired.

Human Metabolism and the First Law of Thermodynamics

Human metabolism is the conversion of food into heat transfer, work, and stored fat. Metabolism is an interesting example of the first law of thermodynamics in action. We now take another look at these topics via the first law of thermodynamics. Considering the body as the system of interest, we can use the first law to examine heat transfer, doing work, and internal energy in activities ranging from sleep to heavy exercise. What are some of the major characteristics of heat transfer, doing work, and energy in the body? For one, body temperature is normally kept constant by heat transfer to the surroundings. This means Q is negative. Another fact is that the body usually does work on the outside world. This means W is positive. In such situations, then, the body loses internal energy, since $\Delta U = Q - W$ is negative.

Now consider the effects of eating. Eating increases the internal energy of the body by adding chemical potential energy (this is an unromantic view of a good steak). The body *metabolizes* all the food we consume. Basically, metabolism is an oxidation process in which the chemical potential energy of food is released. This implies that food input is in the form of work. Food energy is reported in a special unit, known as the Calorie. This energy is measured by burning food in a calorimeter, which is how the units are determined.

In chemistry and biochemistry, one calorie (spelled with a *lowercase* c) is defined as the energy (or heat transfer) required to raise the temperature of one gram of pure water by one degree Celsius. Nutritionists and weight-watchers tend to use the *dietary* calorie, which is frequently called a Calorie (spelled with a *capital* C). One food Calorie is the energy needed to raise the temperature of one *kilogram* of water by one degree Celsius. This means that one dietary Calorie is equal to one kilocalorie for the chemist, and one must be careful to avoid confusion between the two.

Again, consider the internal energy the body has lost. There are three places this internal energy can go—to heat transfer, to doing work, and to stored fat (a tiny fraction also goes to cell repair and growth). Heat transfer and doing work take internal energy out of the body, and food puts it back. If you eat just the right amount of food, then your average internal energy remains constant. Whatever you lose to heat transfer and doing work is replaced by food, so that, in the long run, $\Delta U = 0$. If you overeat repeatedly, then ΔU is always positive, and your body stores this extra internal energy as fat. The reverse is true if you eat too little. If ΔU is negative for a few days, then the body metabolizes its own fat to maintain body temperature and do work that takes energy from the body. This process is how dieting produces weight loss.

Life is not always this simple, as any dieter knows. The body stores fat or metabolizes it only if energy intake changes for a period of several days. Once you have been on a major diet, the next one is less successful because your body alters the way it responds to low energy intake. Your basal metabolic rate (BMR) is the rate at which food is converted into heat transfer and work done while the body is at complete rest. The body adjusts its basal metabolic rate to partially compensate for over-eating or under-eating. The body will decrease the metabolic rate rather than eliminate its own fat to replace lost food intake. You will chill more easily and feel less energetic as a result of the lower metabolic rate, and you will not lose weight as fast as before. Exercise helps to lose weight, because it produces both heat transfer from your body and work, and raises your metabolic rate even when you are at rest. Weight loss is also aided by the quite low efficiency of the body in converting internal energy to work, so that the loss of internal energy resulting from doing work is much greater than the work done.It should be noted, however, that living systems are not in thermalequilibrium.

The body provides us with an excellent indication that many thermodynamic processes are *irreversible*. An irreversible process can go in one direction but not the reverse, under a given set of conditions. For example, although body fat can be converted to do work and produce heat transfer, work done on the body and heat transfer into it cannot be converted to body fat. Otherwise, we could skip lunch by sunning ourselves or by walking down stairs. Another example of an irreversible thermodynamic process is photosynthesis. This process is the intake of one form of energy—light—by plants and its conversion to chemical potential energy. Both applications of the first law of thermodynamics are illustrated in **Figure 15.5**. One great advantage of conservation laws such as the first law of thermodynamics is that they accurately describe the beginning and ending points of complex processes, such as metabolism and photosynthesis, without regard to the complications in between. **Table 15.1** presents a summary of terms relevant to the first law of thermodynamics.

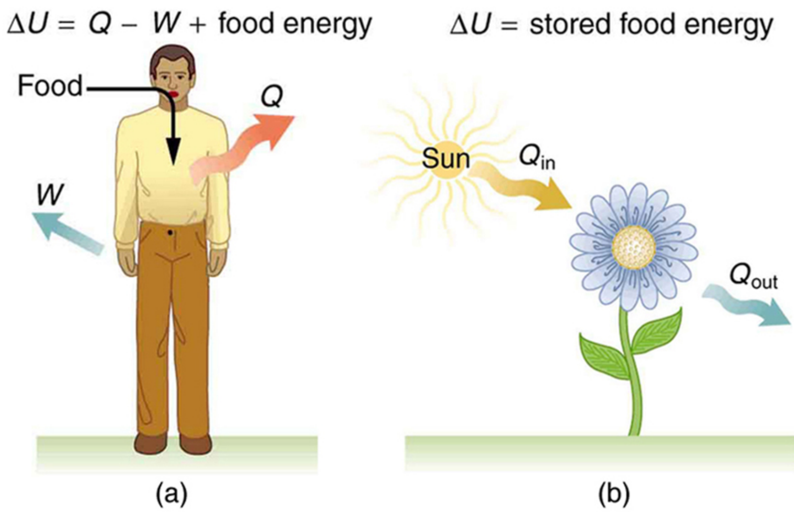


Figure 15.5 (a) The first law of thermodynamics applied to metabolism. Heat transferred out of the body (Q) and work done by the body (W) remove internal energy, while food intake replaces it. (Food intake may be considered as work done on the body.) (b) Plants convert part of the radiant heat transfer in sunlight to stored chemical energy, a process called photosynthesis.

Table 15.1 Summary of Terms for the First Law of Thermodynamics, *ΔU=Q−W*

Term	Definition
U	Internal energy—the sum of the kinetic and potential energies of a system's atoms and molecules. Can be divided into many subcategories, such as thermal and chemical energy. Depends only on the state of a system (such as its P , V , and T), not on how the energy entered the system. Change in internal energy is path independent.
Q	Heat—energy transferred because of a temperature difference. Characterized by random molecular motion. Highly dependent on path. Q entering a system is positive.
W	Work—energy transferred by a force moving through a distance. An organized, orderly process. Path dependent. W done by a system (either against an external force or to increase the volume of the system) is positive.

15.2 The First Law of Thermodynamics and Some Simple Processes

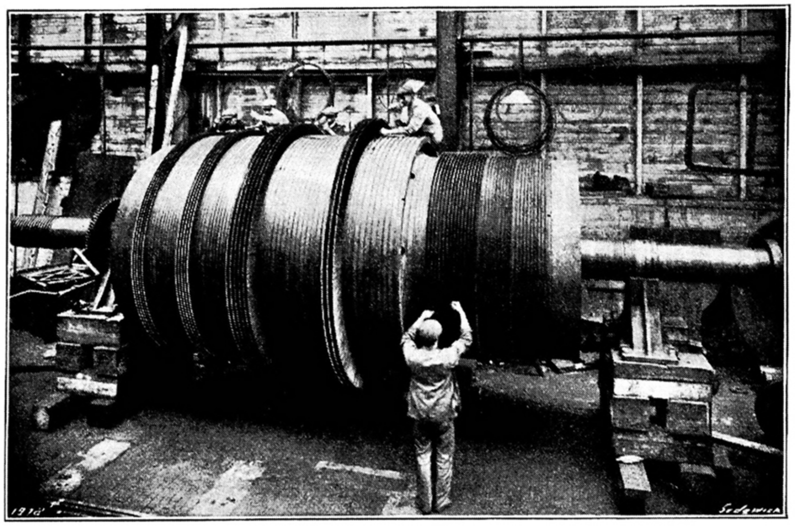


Figure 15.6 Beginning with the Industrial Revolution, humans have harnessed power through the use of the first law of thermodynamics, before we even understood it completely. This photo, of a steam engine at the Turbinia Works, dates from 1911, a mere 61 years after the first explicit statement of the first law of thermodynamics by Rudolph Clausius. (credit: public domain; author unknown)

One of the most important things we can do with heat transfer is to use it to do work for us. Such a device is called a **heat engine**. Car engines and steam turbines that generate electricity are examples of heat engines. **Figure 15.7** shows schematically how the first law of thermodynamics applies to the typical heat engine.

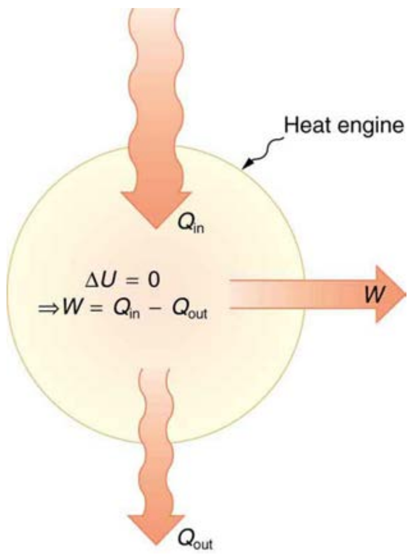


Figure 15.7 Schematic representation of a heat engine, governed, of course, by the first law of thermodynamics. It is impossible to devise a system where $Q_{\text{out}} = 0$, that is, in which no heat transfer occurs to the environment.

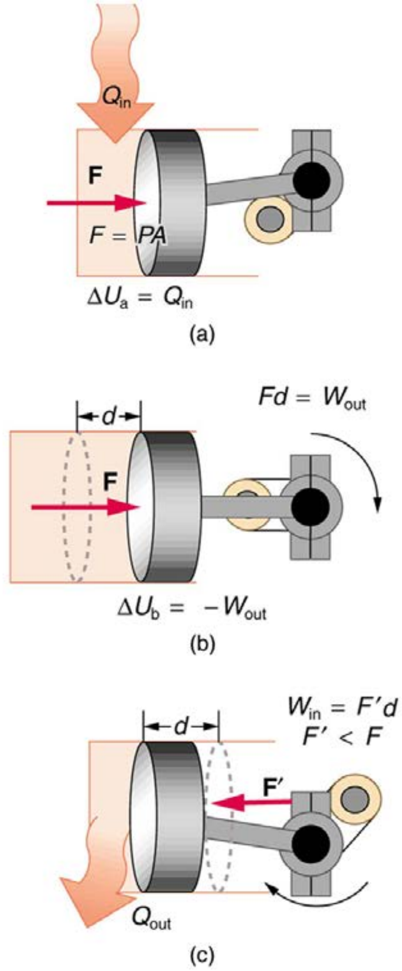


Figure 15.8 (a) Heat transfer to the gas in a cylinder increases the internal energy of the gas, creating higher pressure and temperature. (b) The force exerted on the movable cylinder does work as the gas expands. Gas pressure and temperature decrease when it expands, indicating that the gas's internal energy has been decreased by doing work. (c) Heat transfer to the environment further reduces pressure in the gas so that the piston can be more easily returned to its starting position.

The illustrations above show one of the ways in which heat transfer does work. Fuel combustion produces heat transfer to a gas in a cylinder, increasing the pressure of the gas and thereby the force it exerts on a movable piston. The gas does work on the outside world, as this force moves the piston through some distance. Heat transfer to the gas cylinder results in work being done. To repeat this process, the piston needs to be returned to its starting point. Heat transfer now occurs from the gas to the surroundings so that its pressure decreases, and a force is exerted by the surroundings to push the piston back through some distance. Variations of this process are employed daily in hundreds of millions of heat engines. We will examine heat engines in detail in the next section. In this section, we consider some of the simpler underlying processes on which heat engines are based.

PV Diagrams and their Relationship to Work Done on or by a Gas

A process by which a gas does work on a piston at constant pressure is called an **isobaric process**. Since the pressure is constant, the force exerted is constant and the work done is given as

$$P\Delta V. \tag{15.10}$$

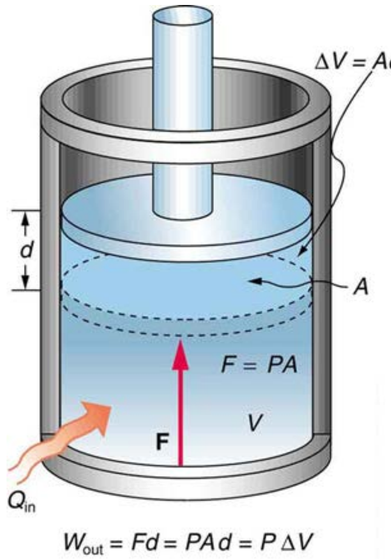


Figure 15.9 An isobaric expansion of a gas requires heat transfer to keep the pressure constant. Since pressure is constant, the work done is $P\Delta V$.

$$W = Fd \tag{15.11}$$

See the symbols as shown in **Figure 15.9**. Now $F = PA$, and so

$$W = PAd. \tag{15.12}$$

Because the volume of a cylinder is its cross-sectional area A times its length d , we see that $Ad = \Delta V$, the change in volume; thus,

$$W = P\Delta V \text{ (isobaric process).} \tag{15.13}$$

Note that if ΔV is positive, then W is positive, meaning that work is done *by* the gas on the outside world.

(Note that the pressure involved in this work that we've called P is the pressure of the gas *inside* the tank. If we call the pressure outside the tank P_{ext} , an expanding gas would be working *against* the external pressure; the work done would therefore be $W = -P_{\text{ext}}\Delta V$ (isobaric process). Many texts use this definition of work, and not the definition based on internal pressure, as the basis of the First Law of Thermodynamics. This definition reverses the sign conventions for work, and results in a statement of the first law that becomes $\Delta U = Q + W$.)

It is not surprising that $W = P\Delta V$, since we have already noted in our treatment of fluids that pressure is a type of potential energy per unit volume and that pressure in fact has units of energy divided by volume. We also noted in our discussion of the ideal gas law that PV has units of energy. In this case, some of the energy associated with pressure becomes work.

Figure 15.10 shows a graph of pressure versus volume (that is, a PV diagram for an isobaric process. You can see in the figure that the work done is the area under the graph. This property of PV diagrams is very useful and broadly applicable: *the work done on or by a system in going from one state to another equals the area under the curve on a* PV *diagram*.

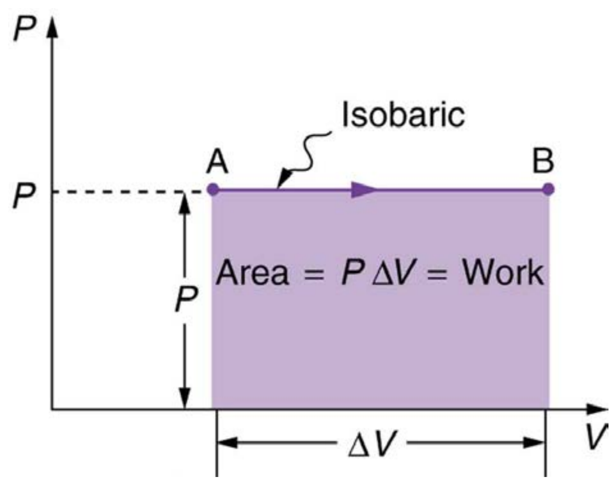


Figure 15.10 A graph of pressure versus volume for a constant-pressure, or isobaric, process, such as the one shown in **Figure 15.9**. The area under the curve equals the work done by the gas, since $W = P\Delta V$.

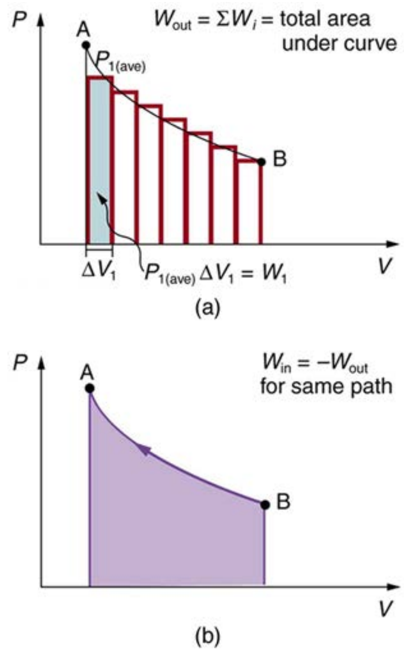


Figure 15.11 (a) A PV diagram in which pressure varies as well as volume. The work done for each interval is its average pressure times the change in volume, or the area under the curve over that interval. Thus the total area under the curve equals the total work done. (b) Work must be done on the system to follow the reverse path. This is interpreted as a negative area under the curve.

We can see where this leads by considering **Figure 15.11**(a), which shows a more general process in which both pressure and volume change. The area under the curve is closely approximated by dividing it into strips, each having an average constant pressure $P_{i(\text{ave})}$. The work done is $W_i = P_{i(\text{ave})}\Delta V_i$ for each strip, and the total work done is the sum of the W_i. Thus the total work done is the total area under the curve. If the path is reversed, as in **Figure 15.11**(b), then work is done on the system. The area under the curve in that case is negative, because ΔV is negative.

PV diagrams clearly illustrate that *the work done depends on the path taken and not just the endpoints*. This path dependence is seen in **Figure 15.12**(a), where more work is done in going from A to C by the path via point B than by the path via point D. The vertical paths, where volume is constant, are called **isochoric** processes. Since volume is constant, $\Delta V = 0$, and no work is done in an isochoric process. Now, if the system follows the cyclical path ABCDA, as in **Figure 15.12**(b), then the total work done is the area inside the loop. The negative area below path CD subtracts, leaving only the area inside the rectangle. In fact, the work done in any cyclical process (one that returns to its starting point) is the area inside the loop it forms on a PV diagram, as **Figure 15.12**(c) illustrates for a general cyclical process. Note that the loop must be traversed in the clockwise direction for work to be positive—that is, for there to be a net work output.

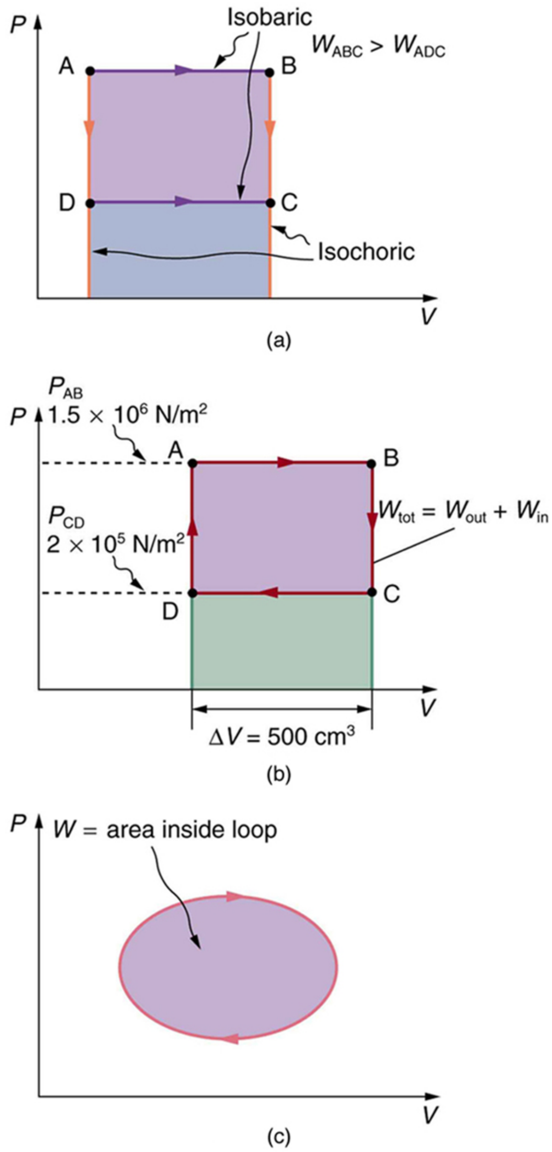


Figure 15.12 (a) The work done in going from A to C depends on path. The work is greater for the path ABC than for the path ADC, because the former is at higher pressure. In both cases, the work done is the area under the path. This area is greater for path ABC. (b) The total work done in the cyclical process ABCDA is the area inside the loop, since the negative area below CD subtracts out, leaving just the area inside the rectangle. (The values given for the pressures and the change in volume are intended for use in the example below.) (c) The area inside any closed loop is the work done in the cyclical process. If the loop is traversed in a clockwise direction, W is positive—it is work done on the outside environment. If the loop is traveled in a counter-clockwise direction, W is negative—it is work that is done to the system.

Example 15.2 Total Work Done in a Cyclical Process Equals the Area Inside the Closed Loop on a *PV* Diagram

Calculate the total work done in the cyclical process ABCDA shown in Figure 15.12(b) by the following two methods to verify that work equals the area inside the closed loop on the PV diagram. (Take the data in the figure to be precise to three significant figures.) (a) Calculate the work done along each segment of the path and add these values to get the total work. (b) Calculate the area inside the rectangle ABCDA.

Strategy

To find the work along any path on a PV diagram, you use the fact that work is pressure times change in volume, or $W = P\Delta V$. So in part (a), this value is calculated for each leg of the path around the closed loop.

Solution for (a)

The work along path AB is

$$\begin{aligned} W_{\text{AB}} &= P_{\text{AB}}\Delta V_{\text{AB}} \\ &= (1.50\times10^6\ \text{N/m}^2)(5.00\times10^{-4}\ \text{m}^3) = 750\ \text{J}. \end{aligned} \tag{15.14}$$

Since the path BC is isochoric, $\Delta V_{\text{BC}} = 0$, and so $W_{\text{BC}} = 0$. The work along path CD is negative, since ΔV_{CD} is negative (the volume decreases). The work is

$$\begin{aligned} W_{CD} &= P_{CD}\Delta V_{CD} \\ &= (2.00\times10^5\ \text{N/m}^2)(-5.00\times10^{-4}\ \text{m}^3) = -100\ \text{J}. \end{aligned} \quad (15.15)$$

Again, since the path DA is isochoric, $\Delta V_{DA} = 0$, and so $W_{DA} = 0$. Now the total work is

$$\begin{aligned} W &= W_{AB} + W_{BC} + W_{CD} + W_{DA} \\ &= 750\ \text{J} + 0 + (-100\text{J}) + 0 = 650\ \text{J}. \end{aligned} \quad (15.16)$$

Solution for (b)

The area inside the rectangle is its height times its width, or

$$\begin{aligned} \text{area} &= (P_{AB} - P_{CD})\Delta V \\ &= \left[(1.50\times10^6\ \text{N/m}^2) - (2.00\times10^5\ \text{N/m}^2)\right](5.00\times10^{-4}\ \text{m}^3) \\ &= 650\ \text{J}. \end{aligned} \quad (15.17)$$

Thus,

$$\text{area} = 650\ \text{J} = W. \quad (15.18)$$

Discussion

The result, as anticipated, is that the area inside the closed loop equals the work done. The area is often easier to calculate than is the work done along each path. It is also convenient to visualize the area inside different curves on PV diagrams in order to see which processes might produce the most work. Recall that work can be done to the system, or by the system, depending on the sign of W. A positive W is work that is done by the system on the outside environment; a negative W represents work done by the environment on the system.

Figure 15.13(a) shows two other important processes on a PV diagram. For comparison, both are shown starting from the same point A. The upper curve ending at point B is an **isothermal** process—that is, one in which temperature is kept constant. If the gas behaves like an ideal gas, as is often the case, and if no phase change occurs, then $PV = nRT$. Since T is constant, PV is a constant for an isothermal process. We ordinarily expect the temperature of a gas to decrease as it expands, and so we correctly suspect that heat transfer must occur from the surroundings to the gas to keep the temperature constant during an isothermal expansion. To show this more rigorously for the special case of a monatomic ideal gas, we note that the average kinetic energy of an atom in such a gas is given by

$$\frac{1}{2}m\bar{v}^2 = \frac{3}{2}kT. \quad (15.19)$$

The kinetic energy of the atoms in a monatomic ideal gas is its only form of internal energy, and so its total internal energy U is

$$U = N\frac{1}{2}m\bar{v}^2 = \frac{3}{2}NkT,\ \text{(monatomic ideal gas)}, \quad (15.20)$$

where N is the number of atoms in the gas. This relationship means that the internal energy of an ideal monatomic gas is constant during an isothermal process—that is, $\Delta U = 0$. If the internal energy does not change, then the net heat transfer into the gas must equal the net work done by the gas. That is, because $\Delta U = Q - W = 0$ here, $Q = W$. We must have just enough heat transfer to replace the work done. An isothermal process is inherently slow, because heat transfer occurs continuously to keep the gas temperature constant at all times and must be allowed to spread through the gas so that there are no hot or cold regions.

Also shown in Figure 15.13(a) is a curve AC for an **adiabatic** process, defined to be one in which there is no heat transfer—that is, $Q = 0$. Processes that are nearly adiabatic can be achieved either by using very effective insulation or by performing the process so fast that there is little time for heat transfer. Temperature must decrease during an adiabatic process, since work is done at the expense of internal energy:

$$U = \frac{3}{2}NkT. \quad (15.21)$$

(You might have noted that a gas released into atmospheric pressure from a pressurized cylinder is substantially colder than the gas in the cylinder.) In fact, because $Q = 0, \Delta U = -W$ for an adiabatic process. Lower temperature results in lower pressure along the way, so that curve AC is lower than curve AB, and less work is done. If the path ABCA could be followed by cooling the gas from B to C at constant volume (isochorically), Figure 15.13(b), there would be a net work output.

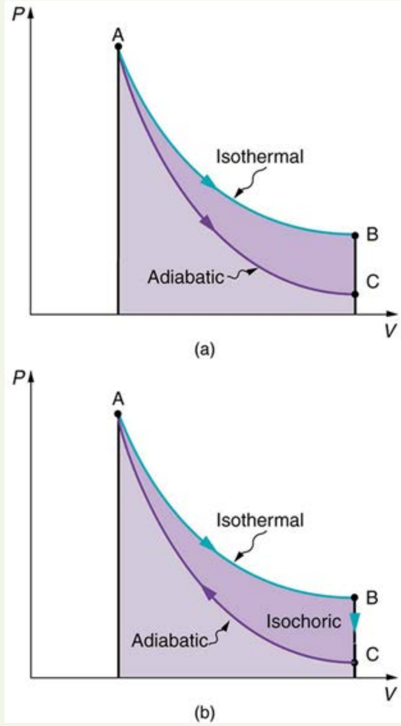


Figure 15.13 (a) The upper curve is an isothermal process ($\Delta T = 0$), whereas the lower curve is an adiabatic process ($Q = 0$). Both start from the same point A, but the isothermal process does more work than the adiabatic because heat transfer into the gas takes place to keep its temperature constant. This keeps the pressure higher all along the isothermal path than along the adiabatic path, producing more work. The adiabatic path thus ends up with a lower pressure and temperature at point C, even though the final volume is the same as for the isothermal process. (b) The cycle ABCA produces a net work output.

Reversible Processes

Both isothermal and adiabatic processes such as shown in **Figure 15.13** are reversible in principle. A **reversible process** is one in which both the system and its environment can return to exactly the states they were in by following the reverse path. The reverse isothermal and adiabatic paths are BA and CA, respectively. Real macroscopic processes are never exactly reversible. In the previous examples, our system is a gas (like that in **Figure 15.9**), and its environment is the piston, cylinder, and the rest of the universe. If there are any energy-dissipating mechanisms, such as friction or turbulence, then heat transfer to the environment occurs for either direction of the piston. So, for example, if the path BA is followed and there is friction, then the gas will be returned to its original state but the environment will not—it will have been heated in both directions. Reversibility requires the direction of heat transfer to reverse for the reverse path. Since dissipative mechanisms cannot be completely eliminated, real processes cannot be reversible.

There must be reasons that real macroscopic processes cannot be reversible. We can imagine them going in reverse. For example, heat transfer occurs spontaneously from hot to cold and never spontaneously the reverse. Yet it would not violate the first law of thermodynamics for this to happen. In fact, all spontaneous processes, such as bubbles bursting, never go in reverse. There is a second thermodynamic law that forbids them from going in reverse. When we study this law, we will learn something about nature and also find that such a law limits the efficiency of heat engines. We will find that heat engines with the greatest possible theoretical efficiency would have to use reversible processes, and even they cannot convert all heat transfer into doing work. **Table 15.2** summarizes the simpler thermodynamic processes and their definitions.

Table 15.2 Summary of Simple Thermodynamic Processes

Isobaric	Constant pressure $W = P\Delta V$
Isochoric	Constant volume $W = 0$
Isothermal	Constant temperature $Q = W$
Adiabatic	No heat transfer $Q = 0$

PhET Explorations: States of Matter

Watch different types of molecules form a solid, liquid, or gas. Add or remove heat and watch the phase change. Change the temperature or volume of a container and see a pressure-temperature diagram respond in real time. Relate the interaction potential to the forces between molecules.

PhET Interactive Simulation

Figure 15.14 States of Matter (http://cnx.org/content/m42233/1.5/states-of-matter_en.jar)

15.3 Introduction to the Second Law of Thermodynamics: Heat Engines and Their Efficiency

Figure 15.15 These ice floes melt during the Arctic summer. Some of them refreeze in the winter, but the second law of thermodynamics predicts that it would be extremely unlikely for the water molecules contained in these particular floes to reform the distinctive alligator-like shape they formed when the picture was taken in the summer of 2009. (credit: Patrick Kelley, U.S. Coast Guard, U.S. Geological Survey)

The second law of thermodynamics deals with the direction taken by spontaneous processes. Many processes occur spontaneously in one direction only—that is, they are irreversible, under a given set of conditions. Although irreversibility is seen in day-to-day life—a broken glass does not resume its original state, for instance—complete irreversibility is a statistical statement that cannot be seen during the lifetime of the universe. More precisely, an **irreversible process** is one that depends on path. If the process can go in only one direction, then the reverse path differs fundamentally and the process cannot be reversible. For example, as noted in the previous section, heat involves the transfer of energy from higher to lower temperature. A cold object in contact with a hot one never gets colder, transferring heat to the hot object and making it hotter. Furthermore, mechanical energy, such as kinetic energy, can be completely converted to thermal energy by friction, but the reverse is impossible. A hot stationary object never spontaneously cools off and starts moving. Yet another example is the expansion of a puff of gas introduced into one corner of a vacuum chamber. The gas expands to fill the chamber, but it never regroups in the corner. The random motion of the gas molecules could take them all back to the corner, but this is never observed to happen. (See **Figure 15.16**.)

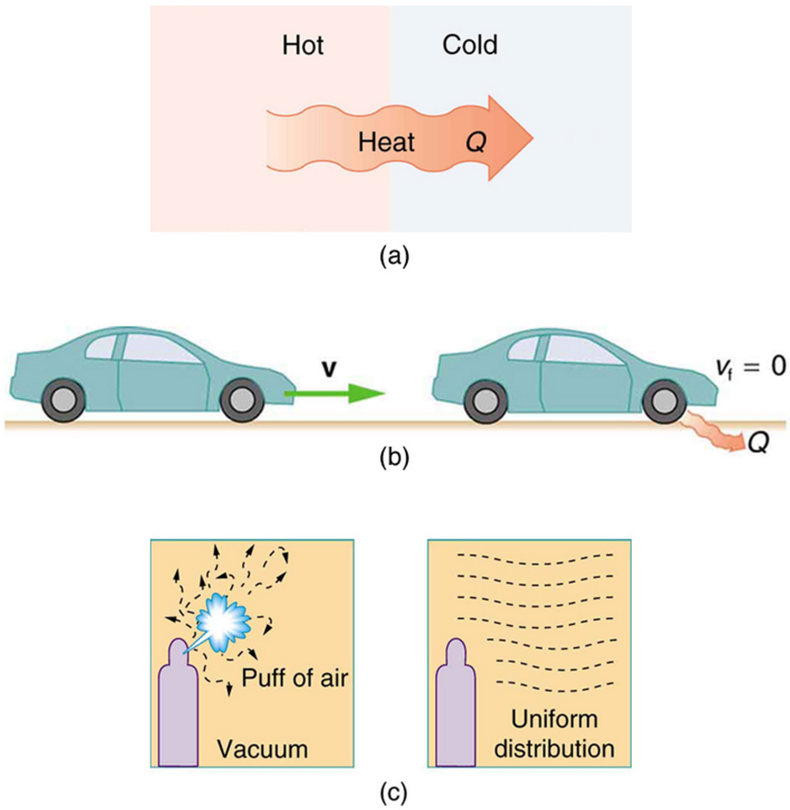


Figure 15.16 Examples of one-way processes in nature. (a) Heat transfer occurs spontaneously from hot to cold and not from cold to hot. (b) The brakes of this car convert its kinetic energy to heat transfer to the environment. The reverse process is impossible. (c) The burst of gas let into this vacuum chamber quickly expands to uniformly fill every part of the chamber. The random motions of the gas molecules will never return them to the corner.

The fact that certain processes never occur suggests that there is a law forbidding them to occur. The first law of thermodynamics would allow them to occur—none of those processes violate conservation of energy. The law that forbids these processes is called the second law of thermodynamics. We shall see that the second law can be stated in many ways that may seem different, but which in fact are equivalent. Like all natural laws, the second law of thermodynamics gives insights into nature, and its several statements imply that it is broadly applicable, fundamentally affecting many apparently disparate processes.

The already familiar direction of heat transfer from hot to cold is the basis of our first version of the **second law of thermodynamics**.

The Second Law of Thermodynamics (first expression)

Heat transfer occurs spontaneously from higher- to lower-temperature bodies but never spontaneously in the reverse direction.

Another way of stating this: It is impossible for any process to have as its sole result heat transfer from a cooler to a hotter object.

Heat Engines

Now let us consider a device that uses heat transfer to do work. As noted in the previous section, such a device is called a heat engine, and one is shown schematically in Figure 15.17(b). Gasoline and diesel engines, jet engines, and steam turbines are all heat engines that do work by using part of the heat transfer from some source. Heat transfer from the hot object (or hot reservoir) is denoted as Q_h, while heat transfer into the cold object (or cold reservoir) is Q_c, and the work done by the engine is W. The temperatures of the hot and cold reservoirs are T_h and T_c, respectively.

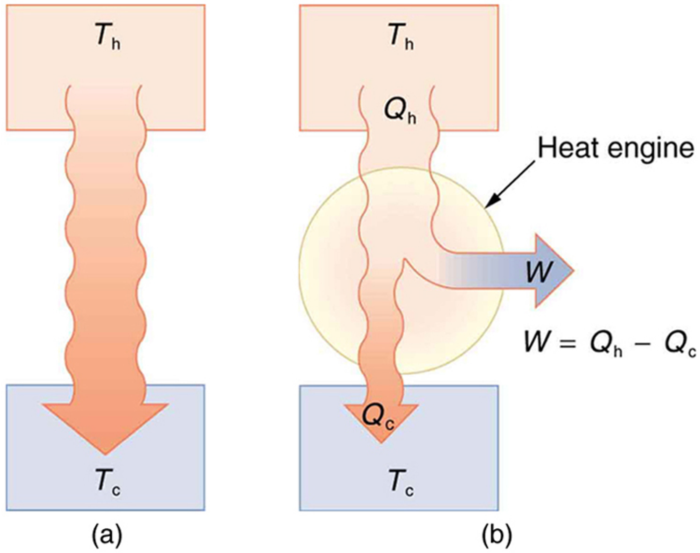


Figure 15.17 (a) Heat transfer occurs spontaneously from a hot object to a cold one, consistent with the second law of thermodynamics. (b) A heat engine, represented here by a circle, uses part of the heat transfer to do work. The hot and cold objects are called the hot and cold reservoirs. Q_h is the heat transfer out of the hot reservoir, W is the work output, and Q_c is the heat transfer into the cold reservoir.

Because the hot reservoir is heated externally, which is energy intensive, it is important that the work is done as efficiently as possible. In fact, we would like W to equal Q_h, and for there to be no heat transfer to the environment ($Q_\text{c} = 0$). Unfortunately, this is impossible. The **second law of thermodynamics** also states, with regard to using heat transfer to do work (the second expression of the second law):

The Second Law of Thermodynamics (second expression)

It is impossible in any system for heat transfer from a reservoir to completely convert to work in a cyclical process in which the system returns to its initial state.

A **cyclical process** brings a system, such as the gas in a cylinder, back to its original state at the end of every cycle. Most heat engines, such as reciprocating piston engines and rotating turbines, use cyclical processes. The second law, just stated in its second form, clearly states that such engines cannot have perfect conversion of heat transfer into work done. Before going into the underlying reasons for the limits on converting heat transfer into work, we need to explore the relationships among W, Q_h, and Q_c, and to define the efficiency of a cyclical heat engine. As noted, a cyclical process brings the system back to its original condition at the end of every cycle. Such a system's internal energy U is the same at the beginning and end of every cycle—that is, $\Delta U = 0$. The first law of thermodynamics states that

$$\Delta U = Q - W, \tag{15.22}$$

where Q is the *net* heat transfer during the cycle ($Q = Q_\text{h} - Q_\text{c}$) and W is the net work done by the system. Since $\Delta U = 0$ for a complete cycle, we have

$$0 = Q - W, \tag{15.23}$$

so that

$$W = Q. \tag{15.24}$$

Thus the net work done by the system equals the net heat transfer into the system, or

$$W = Q_\text{h} - Q_\text{c} \text{ (cyclical process)}, \tag{15.25}$$

just as shown schematically in **Figure 15.17**(b). The problem is that in all processes, there is some heat transfer Q_c to the environment—and usually a very significant amount at that.

In the conversion of energy to work, we are always faced with the problem of getting less out than we put in. We define *conversion efficiency* Eff to be the ratio of useful work output to the energy input (or, in other words, the ratio of what we get to what we spend). In that spirit, we define the efficiency of a heat engine to be its net work output W divided by heat transfer to the engine Q_h; that is,

$$Eff = \frac{W}{Q_\text{h}}. \tag{15.26}$$

Since $W = Q_\text{h} - Q_\text{c}$ in a cyclical process, we can also express this as

$$Eff = \frac{Q_\text{h} - Q_\text{c}}{Q_\text{h}} = 1 - \frac{Q_\text{c}}{Q_\text{h}} \text{ (cyclical process)}, \tag{15.27}$$

making it clear that an efficiency of 1, or 100%, is possible only if there is no heat transfer to the environment ($Q_c = 0$). Note that all Q s are positive. The direction of heat transfer is indicated by a plus or minus sign. For example, Q_c is out of the system and so is preceded by a minus sign.

Example 15.3 Daily Work Done by a Coal-Fired Power Station, Its Efficiency and Carbon Dioxide Emissions

A coal-fired power station is a huge heat engine. It uses heat transfer from burning coal to do work to turn turbines, which are used to generate electricity. In a single day, a large coal power station has 2.50×10^{14} J of heat transfer from coal and 1.48×10^{14} J of heat transfer into the environment. (a) What is the work done by the power station? (b) What is the efficiency of the power station? (c) In the combustion process, the following chemical reaction occurs: $C + O_2 \rightarrow CO_2$. This implies that every 12 kg of coal puts 12 kg + 16 kg + 16 kg = 44 kg of carbon dioxide into the atmosphere. Assuming that 1 kg of coal can provide 2.5×10^6 J of heat transfer upon combustion, how much CO_2 is emitted per day by this power plant?

Strategy for (a)

We can use $W = Q_h - Q_c$ to find the work output W , assuming a cyclical process is used in the power station. In this process, water is boiled under pressure to form high-temperature steam, which is used to run steam turbine-generators, and then condensed back to water to start the cycle again.

Solution for (a)

Work output is given by:

$$W = Q_h - Q_c. \tag{15.28}$$

Substituting the given values:

$$\begin{aligned} W &= 2.50\times10^{14}\text{ J} - 1.48\times10^{14}\text{ J} \\ &= 1.02\times10^{14}\text{ J}. \end{aligned} \tag{15.29}$$

Strategy for (b)

The efficiency can be calculated with $Eff = \frac{W}{Q_h}$ since Q_h is given and work W was found in the first part of this example.

Solution for (b)

Efficiency is given by: $Eff = \frac{W}{Q_h}$. The work W was just found to be 1.02×10^{14} J , and Q_h is given, so the efficiency is

$$\begin{aligned} Eff &= \frac{1.02\times10^{14}\text{ J}}{2.50\times10^{14}\text{ J}} \\ &= 0.408\text{, or } 40.8\% \end{aligned} \tag{15.30}$$

Strategy for (c)

The daily consumption of coal is calculated using the information that each day there is 2.50×10^{14} J of heat transfer from coal. In the combustion process, we have $C + O_2 \rightarrow CO_2$. So every 12 kg of coal puts 12 kg + 16 kg + 16 kg = 44 kg of CO_2 into the atmosphere.

Solution for (c)

The daily coal consumption is

$$\frac{2.50\times10^{14}\text{ J}}{2.50\times10^{6}\text{ J/kg}} = 1.0\times10^{8}\text{ kg}. \tag{15.31}$$

Assuming that the coal is pure and that all the coal goes toward producing carbon dioxide, the carbon dioxide produced per day is

$$1.0\times10^{8}\text{ kg coal}\times\frac{44\text{ kg CO}_2}{12\text{ kg coal}} = 3.7\times10^{8}\text{ kg CO}_2. \tag{15.32}$$

This is 370,000 metric tons of CO_2 produced every day.

Discussion

If all the work output is converted to electricity in a period of one day, the average power output is 1180 MW (this is left to you as an end-of-chapter problem). This value is about the size of a large-scale conventional power plant. The efficiency found is acceptably close to the value of 42% given for coal power stations. It means that fully 59.2% of the energy is heat transfer to the environment, which usually results in warming lakes, rivers, or the ocean near the power station, and is implicated in a warming planet generally. While the laws of thermodynamics limit the efficiency of such plants—including plants fired by nuclear fuel, oil, and natural gas—the heat transfer to the environment could be, and sometimes is, used for heating homes or for industrial processes. The generally low cost of energy has not made it economical to make better use of the waste heat transfer from most heat engines. Coal-fired power plants produce the greatest amount of CO_2 per unit energy output (compared to natural gas or oil), making coal the least efficient fossil fuel.

With the information given in **Example 15.3**, we can find characteristics such as the efficiency of a heat engine without any knowledge of how the heat engine operates, but looking further into the mechanism of the engine will give us greater insight. **Figure 15.18** illustrates the operation of the common four-stroke gasoline engine. The four steps shown complete this heat engine's cycle, bringing the gasoline-air mixture back to its original condition.

The **Otto cycle** shown in **Figure 15.19**(a) is used in four-stroke internal combustion engines, although in fact the true Otto cycle paths do not correspond exactly to the strokes of the engine.

The adiabatic process AB corresponds to the nearly adiabatic compression stroke of the gasoline engine. In both cases, work is done on the system (the gas mixture in the cylinder), increasing its temperature and pressure. Along path BC of the Otto cycle, heat transfer Q_h into the gas occurs at constant volume, causing a further increase in pressure and temperature. This process corresponds to burning fuel in an internal combustion engine, and takes place so rapidly that the volume is nearly constant. Path CD in the Otto cycle is an adiabatic expansion that does work on the outside world, just as the power stroke of an internal combustion engine does in its nearly adiabatic expansion. The work done by the system along path CD is greater than the work done on the system along path AB, because the pressure is greater, and so there is a net work output. Along path DA in the Otto cycle, heat transfer Q_c from the gas at constant volume reduces its temperature and pressure, returning it to its original state. In an internal combustion engine, this process corresponds to the exhaust of hot gases and the intake of an air-gasoline mixture at a considerably lower temperature. In both cases, heat transfer into the environment occurs along this final path.

The net work done by a cyclical process is the area inside the closed path on a PV diagram, such as that inside path ABCDA in **Figure 15.19**. Note that in every imaginable cyclical process, it is absolutely necessary for heat transfer from the system to occur in order to get a net work output. In the Otto cycle, heat transfer occurs along path DA. If no heat transfer occurs, then the return path is the same, and the net work output is zero. The lower the temperature on the path AB, the less work has to be done to compress the gas. The area inside the closed path is then greater, and so the engine does more work and is thus more efficient. Similarly, the higher the temperature along path CD, the more work output there is. (See **Figure 15.20**.) So efficiency is related to the temperatures of the hot and cold reservoirs. In the next section, we shall see what the absolute limit to the efficiency of a heat engine is, and how it is related to temperature.

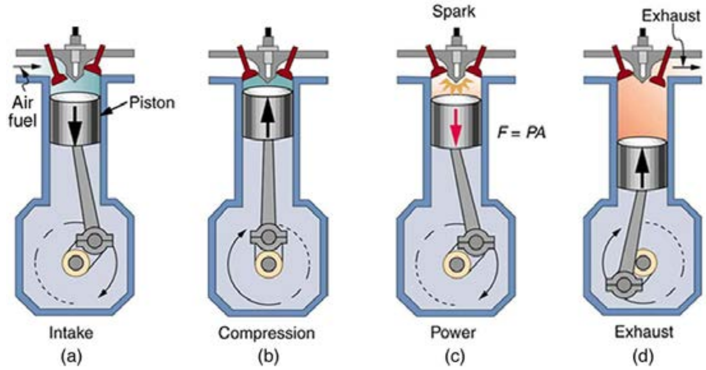


Figure 15.18 In the four-stroke internal combustion gasoline engine, heat transfer into work takes place in the cyclical process shown here. The piston is connected to a rotating crankshaft, which both takes work out of and does work on the gas in the cylinder. (a) Air is mixed with fuel during the intake stroke. (b) During the compression stroke, the air-fuel mixture is rapidly compressed in a nearly adiabatic process, as the piston rises with the valves closed. Work is done on the gas. (c) The power stroke has two distinct parts. First, the air-fuel mixture is ignited, converting chemical potential energy into thermal energy almost instantaneously, which leads to a great increase in pressure. Then the piston descends, and the gas does work by exerting a force through a distance in a nearly adiabatic process. (d) The exhaust stroke expels the hot gas to prepare the engine for another cycle, starting again with the intake stroke.

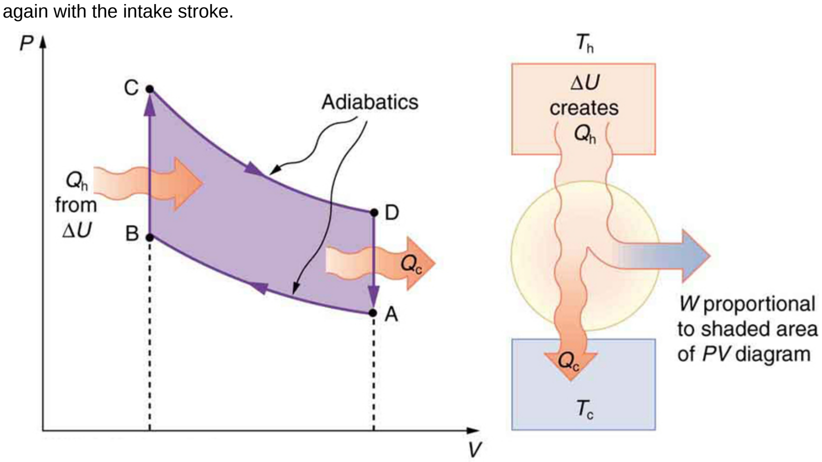


Figure 15.19 PV diagram for a simplified Otto cycle, analogous to that employed in an internal combustion engine. Point A corresponds to the start of the compression stroke of an internal combustion engine. Paths AB and CD are adiabatic and correspond to the compression and power strokes of an internal combustion engine, respectively. Paths BC and DA are isochoric and accomplish similar results to the ignition and exhaust-intake portions, respectively, of the internal combustion engine's cycle. Work is done on the gas along path AB, but more work is done by the gas along path CD, so that there is a net work output.

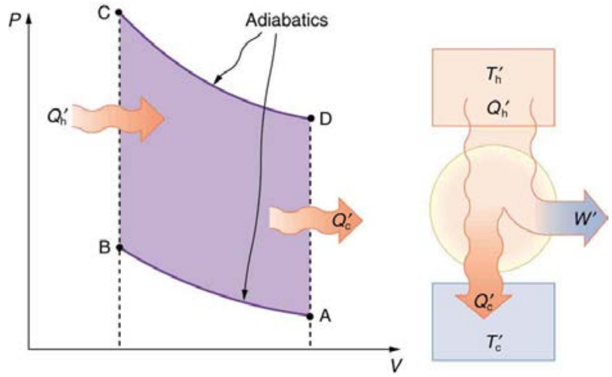


Figure 15.20 This Otto cycle produces a greater work output than the one in **Figure 15.19**, because the starting temperature of path CD is higher and the starting temperature of path AB is lower. The area inside the loop is greater, corresponding to greater net work output.

15.4 Carnot's Perfect Heat Engine: The Second Law of Thermodynamics Restated

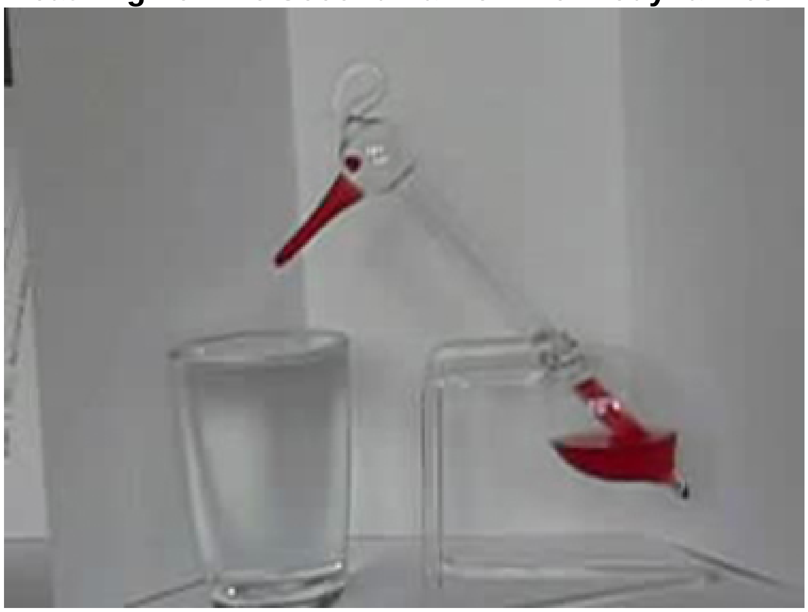


Figure 15.21 This novelty toy, known as the drinking bird, is an example of Carnot's engine. It contains methylene chloride (mixed with a dye) in the abdomen, which boils at a very low temperature—about 100°F . To operate, one gets the bird's head wet. As the water evaporates, fluid moves up into the head, causing the bird to become top-heavy and dip forward back into the water. This cools down the methylene chloride in the head, and it moves back into the abdomen, causing the bird to become bottom heavy and tip up. Except for a very small input of energy—the original head-wetting—the bird becomes a perpetual motion machine of sorts. (credit: Arabesk.nl, Wikimedia Commons)

We know from the second law of thermodynamics that a heat engine cannot be 100% efficient, since there must always be some heat transfer Q_c to the environment, which is often called waste heat. How efficient, then, can a heat engine be? This question was answered at a theoretical level in 1824 by a young French engineer, Sadi Carnot (1796–1832), in his study of the then-emerging heat engine technology crucial to the Industrial Revolution. He devised a theoretical cycle, now called the **Carnot cycle**, which is the most efficient cyclical process possible. The second law of thermodynamics can be restated in terms of the Carnot cycle, and so what Carnot actually discovered was this fundamental law. Any heat engine employing the Carnot cycle is called a **Carnot engine**.

What is crucial to the Carnot cycle—and, in fact, defines it—is that only reversible processes are used. Irreversible processes involve dissipative factors, such as friction and turbulence. This increases heat transfer Q_c to the environment and reduces the efficiency of the engine. Obviously, then, reversible processes are superior.

Carnot Engine

Stated in terms of reversible processes, the **second law of thermodynamics** has a third form:

A Carnot engine operating between two given temperatures has the greatest possible efficiency of any heat engine operating between these two temperatures. Furthermore, all engines employing only reversible processes have this same maximum efficiency when operating between the same given temperatures.

Figure 15.22 shows the PV diagram for a Carnot cycle. The cycle comprises two isothermal and two adiabatic processes. Recall that both isothermal and adiabatic processes are, in principle, reversible.

Carnot also determined the efficiency of a perfect heat engine—that is, a Carnot engine. It is always true that the efficiency of a cyclical heat engine is given by:

$$Eff = \frac{Q_h - Q_c}{Q_h} = 1 - \frac{Q_c}{Q_h}. \tag{15.33}$$

What Carnot found was that for a perfect heat engine, the ratio Q_c / Q_h equals the ratio of the absolute temperatures of the heat reservoirs. That is, $Q_c / Q_h = T_c / T_h$ for a Carnot engine, so that the maximum or **Carnot efficiency** Eff_C is given by

$$Eff_C = 1 - \frac{T_c}{T_h}, \tag{15.34}$$

where T_h and T_c are in kelvins (or any other absolute temperature scale). No real heat engine can do as well as the Carnot efficiency—an actual efficiency of about 0.7 of this maximum is usually the best that can be accomplished. But the ideal Carnot engine, like the drinking bird above, while a fascinating novelty, has zero power. This makes it unrealistic for any applications.

Carnot's interesting result implies that 100% efficiency would be possible only if $T_c = 0\text{ K}$ —that is, only if the cold reservoir were at absolute zero, a practical and theoretical impossibility. But the physical implication is this—the only way to have all heat transfer go into doing work is to remove *all* thermal energy, and this requires a cold reservoir at absolute zero.

It is also apparent that the greatest efficiencies are obtained when the ratio T_c / T_h is as small as possible. Just as discussed for the Otto cycle in the previous section, this means that efficiency is greatest for the highest possible temperature of the hot reservoir and lowest possible temperature of the cold reservoir. (This setup increases the area inside the closed loop on the PV diagram; also, it seems reasonable that the greater the temperature difference, the easier it is to divert the heat transfer to work.) The actual reservoir temperatures of a heat engine are usually related to the type of heat source and the temperature of the environment into which heat transfer occurs. Consider the following example.

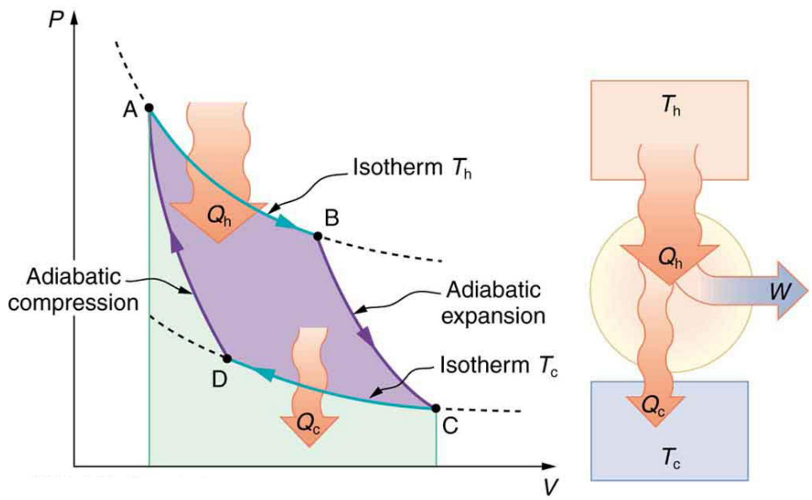


Figure 15.22 PV diagram for a Carnot cycle, employing only reversible isothermal and adiabatic processes. Heat transfer Q_h occurs into the working substance during the isothermal path AB, which takes place at constant temperature T_h . Heat transfer Q_c occurs out of the working substance during the isothermal path CD, which takes place at constant temperature T_c . The net work output W equals the area inside the path ABCDA. Also shown is a schematic of a Carnot engine operating between hot and cold reservoirs at temperatures T_h and T_c . Any heat engine using reversible processes and operating between these two temperatures will have the same maximum efficiency as the Carnot engine.

Example 15.4 Maximum Theoretical Efficiency for a Nuclear Reactor

A nuclear power reactor has pressurized water at 300°C . (Higher temperatures are theoretically possible but practically not, due to limitations with materials used in the reactor.) Heat transfer from this water is a complex process (see Figure 15.23). Steam, produced in the steam generator, is used to drive the turbine-generators. Eventually the steam is condensed to water at 27°C and then heated again to start the cycle over. Calculate the maximum theoretical efficiency for a heat engine operating between these two temperatures.

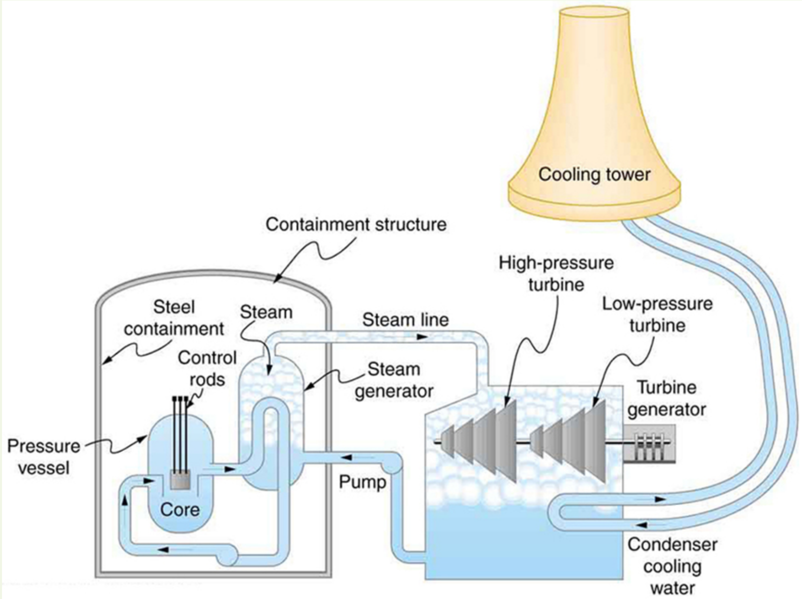


Figure 15.23 Schematic diagram of a pressurized water nuclear reactor and the steam turbines that convert work into electrical energy. Heat exchange is used to generate steam, in part to avoid contamination of the generators with radioactivity. Two turbines are used because this is less expensive than operating a single generator that produces the same amount of electrical energy. The steam is condensed to liquid before being returned to the heat exchanger, to keep exit steam pressure low and aid the flow of steam through the turbines (equivalent to using a lower-temperature cold reservoir). The considerable energy associated with condensation must be dissipated into the local environment; in this example, a cooling tower is used so there is no direct heat transfer to an aquatic environment. (Note that the water going to the cooling tower does not come into contact with the steam flowing over the turbines.)

Strategy

Since temperatures are given for the hot and cold reservoirs of this heat engine, $Eff_{\mathrm{C}} = 1 - \frac{T_{\mathrm{c}}}{T_{\mathrm{h}}}$ can be used to calculate the Carnot (maximum theoretical) efficiency. Those temperatures must first be converted to kelvins.

Solution

The hot and cold reservoir temperatures are given as $300ºC$ and $27.0ºC$, respectively. In kelvins, then, $T_{\mathrm{h}} = 573\text{ K}$ and $T_{\mathrm{c}} = 300\text{ K}$, so that the maximum efficiency is

$$Eff_{\mathrm{C}} = 1 - \frac{T_{\mathrm{c}}}{T_{\mathrm{h}}}. \tag{15.35}$$

Thus,

$$\begin{aligned} Eff_{\mathrm{C}} &= 1 - \frac{300\text{ K}}{573\text{ K}} \\ &= 0.476\text{, or }47.6\%. \end{aligned} \tag{15.36}$$

Discussion

A typical nuclear power station’s actual efficiency is about 35%, a little better than 0.7 times the maximum possible value, a tribute to superior engineering. Electrical power stations fired by coal, oil, and natural gas have greater actual efficiencies (about 42%), because their boilers can reach higher temperatures and pressures. The cold reservoir temperature in any of these power stations is limited by the local environment. **Figure 15.24** shows (a) the exterior of a nuclear power station and (b) the exterior of a coal-fired power station. Both have cooling towers into which water from the condenser enters the tower near the top and is sprayed downward, cooled by evaporation.

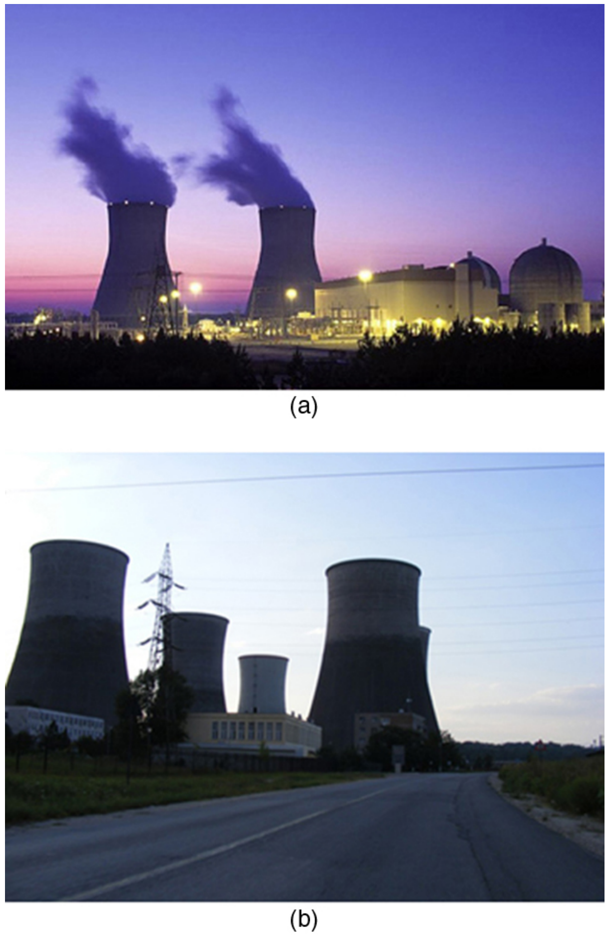


Figure 15.24 (a) A nuclear power station (credit: BlatantWorld.com) and (b) a coal-fired power station. Both have cooling towers in which water evaporates into the environment, representing Q_{c} . The nuclear reactor, which supplies Q_{h} , is housed inside the dome-shaped containment buildings. (credit: Robert & Mihaela Vicol, publicphoto.org)

Since all real processes are irreversible, the actual efficiency of a heat engine can never be as great as that of a Carnot engine, as illustrated in **Figure 15.25**(a). Even with the best heat engine possible, there are always dissipative processes in peripheral equipment, such as electrical transformers or car transmissions. These further reduce the overall efficiency by converting some of the engine's work output back into heat transfer, as shown in **Figure 15.25**(b).

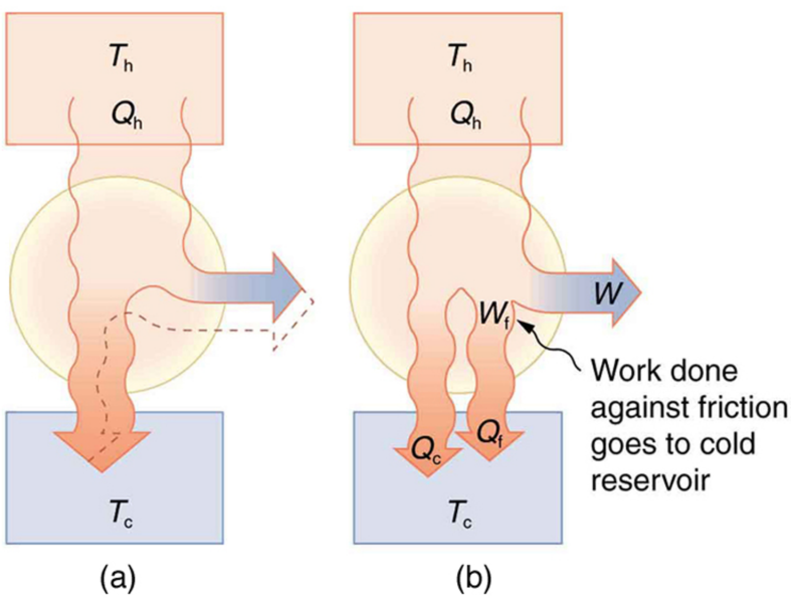


Figure 15.25 Real heat engines are less efficient than Carnot engines. (a) Real engines use irreversible processes, reducing the heat transfer to work. Solid lines represent the actual process; the dashed lines are what a Carnot engine would do between the same two reservoirs. (b) Friction and other dissipative processes in the output mechanisms of a heat engine convert some of its work output into heat transfer to the environment.

15.5 Applications of Thermodynamics: Heat Pumps and Refrigerators

Figure 15.26 Almost every home contains a refrigerator. Most people don't realize they are also sharing their homes with a heat pump. (credit: Id1337x, Wikimedia Commons)

Heat pumps, air conditioners, and refrigerators utilize heat transfer from cold to hot. They are heat engines run backward. We say backward, rather than reverse, because except for Carnot engines, all heat engines, though they can be run backward, cannot truly be reversed. Heat transfer occurs from a cold reservoir Q_c and into a hot one. This requires work input W, which is also converted to heat transfer. Thus the heat transfer to the hot reservoir is $Q_h = Q_c + W$. (Note that Q_h, Q_c, and W are positive, with their directions indicated on schematics rather than by sign.) A heat pump's mission is for heat transfer Q_h to occur into a warm environment, such as a home in the winter. The mission of air conditioners and refrigerators is for heat transfer Q_c to occur from a cool environment, such as chilling a room or keeping food at lower temperatures than the environment. (Actually, a heat pump can be used both to heat and cool a space. It is essentially an air conditioner and a heating unit all in one. In this section we will concentrate on its heating mode.)

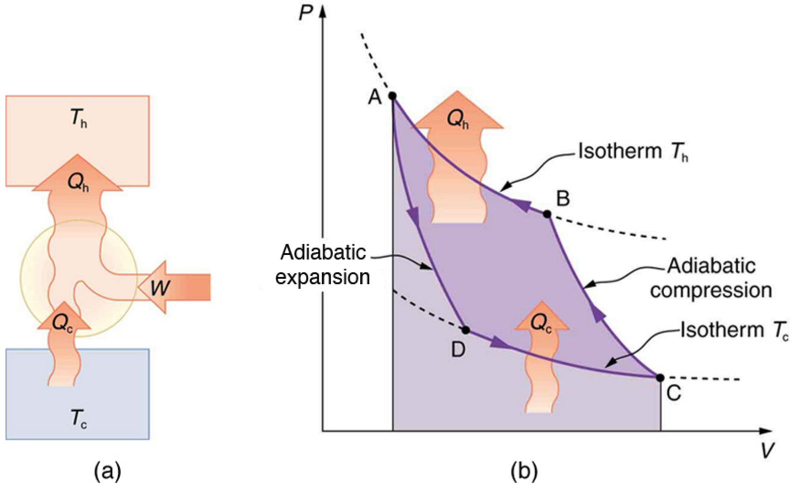


Figure 15.27 Heat pumps, air conditioners, and refrigerators are heat engines operated backward. The one shown here is based on a Carnot (reversible) engine. (a) Schematic diagram showing heat transfer from a cold reservoir to a warm reservoir with a heat pump. The directions of W, Q_h, and Q_c are opposite what they would be in a heat engine. (b) PV diagram for a Carnot cycle similar to that in **Figure 15.28** but reversed, following path ADCBA. The area inside the loop is negative, meaning there is a net work input. There is heat transfer Q_c into the system from a cold reservoir along path DC, and heat transfer Q_h out of the system into a hot reservoir along path BA.

Heat Pumps

The great advantage of using a heat pump to keep your home warm, rather than just burning fuel, is that a heat pump supplies $Q_h = Q_c + W$. Heat transfer is from the outside air, even at a temperature below freezing, to the indoor space. You only pay for W, and you get an additional heat transfer of Q_c from the outside at no cost; in many cases, at least twice as much energy is transferred to the heated space as is used to run the heat pump. When you burn fuel to keep warm, you pay for all of it. The disadvantage is that the work input (required by the second law of thermodynamics) is sometimes more expensive than simply burning fuel, especially if the work is done by electrical energy.

The basic components of a heat pump in its heating mode are shown in **Figure 15.28**. A working fluid such as a non-CFC refrigerant is used. In the outdoor coils (the evaporator), heat transfer Q_c occurs to the working fluid from the cold outdoor air, turning it into a gas.

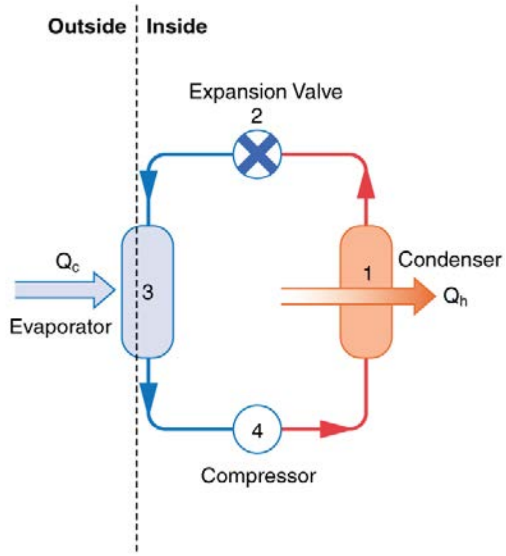


Figure 15.28 A simple heat pump has four basic components: (1) condenser, (2) expansion valve, (3) evaporator, and (4) compressor. In the heating mode, heat transfer Q_c occurs to the working fluid in the evaporator (3) from the colder outdoor air, turning it into a gas. The electrically driven compressor (4) increases the temperature and pressure of the gas and forces it into the condenser coils (1) inside the heated space. Because the temperature of the gas is higher than the temperature in the room, heat transfer from the gas to the room occurs as the gas condenses to a liquid. The working fluid is then cooled as it flows back through an expansion valve (2) to the outdoor evaporator coils.

The electrically driven compressor (work input W) raises the temperature and pressure of the gas and forces it into the condenser coils that are inside the heated space. Because the temperature of the gas is higher than the temperature inside the room, heat transfer to the room occurs and the gas condenses to a liquid. The liquid then flows back through a pressure-reducing valve to the outdoor evaporator coils, being cooled through expansion. (In a cooling cycle, the evaporator and condenser coils exchange roles and the flow direction of the fluid is reversed.)

The quality of a heat pump is judged by how much heat transfer Q_h occurs into the warm space compared with how much work input W is required. In the spirit of taking the ratio of what you get to what you spend, we define a **heat pump's coefficient of performance** (COP_{hp}) to be

$$COP_{hp} = \frac{Q_h}{W}. \tag{15.37}$$

Since the efficiency of a heat engine is $Eff = W / Q_h$, we see that $COP_{hp} = 1 / Eff$, an important and interesting fact. First, since the efficiency of any heat engine is less than 1, it means that COP_{hp} is always greater than 1—that is, a heat pump always has more heat transfer Q_h than work put into it. Second, it means that heat pumps work best when temperature differences are small. The efficiency of a perfect, or Carnot, engine is $Eff_C = 1 - (T_c / T_h)$; thus, the smaller the temperature difference, the smaller the efficiency and the greater the COP_{hp} (because $COP_{hp} = 1 / Eff$). In other words, heat pumps do not work as well in very cold climates as they do in more moderate climates.

Friction and other irreversible processes reduce heat engine efficiency, but they do *not* benefit the operation of a heat pump—instead, they reduce the work input by converting part of it to heat transfer back into the cold reservoir before it gets into the heat pump.

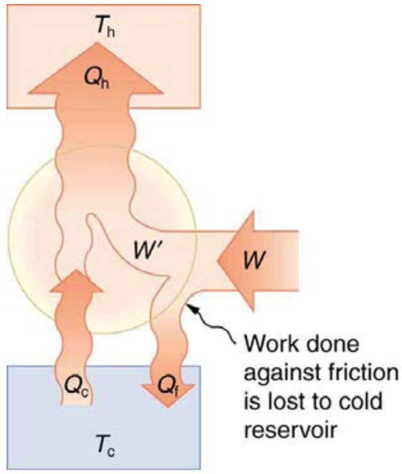


Figure 15.29 When a real heat engine is run backward, some of the intended work input (W) goes into heat transfer before it gets into the heat engine, thereby reducing its coefficient of performance COP_{hp}. In this figure, W' represents the portion of W that goes into the heat pump, while the remainder of W is lost in the form of frictional heat $\left(Q_f\right)$ to the cold reservoir. If all of W had gone into the heat pump, then Q_{h} would have been greater. The best heat pump uses adiabatic and isothermal processes, since, in theory, there would be no dissipative processes to reduce the heat transfer to the hot reservoir.

Example 15.5 The Best *COP* hp of a Heat Pump for Home Use

A heat pump used to warm a home must employ a cycle that produces a working fluid at temperatures greater than typical indoor temperature so that heat transfer to the inside can take place. Similarly, it must produce a working fluid at temperatures that are colder than the outdoor temperature so that heat transfer occurs from outside. Its hot and cold reservoir temperatures therefore cannot be too close, placing a limit on its COP_{hp}. (See **Figure 15.30**.) What is the best coefficient of performance possible for such a heat pump, if it has a hot reservoir temperature of 45.0ºC and a cold reservoir temperature of -15.0ºC?

Strategy

A Carnot engine reversed will give the best possible performance as a heat pump. As noted above, $COP_{\text{hp}} = 1 / \mathit{Eff}$, so that we need to first calculate the Carnot efficiency to solve this problem.

Solution

Carnot efficiency in terms of absolute temperature is given by:

$$\mathit{Eff}_{\text{C}} = 1 - \frac{T_{\text{c}}}{T_{\text{h}}}. \tag{15.38}$$

The temperatures in kelvins are $T_{\text{h}} = 318\text{ K}$ and $T_{\text{c}} = 258\text{ K}$, so that

$$\mathit{Eff}_{\text{C}} = 1 - \frac{258\text{ K}}{318\text{ K}} = 0.1887. \tag{15.39}$$

Thus, from the discussion above,

$$COP_{\text{hp}} = \frac{1}{\mathit{Eff}} = \frac{1}{0.1887} = 5.30, \tag{15.40}$$

or

$$COP_{\text{hp}} = \frac{Q_{\text{h}}}{W} = 5.30, \tag{15.41}$$

so that

$$Q_{\text{h}} = 5.30\text{ W}. \tag{15.42}$$

Discussion

This result means that the heat transfer by the heat pump is 5.30 times as much as the work put into it. It would cost 5.30 times as much for the same heat transfer by an electric room heater as it does for that produced by this heat pump. This is not a violation of conservation of energy. Cold ambient air provides 4.3 J per 1 J of work from the electrical outlet.

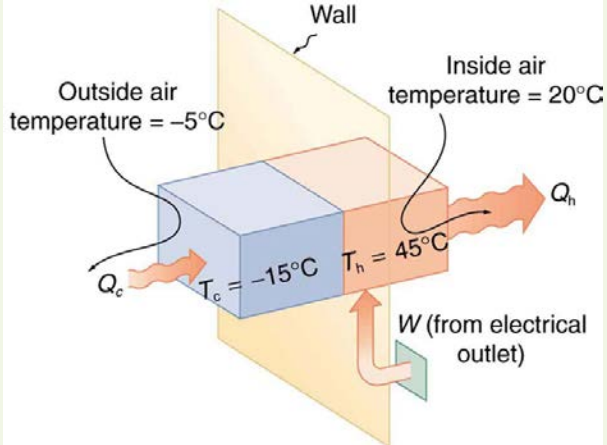


Figure 15.30 Heat transfer from the outside to the inside, along with work done to run the pump, takes place in the heat pump of the example above. Note that the cold temperature produced by the heat pump is lower than the outside temperature, so that heat transfer into the working fluid occurs. The pump's compressor produces a temperature greater than the indoor temperature in order for heat transfer into the house to occur.

Real heat pumps do not perform quite as well as the ideal one in the previous example; their values of COP_{hp} range from about 2 to 4. This range means that the heat transfer Q_{h} from the heat pumps is 2 to 4 times as great as the work W put into them. Their economical feasibility is still limited, however, since W is usually supplied by electrical energy that costs more per joule than heat transfer by burning fuels like natural gas. Furthermore, the initial cost of a heat pump is greater than that of many furnaces, so that a heat pump must last longer for its cost to be recovered. Heat pumps are most likely to be economically superior where winter temperatures are mild, electricity is relatively cheap, and other fuels are relatively expensive. Also, since they can cool as well as heat a space, they have advantages where cooling in summer months is also desired. Thus some of the best locations for heat pumps are in warm summer climates with cool winters. **Figure 15.31** shows a heat pump, called a "*reverse cycle"* or "*split-system cooler"* in some countries.

Figure 15.31 In hot weather, heat transfer occurs from air inside the room to air outside, cooling the room. In cool weather, heat transfer occurs from air outside to air inside, warming the room. This switching is achieved by reversing the direction of flow of the working fluid.

Air Conditioners and Refrigerators

Air conditioners and refrigerators are designed to cool something down in a warm environment. As with heat pumps, work input is required for heat transfer from cold to hot, and this is expensive. The quality of air conditioners and refrigerators is judged by how much heat transfer Q_{c} occurs from a cold environment compared with how much work input W is required. What is considered the benefit in a heat pump is considered waste heat in a refrigerator. We thus define the **coefficient of performance** (COP_{ref}) of an air conditioner or refrigerator to be

$$COP_{\text{ref}} = \frac{Q_{\text{c}}}{W}. \tag{15.43}$$

Noting again that $Q_{\text{h}} = Q_{\text{c}} + W$, we can see that an air conditioner will have a lower coefficient of performance than a heat pump, because $COP_{\text{hp}} = Q_{\text{h}} / W$ and Q_{h} is greater than Q_{c}. In this module's Problems and Exercises, you will show that

$$COP_{\text{ref}} = COP_{\text{hp}} - 1 \tag{15.44}$$

for a heat engine used as either an air conditioner or a heat pump operating between the same two temperatures. Real air conditioners and refrigerators typically do remarkably well, having values of COP_{ref} ranging from 2 to 6. These numbers are better than the COP_{hp} values for the heat pumps mentioned above, because the temperature differences are smaller, but they are less than those for Carnot engines operating between the same two temperatures.

A type of COP rating system called the "energy efficiency rating" (EER) has been developed. This rating is an example where non-SI units are still used and relevant to consumers. To make it easier for the consumer, Australia, Canada, New Zealand, and the U.S. use an Energy Star Rating out of 5 stars—the more stars, the more energy efficient the appliance. EERs are expressed in mixed units of British thermal units (Btu) per hour of heating or cooling divided by the power input in watts. Room air conditioners are readily available with EERs ranging from 6 to 12. Although not the same as the COPs just described, these EERs are good for comparison purposes—the greater the EER, the cheaper an air conditioner is to operate (but the higher its purchase price is likely to be).

The EER of an air conditioner or refrigerator can be expressed as

$$EER = \frac{Q_\text{c} / t_1}{W / t_2}, \tag{15.45}$$

where Q_c is the amount of heat transfer from a cold environment in British thermal units, t_1 is time in hours, W is the work input in joules, and t_2 is time in seconds.

Problem-Solving Strategies for Thermodynamics

1. *Examine the situation to determine whether heat, work, or internal energy are involved.* Look for any system where the primary methods of transferring energy are heat and work. Heat engines, heat pumps, refrigerators, and air conditioners are examples of such systems.
2. *Identify the system of interest and draw a labeled diagram of the system showing energy flow.*
3. *Identify exactly what needs to be determined in the problem (identify the unknowns).* A written list is useful. Maximum efficiency means a Carnot engine is involved. Efficiency is not the same as the coefficient of performance.
4. *Make a list of what is given or can be inferred from the problem as stated (identify the knowns).* Be sure to distinguish heat transfer into a system from heat transfer out of the system, as well as work input from work output. In many situations, it is useful to determine the type of process, such as isothermal or adiabatic.
5. *Solve the appropriate equation for the quantity to be determined (the unknown).*
6. *Substitute the known quantities along with their units into the appropriate equation and obtain numerical solutions complete with units.*
7. *Check the answer to see if it is reasonable: Does it make sense?* For example, efficiency is always less than 1, whereas coefficients of performance are greater than 1.

15.6 Entropy and the Second Law of Thermodynamics: Disorder and the Unavailability of Energy

Figure 15.32 The ice in this drink is slowly melting. Eventually the liquid will reach thermal equilibrium, as predicted by the second law of thermodynamics. (credit: Jon Sullivan, PDPhoto.org)

There is yet another way of expressing the second law of thermodynamics. This version relates to a concept called **entropy**. By examining it, we shall see that the directions associated with the second law—heat transfer from hot to cold, for example—are related to the tendency in nature for systems to become disordered and for less energy to be available for use as work. The entropy of a system can in fact be shown to be a measure of its disorder and of the unavailability of energy to do work.

Making Connections: Entropy, Energy, and Work

Recall that the simple definition of energy is the ability to do work. Entropy is a measure of how much energy is not available to do work. Although all forms of energy are interconvertible, and all can be used to do work, it is not always possible, even in principle, to convert the entire available energy into work. That unavailable energy is of interest in thermodynamics, because the field of thermodynamics arose from efforts to convert heat to work.

We can see how entropy is defined by recalling our discussion of the Carnot engine. We noted that for a Carnot cycle, and hence for any reversible processes, $Q_\text{c} / Q_\text{h} = T_\text{c} / T_\text{h}$. Rearranging terms yields

$$\frac{Q_c}{T_c} = \frac{Q_h}{T_h} \tag{15.46}$$

for any reversible process. Q_c and Q_h are absolute values of the heat transfer at temperatures T_c and T_h, respectively. This ratio of Q/T is defined to be the **change in entropy** ΔS for a reversible process,

$$\Delta S = \left(\frac{Q}{T}\right)_{\text{rev}}, \tag{15.47}$$

where Q is the heat transfer, which is positive for heat transfer into and negative for heat transfer out of, and T is the absolute temperature at which the reversible process takes place. The SI unit for entropy is joules per kelvin (J/K). If temperature changes during the process, then it is usually a good approximation (for small changes in temperature) to take T to be the average temperature, avoiding the need to use integral calculus to find ΔS.

The definition of ΔS is strictly valid only for reversible processes, such as used in a Carnot engine. However, we can find ΔS precisely even for real, irreversible processes. The reason is that the entropy S of a system, like internal energy U, depends only on the state of the system and not how it reached that condition. Entropy is a property of state. Thus the change in entropy ΔS of a system between state 1 and state 2 is the same no matter how the change occurs. We just need to find or imagine a reversible process that takes us from state 1 to state 2 and calculate ΔS for that process. That will be the change in entropy for any process going from state 1 to state 2. (See **Figure 15.33**.)

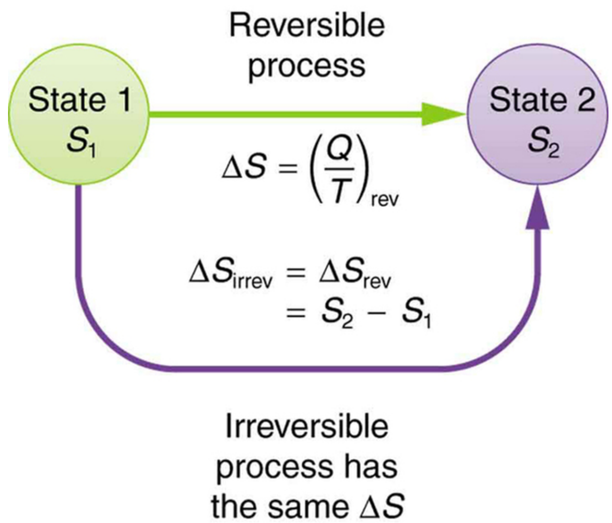


Figure 15.33 When a system goes from state 1 to state 2, its entropy changes by the same amount ΔS, whether a hypothetical reversible path is followed or a real irreversible path is taken.

Now let us take a look at the change in entropy of a Carnot engine and its heat reservoirs for one full cycle. The hot reservoir has a loss of entropy $\Delta S_h = -Q_h/T_h$, because heat transfer occurs out of it (remember that when heat transfers out, then Q has a negative sign). The cold reservoir has a gain of entropy $\Delta S_c = Q_c/T_c$, because heat transfer occurs into it. (We assume the reservoirs are sufficiently large that their temperatures are constant.) So the total change in entropy is

$$\Delta S_{\text{tot}} = \Delta S_h + \Delta S_c. \tag{15.48}$$

Thus, since we know that $Q_h/T_h = Q_c/T_c$ for a Carnot engine,

$$\Delta S_{\text{tot}} = -\frac{Q_h}{T_h} + \frac{Q_c}{T_c} = 0. \tag{15.49}$$

This result, which has general validity, means that *the total change in entropy for a system in any reversible process is zero.*

The entropy of various parts of the system may change, but the total change is zero. Furthermore, the system does not affect the entropy of its surroundings, since heat transfer between them does not occur. Thus the reversible process changes neither the total entropy of the system nor the entropy of its surroundings. Sometimes this is stated as follows: *Reversible processes do not affect the total entropy of the universe.* Real processes are not reversible, though, and they do change total entropy. We can, however, use hypothetical reversible processes to determine the value of entropy in real, irreversible processes. The following example illustrates this point.

Example 15.6 Entropy Increases in an Irreversible (Real) Process

Spontaneous heat transfer from hot to cold is an irreversible process. Calculate the total change in entropy if 4000 J of heat transfer occurs from a hot reservoir at $T_h = 600\text{ K}(327^\circ\text{ C})$ to a cold reservoir at $T_c = 250\text{ K}(-23^\circ\text{ C})$, assuming there is no temperature change in either reservoir. (See **Figure 15.34**.)

Strategy

How can we calculate the change in entropy for an irreversible process when $\Delta S_{\text{tot}} = \Delta S_{\text{h}} + \Delta S_{\text{c}}$ is valid only for reversible processes?

Remember that the total change in entropy of the hot and cold reservoirs will be the same whether a reversible or irreversible process is involved in heat transfer from hot to cold. So we can calculate the change in entropy of the hot reservoir for a hypothetical reversible process in which 4000 J of heat transfer occurs from it; then we do the same for a hypothetical reversible process in which 4000 J of heat transfer occurs to the cold reservoir. This produces the same changes in the hot and cold reservoirs that would occur if the heat transfer were allowed to occur irreversibly between them, and so it also produces the same changes in entropy.

Solution

We now calculate the two changes in entropy using $\Delta S_{\text{tot}} = \Delta S_{\text{h}} + \Delta S_{\text{c}}$. First, for the heat transfer from the hot reservoir,

$$\Delta S_{\text{h}} = \frac{-Q_{\text{h}}}{T_{\text{h}}} = \frac{-4000\text{ J}}{600\text{ K}} = -6.67\text{ J/K.} \tag{15.50}$$

And for the cold reservoir,

$$\Delta S_{\text{c}} = \frac{-Q_{\text{c}}}{T_{\text{c}}} = \frac{4000\text{ J}}{250\text{ K}} = 16.0\text{ J/K.} \tag{15.51}$$

Thus the total is

$$\begin{aligned} \Delta S_{\text{tot}} &= \Delta S_{\text{h}} + \Delta S_{\text{c}} \\ &= (-6.67 + 16.0)\text{ J/K} \\ &= 9.33\text{ J/K.} \end{aligned} \tag{15.52}$$

Discussion

There is an *increase* in entropy for the system of two heat reservoirs undergoing this irreversible heat transfer. We will see that this means there is a loss of ability to do work with this transferred energy. Entropy has increased, and energy has become unavailable to do work.

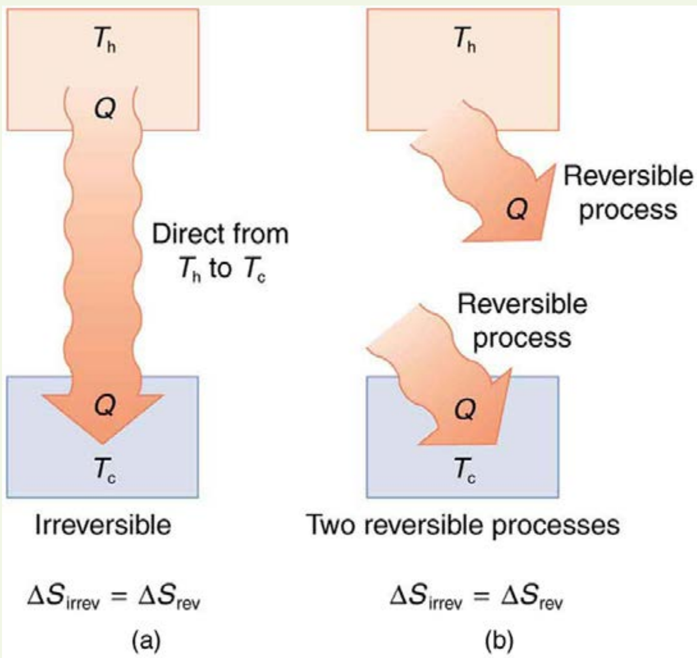


Figure 15.34 (a) Heat transfer from a hot object to a cold one is an irreversible process that produces an overall increase in entropy. (b) The same final state and, thus, the same change in entropy is achieved for the objects if reversible heat transfer processes occur between the two objects whose temperatures are the same as the temperatures of the corresponding objects in the irreversible process.

It is reasonable that entropy increases for heat transfer from hot to cold. Since the change in entropy is Q/T , there is a larger change at lower temperatures. The decrease in entropy of the hot object is therefore less than the increase in entropy of the cold object, producing an overall increase, just as in the previous example. This result is very general:

There is an increase in entropy for any system undergoing an irreversible process.

With respect to entropy, there are only two possibilities: entropy is constant for a reversible process, and it increases for an irreversible process. There is a fourth version of **the second law of thermodynamics stated in terms of entropy**:

The total entropy of a system either increases or remains constant in any process; it never decreases.

For example, heat transfer cannot occur spontaneously from cold to hot, because entropy would decrease.

Entropy is very different from energy. Entropy is *not* conserved but increases in all real processes. Reversible processes (such as in Carnot engines) are the processes in which the most heat transfer to work takes place and are also the ones that keep entropy constant. Thus we are led to make a connection between entropy and the availability of energy to do work.

Entropy and the Unavailability of Energy to Do Work

What does a change in entropy mean, and why should we be interested in it? One reason is that entropy is directly related to the fact that not all heat transfer can be converted into work. The next example gives some indication of how an increase in entropy results in less heat transfer into work.

Example 15.7 Less Work is Produced by a Given Heat Transfer When Entropy Change is Greater

(a) Calculate the work output of a Carnot engine operating between temperatures of 600 K and 100 K for 4000 J of heat transfer to the engine. (b) Now suppose that the 4000 J of heat transfer occurs first from the 600 K reservoir to a 250 K reservoir (without doing any work, and this produces the increase in entropy calculated above) before transferring into a Carnot engine operating between 250 K and 100 K. What work output is produced? (See **Figure 15.35**.)

Strategy

In both parts, we must first calculate the Carnot efficiency and then the work output.

Solution (a)

The Carnot efficiency is given by

$$Eff_{\text{C}} = 1 - \frac{T_{\text{c}}}{T_{\text{h}}}. \tag{15.53}$$

Substituting the given temperatures yields

$$Eff_{\text{C}} = 1 - \frac{100\text{ K}}{600\text{ K}} = 0.833. \tag{15.54}$$

Now the work output can be calculated using the definition of efficiency for any heat engine as given by

$$Eff = \frac{W}{Q_{\text{h}}}. \tag{15.55}$$

Solving for W and substituting known terms gives

$$\begin{aligned} W &= Eff_{\text{C}} Q_{\text{h}} \\ &= (0.833)(4000\text{ J}) = 3333\text{ J}. \end{aligned} \tag{15.56}$$

Solution (b)

Similarly,

$$Eff'_{\text{C}} = 1 - \frac{T_{\text{c}}}{\text{T}'_{\text{c}}} = -\frac{100\text{ K}}{250\text{ K}} = 0.600, \tag{15.57}$$

so that

$$\begin{aligned} W &= Eff'_{\text{C}} Q_h \\ &= (0.600)(4000\text{ J}) = 2400\text{ J}. \end{aligned} \tag{15.58}$$

Discussion

There is 933 J less work from the same heat transfer in the second process. This result is important. The same heat transfer into two perfect engines produces different work outputs, because the entropy change differs in the two cases. In the second case, entropy is greater and less work is produced. Entropy is associated with the *un*availability of energy to do work.

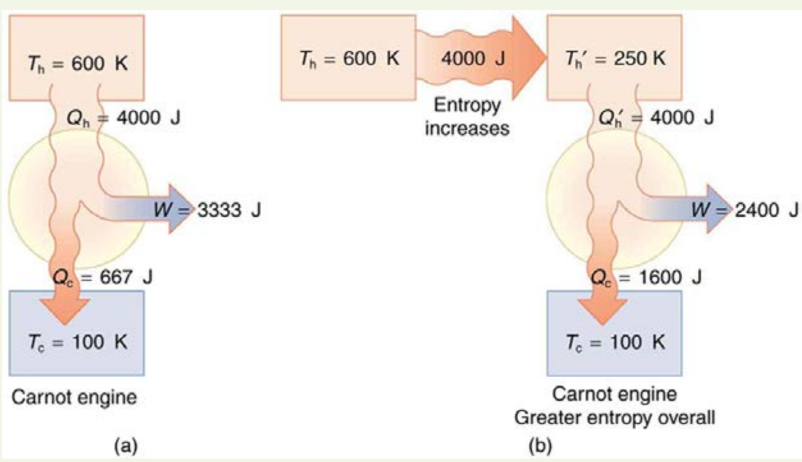


Figure 15.35 (a) A Carnot engine working at between 600 K and 100 K has 4000 J of heat transfer and performs 3333 J of work. (b) The 4000 J of heat transfer occurs first irreversibly to a 250 K reservoir and then goes into a Carnot engine. The increase in entropy caused by the heat transfer to a colder reservoir results in a smaller work output of 2400 J. There is a permanent loss of 933 J of energy for the purpose of doing work.

When entropy increases, a certain amount of energy becomes *permanently* unavailable to do work. The energy is not lost, but its character is changed, so that some of it can never be converted to doing work—that is, to an organized force acting through a distance. For instance, in the previous example, 933 J less work was done after an increase in entropy of 9.33 J/K occurred in the 4000 J heat transfer from the 600 K reservoir to the 250 K reservoir. It can be shown that the amount of energy that becomes unavailable for work is

$$W_{\text{unavail}} = \Delta S \cdot T_0, \tag{15.59}$$

where T_0 is the lowest temperature utilized. In the previous example,

$$W_{\text{unavail}} = (9.33 \text{ J/K})(100 \text{ K}) = 933 \text{ J} \tag{15.60}$$

as found.

Heat Death of the Universe: An Overdose of Entropy

In the early, energetic universe, all matter and energy were easily interchangeable and identical in nature. Gravity played a vital role in the young universe. Although it may have *seemed* disorderly, and therefore, superficially entropic, in fact, there was enormous potential energy available to do work—all the future energy in the universe.

As the universe matured, temperature differences arose, which created more opportunity for work. Stars are hotter than planets, for example, which are warmer than icy asteroids, which are warmer still than the vacuum of the space between them.

Most of these are cooling down from their usually violent births, at which time they were provided with energy of their own—nuclear energy in the case of stars, volcanic energy on Earth and other planets, and so on. Without additional energy input, however, their days are numbered.

As entropy increases, less and less energy in the universe is available to do work. On Earth, we still have great stores of energy such as fossil and nuclear fuels; large-scale temperature differences, which can provide wind energy; geothermal energies due to differences in temperature in Earth's layers; and tidal energies owing to our abundance of liquid water. As these are used, a certain fraction of the energy they contain can never be converted into doing work. Eventually, all fuels will be exhausted, all temperatures will equalize, and it will be impossible for heat engines to function, or for work to be done.

Entropy increases in a closed system, such as the universe. But in parts of the universe, for instance, in the Solar system, it is not a locally closed system. Energy flows from the Sun to the planets, replenishing Earth's stores of energy. The Sun will continue to supply us with energy for about another five billion years. We will enjoy direct solar energy, as well as side effects of solar energy, such as wind power and biomass energy from photosynthetic plants. The energy from the Sun will keep our water at the liquid state, and the Moon's gravitational pull will continue to provide tidal energy. But Earth's geothermal energy will slowly run down and won't be replenished.

But in terms of the universe, and the very long-term, very large-scale picture, the entropy of the universe is increasing, and so the availability of energy to do work is constantly decreasing. Eventually, when all stars have died, all forms of potential energy have been utilized, and all temperatures have equalized (depending on the mass of the universe, either at a very high temperature following a universal contraction, or a very low one, just before all activity ceases) there will be no possibility of doing work.

Either way, the universe is destined for thermodynamic equilibrium—maximum entropy. This is often called the *heat death of the universe*, and will mean the end of all activity. However, whether the universe contracts and heats up, or continues to expand and cools down, the end is not near. Calculations of black holes suggest that entropy can easily continue for at least 10^{100} years.

Order to Disorder

Entropy is related not only to the unavailability of energy to do work—it is also a measure of disorder. This notion was initially postulated by Ludwig Boltzmann in the 1800s. For example, melting a block of ice means taking a highly structured and orderly system of water molecules and converting it into a disorderly liquid in which molecules have no fixed positions. (See **Figure 15.36**.) There is a large increase in entropy in the process, as seen in the following example.

Example 15.8 Entropy Associated with Disorder

Find the increase in entropy of 1.00 kg of ice originally at 0°C that is melted to form water at 0°C.

Strategy

As before, the change in entropy can be calculated from the definition of ΔS once we find the energy Q needed to melt the ice.

Solution

The change in entropy is defined as:

$$\Delta S = \frac{Q}{T}. \tag{15.61}$$

Here Q is the heat transfer necessary to melt 1.00 kg of ice and is given by

$$Q = mL_{\text{f}}, \tag{15.62}$$

where m is the mass and L_{f} is the latent heat of fusion. $L_{\text{f}} = 334 \text{ kJ/kg}$ for water, so that

$$Q = (1.00 \text{ kg})(334 \text{ kJ/kg}) = 3.34\times10^5 \text{ J}. \tag{15.63}$$

Now the change in entropy is positive, since heat transfer occurs into the ice to cause the phase change; thus,

$$\Delta S = \frac{Q}{T} = \frac{3.34\times10^5 \text{ J}}{T}. \tag{15.64}$$

T is the melting temperature of ice. That is, $T = 0^\circ\text{C}{=}273\ \text{K}$. So the change in entropy is

$$\begin{aligned}\Delta S &= \frac{3.34\times10^5\ \text{J}}{273\ \text{K}}\\ &= 1.22\times10^3\ \text{J/K}.\end{aligned} \tag{15.65}$$

Discussion

This is a significant increase in entropy accompanying an increase in disorder.

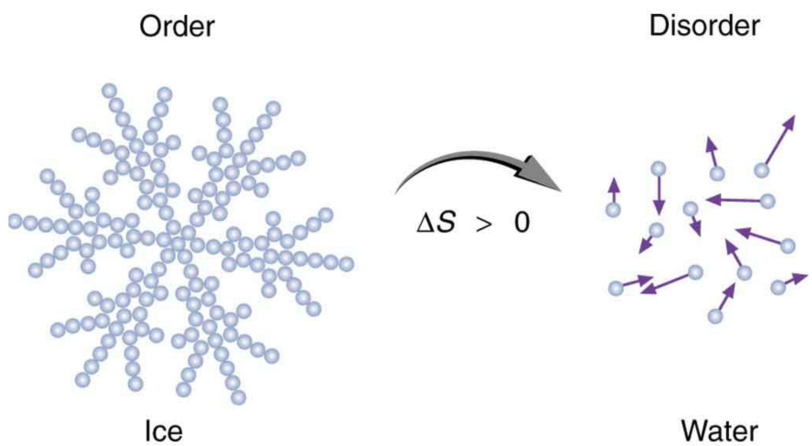


Figure 15.36 When ice melts, it becomes more disordered and less structured. The systematic arrangement of molecules in a crystal structure is replaced by a more random and less orderly movement of molecules without fixed locations or orientations. Its entropy increases because heat transfer occurs into it. Entropy is a measure of disorder.

In another easily imagined example, suppose we mix equal masses of water originally at two different temperatures, say $20.0^\circ\ \text{C}$ and $40.0^\circ\ \text{C}$. The result is water at an intermediate temperature of $30.0^\circ\ \text{C}$. Three outcomes have resulted: entropy has increased, some energy has become unavailable to do work, and the system has become less orderly. Let us think about each of these results.

First, entropy has increased for the same reason that it did in the example above. Mixing the two bodies of water has the same effect as heat transfer from the hot one and the same heat transfer into the cold one. The mixing decreases the entropy of the hot water but increases the entropy of the cold water by a greater amount, producing an overall increase in entropy.

Second, once the two masses of water are mixed, there is only one temperature—you cannot run a heat engine with them. The energy that could have been used to run a heat engine is now unavailable to do work.

Third, the mixture is less orderly, or to use another term, less structured. Rather than having two masses at different temperatures and with different distributions of molecular speeds, we now have a single mass with a uniform temperature.

These three results—entropy, unavailability of energy, and disorder—are not only related but are in fact essentially equivalent.

Life, Evolution, and the Second Law of Thermodynamics

Some people misunderstand the second law of thermodynamics, stated in terms of entropy, to say that the process of the evolution of life violates this law. Over time, complex organisms evolved from much simpler ancestors, representing a large decrease in entropy of the Earth's biosphere. It is a fact that living organisms have evolved to be highly structured, and much lower in entropy than the substances from which they grow. But it is *always* possible for the entropy of one part of the universe to decrease, provided the total change in entropy of the universe increases. In equation form, we can write this as

$$\Delta S_{\text{tot}} = \Delta S_{\text{syst}} + \Delta S_{\text{envir}} > 0. \tag{15.66}$$

Thus ΔS_{syst} can be negative as long as ΔS_{envir} is positive and greater in magnitude.

How is it possible for a system to decrease its entropy? Energy transfer is necessary. If I pick up marbles that are scattered about the room and put them into a cup, my work has decreased the entropy of that system. If I gather iron ore from the ground and convert it into steel and build a bridge, my work has decreased the entropy of that system. Energy coming from the Sun can decrease the entropy of local systems on Earth—that is, ΔS_{syst} is negative. But the overall entropy of the rest of the universe increases by a greater amount—that is, ΔS_{envir} is positive and greater in magnitude. Thus, $\Delta S_{\text{tot}} = \Delta S_{\text{syst}} + \Delta S_{\text{envir}} > 0$, and the second law of thermodynamics is *not* violated.

Every time a plant stores some solar energy in the form of chemical potential energy, or an updraft of warm air lifts a soaring bird, the Earth can be viewed as a heat engine operating between a hot reservoir supplied by the Sun and a cold reservoir supplied by dark outer space—a heat engine of high complexity, causing local decreases in entropy as it uses part of the heat transfer from the Sun into deep space. There is a large total increase in entropy resulting from this massive heat transfer. A small part of this heat transfer is stored in structured systems on Earth, producing much smaller local decreases in entropy. (See **Figure 15.37**.)

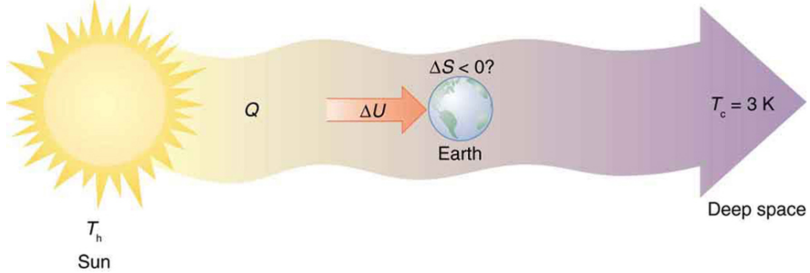


Figure 15.37 Earth's entropy may decrease in the process of intercepting a small part of the heat transfer from the Sun into deep space. Entropy for the entire process increases greatly while Earth becomes more structured with living systems and stored energy in various forms.

PhET Explorations: Reversible Reactions

Watch a reaction proceed over time. How does total energy affect a reaction rate? Vary temperature, barrier height, and potential energies. Record concentrations and time in order to extract rate coefficients. Do temperature dependent studies to extract Arrhenius parameters. This simulation is best used with teacher guidance because it presents an analogy of chemical reactions.

PhET Interactive Simulation

Figure 15.38 Reversible Reactions (http://cnx.org/content/m42237/1.6/reversible-reactions_en.jar)

15.7 Statistical Interpretation of Entropy and the Second Law of Thermodynamics: The Underlying Explanation

Figure 15.39 When you toss a coin a large number of times, heads and tails tend to come up in roughly equal numbers. Why doesn't heads come up 100, 90, or even 80% of the time? (credit: Jon Sullivan, PDPhoto.org)

The various ways of formulating the second law of thermodynamics tell what happens rather than why it happens. Why should heat transfer occur only from hot to cold? Why should energy become ever less available to do work? Why should the universe become increasingly disorderly? The answer is that it is a matter of overwhelming probability. Disorder is simply vastly more likely than order.

When you watch an emerging rain storm begin to wet the ground, you will notice that the drops fall in a disorganized manner both in time and in space. Some fall close together, some far apart, but they never fall in straight, orderly rows. It is not impossible for rain to fall in an orderly pattern, just highly unlikely, because there are many more disorderly ways than orderly ones. To illustrate this fact, we will examine some random processes, starting with coin tosses.

Coin Tosses

What are the possible outcomes of tossing 5 coins? Each coin can land either heads or tails. On the large scale, we are concerned only with the total heads and tails and not with the order in which heads and tails appear. The following possibilities exist:

$$\begin{gathered}\text{5 heads, 0 tails}\\ \text{4 heads, 1 tail}\\ \text{3 heads, 2 tails}\\ \text{2 heads, 3 tails}\\ \text{1 head, 4 tails}\\ \text{0 head, 5 tails}\end{gathered} \tag{15.67}$$

These are what we call macrostates. A **macrostate** is an overall property of a system. It does not specify the details of the system, such as the order in which heads and tails occur or which coins are heads or tails.

Using this nomenclature, a system of 5 coins has the 6 possible macrostates just listed. Some macrostates are more likely to occur than others. For instance, there is only one way to get 5 heads, but there are several ways to get 3 heads and 2 tails, making the latter macrostate more probable. **Table 15.3** lists of all the ways in which 5 coins can be tossed, taking into account the order in which heads and tails occur. Each sequence is called a **microstate**—a detailed description of every element of a system.

Table 15.3 5-Coin Toss

	Individual microstates	Number of microstates
5 heads, 0 tails	HHHHH	1
4 heads, 1 tail	HHHHT, HHHTH, HHTHH, HTHHH, THHHH	5
3 heads, 2 tails	HTHTH, THTHH, HTHHT, THHTH, THHHT HTHTH, THTHH, HTHHT, THHTH, THHHT	10
2 heads, 3 tails	TTTHH, TTHHT, THHTT, HHTTT, TTHTH, THTHT, HTHTT, THTTH, HTTHT, HTTTH	10
1 head, 4 tails	TTTTH, TTTHT, TTHTT, THTTT, HTTTT	5
0 heads, 5 tails	TTTTT	1
		Total: 32

The macrostate of 3 heads and 2 tails can be achieved in 10 ways and is thus 10 times more probable than the one having 5 heads. Not surprisingly, it is equally probable to have the reverse, 2 heads and 3 tails. Similarly, it is equally probable to get 5 tails as it is to get 5 heads. Note that all of these conclusions are based on the crucial assumption that each microstate is equally probable. With coin tosses, this requires that the coins not be asymmetric in a way that favors one side over the other, as with loaded dice. With any system, the assumption that all microstates are equally probable must be valid, or the analysis will be erroneous.

The two most orderly possibilities are 5 heads or 5 tails. (They are more structured than the others.) They are also the least likely, only 2 out of 32 possibilities. The most disorderly possibilities are 3 heads and 2 tails and its reverse. (They are the least structured.) The most disorderly possibilities are also the most likely, with 20 out of 32 possibilities for the 3 heads and 2 tails and its reverse. If we start with an orderly array like 5 heads and toss the coins, it is very likely that we will get a less orderly array as a result, since 30 out of the 32 possibilities are less orderly. So even if you start with an orderly state, there is a strong tendency to go from order to disorder, from low entropy to high entropy. The reverse can happen, but it is unlikely.

Table 15.4 100-Coin Toss

Macrostate		Number of microstates
Heads	Tails	(W)
100	0	1
99	1	1.0×10^{2}
95	5	7.5×10^{7}
90	10	1.7×10^{13}
75	25	2.4×10^{23}
60	40	1.4×10^{28}
55	45	6.1×10^{28}
51	49	9.9×10^{28}
50	50	1.0×10^{29}
49	51	9.9×10^{28}
45	55	6.1×10^{28}
40	60	1.4×10^{28}
25	75	2.4×10^{23}
10	90	1.7×10^{13}
5	95	7.5×10^{7}
1	99	1.0×10^{2}
0	100	1
		Total: 1.27×10^{30}

This result becomes dramatic for larger systems. Consider what happens if you have 100 coins instead of just 5. The most orderly arrangements (most structured) are 100 heads or 100 tails. The least orderly (least structured) is that of 50 heads and 50 tails. There is only 1 way (1 microstate) to get the most orderly arrangement of 100 heads. There are 100 ways (100 microstates) to get the next most orderly arrangement of 99 heads and 1 tail (also 100 to get its reverse). And there are 1.0×10^{29} ways to get 50 heads and 50 tails, the least orderly arrangement. **Table 15.4** is an abbreviated list of the various macrostates and the number of microstates for each macrostate. The total number of microstates—the total number of different ways 100 coins can be tossed—is an impressively large 1.27×10^{30}. Now, if we start with an orderly macrostate like 100 heads and toss the coins, there is a virtual certainty that we will get a less orderly macrostate. If we keep tossing the coins, it is possible, but exceedingly unlikely, that we will ever get back to the most orderly macrostate. If you tossed the coins once each second, you could expect to get either 100 heads or 100 tails once in 2×10^{22} years! This period is 1 trillion (10^{12}) times longer than the age of the universe, and so the chances are essentially zero. In contrast, there is an 8% chance of getting 50 heads, a 73% chance of getting from 45 to 55 heads, and a 96% chance of getting from 40 to 60 heads. Disorder is highly likely.

Disorder in a Gas

The fantastic growth in the odds favoring disorder that we see in going from 5 to 100 coins continues as the number of entities in the system increases. Let us now imagine applying this approach to perhaps a small sample of gas. Because counting microstates and macrostates involves statistics, this is called **statistical analysis**. The macrostates of a gas correspond to its macroscopic properties, such as volume, temperature, and pressure; and its microstates correspond to the detailed description of the positions and velocities of its atoms. Even a small amount of gas has a huge number of atoms: $1.0\ \text{cm}^3$ of an ideal gas at 1.0 atm and $0^\text{o}\ \text{C}$ has 2.7×10^{19} atoms. So each macrostate has an immense number of microstates. In plain language, this means that there are an immense number of ways in which the atoms in a gas can be arranged, while still having the same pressure, temperature, and so on.

The most likely conditions (or macrostates) for a gas are those we see all the time—a random distribution of atoms in space with a Maxwell-Boltzmann distribution of speeds in random directions, as predicted by kinetic theory. This is the most disorderly and least structured condition we can imagine. In contrast, one type of very orderly and structured macrostate has all of the atoms in one corner of a container with identical velocities. There are very few ways to accomplish this (very few microstates corresponding to it), and so it is exceedingly unlikely ever to occur. (See **Figure 15.40**(b).) Indeed, it is so unlikely that we have a law saying that it is impossible, which has never been observed to be violated—the second law of thermodynamics.

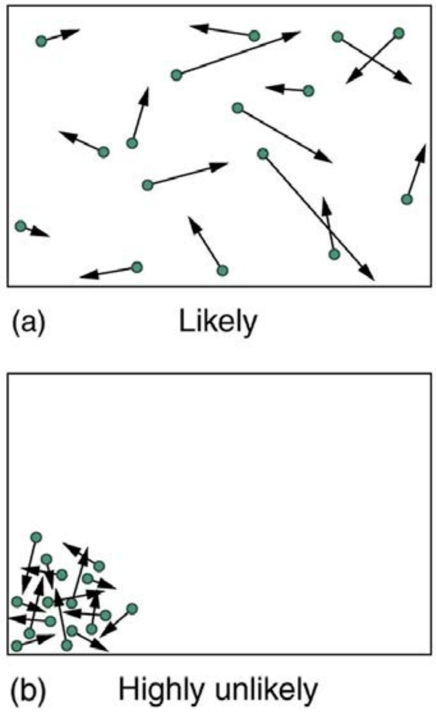


Figure 15.40 (a) The ordinary state of gas in a container is a disorderly, random distribution of atoms or molecules with a Maxwell-Boltzmann distribution of speeds. It is so unlikely that these atoms or molecules would ever end up in one corner of the container that it might as well be impossible. (b) With energy transfer, the gas can be forced into one corner and its entropy greatly reduced. But left alone, it will spontaneously increase its entropy and return to the normal conditions, because they are immensely more likely.

The disordered condition is one of high entropy, and the ordered one has low entropy. With a transfer of energy from another system, we could force all of the atoms into one corner and have a local decrease in entropy, but at the cost of an overall increase in entropy of the universe. If the atoms start out in one corner, they will quickly disperse and become uniformly distributed and will never return to the orderly original state (Figure 15.40(b)). Entropy will increase. With such a large sample of atoms, it is possible—but unimaginably unlikely—for entropy to decrease. Disorder is vastly more likely than order.

The arguments that disorder and high entropy are the most probable states are quite convincing. The great Austrian physicist Ludwig Boltzmann (1844–1906)—who, along with Maxwell, made so many contributions to kinetic theory—proved that the entropy of a system in a given state (a macrostate) can be written as

$$S = k \ln W, \tag{15.68}$$

where $k = 1.38\times10^{-23}\text{ J/K}$ is Boltzmann's constant, and $\ln W$ is the natural logarithm of the number of microstates W corresponding to the given macrostate. W is proportional to the probability that the macrostate will occur. Thus entropy is directly related to the probability of a state—the more likely the state, the greater its entropy. Boltzmann proved that this expression for S is equivalent to the definition $\Delta S = Q / T$, which we have used extensively.

Thus the second law of thermodynamics is explained on a very basic level: entropy either remains the same or increases in every process. This phenomenon is due to the extraordinarily small probability of a decrease, based on the extraordinarily larger number of microstates in systems with greater entropy. Entropy *can* decrease, but for any macroscopic system, this outcome is so unlikely that it will never be observed.

Example 15.9 Entropy Increases in a Coin Toss

Suppose you toss 100 coins starting with 60 heads and 40 tails, and you get the most likely result, 50 heads and 50 tails. What is the change in entropy?

Strategy

Noting that the number of microstates is labeled W in Table 15.4 for the 100-coin toss, we can use $\Delta S = S_\text{f} - S_\text{i} = k\ln W_\text{f}$ - $k\ln W_\text{i}$ to calculate the change in entropy.

Solution

The change in entropy is

$$\Delta S = S_\text{f} - S_\text{i} = k\ln W_\text{f} - k\ln W_{\text{i},} \tag{15.69}$$

where the subscript i stands for the initial 60 heads and 40 tails state, and the subscript f for the final 50 heads and 50 tails state. Substituting the values for W from Table 15.4 gives

$$\begin{aligned} \Delta S &= (1.38\times10^{-23}\text{ J/K})[\ln(1.0\times10^{29}) - \ln(1.4\times10^{28})] \\ &= 2.7\times10^{-23}\text{ J/K} \end{aligned} \tag{15.70}$$

Discussion

This increase in entropy means we have moved to a less orderly situation. It is not impossible for further tosses to produce the initial state of 60 heads and 40 tails, but it is less likely. There is about a 1 in 90 chance for that decrease in entropy ($-2.7\times10^{-23}\ \text{J/K}$) to occur. If we calculate the decrease in entropy to move to the most orderly state, we get $\Delta S = -92\times10^{-23}\ \text{J/K}$. There is about a $1 \text{ in } 10^{30}$ chance of this change occurring. So while very small decreases in entropy are unlikely, slightly greater decreases are impossibly unlikely. These probabilities imply, again, that for a macroscopic system, a decrease in entropy is impossible. For example, for heat transfer to occur spontaneously from 1.00 kg of 0°C ice to its 0°C environment, there would be a decrease in entropy of $1.22\times10^{3}\ \text{J/K}$. Given that a ΔS of $10^{-21}\ \text{J/K}$ corresponds to about a $1 \text{ in } 10^{30}$ chance, a decrease of this size ($10^{3}\ \text{J/K}$) is an *utter* impossibility. Even for a milligram of melted ice to spontaneously refreeze is impossible.

Problem-Solving Strategies for Entropy

1. *Examine the situation to determine if entropy is involved.*
2. *Identify the system of interest and draw a labeled diagram of the system showing energy flow.*
3. *Identify exactly what needs to be determined in the problem (identify the unknowns).* A written list is useful.
4. *Make a list of what is given or can be inferred from the problem as stated (identify the knowns).* You must carefully identify the heat transfer, if any, and the temperature at which the process takes place. It is also important to identify the initial and final states.
5. *Solve the appropriate equation for the quantity to be determined (the unknown).* Note that the change in entropy can be determined between any states by calculating it for a reversible process.
6. *Substitute the known value along with their units into the appropriate equation, and obtain numerical solutions complete with units.*
7. *To see if it is reasonable: Does it make sense?* For example, total entropy should increase for any real process or be constant for a reversible process. Disordered states should be more probable and have greater entropy than ordered states.

Glossary

adiabatic process: a process in which no heat transfer takes place

Carnot cycle: a cyclical process that uses only reversible processes, the adiabatic and isothermal processes

Carnot efficiency: the maximum theoretical efficiency for a heat engine

Carnot engine: a heat engine that uses a Carnot cycle

change in entropy: the ratio of heat transfer to temperature Q/T

coefficient of performance: for a heat pump, it is the ratio of heat transfer at the output (the hot reservoir) to the work supplied; for a refrigerator or air conditioner, it is the ratio of heat transfer from the cold reservoir to the work supplied

cyclical process: a process in which the path returns to its original state at the end of every cycle

entropy: a measurement of a system's disorder and its inability to do work in a system

first law of thermodynamics: states that the change in internal energy of a system equals the net heat transfer *into* the system minus the net work done *by* the system

heat engine: a machine that uses heat transfer to do work

heat pump: a machine that generates heat transfer from cold to hot

human metabolism: conversion of food into heat transfer, work, and stored fat

internal energy: the sum of the kinetic and potential energies of a system’s atoms and molecules

irreversible process: any process that depends on path direction

isobaric process: constant-pressure process in which a gas does work

isochoric process: a constant-volume process

isothermal process: a constant-temperature process

macrostate: an overall property of a system

microstate: each sequence within a larger macrostate

Otto cycle: a thermodynamic cycle, consisting of a pair of adiabatic processes and a pair of isochoric processes, that converts heat into work, e.g., the four-stroke engine cycle of intake, compression, ignition, and exhaust

reversible process: a process in which both the heat engine system and the external environment theoretically can be returned to their original states

second law of thermodynamics stated in terms of entropy: the total entropy of a system either increases or remains constant; it never decreases

second law of thermodynamics: heat transfer flows from a hotter to a cooler object, never the reverse, and some heat energy in any process is lost to available work in a cyclical process

statistical analysis: using statistics to examine data, such as counting microstates and macrostates

Section Summary

15.1 The First Law of Thermodynamics

- The first law of thermodynamics is given as $\Delta U = Q - W$, where ΔU is the change in internal energy of a system, Q is the net heat transfer (the sum of all heat transfer into and out of the system), and W is the net work done (the sum of all work done on or by the system).
- Both Q and W are energy in transit; only ΔU represents an independent quantity capable of being stored.
- The internal energy U of a system depends only on the state of the system and not how it reached that state.
- Metabolism of living organisms, and photosynthesis of plants, are specialized types of heat transfer, doing work, and internal energy of systems.

15.2 The First Law of Thermodynamics and Some Simple Processes

- One of the important implications of the first law of thermodynamics is that machines can be harnessed to do work that humans previously did by hand or by external energy supplies such as running water or the heat of the Sun. A machine that uses heat transfer to do work is known as a heat engine.
- There are several simple processes, used by heat engines, that flow from the first law of thermodynamics. Among them are the isobaric, isochoric, isothermal and adiabatic processes.
- These processes differ from one another based on how they affect pressure, volume, temperature, and heat transfer.
- If the work done is performed on the outside environment, work (W) will be a positive value. If the work done is done to the heat engine system, work (W) will be a negative value.
- Some thermodynamic processes, including isothermal and adiabatic processes, are reversible in theory; that is, both the thermodynamic system and the environment can be returned to their initial states. However, because of loss of energy owing to the second law of thermodynamics, complete reversibility does not work in practice.

15.3 Introduction to the Second Law of Thermodynamics: Heat Engines and Their Efficiency

- The two expressions of the second law of thermodynamics are: (i) Heat transfer occurs spontaneously from higher- to lower-temperature bodies but never spontaneously in the reverse direction; and (ii) It is impossible in any system for heat transfer from a reservoir to completely convert to work in a cyclical process in which the system returns to its initial state.
- Irreversible processes depend on path and do not return to their original state. Cyclical processes are processes that return to their original state at the end of every cycle.
- In a cyclical process, such as a heat engine, the net work done by the system equals the net heat transfer into the system, or $W = Q_{\mathrm{h}} - Q_{\mathrm{c}}$, where Q_{h} is the heat transfer from the hot object (hot reservoir), and Q_{c} is the heat transfer into the cold object (cold reservoir).
- Efficiency can be expressed as $Eff = \frac{W}{Q_{\mathrm{h}}}$, the ratio of work output divided by the amount of energy input.
- The four-stroke gasoline engine is often explained in terms of the Otto cycle, which is a repeating sequence of processes that convert heat into work.

15.4 Carnot's Perfect Heat Engine: The Second Law of Thermodynamics Restated

- The Carnot cycle is a theoretical cycle that is the most efficient cyclical process possible. Any engine using the Carnot cycle, which uses only reversible processes (adiabatic and isothermal), is known as a Carnot engine.
- Any engine that uses the Carnot cycle enjoys the maximum theoretical efficiency.
- While Carnot engines are ideal engines, in reality, no engine achieves Carnot's theoretical maximum efficiency, since dissipative processes, such as friction, play a role. Carnot cycles without heat loss may be possible at absolute zero, but this has never been seen in nature.

15.5 Applications of Thermodynamics: Heat Pumps and Refrigerators

- An artifact of the second law of thermodynamics is the ability to heat an interior space using a heat pump. Heat pumps compress cold ambient air and, in so doing, heat it to room temperature without violation of conservation principles.
- To calculate the heat pump's coefficient of performance, use the equation $COP_{\mathrm{hp}} = \frac{Q_{\mathrm{h}}}{W}$.
- A refrigerator is a heat pump; it takes warm ambient air and expands it to chill it.

15.6 Entropy and the Second Law of Thermodynamics: Disorder and the Unavailability of Energy

- Entropy is the loss of energy available to do work.
- Another form of the second law of thermodynamics states that the total entropy of a system either increases or remains constant; it never decreases.
- Entropy is zero in a reversible process; it increases in an irreversible process.
- The ultimate fate of the universe is likely to be thermodynamic equilibrium, where the universal temperature is constant and no energy is available to do work.
- Entropy is also associated with the tendency toward disorder in a closed system.

15.7 Statistical Interpretation of Entropy and the Second Law of Thermodynamics: The Underlying Explanation

- Disorder is far more likely than order, which can be seen statistically.

- The entropy of a system in a given state (a macrostate) can be written as
$$S = k\ln W,$$
where $k = 1.38\times10^{-23}\ \text{J/K}$ is Boltzmann's constant, and $\ln W$ is the natural logarithm of the number of microstates W corresponding to the given macrostate.

Conceptual Questions

15.1 The First Law of Thermodynamics

1. Describe the photo of the tea kettle at the beginning of this section in terms of heat transfer, work done, and internal energy. How is heat being transferred? What is the work done and what is doing it? How does the kettle maintain its internal energy?

2. The first law of thermodynamics and the conservation of energy, as discussed in **Conservation of Energy**, are clearly related. How do they differ in the types of energy considered?

3. Heat transfer Q and work done W are always energy in transit, whereas internal energy U is energy stored in a system. Give an example of each type of energy, and state specifically how it is either in transit or resides in a system.

4. How do heat transfer and internal energy differ? In particular, which can be stored as such in a system and which cannot?

5. If you run down some stairs and stop, what happens to your kinetic energy and your initial gravitational potential energy?

6. Give an explanation of how food energy (calories) can be viewed as molecular potential energy (consistent with the atomic and molecular definition of internal energy).

7. Identify the type of energy transferred to your body in each of the following as either internal energy, heat transfer, or doing work: (a) basking in sunlight; (b) eating food; (c) riding an elevator to a higher floor.

15.2 The First Law of Thermodynamics and Some Simple Processes

8. A great deal of effort, time, and money has been spent in the quest for the so-called perpetual-motion machine, which is defined as a hypothetical machine that operates or produces useful work indefinitely and/or a hypothetical machine that produces more work or energy than it consumes. Explain, in terms of heat engines and the first law of thermodynamics, why or why not such a machine is likely to be constructed.

9. One method of converting heat transfer into doing work is for heat transfer into a gas to take place, which expands, doing work on a piston, as shown in the figure below. (a) Is the heat transfer converted directly to work in an isobaric process, or does it go through another form first? Explain your answer. (b) What about in an isothermal process? (c) What about in an adiabatic process (where heat transfer occurred prior to the adiabatic process)?

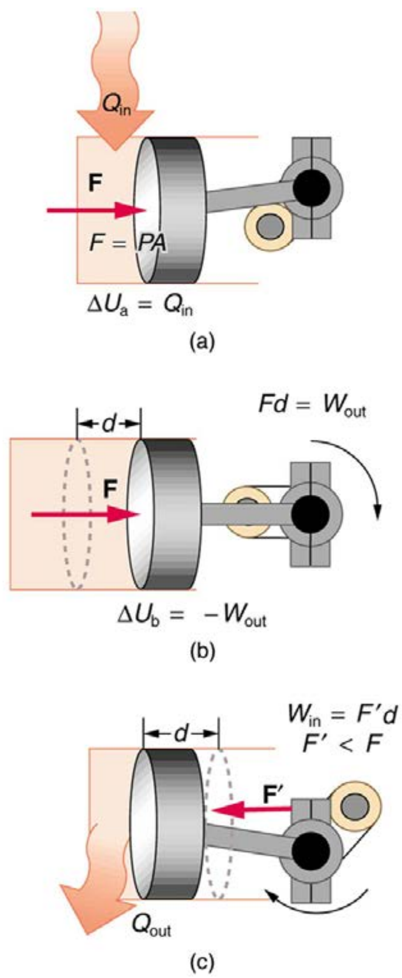


Figure 15.41

10. Would the previous question make any sense for an isochoric process? Explain your answer.

11. We ordinarily say that $\Delta U = 0$ for an isothermal process. Does this assume no phase change takes place? Explain your answer.

12. The temperature of a rapidly expanding gas decreases. Explain why in terms of the first law of thermodynamics. (Hint: Consider whether the gas does work and whether heat transfer occurs rapidly into the gas through conduction.)

13. Which cyclical process represented by the two closed loops, ABCFA and ABDEA, on the PV diagram in the figure below produces the greatest *net* work? Is that process also the one with the smallest work input required to return it to point A? Explain your responses.

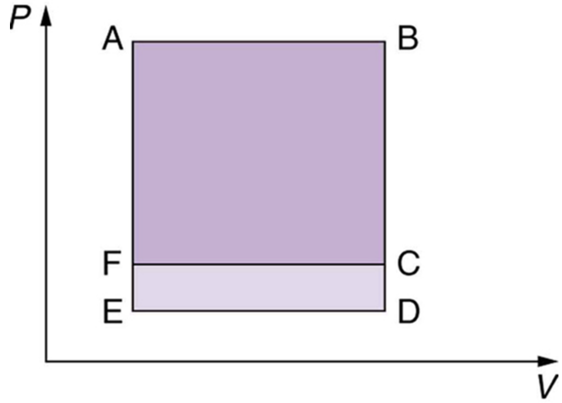


Figure 15.42 The two cyclical processes shown on this PV diagram start with and return the system to the conditions at point A, but they follow different paths and produce different amounts of work.

14. A real process may be nearly adiabatic if it occurs over a very short time. How does the short time span help the process to be adiabatic?

15. It is unlikely that a process can be isothermal unless it is a very slow process. Explain why. Is the same true for isobaric and isochoric processes? Explain your answer.

15.3 Introduction to the Second Law of Thermodynamics: Heat Engines and Their Efficiency

16. Imagine you are driving a car up Pike's Peak in Colorado. To raise a car weighing 1000 kilograms a distance of 100 meters would require about a million joules. You could raise a car 12.5 kilometers with the energy in a gallon of gas. Driving up Pike's Peak (a mere 3000-meter climb) should consume a little less than a quart of gas. But other considerations have to be taken into account. Explain, in terms of efficiency, what factors may keep you from realizing your ideal energy use on this trip.

17. Is a temperature difference necessary to operate a heat engine? State why or why not.

18. Definitions of efficiency vary depending on how energy is being converted. Compare the definitions of efficiency for the human body and heat engines. How does the definition of efficiency in each relate to the type of energy being converted into doing work?

19. Why—other than the fact that the second law of thermodynamics says reversible engines are the most efficient—should heat engines employing reversible processes be more efficient than those employing irreversible processes? Consider that dissipative mechanisms are one cause of irreversibility.

15.4 Carnot's Perfect Heat Engine: The Second Law of Thermodynamics Restated

20. Think about the drinking bird at the beginning of this section (**Figure 15.21**). Although the bird enjoys the theoretical maximum efficiency possible, if left to its own devices over time, the bird will cease "drinking." What are some of the dissipative processes that might cause the bird's motion to cease?

21. Can improved engineering and materials be employed in heat engines to reduce heat transfer into the environment? Can they eliminate heat transfer into the environment entirely?

22. Does the second law of thermodynamics alter the conservation of energy principle?

15.5 Applications of Thermodynamics: Heat Pumps and Refrigerators

23. Explain why heat pumps do not work as well in very cold climates as they do in milder ones. Is the same true of refrigerators?

24. In some Northern European nations, homes are being built without heating systems of any type. They are very well insulated and are kept warm by the body heat of the residents. However, when the residents are not at home, it is still warm in these houses. What is a possible explanation?

25. Why do refrigerators, air conditioners, and heat pumps operate most cost-effectively for cycles with a small difference between T_{h} and T_{c}? (Note that the temperatures of the cycle employed are crucial to its COP.)

26. Grocery store managers contend that there is *less* total energy consumption in the summer if the store is kept at a *low* temperature. Make arguments to support or refute this claim, taking into account that there are numerous refrigerators and freezers in the store.

27. Can you cool a kitchen by leaving the refrigerator door open?

15.6 Entropy and the Second Law of Thermodynamics: Disorder and the Unavailability of Energy

28. A woman shuts her summer cottage up in September and returns in June. No one has entered the cottage in the meantime. Explain what she is likely to find, in terms of the second law of thermodynamics.

29. Consider a system with a certain energy content, from which we wish to extract as much work as possible. Should the system's entropy be high or low? Is this orderly or disorderly? Structured or uniform? Explain briefly.

30. Does a gas become more orderly when it liquefies? Does its entropy change? If so, does the entropy increase or decrease? Explain your answer.

31. Explain how water's entropy can decrease when it freezes without violating the second law of thermodynamics. Specifically, explain what happens to the entropy of its surroundings.

32. Is a uniform-temperature gas more or less orderly than one with several different temperatures? Which is more structured? In which can heat transfer result in work done without heat transfer from another system?

33. Give an example of a spontaneous process in which a system becomes less ordered and energy becomes less available to do work. What happens to the system's entropy in this process?

34. What is the change in entropy in an adiabatic process? Does this imply that adiabatic processes are reversible? Can a process be precisely adiabatic for a macroscopic system?

35. Does the entropy of a star increase or decrease as it radiates? Does the entropy of the space into which it radiates (which has a temperature of about 3 K) increase or decrease? What does this do to the entropy of the universe?

36. Explain why a building made of bricks has smaller entropy than the same bricks in a disorganized pile. Do this by considering the number of ways that each could be formed (the number of microstates in each macrostate).

15.7 Statistical Interpretation of Entropy and the Second Law of Thermodynamics: The Underlying Explanation

37. Explain why a building made of bricks has smaller entropy than the same bricks in a disorganized pile. Do this by considering the number of ways that each could be formed (the number of microstates in each macrostate).

Problems & Exercises

15.1 The First Law of Thermodynamics

38. What is the change in internal energy of a car if you put 12.0 gal of gasoline into its tank? The energy content of gasoline is $1.3\times10^{8}\ \text{J/gal}$. All other factors, such as the car's temperature, are constant.

39. How much heat transfer occurs from a system, if its internal energy decreased by 150 J while it was doing 30.0 J of work?

40. A system does $1.80\times10^{8}\ \text{J}$ of work while $7.50\times10^{8}\ \text{J}$ of heat transfer occurs to the environment. What is the change in internal energy of the system assuming no other changes (such as in temperature or by the addition of fuel)?

41. What is the change in internal energy of a system which does $4.50\times10^{5}\ \text{J}$ of work while $3.00\times10^{6}\ \text{J}$ of heat transfer occurs into the system, and $8.00\times10^{6}\ \text{J}$ of heat transfer occurs to the environment?

42. Suppose a woman does 500 J of work and 9500 J of heat transfer occurs into the environment in the process. (a) What is the decrease in her internal energy, assuming no change in temperature or consumption of food? (That is, there is no other energy transfer.) (b) What is her efficiency?

43. (a) How much food energy will a man metabolize in the process of doing 35.0 kJ of work with an efficiency of 5.00%? (b) How much heat transfer occurs to the environment to keep his temperature constant? Explicitly show how you follow the steps in the Problem-Solving Strategy for thermodynamics found in **Problem-Solving Strategies for Thermodynamics**.

44. (a) What is the average metabolic rate in watts of a man who metabolizes 10,500 kJ of food energy in one day? (b) What is the maximum amount of work in joules he can do without breaking down fat, assuming a maximum efficiency of 20.0%? (c) Compare his work output with the daily output of a 187-W (0.250-horsepower) motor.

45. (a) How long will the energy in a 1470-kJ (350-kcal) cup of yogurt last in a woman doing work at the rate of 150 W with an efficiency of 20.0% (such as in leisurely climbing stairs)? (b) Does the time found in part (a) imply that it is easy to consume more food energy than you can reasonably expect to work off with exercise?

46. (a) A woman climbing the Washington Monument metabolizes $6.00\times10^{2}\ \text{kJ}$ of food energy. If her efficiency is 18.0%, how much heat transfer occurs to the environment to keep her temperature constant? (b) Discuss the amount of heat transfer found in (a). Is it consistent with the fact that you quickly warm up when exercising?

15.2 The First Law of Thermodynamics and Some Simple Processes

47. A car tire contains $0.0380\ \text{m}^3$ of air at a pressure of $2.20\times10^{5}\ \text{N/m}^2$ (about 32 psi). How much more internal energy does this gas have than the same volume has at zero gauge pressure (which is equivalent to normal atmospheric pressure)?

48. A helium-filled toy balloon has a gauge pressure of 0.200 atm and a volume of 10.0 L. How much greater is the internal energy of the helium in the balloon than it would be at zero gauge pressure?

49. Steam to drive an old-fashioned steam locomotive is supplied at a constant gauge pressure of $1.75\times10^{6}\ \text{N/m}^2$ (about 250 psi) to a piston with a 0.200-m radius. (a) By calculating $P\Delta V$, find the work done by the steam when the piston moves 0.800 m. Note that this is the net work output, since gauge pressure is used. (b) Now find the amount of work by calculating the force exerted times the distance traveled. Is the answer the same as in part (a)?

50. A hand-driven tire pump has a piston with a 2.50-cm diameter and a maximum stroke of 30.0 cm. (a) How much work do you do in one stroke if the average gauge pressure is $2.40\times10^{5}\ \text{N/m}^2$ (about 35 psi)? (b) What average force do you exert on the piston, neglecting friction and gravitational force?

51. Calculate the net work output of a heat engine following path ABCDA in the figure below.

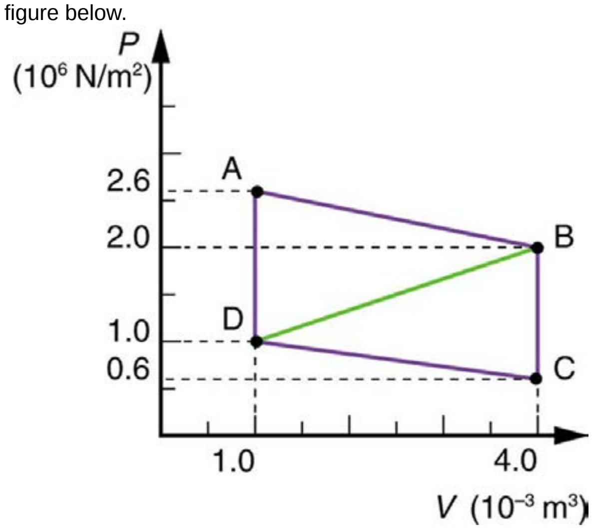


Figure 15.43

52. What is the net work output of a heat engine that follows path ABDA in the figure above, with a straight line from B to D? Why is the work output less than for path ABCDA? Explicitly show how you follow the steps in the **Problem-Solving Strategies for Thermodynamics**.

53. Unreasonable Results

What is wrong with the claim that a cyclical heat engine does 4.00 kJ of work on an input of 24.0 kJ of heat transfer while 16.0 kJ of heat transfers to the environment?

54. (a) A cyclical heat engine, operating between temperatures of $450^{\circ}\ \text{C}$ and $150^{\circ}\ \text{C}$ produces 4.00 MJ of work on a heat transfer of 5.00 MJ into the engine. How much heat transfer occurs to the environment? (b) What is unreasonable about the engine? (c) Which premise is unreasonable?

55. Construct Your Own Problem

Consider a car's gasoline engine. Construct a problem in which you calculate the maximum efficiency this engine can have. Among the things to consider are the effective hot and cold reservoir temperatures. Compare your calculated efficiency with the actual efficiency of car engines.

56. Construct Your Own Problem

Consider a car trip into the mountains. Construct a problem in which you calculate the overall efficiency of the car for the trip as a ratio of kinetic and potential energy gained to fuel consumed. Compare this efficiency to the thermodynamic efficiency quoted for gasoline engines and discuss why the thermodynamic efficiency is so much greater. Among the factors to be considered are the gain in altitude and speed, the mass of the car, the distance traveled, and typical fuel economy.

15.3 Introduction to the Second Law of Thermodynamics: Heat Engines and Their Efficiency

57. A certain heat engine does 10.0 kJ of work and 8.50 kJ of heat transfer occurs to the environment in a cyclical process. (a) What was the heat transfer into this engine? (b) What was the engine's efficiency?

58. With $2.56\times10^{6}\ \text{J}$ of heat transfer into this engine, a given cyclical heat engine can do only $1.50\times10^{5}\ \text{J}$ of work. (a) What is the engine's efficiency? (b) How much heat transfer to the environment takes place?

59. (a) What is the work output of a cyclical heat engine having a 22.0% efficiency and $6.00\times10^{9}\ \text{J}$ of heat transfer into the engine? (b) How much heat transfer occurs to the environment?

60. (a) What is the efficiency of a cyclical heat engine in which 75.0 kJ of heat transfer occurs to the environment for every 95.0 kJ of heat transfer into the engine? (b) How much work does it produce for 100 kJ of heat transfer into the engine?

61. The engine of a large ship does $2.00\times10^{8}\text{ J}$ of work with an efficiency of 5.00%. (a) How much heat transfer occurs to the environment? (b) How many barrels of fuel are consumed, if each barrel produces $6.00\times10^{9}\text{ J}$ of heat transfer when burned?

62. (a) How much heat transfer occurs to the environment by an electrical power station that uses $1.25\times10^{14}\text{ J}$ of heat transfer into the engine with an efficiency of 42.0%? (b) What is the ratio of heat transfer to the environment to work output? (c) How much work is done?

63. Assume that the turbines at a coal-powered power plant were upgraded, resulting in an improvement in efficiency of 3.32%. Assume that prior to the upgrade the power station had an efficiency of 36% and that the heat transfer into the engine in one day is still the same at $2.50\times10^{14}\text{ J}$. (a) How much more electrical energy is produced due to the upgrade? (b) How much less heat transfer occurs to the environment due to the upgrade?

64. This problem compares the energy output and heat transfer to the environment by two different types of nuclear power stations—one with the normal efficiency of 34.0%, and another with an improved efficiency of 40.0%. Suppose both have the same heat transfer into the engine in one day, $2.50\times10^{14}\text{ J}$. (a) How much more electrical energy is produced by the more efficient power station? (b) How much less heat transfer occurs to the environment by the more efficient power station? (One type of more efficient nuclear power station, the gas-cooled reactor, has not been reliable enough to be economically feasible in spite of its greater efficiency.)

15.4 Carnot's Perfect Heat Engine: The Second Law of Thermodynamics Restated

65. A certain gasoline engine has an efficiency of 30.0%. What would the hot reservoir temperature be for a Carnot engine having that efficiency, if it operates with a cold reservoir temperature of 200°C?

66. A gas-cooled nuclear reactor operates between hot and cold reservoir temperatures of 700°C and 27.0°C. (a) What is the maximum efficiency of a heat engine operating between these temperatures? (b) Find the ratio of this efficiency to the Carnot efficiency of a standard nuclear reactor (found in **Example 15.4**).

67. (a) What is the hot reservoir temperature of a Carnot engine that has an efficiency of 42.0% and a cold reservoir temperature of 27.0°C? (b) What must the hot reservoir temperature be for a real heat engine that achieves 0.700 of the maximum efficiency, but still has an efficiency of 42.0% (and a cold reservoir at 27.0°C)? (c) Does your answer imply practical limits to the efficiency of car gasoline engines?

68. Steam locomotives have an efficiency of 17.0% and operate with a hot steam temperature of 425°C. (a) What would the cold reservoir temperature be if this were a Carnot engine? (b) What would the maximum efficiency of this steam engine be if its cold reservoir temperature were 150°C?

69. Practical steam engines utilize 450°C steam, which is later exhausted at 270°C. (a) What is the maximum efficiency that such a heat engine can have? (b) Since 270°C steam is still quite hot, a second steam engine is sometimes operated using the exhaust of the first. What is the maximum efficiency of the second engine if its exhaust has a temperature of 150°C? (c) What is the overall efficiency of the two engines? (d) Show that this is the same efficiency as a single Carnot engine operating between 450°C and 150°C. Explicitly show how you follow the steps in the **Problem-Solving Strategies for Thermodynamics**.

70. A coal-fired electrical power station has an efficiency of 38%. The temperature of the steam leaving the boiler is 550°C. What percentage of the maximum efficiency does this station obtain? (Assume the temperature of the environment is 20°C.)

71. Would you be willing to financially back an inventor who is marketing a device that she claims has 25 kJ of heat transfer at 600 K, has heat transfer to the environment at 300 K, and does 12 kJ of work? Explain your answer.

72. Unreasonable Results

(a) Suppose you want to design a steam engine that has heat transfer to the environment at 270°C and has a Carnot efficiency of 0.800. What temperature of hot steam must you use? (b) What is unreasonable about the temperature? (c) Which premise is unreasonable?

73. Unreasonable Results

Calculate the cold reservoir temperature of a steam engine that uses hot steam at 450°C and has a Carnot efficiency of 0.700. (b) What is unreasonable about the temperature? (c) Which premise is unreasonable?

15.5 Applications of Thermodynamics: Heat Pumps and Refrigerators

74. What is the coefficient of performance of an ideal heat pump that has heat transfer from a cold temperature of -25.0°C to a hot temperature of 40.0°C?

75. Suppose you have an ideal refrigerator that cools an environment at -20.0°C and has heat transfer to another environment at 50.0°C. What is its coefficient of performance?

76. What is the best coefficient of performance possible for a hypothetical refrigerator that could make liquid nitrogen at -200°C and has heat transfer to the environment at 35.0°C?

77. In a very mild winter climate, a heat pump has heat transfer from an environment at 5.00°C to one at 35.0°C. What is the best possible coefficient of performance for these temperatures? Explicitly show how you follow the steps in the **Problem-Solving Strategies for Thermodynamics**.

78. (a) What is the best coefficient of performance for a heat pump that has a hot reservoir temperature of 50.0°C and a cold reservoir temperature of -20.0°C? (b) How much heat transfer occurs into the warm environment if $3.60\times10^{7}\text{ J}$ of work ($10.0\text{kW}\cdot\text{h}$) is put into it? (c) If the cost of this work input is $10.0\text{ cents/kW}\cdot\text{h}$, how does its cost compare with the direct heat transfer achieved by burning natural gas at a cost of 85.0 cents per therm. (A therm is a common unit of energy for natural gas and equals $1.055\times10^{8}\text{ J}$.)

79. (a) What is the best coefficient of performance for a refrigerator that cools an environment at -30.0°C and has heat transfer to another environment at 45.0°C? (b) How much work in joules must be done for a heat transfer of 4186 kJ from the cold environment? (c) What is the cost of doing this if the work costs 10.0 cents per $3.60\times10^{6}\text{ J}$ (a kilowatt-hour)? (d) How many kJ of heat transfer occurs into the warm environment? (e) Discuss what type of refrigerator might operate between these temperatures.

80. Suppose you want to operate an ideal refrigerator with a cold temperature of -10.0°C, and you would like it to have a coefficient of performance of 7.00. What is the hot reservoir temperature for such a refrigerator?

81. An ideal heat pump is being considered for use in heating an environment with a temperature of 22.0°C. What is the cold reservoir temperature if the pump is to have a coefficient of performance of 12.0?

82. A 4-ton air conditioner removes $5.06\times10^{7}\text{ J}$ (48,000 British thermal units) from a cold environment in 1.00 h. (a) What energy input in joules is necessary to do this if the air conditioner has an energy efficiency rating (EER) of 12.0? (b) What is the cost of doing this if the work costs 10.0 cents per $3.60\times10^{6}\text{ J}$ (one kilowatt-hour)? (c) Discuss whether this cost seems realistic. Note that the energy efficiency rating (EER) of an air conditioner or refrigerator is defined to be the number of British thermal units of heat transfer from a cold environment per hour divided by the watts of power input.

83. Show that the coefficients of performance of refrigerators and heat pumps are related by $COP_{\text{ref}} = COP_{\text{hp}} - 1$.

Start with the definitions of the COP s and the conservation of energy relationship between Q_{h} , Q_{c} , and W .

15.6 Entropy and the Second Law of Thermodynamics: Disorder and the Unavailability of Energy

84. (a) On a winter day, a certain house loses $5.00\times10^{8}\text{ J}$ of heat to the outside (about 500,000 Btu). What is the total change in entropy due to this heat transfer alone, assuming an average indoor temperature of 21.0° C and an average outdoor temperature of 5.00° C ? (b) This large change in entropy implies a large amount of energy has become unavailable to do work. Where do we find more energy when such energy is lost to us?

85. On a hot summer day, $4.00\times10^{6}\text{ J}$ of heat transfer into a parked car takes place, increasing its temperature from 35.0° C to 45.0° C . What is the increase in entropy of the car due to this heat transfer alone?

86. A hot rock ejected from a volcano's lava fountain cools from 1100° C to 40.0° C , and its entropy decreases by 950 J/K. How much heat transfer occurs from the rock?

87. When $1.60\times10^{5}\text{ J}$ of heat transfer occurs into a meat pie initially at 20.0° C , its entropy increases by 480 J/K. What is its final temperature?

88. The Sun radiates energy at the rate of $3.80\times10^{26}\text{ W}$ from its 5500° C surface into dark empty space (a negligible fraction radiates onto Earth and the other planets). The effective temperature of deep space is -270° C . (a) What is the increase in entropy in one day due to this heat transfer? (b) How much work is made unavailable?

89. (a) In reaching equilibrium, how much heat transfer occurs from 1.00 kg of water at 40.0° C when it is placed in contact with 1.00 kg of 20.0° C water in reaching equilibrium? (b) What is the change in entropy due to this heat transfer? (c) How much work is made unavailable, taking the lowest temperature to be 20.0° C ? Explicitly show how you follow the steps in the **Problem-Solving Strategies for Entropy**.

90. What is the decrease in entropy of 25.0 g of water that condenses on a bathroom mirror at a temperature of 35.0° C , assuming no change in temperature and given the latent heat of vaporization to be 2450 kJ/kg?

91. Find the increase in entropy of 1.00 kg of liquid nitrogen that starts at its boiling temperature, boils, and warms to 20.0° C at constant pressure.

92. A large electrical power station generates 1000 MW of electricity with an efficiency of 35.0%. (a) Calculate the heat transfer to the power station, Q_{h} , in one day. (b) How much heat transfer Q_{c} occurs to the environment in one day? (c) If the heat transfer in the cooling towers is from 35.0° C water into the local air mass, which increases in temperature from 18.0° C to 20.0° C , what is the total increase in entropy due to this heat transfer? (d) How much energy becomes unavailable to do work because of this increase in entropy, assuming an 18.0° C lowest temperature? (Part of Q_{c} could be utilized to operate heat engines or for simply heating the surroundings, but it rarely is.)

93. (a) How much heat transfer occurs from 20.0 kg of 90.0° C water placed in contact with 20.0 kg of 10.0° C water, producing a final temperature of 50.0° C ? (b) How much work could a Carnot engine do with this heat transfer, assuming it operates between two reservoirs at constant temperatures of 90.0° C and 10.0° C ? (c) What increase in entropy is produced by mixing 20.0 kg of 90.0° C water with 20.0 kg of 10.0° C water? (d) Calculate the amount of work made unavailable by this mixing using a low temperature of 10.0° C , and compare it with the work done by the Carnot engine. Explicitly show how you follow the steps in the **Problem-Solving Strategies for Entropy**. (e) Discuss how everyday processes make increasingly more energy unavailable to do work, as implied by this problem.

15.7 Statistical Interpretation of Entropy and the Second Law of Thermodynamics: The Underlying Explanation

94. Using **Table 15.4**, verify the contention that if you toss 100 coins each second, you can expect to get 100 heads or 100 tails once in 2×10^{22} years; calculate the time to two-digit accuracy.

95. What percent of the time will you get something in the range from 60 heads and 40 tails through 40 heads and 60 tails when tossing 100 coins? The total number of microstates in that range is 1.22×10^{30} . (Consult **Table 15.4**.)

96. (a) If tossing 100 coins, how many ways (microstates) are there to get the three most likely macrostates of 49 heads and 51 tails, 50 heads and 50 tails, and 51 heads and 49 tails? (b) What percent of the total possibilities is this? (Consult **Table 15.4**.)

97. (a) What is the change in entropy if you start with 100 coins in the 45 heads and 55 tails macrostate, toss them, and get 51 heads and 49 tails? (b) What if you get 75 heads and 25 tails? (c) How much more likely is 51 heads and 49 tails than 75 heads and 25 tails? (d) Does either outcome violate the second law of thermodynamics?

98. (a) What is the change in entropy if you start with 10 coins in the 5 heads and 5 tails macrostate, toss them, and get 2 heads and 8 tails? (b) How much more likely is 5 heads and 5 tails than 2 heads and 8 tails? (Take the ratio of the number of microstates to find out.) (c) If you were betting on 2 heads and 8 tails would you accept odds of 252 to 45? Explain why or why not.

Table 15.5 10-Coin Toss

Macrostate		Number of Microstates (W)
Heads	Tails	
10	0	1
9	1	10
8	2	45
7	3	120
6	4	210
5	5	252
4	6	210
3	7	120
2	8	45
1	9	10
0	10	1
		Total: 1024

99. (a) If you toss 10 coins, what percent of the time will you get the three most likely macrostates (6 heads and 4 tails, 5 heads and 5 tails, 4 heads and 6 tails)? (b) You can realistically toss 10 coins and count the number of heads and tails about twice a minute. At that rate, how long will it take on average to get either 10 heads and 0 tails or 0 heads and 10 tails?

100. (a) Construct a table showing the macrostates and all of the individual microstates for tossing 6 coins. (Use **Table 15.5** as a guide.) (b) How many macrostates are there? (c) What is the total number of microstates? (d) What percent chance is there of tossing 5 heads and 1 tail? (e) How much more likely are you to toss 3 heads and 3 tails than 5 heads and 1 tail? (Take the ratio of the number of microstates to find out.)

101. In an air conditioner, 12.65 MJ of heat transfer occurs from a cold environment in 1.00 h. (a) What mass of ice melting would involve the same heat transfer? (b) How many hours of operation would be equivalent to melting 900 kg of ice? (c) If ice costs 20 cents per kg, do you think the air conditioner could be operated more cheaply than by simply using ice? Describe in detail how you evaluate the relative costs.

16 OSCILLATORY MOTION AND WAVES

Figure 16.1 There are at least four types of waves in this picture—only the water waves are evident. There are also sound waves, light waves, and waves on the guitar strings. (credit: John Norton)

Learning Objectives

Introduction to Oscillatory Motion and Waves

What do an ocean buoy, a child in a swing, the cone inside a speaker, a guitar, atoms in a crystal, the motion of chest cavities, and the beating of hearts all have in common? They all **oscillate**—-that is, they move back and forth between two points. Many systems oscillate, and they have certain characteristics in common. All oscillations involve force and energy. You push a child in a swing to get the motion started. The energy of atoms vibrating in a crystal can be increased with heat. You put energy into a guitar string when you pluck it.

Some oscillations create **waves**. A guitar creates sound waves. You can make water waves in a swimming pool by slapping the water with your hand. You can no doubt think of other types of waves. Some, such as water waves, are visible. Some, such as sound waves, are not. But *every wave is a disturbance that moves from its source and carries energy*. Other examples of waves include earthquakes and visible light. Even subatomic particles, such as electrons, can behave like waves.

By studying oscillatory motion and waves, we shall find that a small number of underlying principles describe all of them and that wave phenomena are more common than you have ever imagined. We begin by studying the type of force that underlies the simplest oscillations and waves. We will then expand our exploration of oscillatory motion and waves to include concepts such as simple harmonic motion, uniform circular motion, and damped harmonic motion. Finally, we will explore what happens when two or more waves share the same space, in the phenomena known as superposition and interference.

16.1 Hooke's Law: Stress and Strain Revisited

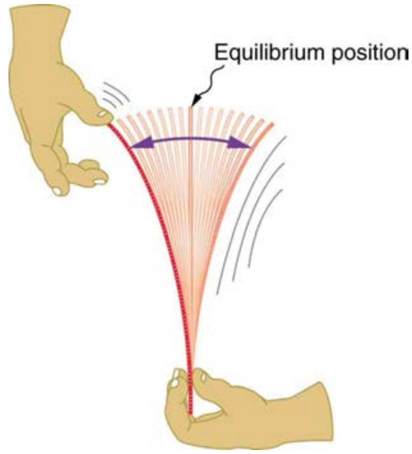


Figure 16.2 When displaced from its vertical equilibrium position, this plastic ruler oscillates back and forth because of the restoring force opposing displacement. When the ruler is on the left, there is a force to the right, and vice versa.

Newton's first law implies that an object oscillating back and forth is experiencing forces. Without force, the object would move in a straight line at a constant speed rather than oscillate. Consider, for example, plucking a plastic ruler to the left as shown in **Figure 16.2**. The deformation of the ruler creates a force in the opposite direction, known as a **restoring force**. Once released, the restoring force causes the ruler to move back toward its stable equilibrium position, where the net force on it is zero. However, by the time the ruler gets there, it gains momentum and continues to move to the right, producing the opposite deformation. It is then forced to the left, back through equilibrium, and the process is repeated until dissipative forces dampen the motion. These forces remove mechanical energy from the system, gradually reducing the motion until the ruler comes to rest.

The simplest oscillations occur when the restoring force is directly proportional to displacement. When stress and strain were covered in **Newton's Third Law of Motion**, the name was given to this relationship between force and displacement was Hooke's law:

$$F=-kx. \tag{16.1}$$

Here, F is the restoring force, x is the displacement from equilibrium or **deformation**, and k is a constant related to the difficulty in deforming the system. The minus sign indicates the restoring force is in the direction opposite to the displacement.

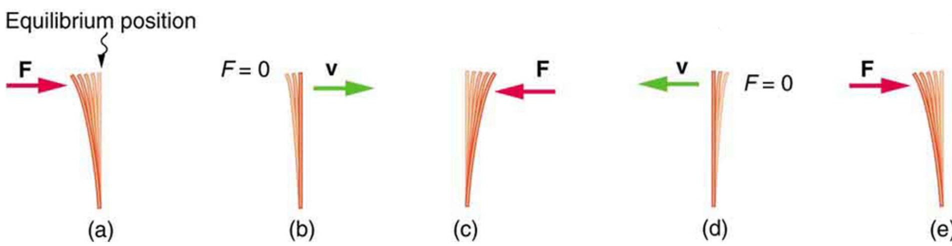


Figure 16.3 (a) The plastic ruler has been released, and the restoring force is returning the ruler to its equilibrium position. (b) The net force is zero at the equilibrium position, but the ruler has momentum and continues to move to the right. (c) The restoring force is in the opposite direction. It stops the ruler and moves it back toward equilibrium again. (d) Now the ruler has momentum to the left. (e) In the absence of damping (caused by frictional forces), the ruler reaches its original position. From there, the motion will repeat itself.

The **force constant** k is related to the rigidity (or stiffness) of a system—the larger the force constant, the greater the restoring force, and the stiffer the system. The units of k are newtons per meter (N/m). For example, k is directly related to Young's modulus when we stretch a string. **Figure 16.4** shows a graph of the absolute value of the restoring force versus the displacement for a system that can be described by Hooke's law—a simple spring in this case. The slope of the graph equals the force constant k in newtons per meter. A common physics laboratory exercise is to measure restoring forces created by springs, determine if they follow Hooke's law, and calculate their force constants if they do.

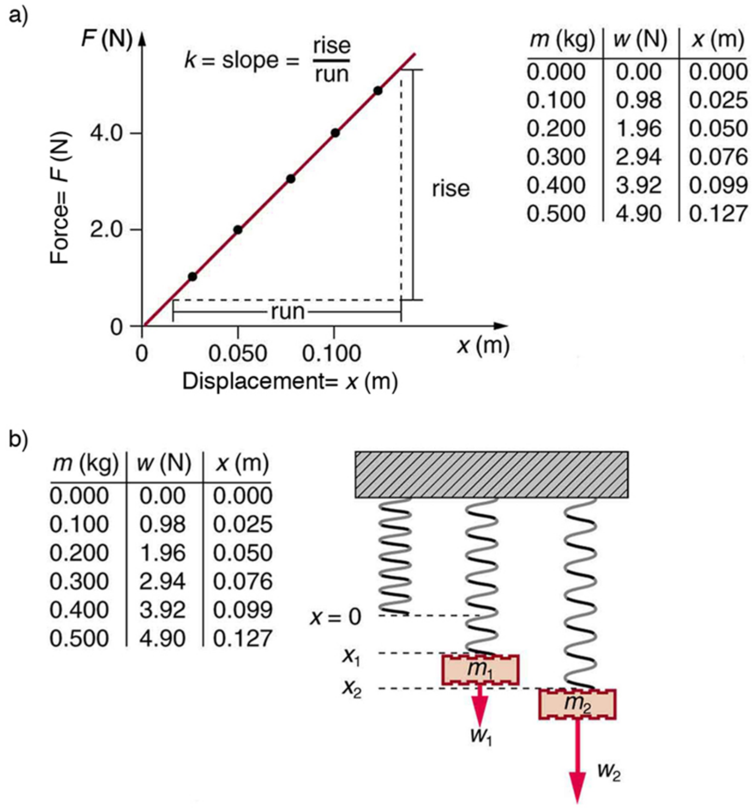


Figure 16.4 (a) A graph of absolute value of the restoring force versus displacement is displayed. The fact that the graph is a straight line means that the system obeys Hooke's law. The slope of the graph is the force constant k . (b) The data in the graph were generated by measuring the displacement of a spring from equilibrium while supporting various weights. The restoring force equals the weight supported, if the mass is stationary.

Example 16.1 How Stiff Are Car Springs?

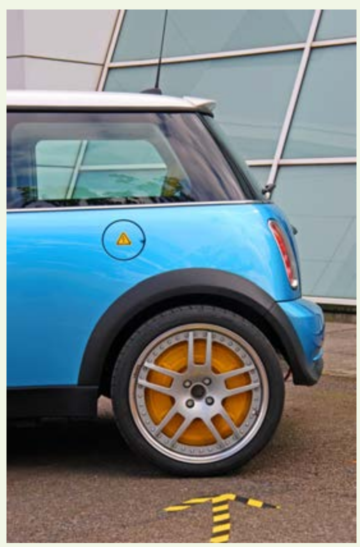


Figure 16.5 The mass of a car increases due to the introduction of a passenger. This affects the displacement of the car on its suspension system. (credit: exfordy on Flickr)

What is the force constant for the suspension system of a car that settles 1.20 cm when an 80.0-kg person gets in?

Strategy

Consider the car to be in its equilibrium position $x=0$ before the person gets in. The car then settles down 1.20 cm, which means it is displaced to a position $x=-1.20\times10^{-2}\text{ m}$. At that point, the springs supply a restoring force F equal to the person's weight $w=mg=(80.0\text{ kg})\left(9.80\text{ m/s}^2\right)=784\text{ N}$. We take this force to be F in Hooke's law. Knowing F and x , we can then solve the force constant k .

Solution

1. Solve Hooke's law, $F=-kx$, for k :

$$k = -\frac{F}{x}. \tag{16.2}$$

Substitute known values and solve k:

$$\begin{aligned} k &= -\frac{784\ \text{N}}{-1.20\times10^{-2}\ \text{m}} \\ &= 6.53\times10^{4}\ \text{N/m}. \end{aligned} \tag{16.3}$$

Discussion

Note that F and x have opposite signs because they are in opposite directions—the restoring force is up, and the displacement is down. Also, note that the car would oscillate up and down when the person got in if it were not for damping (due to frictional forces) provided by shock absorbers. Bouncing cars are a sure sign of bad shock absorbers.

Energy in Hooke's Law of Deformation

In order to produce a deformation, work must be done. That is, a force must be exerted through a distance, whether you pluck a guitar string or compress a car spring. If the only result is deformation, and no work goes into thermal, sound, or kinetic energy, then all the work is initially stored in the deformed object as some form of potential energy. The potential energy stored in a spring is $\text{PE}_{\text{el}} = \frac{1}{2}kx^2$. Here, we generalize the idea to elastic potential energy for a deformation of any system that can be described by Hooke's law. Hence,

$$\text{PE}_{\text{el}} = \frac{1}{2}kx^2, \tag{16.4}$$

where PE_{el} is the **elastic potential energy** stored in any deformed system that obeys Hooke's law and has a displacement x from equilibrium and a force constant k .

It is possible to find the work done in deforming a system in order to find the energy stored. This work is performed by an applied force F_{app} . The applied force is exactly opposite to the restoring force (action-reaction), and so $F_{\text{app}} = kx$. Figure 16.6 shows a graph of the applied force versus deformation x for a system that can be described by Hooke's law. Work done on the system is force multiplied by distance, which equals the area under the curve or $(1/2)kx^2$ (Method A in the figure). Another way to determine the work is to note that the force increases linearly from 0 to kx , so that the average force is $(1/2)kx$, the distance moved is x , and thus $W = F_{\text{app}}d = [(1/2)kx](x) = (1/2)kx^2$ (Method B in the figure).

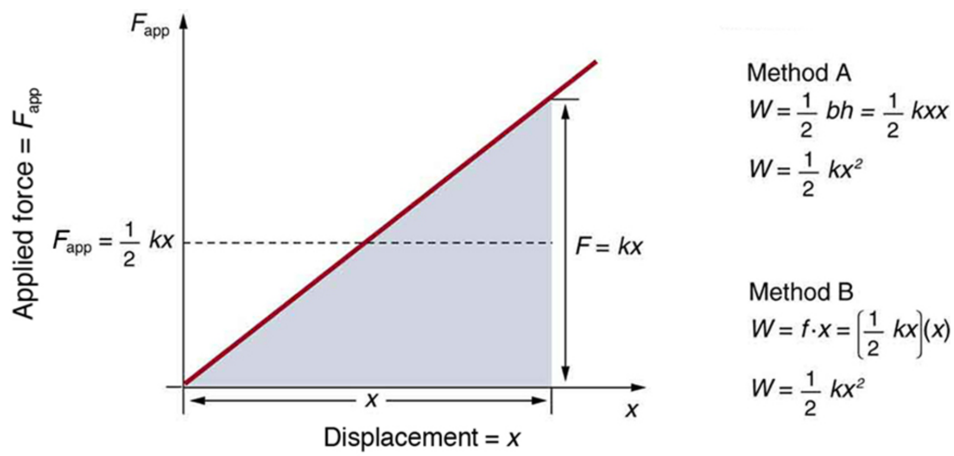


Figure 16.6 A graph of applied force versus distance for the deformation of a system that can be described by Hooke's law is displayed. The work done on the system equals the area under the graph or the area of the triangle, which is half its base multiplied by its height, or $W = (1/2)kx^2$.

Example 16.2 Calculating Stored Energy: A Tranquilizer Gun Spring

We can use a toy gun's spring mechanism to ask and answer two simple questions: (a) How much energy is stored in the spring of a tranquilizer gun that has a force constant of 50.0 N/m and is compressed 0.150 m? (b) If you neglect friction and the mass of the spring, at what speed will a 2.00-g projectile be ejected from the gun?

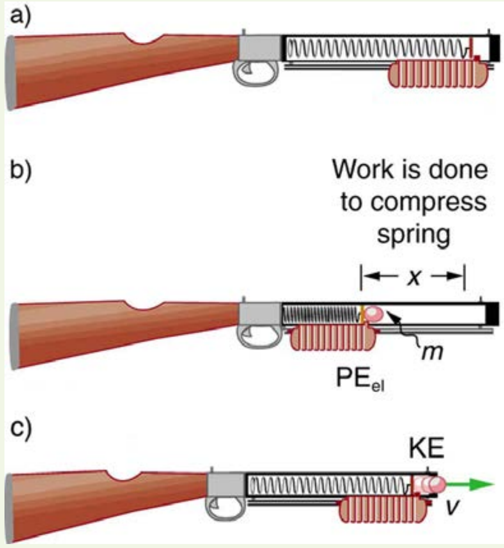


Figure 16.7 (a) In this image of the gun, the spring is uncompressed before being cocked. (b) The spring has been compressed a distance x , and the projectile is in place. (c) When released, the spring converts elastic potential energy PE_{el} into kinetic energy.

Strategy for a

(a): The energy stored in the spring can be found directly from elastic potential energy equation, because k and x are given.

Solution for a

Entering the given values for k and x yields

$$\begin{aligned}\text{PE}_{\text{el}} &= \frac{1}{2}kx^2 = \frac{1}{2}(50.0\text{ N/m})(0.150\text{ m})^2 = 0.563\text{ N}\cdot\text{m} \\ &= 0.563\text{ J}\end{aligned} \tag{16.5}$$

Strategy for b

Because there is no friction, the potential energy is converted entirely into kinetic energy. The expression for kinetic energy can be solved for the projectile's speed.

Solution for b

1. Identify known quantities:

$$\text{KE}_{\text{f}} = \text{PE}_{\text{el}} \text{ or } 1/2mv^2 = (1/2)kx^2 = \text{PE}_{\text{el}} = 0.563\text{ J} \tag{16.6}$$

2. Solve for v :

$$v = \left[\frac{2\text{PE}_{\text{el}}}{m}\right]^{1/2} = \left[\frac{2(0.563\text{ J})}{0.002\text{ kg}}\right]^{1/2} = 23.7(\text{J/kg})^{1/2} \tag{16.7}$$

3. Convert units: 23.7 m/s

Discussion

(a) and (b): This projectile speed is impressive for a tranquilizer gun (more than 80 km/h). The numbers in this problem seem reasonable. The force needed to compress the spring is small enough for an adult to manage, and the energy imparted to the dart is small enough to limit the damage it might do. Yet, the speed of the dart is great enough for it to travel an acceptable distance.

Check your Understanding

Envision holding the end of a ruler with one hand and deforming it with the other. When you let go, you can see the oscillations of the ruler. In what way could you modify this simple experiment to increase the rigidity of the system?

Solution

You could hold the ruler at its midpoint so that the part of the ruler that oscillates is half as long as in the original experiment.

Check your Understanding

If you apply a deforming force on an object and let it come to equilibrium, what happened to the work you did on the system?

Solution

It was stored in the object as potential energy.

16.2 Period and Frequency in Oscillations

Figure 16.8 The strings on this guitar vibrate at regular time intervals. (credit: JAR)

When you pluck a guitar string, the resulting sound has a steady tone and lasts a long time. Each successive vibration of the string takes the same time as the previous one. We define **periodic motion** to be a motion that repeats itself at regular time intervals, such as exhibited by the guitar string or by an object on a spring moving up and down. The time to complete one oscillation remains constant and is called the **period** T. Its units are usually seconds, but may be any convenient unit of time. The word period refers to the time for some event whether repetitive or not; but we shall be primarily interested in periodic motion, which is by definition repetitive. A concept closely related to period is the frequency of an event. For example, if you get a paycheck twice a month, the frequency of payment is two per month and the period between checks is half a month. **Frequency** f is defined to be the number of events per unit time. For periodic motion, frequency is the number of oscillations per unit time. The relationship between frequency and period is

$$f = \frac{1}{T}. \tag{16.8}$$

The SI unit for frequency is the *cycle per second*, which is defined to be a *hertz* (Hz):

$$1 \text{ Hz} = 1\frac{\text{cycle}}{\text{sec}} \text{or } 1 \text{ Hz} = \frac{1}{\text{s}} \tag{16.9}$$

A cycle is one complete oscillation. Note that a vibration can be a single or multiple event, whereas oscillations are usually repetitive for a significant number of cycles.

Example 16.3 Determine the Frequency of Two Oscillations: Medical Ultrasound and the Period of Middle C

We can use the formulas presented in this module to determine both the frequency based on known oscillations and the oscillation based on a known frequency. Let's try one example of each. (a) A medical imaging device produces ultrasound by oscillating with a period of 0.400 µs. What is the frequency of this oscillation? (b) The frequency of middle C on a typical musical instrument is 264 Hz. What is the time for one complete oscillation?

Strategy

Both questions (a) and (b) can be answered using the relationship between period and frequency. In question (a), the period T is given and we are asked to find frequency f. In question (b), the frequency f is given and we are asked to find the period T.

Solution a

1. Substitute $0.400\ \mu\text{s}$ for T in $f = \frac{1}{T}$:

$$f = \frac{1}{T} = \frac{1}{0.400\times10^{-6}\text{ s}}. \tag{16.10}$$

Solve to find

$$f = 2.50\times10^{6}\text{ Hz}. \tag{16.11}$$

Discussion a

The frequency of sound found in (a) is much higher than the highest frequency that humans can hear and, therefore, is called ultrasound. Appropriate oscillations at this frequency generate ultrasound used for noninvasive medical diagnoses, such as observations of a fetus in the womb.

Solution b

1. Identify the known values:
The time for one complete oscillation is the period T:

$$f = \frac{1}{T}. \tag{16.12}$$

2. Solve for T:

$$T = \frac{1}{f}. \tag{16.13}$$

3. Substitute the given value for the frequency into the resulting expression:

$$T = \frac{1}{f} = \frac{1}{264\text{ Hz}} = \frac{1}{264\text{ cycles/s}} = 3.79\times10^{-3}\text{ s} = 3.79\text{ ms.} \tag{16.14}$$

Discussion

The period found in (b) is the time per cycle, but this value is often quoted as simply the time in convenient units (ms or milliseconds in this case).

Check your Understanding

Identify an event in your life (such as receiving a paycheck) that occurs regularly. Identify both the period and frequency of this event.

Solution

I visit my parents for dinner every other Sunday. The frequency of my visits is 26 per calendar year. The period is two weeks.

16.3 Simple Harmonic Motion: A Special Periodic Motion

The oscillations of a system in which the net force can be described by Hooke's law are of special importance, because they are very common. They are also the simplest oscillatory systems. **Simple Harmonic Motion** (SHM) is the name given to oscillatory motion for a system where the net force can be described by Hooke's law, and such a system is called a **simple harmonic oscillator**. If the net force can be described by Hooke's law and there is no *damping* (by friction or other non-conservative forces), then a simple harmonic oscillator will oscillate with equal displacement on either side of the equilibrium position, as shown for an object on a spring in Figure 16.9. The maximum displacement from equilibrium is called the **amplitude** X. The units for amplitude and displacement are the same, but depend on the type of oscillation. For the object on the spring, the units of amplitude and displacement are meters; whereas for sound oscillations, they have units of pressure (and other types of oscillations have yet other units). Because amplitude is the maximum displacement, it is related to the energy in the oscillation.

Take-Home Experiment: SHM and the Marble

Find a bowl or basin that is shaped like a hemisphere on the inside. Place a marble inside the bowl and tilt the bowl periodically so the marble rolls from the bottom of the bowl to equally high points on the sides of the bowl. Get a feel for the force required to maintain this periodic motion. What is the restoring force and what role does the force you apply play in the simple harmonic motion (SHM) of the marble?

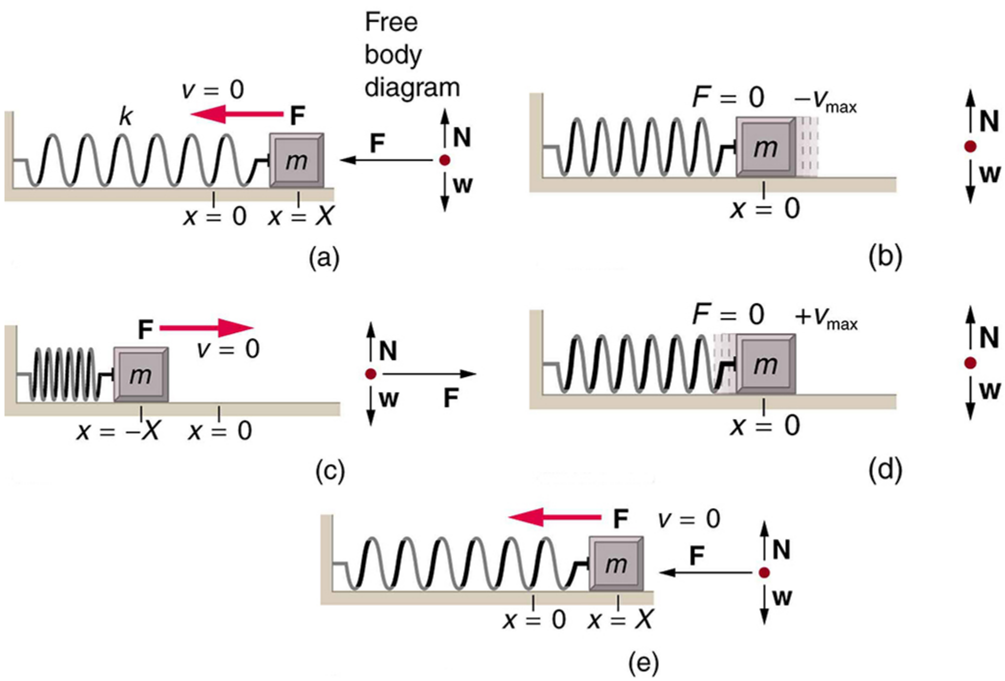


Figure 16.9 An object attached to a spring sliding on a frictionless surface is an uncomplicated simple harmonic oscillator. When displaced from equilibrium, the object performs simple harmonic motion that has an amplitude X and a period T. The object's maximum speed occurs as it passes through equilibrium. The stiffer the spring is, the smaller the period T. The greater the mass of the object is, the greater the period T.

What is so significant about simple harmonic motion? One special thing is that the period T and frequency f of a simple harmonic oscillator are independent of amplitude. The string of a guitar, for example, will oscillate with the same frequency whether plucked gently or hard. Because the period is constant, a simple harmonic oscillator can be used as a clock.

Two important factors do affect the period of a simple harmonic oscillator. The period is related to how stiff the system is. A very stiff object has a large force constant k, which causes the system to have a smaller period. For example, you can adjust a diving board's stiffness—the stiffer it is, the faster it vibrates, and the shorter its period. Period also depends on the mass of the oscillating system. The more massive the system is, the longer the period. For example, a heavy person on a diving board bounces up and down more slowly than a light one.

In fact, the mass m and the force constant k are the *only* factors that affect the period and frequency of simple harmonic motion.

Period of Simple Harmonic Oscillator

The *period of a simple harmonic oscillator* is given by

$$T = 2\pi\sqrt{\frac{m}{k}} \tag{16.15}$$

and, because $f = 1 / T$, the *frequency of a simple harmonic oscillator* is

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}. \tag{16.16}$$

Note that neither T nor f has any dependence on amplitude.

Take-Home Experiment: Mass and Ruler Oscillations

Find two identical wooden or plastic rulers. Tape one end of each ruler firmly to the edge of a table so that the length of each ruler that protrudes from the table is the same. On the free end of one ruler tape a heavy object such as a few large coins. Pluck the ends of the rulers at the same time and observe which one undergoes more cycles in a time period, and measure the period of oscillation of each of the rulers.

Example 16.4 Calculate the Frequency and Period of Oscillations: Bad Shock Absorbers in a Car

If the shock absorbers in a car go bad, then the car will oscillate at the least provocation, such as when going over bumps in the road and after stopping (See **Figure 16.10**). Calculate the frequency and period of these oscillations for such a car if the car's mass (including its load) is 900 kg and the force constant (k) of the suspension system is $6.53\times10^4\ \text{N/m}$.

Strategy

The frequency of the car's oscillations will be that of a simple harmonic oscillator as given in the equation $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$. The mass and the force constant are both given.

Solution

1. Enter the known values of *k* and *m*:

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{6.53\times10^4\ \text{N/m}}{900\ \text{kg}}}. \tag{16.17}$$

2. Calculate the frequency:

$$\frac{1}{2\pi}\sqrt{72.6 / \text{s}^{-2}} = 1.3656 / \text{s}^{-1} \approx 1.36 / \text{s}^{-1} = 1.36\ \text{Hz}. \tag{16.18}$$

3. You could use $T = 2\pi\sqrt{\frac{m}{k}}$ to calculate the period, but it is simpler to use the relationship $T = 1 / f$ and substitute the value just found for f :

$$T = \frac{1}{f} = \frac{1}{1.356\ \text{Hz}} = 0.738\ \text{s}. \tag{16.19}$$

Discussion

The values of T and f both seem about right for a bouncing car. You can observe these oscillations if you push down hard on the end of a car and let go.

The Link between Simple Harmonic Motion and Waves

If a time-exposure photograph of the bouncing car were taken as it drove by, the headlight would make a wavelike streak, as shown in **Figure 16.10**. Similarly, **Figure 16.11** shows an object bouncing on a spring as it leaves a wavelike "trace of its position on a moving strip of paper. Both waves are sine functions. All simple harmonic motion is intimately related to sine and cosine waves.

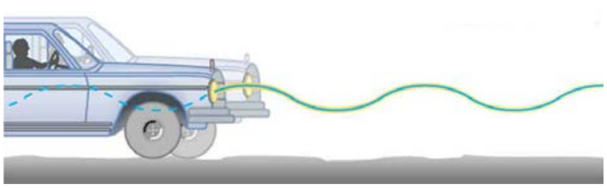


Figure 16.10 The bouncing car makes a wavelike motion. If the restoring force in the suspension system can be described only by Hooke's law, then the wave is a sine function. (The wave is the trace produced by the headlight as the car moves to the right.)

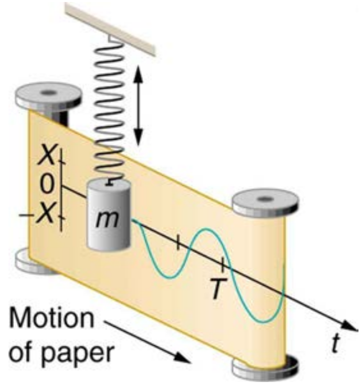


Figure 16.11 The vertical position of an object bouncing on a spring is recorded on a strip of moving paper, leaving a sine wave.

The displacement as a function of time *t* in any simple harmonic motion—that is, one in which the net restoring force can be described by Hooke's law, is given by

$$x(t) = X \cos\frac{2\pi t}{T}, \tag{16.20}$$

where X is amplitude. At $t = 0$, the initial position is $x_0 = X$, and the displacement oscillates back and forth with a period T. (When $t = T$, we get $x = X$ again because $\cos 2\pi = 1$.). Furthermore, from this expression for x, the velocity v as a function of time is given by:

$$v(t) = -v_{\text{max}} \sin\left(\frac{2\pi t}{T}\right), \tag{16.21}$$

where $v_{\text{max}} = 2\pi X / T = X\sqrt{k / m}$. The object has zero velocity at maximum displacement—for example, $v = 0$ when $t = 0$, and at that time $x = X$. The minus sign in the first equation for $v(t)$ gives the correct direction for the velocity. Just after the start of the motion, for instance, the velocity is negative because the system is moving back toward the equilibrium point. Finally, we can get an expression for acceleration using Newton's second law. [Then we have $x(t), v(t), t,$ and $a(t)$, the quantities needed for kinematics and a description of simple harmonic motion.] According to Newton's second law, the acceleration is $a = F / m = kx / m$. So, $a(t)$ is also a cosine function:

$$a(t) = -\frac{kX}{m}\cos\frac{2\pi t}{T}. \tag{16.22}$$

Hence, $a(t)$ is directly proportional to and in the opposite direction to $a(t)$.

Figure 16.12 shows the simple harmonic motion of an object on a spring and presents graphs of $x(t), v(t),$ and $a(t)$ versus time.

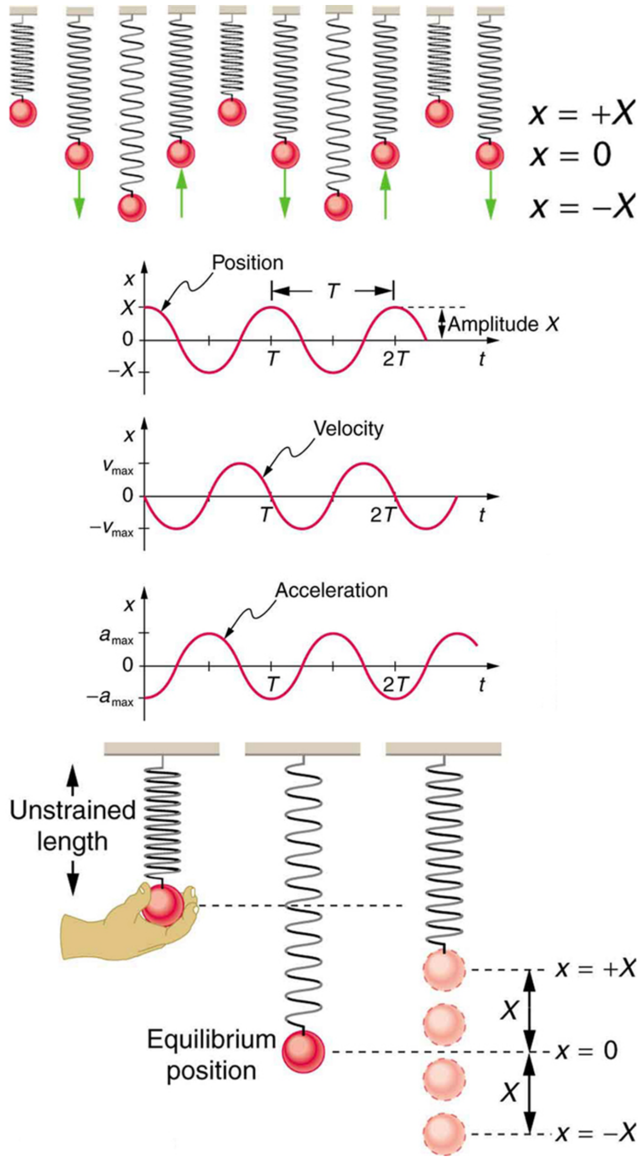


Figure 16.12 Graphs of $x(t)$, $v(t)$, and $a(t)$ versus t for the motion of an object on a spring. The net force on the object can be described by Hooke's law, and so the object undergoes simple harmonic motion. Note that the initial position has the vertical displacement at its maximum value X; v is initially zero and then negative as the object moves down; and the initial acceleration is negative, back toward the equilibrium position and becomes zero at that point.

The most important point here is that these equations are mathematically straightforward and are valid for all simple harmonic motion. They are very useful in visualizing waves associated with simple harmonic motion, including visualizing how waves add with one another.

Check Your Understanding

Suppose you pluck a banjo string. You hear a single note that starts out loud and slowly quiets over time. Describe what happens to the sound waves in terms of period, frequency and amplitude as the sound decreases in volume.

Solution
Frequency and period remain essentially unchanged. Only amplitude decreases as volume decreases.

Check Your Understanding

A babysitter is pushing a child on a swing. At the point where the swing reaches x, where would the corresponding point on a wave of this motion be located?

Solution
x is the maximum deformation, which corresponds to the amplitude of the wave. The point on the wave would either be at the very top or the very bottom of the curve.

PhET Explorations: Masses and Springs

A realistic mass and spring laboratory. Hang masses from springs and adjust the spring stiffness and damping. You can even slow time. Transport the lab to different planets. A chart shows the kinetic, potential, and thermal energy for each spring.

PhET Interactive Simulation

Figure 16.13 Masses and Springs (http://cnx.org/content/m42242/1.6/mass-spring-lab_en.jar)

16.4 The Simple Pendulum

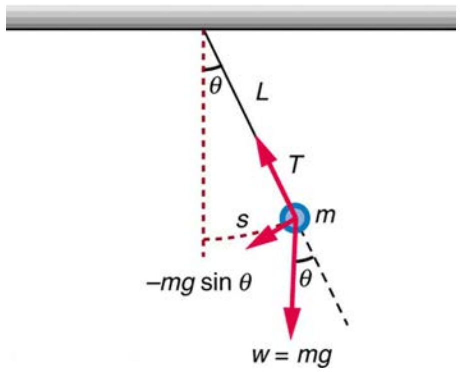


Figure 16.14 A simple pendulum has a small-diameter bob and a string that has a very small mass but is strong enough not to stretch appreciably. The linear displacement from equilibrium is s, the length of the arc. Also shown are the forces on the bob, which result in a net force of $-mg\ \sin\theta$ toward the equilibrium position—that is, a restoring force.

Pendulums are in common usage. Some have crucial uses, such as in clocks; some are for fun, such as a child's swing; and some are just there, such as the sinker on a fishing line. For small displacements, a pendulum is a simple harmonic oscillator. A **simple pendulum** is defined to have an object that has a small mass, also known as the pendulum bob, which is suspended from a light wire or string, such as shown in Figure 16.14. Exploring the simple pendulum a bit further, we can discover the conditions under which it performs simple harmonic motion, and we can derive an interesting expression for its period.

We begin by defining the displacement to be the arc length s. We see from Figure 16.14 that the net force on the bob is tangent to the arc and equals $-mg\ \sin\theta$. (The weight mg has components $mg\ \cos\theta$ along the string and $mg\ \sin\theta$ tangent to the arc.) Tension in the string exactly cancels the component $mg\ \cos\theta$ parallel to the string. This leaves a *net* restoring force back toward the equilibrium position at $\theta = 0$.

Now, if we can show that the restoring force is directly proportional to the displacement, then we have a simple harmonic oscillator. In trying to determine if we have a simple harmonic oscillator, we should note that for small angles (less than about 15^{o}), $\sin\theta \approx \theta$ ($\sin\theta$ and θ differ by about 1% or less at smaller angles). Thus, for angles less than about 15^{o}, the restoring force F is

$$F \approx -mg\theta. \tag{16.23}$$

The displacement s is directly proportional to θ. When θ is expressed in radians, the arc length in a circle is related to its radius (L in this instance) by:

$$s = L\theta, \tag{16.24}$$

so that

$$\theta = \frac{s}{L}. \tag{16.25}$$

For small angles, then, the expression for the restoring force is:

$$F \approx -\frac{mg}{L}s \tag{16.26}$$

This expression is of the form:

$$F = -kx, \tag{16.27}$$

where the force constant is given by $k = mg / L$ and the displacement is given by $x = s$. For angles less than about 15^{o}, the restoring force is directly proportional to the displacement, and the simple pendulum is a simple harmonic oscillator.

Using this equation, we can find the period of a pendulum for amplitudes less than about 15^{o}. For the simple pendulum:

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{mg / L}}. \tag{16.28}$$

Thus,

$$T = 2\pi\sqrt{\frac{L}{g}} \tag{16.29}$$

for the period of a simple pendulum. This result is interesting because of its simplicity. The only things that affect the period of a simple pendulum are its length and the acceleration due to gravity. The period is completely independent of other factors, such as mass. As with simple harmonic oscillators, the period T for a pendulum is nearly independent of amplitude, especially if θ is less than about 15^{o}. Even simple pendulum clocks can be finely adjusted and accurate.

Note the dependence of T on g. If the length of a pendulum is precisely known, it can actually be used to measure the acceleration due to gravity. Consider the following example.

Example 16.5 Measuring Acceleration due to Gravity: The Period of a Pendulum

What is the acceleration due to gravity in a region where a simple pendulum having a length 75.000 cm has a period of 1.7357 s?

Strategy

We are asked to find g given the period T and the length L of a pendulum. We can solve $T = 2\pi\sqrt{\frac{L}{g}}$ for g, assuming only that the angle of deflection is less than 15^{o}.

Solution

1. Square $T = 2\pi\sqrt{\frac{L}{g}}$ and solve for g:

$$g = 4\pi^2\frac{L}{T^2}. \tag{16.30}$$

2. Substitute known values into the new equation:

$$g = 4\pi^2\frac{0.75000\text{ m}}{(1.7357\text{ s})^2}. \tag{16.31}$$

3. Calculate to find g:

$$g = 9.8281\text{ m/s}^2. \tag{16.32}$$

Discussion

This method for determining g can be very accurate. This is why length and period are given to five digits in this example. For the precision of the approximation $\sin\theta \approx \theta$ to be better than the precision of the pendulum length and period, the maximum displacement angle should be kept below about 0.5^{o}.

Making Career Connections

Knowing g can be important in geological exploration; for example, a map of g over large geographical regions aids the study of plate tectonics and helps in the search for oil fields and large mineral deposits.

Take Home Experiment: Determining g

Use a simple pendulum to determine the acceleration due to gravity g in your own locale. Cut a piece of a string or dental floss so that it is about 1 m long. Attach a small object of high density to the end of the string (for example, a metal nut or a car key). Starting at an angle of less than 10^{o}, allow the pendulum to swing and measure the pendulum's period for 10 oscillations using a stopwatch. Calculate g. How accurate is this measurement? How might it be improved?

Check Your Understanding

An engineer builds two simple pendula. Both are suspended from small wires secured to the ceiling of a room. Each pendulum hovers 2 cm above the floor. Pendulum 1 has a bob with a mass of 10 kg. Pendulum 2 has a bob with a mass of 100 kg. Describe how the motion of the pendula will differ if the bobs are both displaced by 12^{o}.

Solution

The movement of the pendula will not differ at all because the mass of the bob has no effect on the motion of a simple pendulum. The pendula are only affected by the period (which is related to the pendulum's length) and by the acceleration due to gravity.

PhET Explorations: Pendulum Lab

Play with one or two pendulums and discover how the period of a simple pendulum depends on the length of the string, the mass of the pendulum bob, and the amplitude of the swing. It's easy to measure the period using the photogate timer. You can vary friction and the strength of gravity. Use the pendulum to find the value of g on planet X. Notice the anharmonic behavior at large amplitude.

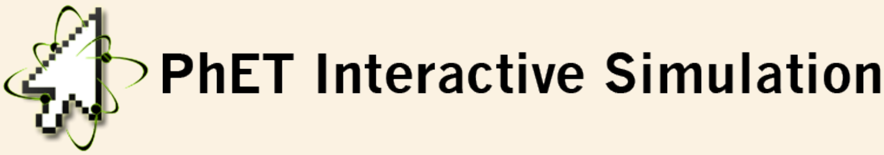


Figure 16.15 Pendulum Lab (http://cnx.org/content/m42243/1.5/pendulum-lab_en.jar)

16.5 Energy and the Simple Harmonic Oscillator

To study the energy of a simple harmonic oscillator, we first consider all the forms of energy it can have We know from **Hooke's Law: Stress and Strain Revisited** that the energy stored in the deformation of a simple harmonic oscillator is a form of potential energy given by:

$$\text{PE}_{\text{el}} = \frac{1}{2}kx^2. \tag{16.33}$$

Because a simple harmonic oscillator has no dissipative forces, the other important form of energy is kinetic energy KE. Conservation of energy for these two forms is:

$$\text{KE} + \text{PE}_{\text{el}} = \text{constant} \tag{16.34}$$

or

$$\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \text{constant}. \tag{16.35}$$

This statement of conservation of energy is valid for *all* simple harmonic oscillators, including ones where the gravitational force plays a role

Namely, for a simple pendulum we replace the velocity with $v = L\omega$, the spring constant with $k = mg/L$, and the displacement term with $x = L\theta$. Thus

$$\frac{1}{2}mL^2\omega^2 + \frac{1}{2}mgL\theta^2 = \text{constant}. \tag{16.36}$$

In the case of undamped simple harmonic motion, the energy oscillates back and forth between kinetic and potential, going completely from one to the other as the system oscillates. So for the simple example of an object on a frictionless surface attached to a spring, as shown again in **Figure 16.16**, the motion starts with all of the energy stored in the spring. As the object starts to move, the elastic potential energy is converted to kinetic energy, becoming entirely kinetic energy at the equilibrium position. It is then converted back into elastic potential energy by the spring, the velocity becomes zero when the kinetic energy is completely converted, and so on. This concept provides extra insight here and in later applications of simple harmonic motion, such as alternating current circuits.

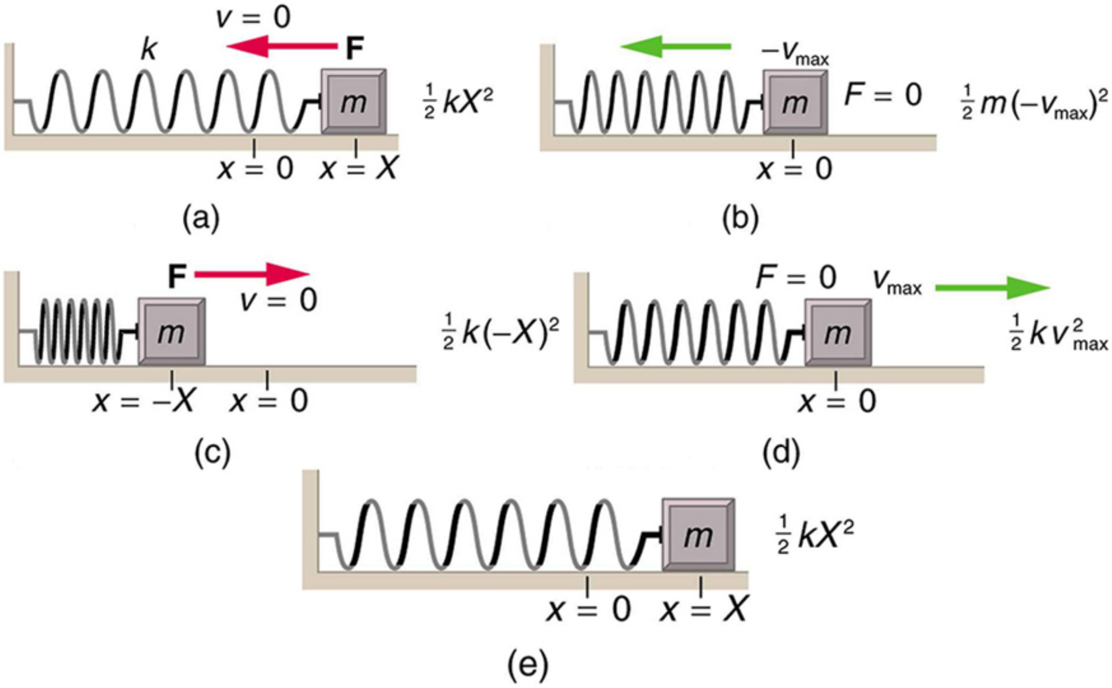


Figure 16.16 The transformation of energy in simple harmonic motion is illustrated for an object attached to a spring on a frictionless surface.

The conservation of energy principle can be used to derive an expression for velocity v. If we start our simple harmonic motion with zero velocity and maximum displacement ($x = X$), then the total energy is

$$\frac{1}{2}kX^2. \quad (16.37)$$

This total energy is constant and is shifted back and forth between kinetic energy and potential energy, at most times being shared by each. The conservation of energy for this system in equation form is thus:

$$\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kX^2. \quad (16.38)$$

Solving this equation for v yields:

$$v = \pm\sqrt{\frac{k}{m}\left(X^2 - x^2\right)}. \quad (16.39)$$

Manipulating this expression algebraically gives:

$$v = \pm\sqrt{\frac{k}{m}}X\sqrt{1 - \frac{x^2}{X^2}} \quad (16.40)$$

and so

$$v = \pm v_{\text{max}}\sqrt{1 - \frac{x^2}{X^2}}, \quad (16.41)$$

where

$$v_{\text{max}} = \sqrt{\frac{k}{m}}X. \quad (16.42)$$

From this expression, we see that the velocity is a maximum (v_{max}) at $x = 0$, as stated earlier in $v(t) = -v_{\text{max}}\sin\frac{2\pi t}{T}$. Notice that the maximum velocity depends on three factors. Maximum velocity is directly proportional to amplitude. As you might guess, the greater the maximum displacement the greater the maximum velocity. Maximum velocity is also greater for stiffer systems, because they exert greater force for the same displacement. This observation is seen in the expression for $v_{\text{max}};$ it is proportional to the square root of the force constant k. Finally, the maximum velocity is smaller for objects that have larger masses, because the maximum velocity is inversely proportional to the square root of m. For a given force, objects that have large masses accelerate more slowly.

A similar calculation for the simple pendulum produces a similar result, namely:

$$\omega_{\text{max}} = \sqrt{\frac{g}{L}}\theta_{\text{max}}. \quad (16.43)$$

Example 16.6 Determine the Maximum Speed of an Oscillating System: A Bumpy Road

Suppose that a car is 900 kg and has a suspension system that has a force constant $k = 6.53\times10^4\ \text{N/m}$. The car hits a bump and bounces with an amplitude of 0.100 m. What is its maximum vertical velocity if you assume no damping occurs?

Strategy

We can use the expression for v_{max} given in $v_{\text{max}} = \sqrt{\frac{k}{m}}X$ to determine the maximum vertical velocity. The variables m and k are given in the problem statement, and the maximum displacement X is 0.100 m.

Solution

1. Identify known.
2. Substitute known values into $v_{\text{max}} = \sqrt{\frac{k}{m}}X$:

$$v_{\text{max}} = \sqrt{\frac{6.53\times10^4\ \text{N/m}}{900\ \text{kg}}}(0.100\ \text{m}). \quad (16.44)$$

3. Calculate to find $v_{\text{max}} = 0.852\ \text{m/s}.$

Discussion

This answer seems reasonable for a bouncing car. There are other ways to use conservation of energy to find v_{max}. We could use it directly, as was done in the example featured in **Hooke's Law: Stress and Strain Revisited**.

The small vertical displacement y of an oscillating simple pendulum, starting from its equilibrium position, is given as

$$y(t) = a\sin\omega t, \quad (16.45)$$

where a is the amplitude, ω is the angular velocity and t is the time taken. Substituting $\omega = \frac{2\pi}{T}$, we have

$$yt = a \sin\left(\frac{2\pi t}{T}\right). \tag{16.46}$$

Thus, the displacement of pendulum is a function of time as shown above.

Also the velocity of the pendulum is given by

$$v(t) = \frac{2a\pi}{T} \cos\left(\frac{2\pi t}{T}\right), \tag{16.47}$$

so the motion of the pendulum is a function of time.

Check Your Understanding

Why does it hurt more if your hand is snapped with a ruler than with a loose spring, even if the displacement of each system is equal?

Solution

The ruler is a stiffer system, which carries greater force for the same amount of displacement. The ruler snaps your hand with greater force, which hurts more.

Check Your Understanding

You are observing a simple harmonic oscillator. Identify one way you could decrease the maximum velocity of the system.

Solution

You could increase the mass of the object that is oscillating.

16.6 Uniform Circular Motion and Simple Harmonic Motion

Figure 16.17 The horses on this merry-go-round exhibit uniform circular motion. (credit: Wonderlane, Flickr)

There is an easy way to produce simple harmonic motion by using uniform circular motion. **Figure 16.18** shows one way of using this method. A ball is attached to a uniformly rotating vertical turntable, and its shadow is projected on the floor as shown. The shadow undergoes simple harmonic motion. Hooke's law usually describes uniform circular motions (ω constant) rather than systems that have large visible displacements. So observing the projection of uniform circular motion, as in **Figure 16.18**, is often easier than observing a precise large-scale simple harmonic oscillator. If studied in sufficient depth, simple harmonic motion produced in this manner can give considerable insight into many aspects of oscillations and waves and is very useful mathematically. In our brief treatment, we shall indicate some of the major features of this relationship and how they might be useful.

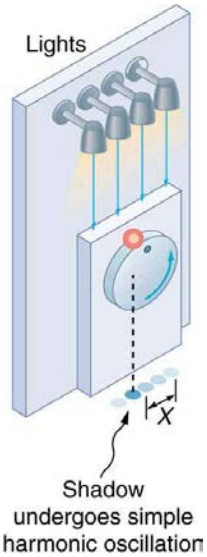


Figure 16.18 The shadow of a ball rotating at constant angular velocity ω on a turntable goes back and forth in precise simple harmonic motion.

Figure 16.19 shows the basic relationship between uniform circular motion and simple harmonic motion. The point P travels around the circle at constant angular velocity ω. The point P is analogous to an object on the merry-go-round. The projection of the position of P onto a fixed axis undergoes simple harmonic motion and is analogous to the shadow of the object. At the time shown in the figure, the projection has position x and moves to the left with velocity v. The velocity of the point P around the circle equals $\bar{v}_{\text{max}}$. The projection of $\bar{v}_{\text{max}}$ on the x-axis is the velocity v of the simple harmonic motion along the x-axis.

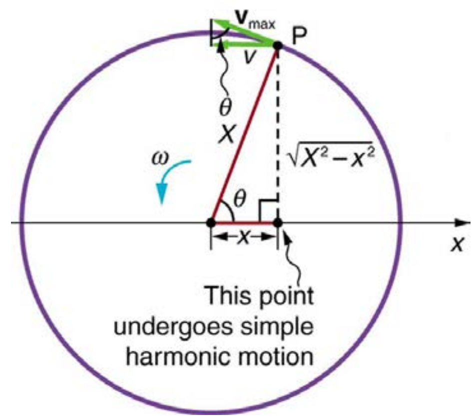


Figure 16.19 A point P moving on a circular path with a constant angular velocity ω is undergoing uniform circular motion. Its projection on the x-axis undergoes simple harmonic motion. Also shown is the velocity of this point around the circle, $\bar{v}_{\text{max}}$, and its projection, which is v. Note that these velocities form a similar triangle to the displacement triangle.

To see that the projection undergoes simple harmonic motion, note that its position x is given by

$$x = X \cos \theta, \tag{16.48}$$

where $\theta = \omega t$, ω is the constant angular velocity, and X is the radius of the circular path. Thus,

$$x = X \cos \omega t. \tag{16.49}$$

The angular velocity ω is in radians per unit time; in this case 2π radians is the time for one revolution T. That is, $\omega = 2\pi / T$. Substituting this expression for ω, we see that the position x is given by:

$$x(t) = \cos\left(\frac{2\pi t}{T}\right). \tag{16.50}$$

This expression is the same one we had for the position of a simple harmonic oscillator in **Simple Harmonic Motion: A Special Periodic Motion**. If we make a graph of position versus time as in **Figure 16.20**, we see again the wavelike character (typical of simple harmonic motion) of the projection of uniform circular motion onto the x-axis.

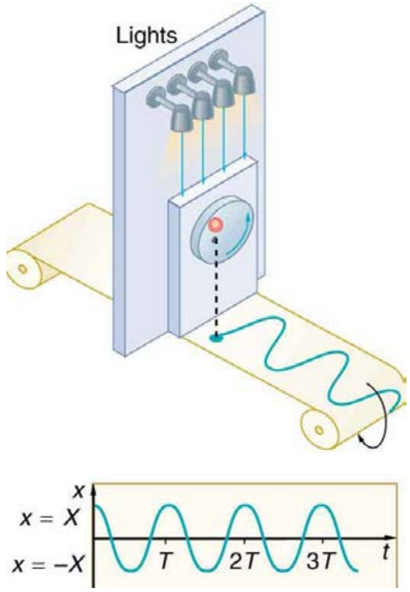


Figure 16.20 The position of the projection of uniform circular motion performs simple harmonic motion, as this wavelike graph of x versus t indicates.

Now let us use **Figure 16.19** to do some further analysis of uniform circular motion as it relates to simple harmonic motion. The triangle formed by the velocities in the figure and the triangle formed by the displacements ($X, x,$ and $\sqrt{X^2 - x^2}$) are similar right triangles. Taking ratios of similar sides, we see that

$$\frac{v}{v_{\text{max}}} = \frac{\sqrt{X^2 - x^2}}{X} = \sqrt{1 - \frac{x^2}{X^2}}. \tag{16.51}$$

We can solve this equation for the speed v or

$$v = v_{\text{max}}\sqrt{1 - \frac{x^2}{X^2}}. \tag{16.52}$$

This expression for the speed of a simple harmonic oscillator is exactly the same as the equation obtained from conservation of energy considerations in **Energy and the Simple Harmonic Oscillator**.You can begin to see that it is possible to get all of the characteristics of simple harmonic motion from an analysis of the projection of uniform circular motion.

Finally, let us consider the period T of the motion of the projection. This period is the time it takes the point P to complete one revolution. That time is the circumference of the circle $2\pi X$ divided by the velocity around the circle, v_{max} . Thus, the period T is

$$T = \frac{2\pi X}{v_{\text{max}}}. \tag{16.53}$$

We know from conservation of energy considerations that

$$v_{\text{max}} = \sqrt{\frac{k}{m}}X. \tag{16.54}$$

Solving this equation for X / v_{max} gives

$$\frac{X}{v_{\text{max}}} = \sqrt{\frac{m}{k}}. \tag{16.55}$$

Substituting this expression into the equation for T yields

$$T = 2\pi\sqrt{\frac{m}{k}}. \tag{16.56}$$

Thus, the period of the motion is the same as for a simple harmonic oscillator. We have determined the period for any simple harmonic oscillator using the relationship between uniform circular motion and simple harmonic motion.

Some modules occasionally refer to the connection between uniform circular motion and simple harmonic motion. Moreover, if you carry your study of physics and its applications to greater depths, you will find this relationship useful. It can, for example, help to analyze how waves add when they are superimposed.

Check Your Understanding

Identify an object that undergoes uniform circular motion. Describe how you could trace the simple harmonic motion of this object as a wave.

Solution

A record player undergoes uniform circular motion. You could attach dowel rod to one point on the outside edge of the turntable and attach a pen to the other end of the dowel. As the record player turns, the pen will move. You can drag a long piece of paper under the pen, capturing its motion as a wave.

16.7 Damped Harmonic Motion

Figure 16.21 In order to counteract dampening forces, this dad needs to keep pushing the swing. (credit: Erik A. Johnson, Flickr)

A guitar string stops oscillating a few seconds after being plucked. To keep a child happy on a swing, you must keep pushing. Although we can often make friction and other non-conservative forces negligibly small, completely undamped motion is rare. In fact, we may even want to damp oscillations, such as with car shock absorbers.

For a system that has a small amount of damping, the period and frequency are nearly the same as for simple harmonic motion, but the amplitude gradually decreases as shown in **Figure 16.22**. This occurs because the non-conservative damping force removes energy from the system, usually in the form of thermal energy. In general, energy removal by non-conservative forces is described as

$$W_{\text{nc}} = \Delta(\text{KE} + \text{PE}), \tag{16.57}$$

where W_{nc} is work done by a non-conservative force (here the damping force). For a damped harmonic oscillator, W_{nc} is negative because it removes mechanical energy (KE + PE) from the system.

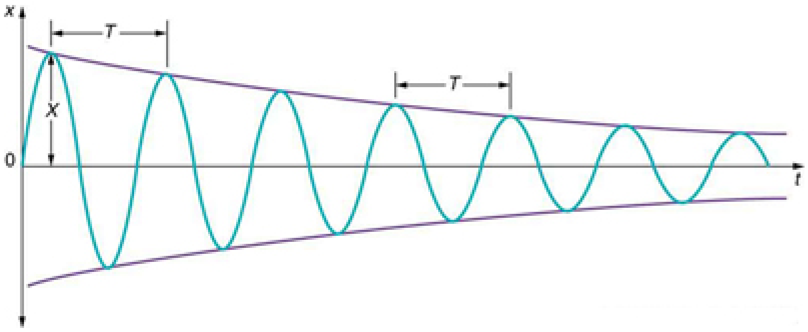


Figure 16.22 In this graph of displacement versus time for a harmonic oscillator with a small amount of damping, the amplitude slowly decreases, but the period and frequency are nearly the same as if the system were completely undamped.

If you gradually *increase* the amount of damping in a system, the period and frequency begin to be affected, because damping opposes and hence slows the back and forth motion. (The net force is smaller in both directions.) If there is very large damping, the system does not even oscillate—it slowly moves toward equilibrium. **Figure 16.23** shows the displacement of a harmonic oscillator for different amounts of damping. When we want to damp out oscillations, such as in the suspension of a car, we may want the system to return to equilibrium as quickly as possible **Critical damping** is defined as the condition in which the damping of an oscillator results in it returning as quickly as possible to its equilibrium position The critically damped system may overshoot the equilibrium position, but if it does, it will do so only once. Critical damping is represented by Curve A in **Figure 16.23**. With less-than critical damping, the system will return to equilibrium faster but will overshoot and cross over one or more times. Such a system is **underdamped**; its displacement is represented by the curve in **Figure 16.22**. Curve B in **Figure 16.23** represents an **overdamped** system. As with critical damping, it too may overshoot the equilibrium position, but will reach equilibrium over a longer period of time.

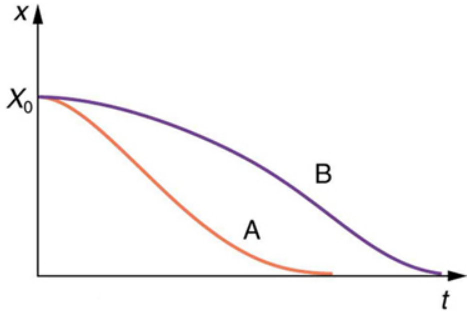


Figure 16.23 Displacement versus time for a critically damped harmonic oscillator (A) and an overdamped harmonic oscillator (B). The critically damped oscillator returns to equilibrium at $X = 0$ in the smallest time possible without overshooting.

Critical damping is often desired, because such a system returns to equilibrium rapidly and remains at equilibrium as well. In addition, a constant force applied to a critically damped system moves the system to a new equilibrium position in the shortest time possible without overshooting or oscillating about the new position. For example, when you stand on bathroom scales that have a needle gauge, the needle moves to its equilibrium position without oscillating. It would be quite inconvenient if the needle oscillated about the new equilibrium position for a long time before settling. Damping forces can vary greatly in character. Friction, for example, is sometimes independent of velocity (as assumed in most places in this text). But many damping forces depend on velocity—sometimes in complex ways, sometimes simply being proportional to velocity.

Example 16.7 Damping an Oscillatory Motion: Friction on an Object Connected to a Spring

Damping oscillatory motion is important in many systems, and the ability to control the damping is even more so. This is generally attained using non-conservative forces such as the friction between surfaces, and viscosity for objects moving through fluids. The following example considers friction. Suppose a 0.200-kg object is connected to a spring as shown in **Figure 16.24**, but there is simple friction between the object and the surface, and the coefficient of friction μ_k is equal to 0.0800. (a) What is the frictional force between the surfaces? (b) What total distance does the object travel if it is released 0.100 m from equilibrium, starting at $v = 0$? The force constant of the spring is $k = 50.0\ \text{N/m}$.

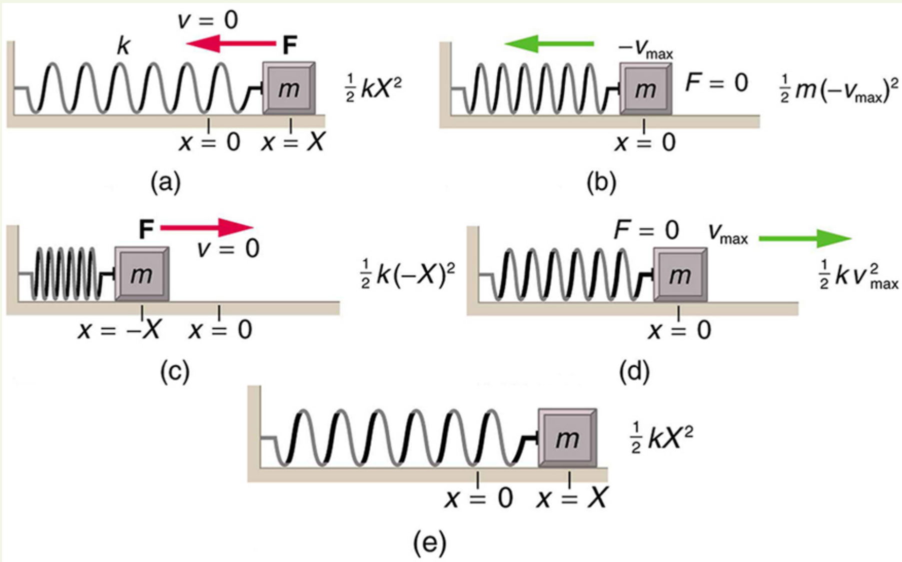


Figure 16.24 The transformation of energy in simple harmonic motion is illustrated for an object attached to a spring on a frictionless surface.

Strategy

This problem requires you to integrate your knowledge of various concepts regarding waves, oscillations, and damping. To solve an integrated concept problem, you must first identify the physical principles involved. Part (a) is about the frictional force. This is a topic involving the application of Newton's Laws. Part (b) requires an understanding of work and conservation of energy, as well as some understanding of horizontal oscillatory systems.

Now that we have identified the principles we must apply in order to solve the problems, we need to identify the knowns and unknowns for each part of the question, as well as the quantity that is constant in Part (a) and Part (b) of the question.

Solution a

1. Choose the proper equation: Friction is $f = \mu_\text{k}mg$.
2. Identify the known values.
3. Enter the known values into the equation:

$$f = (0.0800)(0.200\ \text{kg})(9.80\ \text{m/s}^2). \tag{16.58}$$

4. Calculate and convert units: $f = 0.157\ \text{N}.$

Discussion a

The force here is small because the system and the coefficients are small.

Solution b

Identify the known:

- The system involves elastic potential energy as the spring compresses and expands, friction that is related to the work done, and the kinetic energy as the body speeds up and slows down.
- Energy is not conserved as the mass oscillates because friction is a non-conservative force.
- The motion is horizontal, so gravitational potential energy does not need to be considered.
- Because the motion starts from rest, the energy in the system is initially $\text{PE}_{\text{el,i}} = (1/2)kX^2$. This energy is removed by work done by friction $W_\text{nc} = -fd$, where d is the total distance traveled and $f = \mu_\text{k}mg$ is the force of friction. When the system stops moving, the friction force will balance the force exerted by the spring, so $\text{PE}_{\text{e1,f}} = (1/2)kx^2$ where x is the final position and is given by

$$\begin{aligned} F_{\text{el}} &= f \\ kx &= \mu_{\text{k}} mg \\ x &= \frac{\mu_{\text{k}} mg}{k} \end{aligned}. \tag{16.59}$$

1. By equating the work done to the energy removed, solve for the distance d .
2. The work done by the non-conservative forces equals the initial, stored elastic potential energy. Identify the correct equation to use:

$$\text{W}_{\text{nc}} = \Delta(\text{KE} + \text{PE}) = \text{PE}_{\text{el,f}} - \text{PE}_{\text{el,i}} = \frac{1}{2}k\left(\left(\frac{\mu_{\text{k}} mg}{k}\right)^2 - X^2\right). \tag{16.60}$$

3. Recall that $W_{\text{nc}} = -fd$.
4. Enter the friction as $f = \mu_{\text{k}} mg$ into $W_{\text{nc}} = -fd$, thus

$$W_{\text{nc}} = -\mu_{\text{k}} mgd. \tag{16.61}$$

5. Combine these two equations to find

$$\frac{1}{2}k\left(\left(\frac{\mu_k mg}{k}\right)^2 - X^2\right) = -\mu_{\text{k}} mgd. \tag{16.62}$$

6. Solve the equation for d :

$$d = \frac{k}{2\mu_{\text{k}} mg}\left(X^2 - \left(\frac{\mu_{\text{k}} mg}{k}\right)^2\right). \tag{16.63}$$

7. Enter the known values into the resulting equation:

$$d = \frac{50.0\text{ N/m}}{2(0.0800)(0.200\text{ kg})\left(9.80\text{ m/s}^2\right)}\left((0.100\text{ m})^2 - \left(\frac{(0.0800)(0.200\text{ kg})\left(9.80\text{ m/s}^2\right)}{50.0\text{ N/m}}\right)^2\right). \tag{16.64}$$

8. Calculate d and convert units:

$$d = 1.59\text{ m}. \tag{16.65}$$

Discussion b

This is the total distance traveled back and forth across $x = 0$, which is the undamped equilibrium position. The number of oscillations about the equilibrium position will be more than $d / X = (1.59\text{ m}) / (0.100\text{ m}) = 15.9$ because the amplitude of the oscillations is decreasing with time. At the end of the motion, this system will not return to $x = 0$ for this type of damping force, because static friction will exceed the restoring force. This system is underdamped. In contrast, an overdamped system with a simple constant damping force would not cross the equilibrium position $x = 0$ a single time. For example, if this system had a damping force 20 times greater, it would only move 0.0484 m toward the equilibrium position from its original 0.100-m position.

This worked example illustrates how to apply problem-solving strategies to situations that integrate the different concepts you have learned. The first step is to identify the physical principles involved in the problem. The second step is to solve for the unknowns using familiar problem-solving strategies. These are found throughout the text, and many worked examples show how to use them for single topics. In this integrated concepts example, you can see how to apply them across several topics. You will find these techniques useful in applications of physics outside a physics course, such as in your profession, in other science disciplines, and in everyday life.

Check Your Understanding

Why are completely undamped harmonic oscillators so rare?

Solution
Friction often comes into play whenever an object is moving. Friction causes damping in a harmonic oscillator.

Check Your Understanding

Describe the difference between overdamping, underdamping, and critical damping.

Solution
An overdamped system moves slowly toward equilibrium. An underdamped system moves quickly to equilibrium, but will oscillate about the equilibrium point as it does so. A critically damped system moves as quickly as possible toward equilibrium without oscillating about the equilibrium.

16.8 Forced Oscillations and Resonance

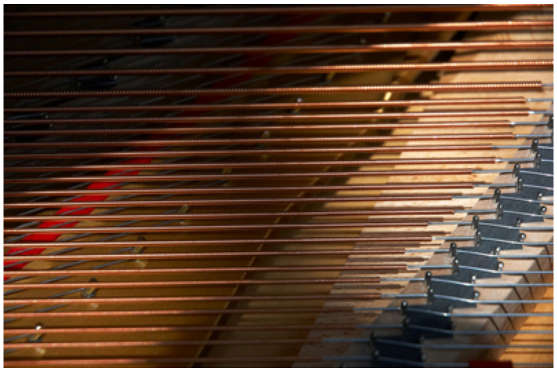


Figure 16.25 You can cause the strings in a piano to vibrate simply by producing sound waves from your voice. (credit: Matt Billings, Flickr)

Sit in front of a piano sometime and sing a loud brief note at it with the dampers off its strings. It will sing the same note back at you—the strings, having the same frequencies as your voice, are resonating in response to the forces from the sound waves that you sent to them. Your voice and a piano's strings is a good example of the fact that objects—in this case, piano strings—can be forced to oscillate but oscillate best at their natural frequency. In this section, we shall briefly explore applying a *periodic driving force* acting on a simple harmonic oscillator. The driving force puts energy into the system at a certain frequency, not necessarily the same as the natural frequency of the system. The **natural frequency** is the frequency at which a system would oscillate if there were no driving and no damping force.

Most of us have played with toys involving an object supported on an elastic band, something like the paddle ball suspended from a finger in Figure 16.26. Imagine the finger in the figure is your finger. At first you hold your finger steady, and the ball bounces up and down with a small amount of damping. If you move your finger up and down slowly, the ball will follow along without bouncing much on its own. As you increase the frequency at which you move your finger up and down, the ball will respond by oscillating with increasing amplitude. When you drive the ball at its natural frequency, the ball's oscillations increase in amplitude with each oscillation for as long as you drive it. The phenomenon of driving a system with a frequency equal to its natural frequency is called **resonance**. A system being driven at its natural frequency is said to **resonate**. As the driving frequency gets progressively higher than the resonant or natural frequency, the amplitude of the oscillations becomes smaller, until the oscillations nearly disappear and your finger simply moves up and down with little effect on the ball.

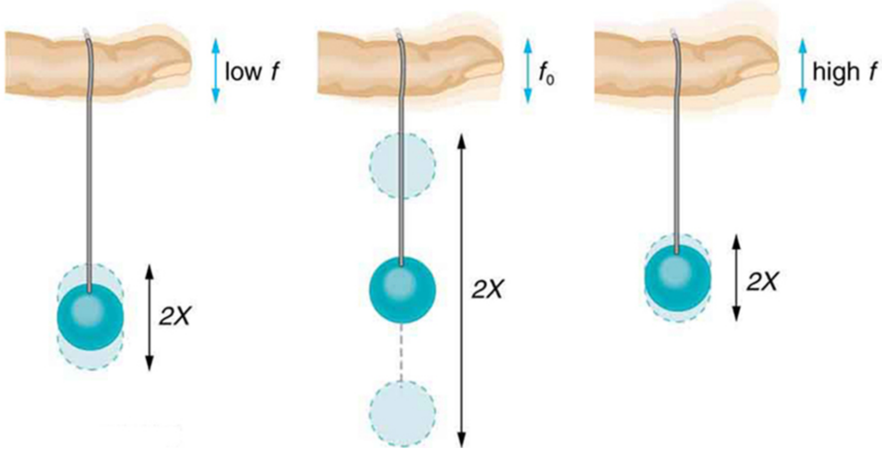


Figure 16.26 The paddle ball on its rubber band moves in response to the finger supporting it. If the finger moves with the natural frequency f_0 of the ball on the rubber band, then a resonance is achieved, and the amplitude of the ball's oscillations increases dramatically. At higher and lower driving frequencies, energy is transferred to the ball less efficiently, and it responds with lower-amplitude oscillations.

Figure 16.27 shows a graph of the amplitude of a damped harmonic oscillator as a function of the frequency of the periodic force driving it. There are three curves on the graph, each representing a different amount of damping. All three curves peak at the point where the frequency of the driving force equals the natural frequency of the harmonic oscillator. The highest peak, or greatest response, is for the least amount of damping, because less energy is removed by the damping force.

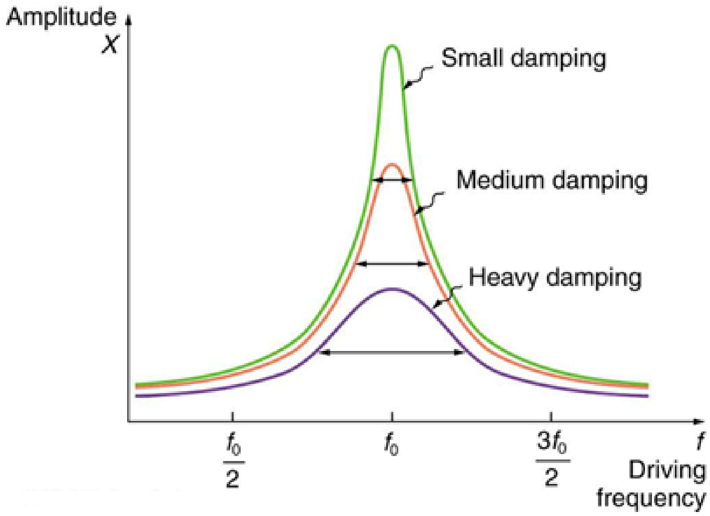


Figure 16.27 Amplitude of a harmonic oscillator as a function of the frequency of the driving force. The curves represent the same oscillator with the same natural frequency but with different amounts of damping. Resonance occurs when the driving frequency equals the natural frequency, and the greatest response is for the least amount of damping. The narrowest response is also for the least damping.

It is interesting that the widths of the resonance curves shown in **Figure 16.27** depend on damping: the less the damping, the narrower the resonance. The message is that if you want a driven oscillator to resonate at a very specific frequency, you need as little damping as possible. Little damping is the case for piano strings and many other musical instruments. Conversely, if you want small-amplitude oscillations, such as in a car's suspension system, then you want heavy damping. Heavy damping reduces the amplitude, but the tradeoff is that the system responds at more frequencies.

These features of driven harmonic oscillators apply to a huge variety of systems. When you tune a radio, for example, you are adjusting its resonant frequency so that it only oscillates to the desired station's broadcast (driving) frequency. The more selective the radio is in discriminating between stations, the smaller its damping. Magnetic resonance imaging (MRI) is a widely used medical diagnostic tool in which atomic nuclei (mostly hydrogen nuclei) are made to resonate by incoming radio waves (on the order of 100 MHz). A child on a swing is driven by a parent at the swing's natural frequency to achieve maximum amplitude. In all of these cases, the efficiency of energy transfer from the driving force into the oscillator is best at resonance. Speed bumps and gravel roads prove that even a car's suspension system is not immune to resonance. In spite of finely engineered shock absorbers, which ordinarily convert mechanical energy to thermal energy almost as fast as it comes in, speed bumps still cause a large-amplitude oscillation. On gravel roads that are corrugated, you may have noticed that if you travel at the "wrong" speed, the bumps are very noticeable whereas at other speeds you may hardly feel the bumps at all. **Figure 16.28** shows a photograph of a famous example (the Tacoma Narrows Bridge) of the destructive effects of a driven harmonic oscillation. The Millennium Bridge in London was closed for a short period of time for the same reason while inspections were carried out.

In our bodies, the chest cavity is a clear example of a system at resonance. The diaphragm and chest wall drive the oscillations of the chest cavity which result in the lungs inflating and deflating. The system is critically damped and the muscular diaphragm oscillates at the resonant value for the system, making it highly efficient.

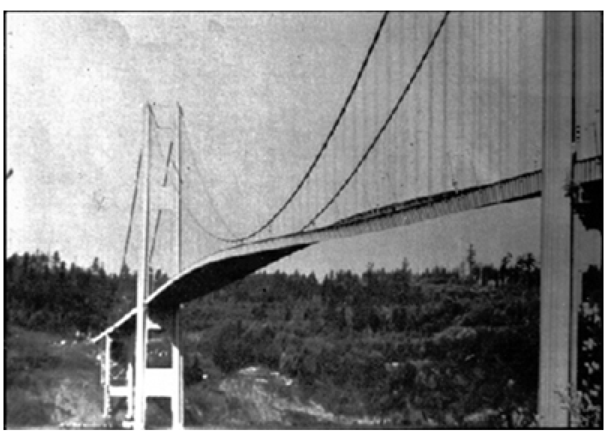


Figure 16.28 In 1940, the Tacoma Narrows Bridge in Washington state collapsed. Heavy cross winds drove the bridge into oscillations at its resonant frequency. Damping decreased when support cables broke loose and started to slip over the towers, allowing increasingly greater amplitudes until the structure failed (credit: PRI's *Studio 360*, via Flickr)

Check Your Understanding

A famous magic trick involves a performer singing a note toward a crystal glass until the glass shatters. Explain why the trick works in terms of resonance and natural frequency.

Solution
The performer must be singing a note that corresponds to the natural frequency of the glass. As the sound wave is directed at the glass, the glass responds by resonating at the same frequency as the sound wave. With enough energy introduced into the system, the glass begins to vibrate and eventually shatters.

16.9 Waves

Figure 16.29 Waves in the ocean behave similarly to all other types of waves. (credit: Steve Jurveston, Flickr)

What do we mean when we say something is a wave? The most intuitive and easiest wave to imagine is the familiar water wave. More precisely, a **wave** is a disturbance that propagates, or moves from the place it was created. For water waves, the disturbance is in the surface of the water, perhaps created by a rock thrown into a pond or by a swimmer splashing the surface repeatedly. For sound waves, the disturbance is a change in air pressure, perhaps created by the oscillating cone inside a speaker. For earthquakes, there are several types of disturbances, including disturbance of Earth's surface and pressure disturbances under the surface. Even radio waves are most easily understood using an analogy with water waves. Visualizing water waves is useful because there is more to it than just a mental image. Water waves exhibit characteristics common to all waves, such as amplitude, period, frequency and energy. All wave characteristics can be described by a small set of underlying principles.

A wave is a disturbance that propagates, or moves from the place it was created. The simplest waves repeat themselves for several cycles and are associated with simple harmonic motion. Let us start by considering the simplified water wave in Figure 16.30. The wave is an up and down disturbance of the water surface. It causes a sea gull to move up and down in simple harmonic motion as the wave crests and troughs (peaks and valleys) pass under the bird. The time for one complete up and down motion is the wave's period T. The wave's frequency is $f = 1 / T$, as usual.

The wave itself moves to the right in the figure. This movement of the wave is actually the disturbance moving to the right, not the water itself (or the bird would move to the right). We define **wave velocity** v_{w} to be the speed at which the disturbance moves. Wave velocity is sometimes also called the *propagation velocity or propagation speed,* because the disturbance propagates from one location to another.

Misconception Alert

Many people think that water waves push water from one direction to another. In fact, the particles of water tend to stay in one location, save for moving up and down due to the energy in the wave. The energy moves forward through the water, but the water stays in one place. If you feel yourself pushed in an ocean, what you feel is the energy of the wave, not a rush of water.

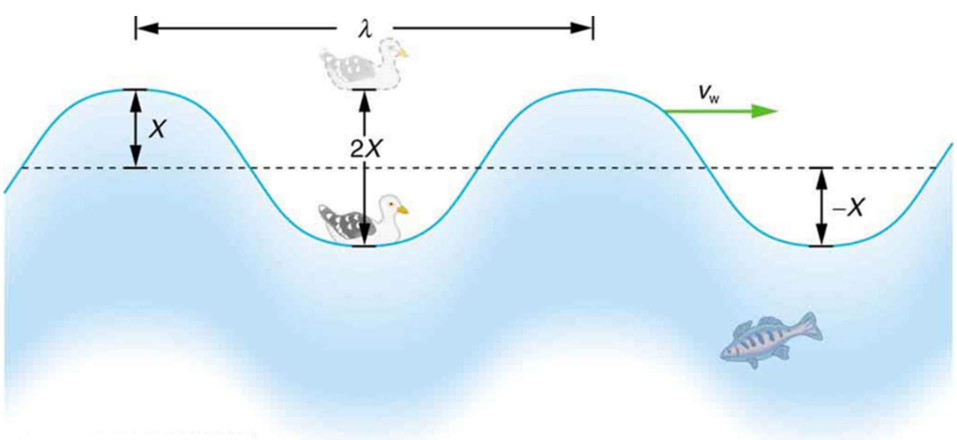


Figure 16.30 An idealized ocean wave passes under a sea gull that bobs up and down in simple harmonic motion. The wave has a wavelength λ, which is the distance between adjacent identical parts of the wave. The up and down disturbance of the surface propagates parallel to the surface at a speed v_{w}.

The water wave in the figure also has a length associated with it, called its **wavelength** λ, the distance between adjacent identical parts of a wave. (λ is the distance parallel to the direction of propagation.) The speed of propagation v_{w} is the distance the wave travels in a given time, which is one wavelength in the time of one period. In equation form, that is

$$v_{\text{w}} = \frac{\lambda}{T} \tag{16.66}$$

or

$$v_{\text{w}} = f\lambda. \tag{16.67}$$

This fundamental relationship holds for all types of waves. For water waves, v_w is the speed of a surface wave; for sound, v_w is the speed of sound; and for visible light, v_w is the speed of light, for example.

Take-Home Experiment: Waves in a Bowl

Fill a large bowl or basin with water and wait for the water to settle so there are no ripples. Gently drop a cork into the middle of the bowl. Estimate the wavelength and period of oscillation of the water wave that propagates away from the cork. Remove the cork from the bowl and wait for the water to settle again. Gently drop the cork at a height that is different from the first drop. Does the wavelength depend upon how high above the water the cork is dropped?

Example 16.8 Calculate the Velocity of Wave Propagation: Gull in the Ocean

Calculate the wave velocity of the ocean wave in **Figure 16.30** if the distance between wave crests is 10.0 m and the time for a sea gull to bob up and down is 5.00 s.

Strategy

We are asked to find v_w. The given information tells us that $\lambda = 10.0\text{ m}$ and $T = 5.00\text{ s}$. Therefore, we can use $v_w = \frac{\lambda}{T}$ to find the wave velocity.

Solution

1. Enter the known values into $v_w = \frac{\lambda}{T}$:

$$v_w = \frac{10.0\text{ m}}{5.00\text{ s}}. \tag{16.68}$$

2. Solve for v_w to find $v_w = 2.00\text{ m/s}.$

Discussion

This slow speed seems reasonable for an ocean wave. Note that the wave moves to the right in the figure at this speed, not the varying speed at which the sea gull moves up and down.

Transverse and Longitudinal Waves

A simple wave consists of a periodic disturbance that propagates from one place to another. The wave in **Figure 16.31** propagates in the horizontal direction while the surface is disturbed in the vertical direction. Such a wave is called a **transverse wave** or shear wave; in such a wave, the disturbance is perpendicular to the direction of propagation. In contrast, in a **longitudinal wave** or compressional wave, the disturbance is parallel to the direction of propagation. **Figure 16.32** shows an example of a longitudinal wave. The size of the disturbance is its amplitude *X* and is completely independent of the speed of propagation v_w.

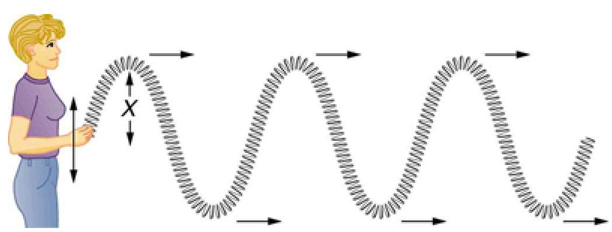


Figure 16.31 In this example of a transverse wave, the wave propagates horizontally, and the disturbance in the cord is in the vertical direction.

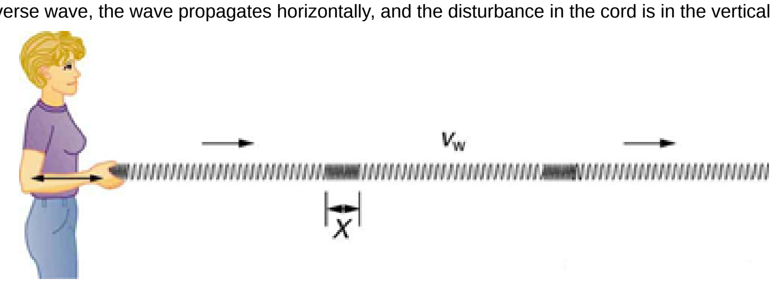


Figure 16.32 In this example of a longitudinal wave, the wave propagates horizontally, and the disturbance in the cord is also in the horizontal direction.

Waves may be transverse, longitudinal, or *a combination of the two*. (Water waves are actually a combination of transverse and longitudinal. The simplified water wave illustrated in **Figure 16.30** shows no longitudinal motion of the bird.) The waves on the strings of musical instruments are transverse—so are electromagnetic waves, such as visible light.

Sound waves in air and water are longitudinal. Their disturbances are periodic variations in pressure that are transmitted in fluids. Fluids do not have appreciable shear strength, and thus the sound waves in them must be longitudinal or compressional. Sound in solids can be both longitudinal and transverse.

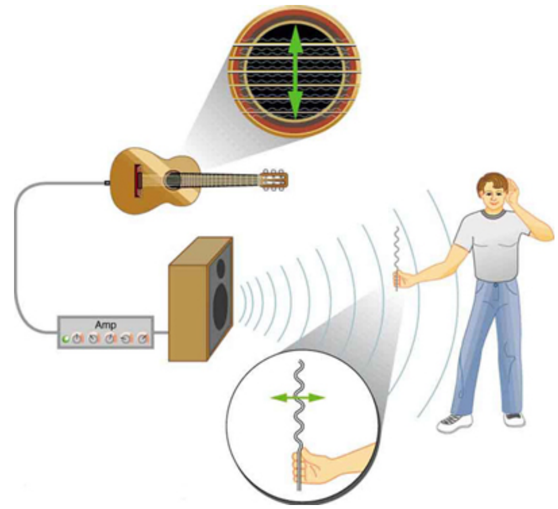


Figure 16.33 The wave on a guitar string is transverse. The sound wave rattles a sheet of paper in a direction that shows the sound wave is longitudinal.

Earthquake waves under Earth's surface also have both longitudinal and transverse components (called compressional or P-waves and shear or S-waves, respectively). These components have important individual characteristics—they propagate at different speeds, for example. Earthquakes also have surface waves that are similar to surface waves on water.

Check Your Understanding

Why is it important to differentiate between longitudinal and transverse waves?

Solution
In the different types of waves, energy can propagate in a different direction relative to the motion of the wave. This is important to understand how different types of waves affect the materials around them.

PhET Explorations: Wave on a String

Watch a string vibrate in slow motion. Wiggle the end of the string and make waves, or adjust the frequency and amplitude of an oscillator. Adjust the damping and tension. The end can be fixed, loose, or open.

PhET Interactive Simulation

Figure 16.34 Wave on a String (http://cnx.org/content/m42248/1.5/wave-on-a-string_en.jar)

16.10 Superposition and Interference

Figure 16.35 These waves result from the superposition of several waves from different sources, producing a complex pattern. (credit: waterborough, Wikimedia Commons)

Most waves do not look very simple. They look more like the waves in **Figure 16.35** than like the simple water wave considered in **Waves**. (Simple waves may be created by a simple harmonic oscillation, and thus have a sinusoidal shape). Complex waves are more interesting, even beautiful, but they look formidable. Most waves appear complex because they result from several simple waves adding together. Luckily, the rules for adding waves are quite simple.

When two or more waves arrive at the same point, they superimpose themselves on one another. More specifically, the disturbances of waves are superimposed when they come together—a phenomenon called **superposition**. Each disturbance corresponds to a force, and forces add. If the disturbances are along the same line, then the resulting wave is a simple addition of the disturbances of the individual waves—that is, their amplitudes add. **Figure 16.36** and **Figure 16.37** illustrate superposition in two special cases, both of which produce simple results.

Figure 16.36 shows two identical waves that arrive at the same point exactly in phase. The crests of the two waves are precisely aligned, as are the troughs. This superposition produces pure **constructive interference**. Because the disturbances add, pure constructive interference produces a wave that has twice the amplitude of the individual waves, but has the same wavelength.

Figure 16.37 shows two identical waves that arrive exactly out of phase—that is, precisely aligned crest to trough—producing pure **destructive interference**. Because the disturbances are in the opposite direction for this superposition, the resulting amplitude is zero for pure destructive interference—the waves completely cancel.

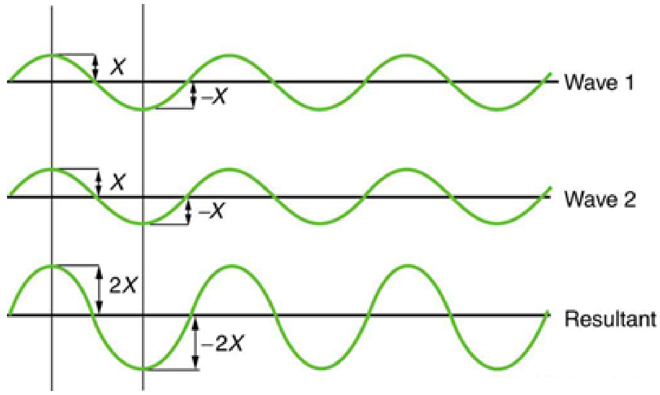


Figure 16.36 Pure constructive interference of two identical waves produces one with twice the amplitude, but the same wavelength.

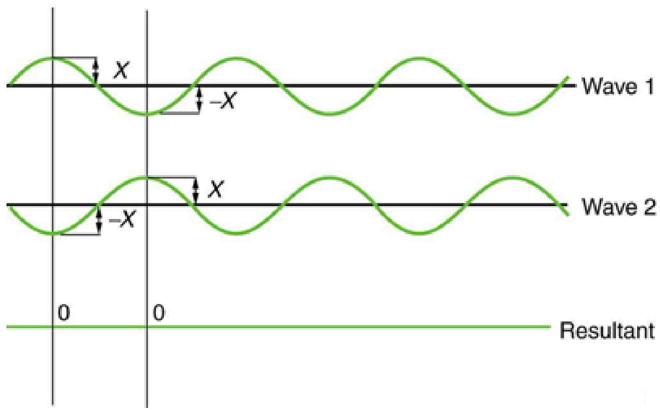


Figure 16.37 Pure destructive interference of two identical waves produces zero amplitude, or complete cancellation.

While pure constructive and pure destructive interference do occur, they require precisely aligned identical waves. The superposition of most waves produces a combination of constructive and destructive interference and can vary from place to place and time to time. Sound from a stereo, for example, can be loud in one spot and quiet in another. Varying loudness means the sound waves add partially constructively and partially destructively at different locations. A stereo has at least two speakers creating sound waves, and waves can reflect from walls. All these waves superimpose. An example of sounds that vary over time from constructive to destructive is found in the combined whine of airplane jets heard by a stationary passenger. The combined sound can fluctuate up and down in volume as the sound from the two engines varies in time from constructive to destructive. These examples are of waves that are similar.

An example of the superposition of two dissimilar waves is shown in **Figure 16.38**. Here again, the disturbances add and subtract, producing a more complicated looking wave.

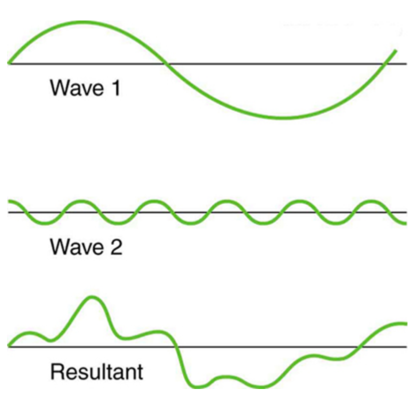


Figure 16.38 Superposition of non-identical waves exhibits both constructive and destructive interference.

Standing Waves

Sometimes waves do not seem to move; rather, they just vibrate in place. Unmoving waves can be seen on the surface of a glass of milk in a refrigerator, for example. Vibrations from the refrigerator motor create waves on the milk that oscillate up and down but do not seem to move across the surface. These waves are formed by the superposition of two or more moving waves, such as illustrated in **Figure 16.39** for two identical waves moving in opposite directions. The waves move through each other with their disturbances adding as they go by. If the two waves have the same amplitude and wavelength, then they alternate between constructive and destructive interference. The resultant looks like a wave standing in place and, thus, is called a **standing wave**. Waves on the glass of milk are one example of standing waves. There are other standing waves, such as on guitar strings and in organ pipes. With the glass of milk, the two waves that produce standing waves may come from reflections from the side of the glass.

A closer look at earthquakes provides evidence for conditions appropriate for resonance, standing waves, and constructive and destructive interference. A building may be vibrated for several seconds with a driving frequency matching that of the natural frequency of vibration of the building—producing a resonance resulting in one building collapsing while neighboring buildings do not. Often buildings of a certain height are devastated while other taller buildings remain intact. The building height matches the condition for setting up a standing wave for that particular height. As the earthquake waves travel along the surface of Earth and reflect off denser rocks, constructive interference occurs at certain points. Often areas closer to the epicenter are not damaged while areas farther away are damaged.

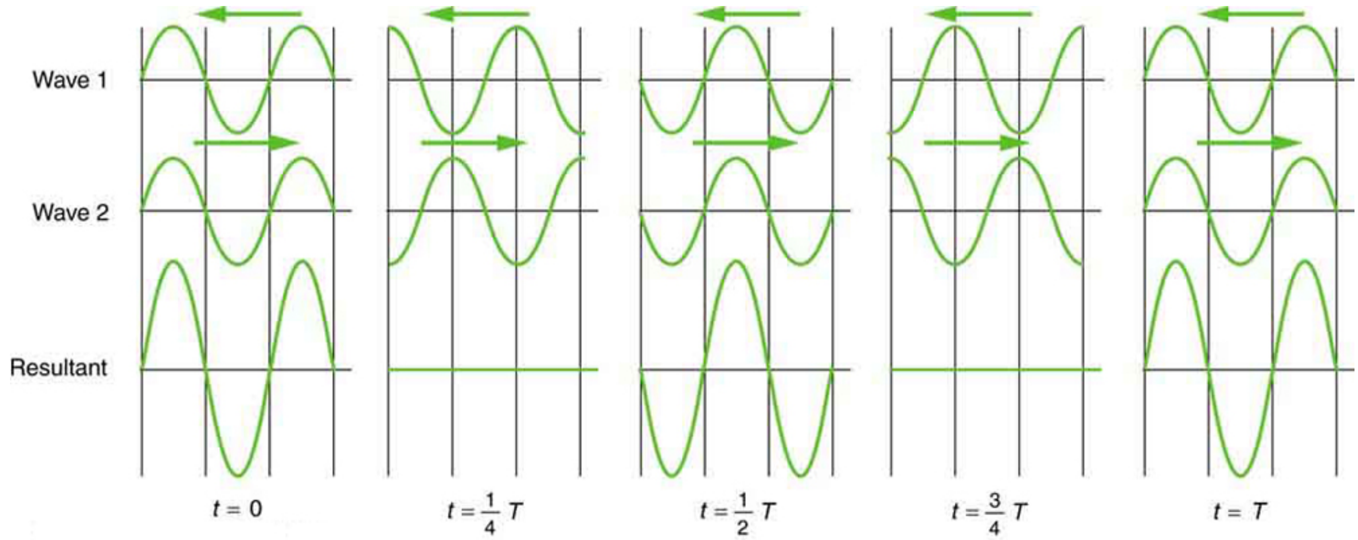


Figure 16.39 Standing wave created by the superposition of two identical waves moving in opposite directions. The oscillations are at fixed locations in space and result from alternately constructive and destructive interference.

Standing waves are also found on the strings of musical instruments and are due to reflections of waves from the ends of the string. **Figure 16.40** and **Figure 16.41** show three standing waves that can be created on a string that is fixed at both ends. **Nodes** are the points where the string does not move; more generally, nodes are where the wave disturbance is zero in a standing wave. The fixed ends of strings must be nodes, too, because the string cannot move there. The word **antinode** is used to denote the location of maximum amplitude in standing waves. Standing waves on strings have a frequency that is related to the propagation speed v_{w} of the disturbance on the string. The wavelength λ is determined by the distance between the points where the string is fixed in place.

The lowest frequency, called the **fundamental frequency**, is thus for the longest wavelength, which is seen to be $\lambda_1 = 2L$. Therefore, the fundamental frequency is $f_1 = v_{\text{w}} / \lambda_1 = v_{\text{w}} / 2L$. In this case, the **overtones** or harmonics are multiples of the fundamental frequency. As seen in **Figure 16.41**, the first harmonic can easily be calculated since $\lambda_2 = L$. Thus, $f_2 = v_{\text{w}} / \lambda_2 = v_{\text{w}} / 2L = 2f_1$. Similarly, $f_3 = 3f_1$, and so on. All of these frequencies can be changed by adjusting the tension in the string. The greater the tension, the greater v_{w} is and the higher the frequencies. This observation is familiar to anyone who has ever observed a string instrument being tuned. We will see in later chapters that standing waves are crucial to many resonance phenomena, such as in sounding boxes on string instruments.

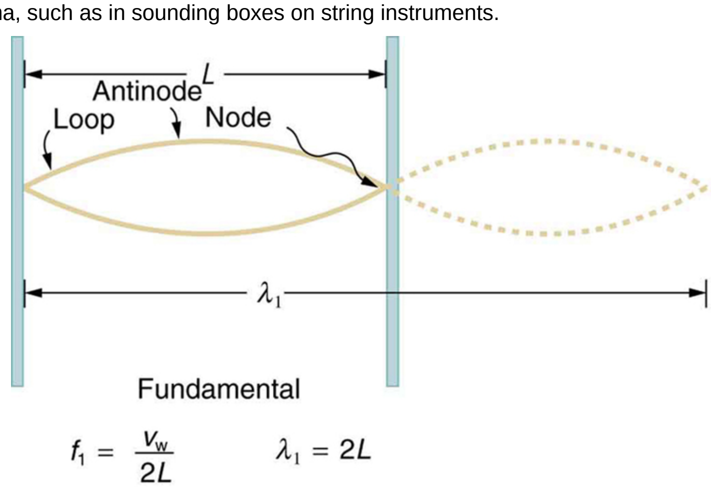


Figure 16.40 The figure shows a string oscillating at its fundamental frequency.

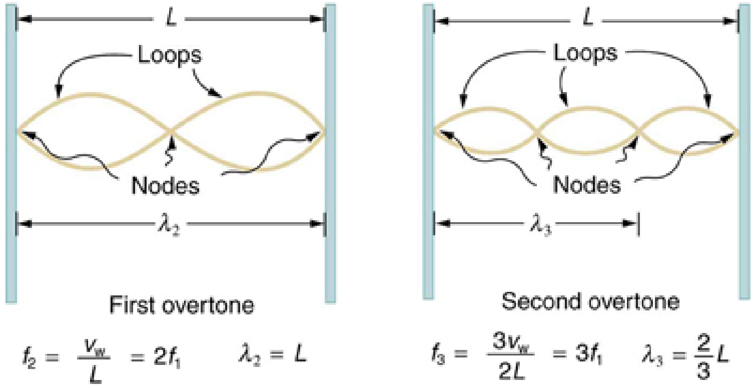


Figure 16.41 First and second harmonic frequencies are shown.

Beats

Striking two adjacent keys on a piano produces a warbling combination usually considered to be unpleasant. The superposition of two waves of similar but not identical frequencies is the culprit. Another example is often noticeable in jet aircraft, particularly the two-engine variety, while taxiing. The combined sound of the engines goes up and down in loudness. This varying loudness happens because the sound waves have similar but not identical frequencies. The discordant warbling of the piano and the fluctuating loudness of the jet engine noise are both due to alternately constructive and destructive interference as the two waves go in and out of phase. **Figure 16.42** illustrates this graphically.

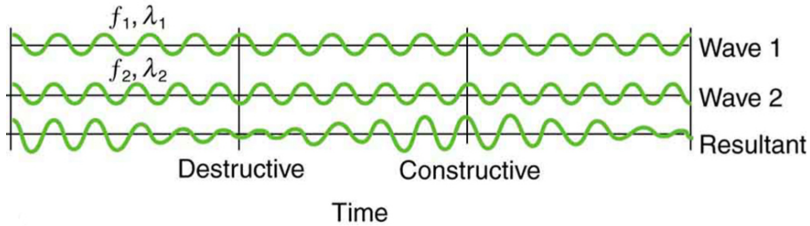


Figure 16.42 Beats are produced by the superposition of two waves of slightly different frequencies but identical amplitudes. The waves alternate in time between constructive interference and destructive interference, giving the resulting wave a time-varying amplitude.

The wave resulting from the superposition of two similar-frequency waves has a frequency that is the average of the two. This wave fluctuates in amplitude, or *beats*, with a frequency called the **beat frequency**. We can determine the beat frequency by adding two waves together mathematically. Note that a wave can be represented at one point in space as

$$x = X\cos\left(\frac{2\pi\, t}{T}\right) = X\cos(2\pi\, ft), \tag{16.69}$$

where $f = 1/T$ is the frequency of the wave. Adding two waves that have different frequencies but identical amplitudes produces a resultant

$$x = x_1 + x_2. \tag{16.70}$$

More specifically,

$$x = X\cos(2\pi\, f_1\, t) + X\cos(2\pi\, f_2\, t). \tag{16.71}$$

Using a trigonometric identity, it can be shown that

$$x = 2X\cos(\pi\, f_B\, t)\cos(2\pi\, f_{ave}\, t), \tag{16.72}$$

where

$$f_B = |\, f_1 - f_2\, | \tag{16.73}$$

is the beat frequency, and f_{ave} is the average of f_1 and f_2. These results mean that the resultant wave has twice the amplitude and the average frequency of the two superimposed waves, but it also fluctuates in overall amplitude at the beat frequency f_B. The first cosine term in the expression effectively causes the amplitude to go up and down. The second cosine term is the wave with frequency f_{ave}. This result is valid for all types of waves. However, if it is a sound wave, providing the two frequencies are similar, then what we hear is an average frequency that gets louder and softer (or warbles) at the beat frequency.

Making Career Connections

Piano tuners use beats routinely in their work. When comparing a note with a tuning fork, they listen for beats and adjust the string until the beats go away (to zero frequency). For example, if the tuning fork has a 256 Hz frequency and two beats per second are heard, then the other frequency is either 254 or 258 Hz. Most keys hit multiple strings, and these strings are actually adjusted until they have nearly the same frequency and give a slow beat for richness. Twelve-string guitars and mandolins are also tuned using beats.

While beats may sometimes be annoying in audible sounds, we will find that beats have many applications. Observing beats is a very useful way to compare similar frequencies. There are applications of beats as apparently disparate as in ultrasonic imaging and radar speed traps.

Check Your Understanding

Imagine you are holding one end of a jump rope, and your friend holds the other. If your friend holds her end still, you can move your end up and down, creating a transverse wave. If your friend then begins to move her end up and down, generating a wave in the opposite direction, what resultant wave forms would you expect to see in the jump rope?

Solution

The rope would alternate between having waves with amplitudes two times the original amplitude and reaching equilibrium with no amplitude at all. The wavelengths will result in both constructive and destructive interference

Check Your Understanding

Define nodes and antinodes.

Solution

Nodes are areas of wave interference where there is no motion. Antinodes are areas of wave interference where the motion is at its maximum point.

Check Your Understanding

You hook up a stereo system. When you test the system, you notice that in one corner of the room, the sounds seem dull. In another area, the sounds seem excessively loud. Describe how the sound moving about the room could result in these effects.

Solution

With multiple speakers putting out sounds into the room, and these sounds bouncing off walls, there is bound to be some wave interference. In the dull areas, the interference is probably mostly destructive. In the louder areas, the interference is probably mostly constructive.

PhET Explorations: Wave Interference

Make waves with a dripping faucet, audio speaker, or laser! Add a second source or a pair of slits to create an interference pattern.

PhET Interactive Simulation

Figure 16.43 Wave Interference (http://cnx.org/content/m42249/1.5/wave-interference_en.jar)

16.11 Energy in Waves: Intensity

Figure 16.44 The destructive effect of an earthquake is palpable evidence of the energy carried in these waves. The Richter scale rating of earthquakes is related to both their amplitude and the energy they carry. (credit: Petty Officer 2nd Class Candice Villarreal, U.S. Navy)

All waves carry energy. The energy of some waves can be directly observed. Earthquakes can shake whole cities to the ground, performing the work of thousands of wrecking balls.

Loud sounds pulverize nerve cells in the inner ear, causing permanent hearing loss. Ultrasound is used for deep-heat treatment of muscle strains. A laser beam can burn away a malignancy. Water waves chew up beaches.

The amount of energy in a wave is related to its amplitude. Large-amplitude earthquakes produce large ground displacements. Loud sounds have higher pressure amplitudes and come from larger-amplitude source vibrations than soft sounds. Large ocean breakers churn up the shore more than small ones. More quantitatively, a wave is a displacement that is resisted by a restoring force. The larger the displacement x, the larger the force

$F = kx$ needed to create it. Because work W is related to force multiplied by distance (Fx) and energy is put into the wave by the work done to create it, the energy in a wave is related to amplitude. In fact, a wave's energy is directly proportional to its amplitude squared because

$$W \propto Fx = kx^2. \tag{16.74}$$

The energy effects of a wave depend on time as well as amplitude. For example, the longer deep-heat ultrasound is applied, the more energy it transfers. Waves can also be concentrated or spread out. Sunlight, for example, can be focused to burn wood. Earthquakes spread out, so they do less damage the farther they get from the source. In both cases, changing the area the waves cover has important effects. All these pertinent factors are included in the definition of **intensity** I as power per unit area:

$$I = \frac{P}{A} \tag{16.75}$$

where P is the power carried by the wave through area A. The definition of intensity is valid for any energy in transit, including that carried by waves. The SI unit for intensity is watts per square meter (W/m^2). For example, infrared and visible energy from the Sun impinge on Earth at an intensity of $1300\ \text{W/m}^2$ just above the atmosphere. There are other intensity-related units in use, too. The most common is the decibel. For example, a 90 decibel sound level corresponds to an intensity of $10^{-3}\ \text{W/m}^2$. (This quantity is not much power per unit area considering that 90 decibels is a relatively high sound level. Decibels will be discussed in some detail in a later chapter.

Example 16.9 Calculating intensity and power: How much energy is in a ray of sunlight?

The average intensity of sunlight on Earth's surface is about $700\ \text{W/m}^2$.

(a) Calculate the amount of energy that falls on a solar collector having an area of $0.500\ \text{m}^2$ in $4.00\ \text{h}$.

(b) What intensity would such sunlight have if concentrated by a magnifying glass onto an area 200 times smaller than its own?

Strategy a

Because power is energy per unit time or $P = \frac{E}{t}$, the definition of intensity can be written as $I = \frac{P}{A} = \frac{E/t}{A}$, and this equation can be solved for E with the given information.

Solution a

1. Begin with the equation that states the definition of intensity:

$$I = \frac{P}{A}. \tag{16.76}$$

2. Replace P with its equivalent E/t:

$$I = \frac{E/t}{A}. \tag{16.77}$$

3. Solve for E:

$$E = IAt. \tag{16.78}$$

4. Substitute known values into the equation:

$$E = \left(700\ \text{W/m}^2\right)\left(0.500\ \text{m}^2\right)\left[(4.00\ \text{h})(3600\ \text{s/h})\right]. \tag{16.79}$$

5. Calculate to find E and convert units:

$$5.04 \times 10^6\ \text{J}, \tag{16.80}$$

Discussion a

The energy falling on the solar collector in 4 h in part is enough to be useful—for example, for heating a significant amount of water.

Strategy b

Taking a ratio of new intensity to old intensity and using primes for the new quantities, we will find that it depends on the ratio of the areas. All other quantities will cancel.

Solution b

1. Take the ratio of intensities, which yields:

$$\frac{I'}{I} = \frac{P'/A'}{P/A} = \frac{A}{A'}\left(\text{The powers cancel because } P' = P\right). \tag{16.81}$$

2. Identify the knowns:

$$A = 200A', \tag{16.82}$$

$$\frac{I'}{I} = 200. \tag{16.83}$$

3. Substitute known quantities:

$$I' = 200I = 200\left(700\ \text{W/m}^2\right). \tag{16.84}$$

4. Calculate to find I':

$$I' = 1.40 \times 10^5\ \text{W/m}^2. \tag{16.85}$$

Discussion b

Decreasing the area increases the intensity considerably. The intensity of the concentrated sunlight could even start a fire.

Example 16.10 Determine the combined intensity of two waves: Perfect constructive interference

If two identical waves, each having an intensity of $1.00\ \text{W/m}^2$, interfere perfectly constructively, what is the intensity of the resulting wave?

Strategy

We know from **Superposition and Interference** that when two identical waves, which have equal amplitudes X , interfere perfectly constructively, the resulting wave has an amplitude of $2X$. Because a wave's intensity is proportional to amplitude squared, the intensity of the resulting wave is four times as great as in the individual waves.

Solution

1. Recall that intensity is proportional to amplitude squared.
2. Calculate the new amplitude:

$$I\prime \propto (X\prime)^2 = (2X)^2 = 4X^2. \tag{16.86}$$

3. Recall that the intensity of the old amplitude was:

$$I \propto X^2. \tag{16.87}$$

4. Take the ratio of new intensity to the old intensity. This gives:

$$\frac{I\prime}{I} = 4. \tag{16.88}$$

5. Calculate to find $I\prime$:

$$I\prime = 4I = 4.00\ \text{W/m}^2. \tag{16.89}$$

Discussion

The intensity goes up by a factor of 4 when the amplitude doubles. This answer is a little disquieting. The two individual waves each have intensities of $1.00\ \text{W/m}^2$, yet their sum has an intensity of $4.00\ \text{W/m}^2$, which may appear to violate conservation of energy. This violation, of course, cannot happen. What does happen is intriguing. The area over which the intensity is $4.00\ \text{W/m}^2$ is much less than the area covered by the two waves before they interfered. There are other areas where the intensity is zero. The addition of waves is not as simple as our first look in **Superposition and Interference** suggested. We actually get a pattern of both constructive interference and destructive interference whenever two waves are added. For example, if we have two stereo speakers putting out $1.00\ \text{W/m}^2$ each, there will be places in the room where the intensity is $4.00\ \text{W/m}^2$, other places where the intensity is zero, and others in between. **Figure 16.45** shows what this interference might look like. We will pursue interference patterns elsewhere in this text.

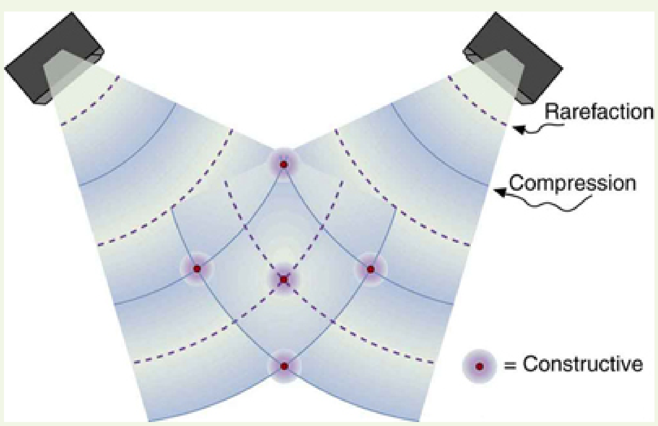


Figure 16.45 These stereo speakers produce both constructive interference and destructive interference in the room, a property common to the superposition of all types of waves. The shading is proportional to intensity.

Check Your Understanding

Which measurement of a wave is most important when determining the wave's intensity?

Solution

Amplitude, because a wave's energy is directly proportional to its amplitude squared.

Glossary

amplitude: the maximum displacement from the equilibrium position of an object oscillating around the equilibrium position

antinode: the location of maximum amplitude in standing waves

beat frequency: the frequency of the amplitude fluctuations of a wave

constructive interference: when two waves arrive at the same point exactly in phase; that is, the crests of the two waves are precisely aligned, as are the troughs

critical damping: the condition in which the damping of an oscillator causes it to return as quickly as possible to its equilibrium position without oscillating back and forth about this position

deformation: displacement from equilibrium

destructive interference: when two identical waves arrive at the same point exactly out of phase; that is, precisely aligned crest to trough

elastic potential energy: potential energy stored as a result of deformation of an elastic object, such as the stretching of a spring

force constant: a constant related to the rigidity of a system: the larger the force constant, the more rigid the system; the force constant is represented by *k*

frequency: number of events per unit of time

fundamental frequency: the lowest frequency of a periodic waveform

intensity: power per unit area

longitudinal wave: a wave in which the disturbance is parallel to the direction of propagation

natural frequency: the frequency at which a system would oscillate if there were no driving and no damping forces

nodes: the points where the string does not move; more generally, nodes are where the wave disturbance is zero in a standing wave

oscillate: moving back and forth regularly between two points

over damping: the condition in which damping of an oscillator causes it to return to equilibrium without oscillating; oscillator moves more slowly toward equilibrium than in the critically damped system

overtones: multiples of the fundamental frequency of a sound

periodic motion: motion that repeats itself at regular time intervals

period: time it takes to complete one oscillation

resonance: the phenomenon of driving a system with a frequency equal to the system's natural frequency

resonate: a system being driven at its natural frequency

restoring force: force acting in opposition to the force caused by a deformation

simple harmonic motion: the oscillatory motion in a system where the net force can be described by Hooke's law

simple harmonic oscillator: a device that implements Hooke's law, such as a mass that is attached to a spring, with the other end of the spring being connected to a rigid support such as a wall

simple pendulum: an object with a small mass suspended from a light wire or string

superposition: the phenomenon that occurs when two or more waves arrive at the same point

transverse wave: a wave in which the disturbance is perpendicular to the direction of propagation

under damping: the condition in which damping of an oscillator causes it to return to equilibrium with the amplitude gradually decreasing to zero; system returns to equilibrium faster but overshoots and crosses the equilibrium position one or more times

wave velocity: the speed at which the disturbance moves. Also called the propagation velocity or propagation speed

wavelength: the distance between adjacent identical parts of a wave

wave: a disturbance that moves from its source and carries energy

Section Summary

16.1 Hooke's Law: Stress and Strain Revisited

- An oscillation is a back and forth motion of an object between two points of deformation.
- An oscillation may create a wave, which is a disturbance that propagates from where it was created.
- The simplest type of oscillations and waves are related to systems that can be described by Hooke's law:

$$F = -kx,$$

  where F is the restoring force, x is the displacement from equilibrium or deformation, and k is the force constant of the system.
- Elastic potential energy $\mathrm{PE_{el}}$ stored in the deformation of a system that can be described by Hooke's law is given by

$$PE_{el} = (1/2)kx^2.$$

16.2 Period and Frequency in Oscillations

- Periodic motion is a repetitious oscillation.
- The time for one oscillation is the period T.
- The number of oscillations per unit time is the frequency f.
- These quantities are related by

$$f = \frac{1}{T}.$$

16.3 Simple Harmonic Motion: A Special Periodic Motion

- Simple harmonic motion is oscillatory motion for a system that can be described only by Hooke's law. Such a system is also called a simple harmonic oscillator.
- Maximum displacement is the amplitude X. The period T and frequency f of a simple harmonic oscillator are given by

  $T = 2\pi\sqrt{\frac{m}{k}}$ and $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$, where m is the mass of the system.
- Displacement in simple harmonic motion as a function of time is given by $x(t) = X\cos\frac{2\pi t}{T}$.
- The velocity is given by $v(t) = -v_{max}\sin\frac{2\pi t}{T}$, where $v_{max} = \sqrt{k/m}X$.
- The acceleration is found to be $a(t) = -\frac{kX}{m}\cos\frac{2\pi t}{T}$.

16.4 The Simple Pendulum

- A mass m suspended by a wire of length L is a simple pendulum and undergoes simple harmonic motion for amplitudes less than about 15°.

  The period of a simple pendulum is

$$T = 2\pi\sqrt{\frac{L}{g}},$$

  where L is the length of the string and g is the acceleration due to gravity.

16.5 Energy and the Simple Harmonic Oscillator

- Energy in the simple harmonic oscillator is shared between elastic potential energy and kinetic energy, with the total being constant:

$$\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \text{constant}.$$

- Maximum velocity depends on three factors: it is directly proportional to amplitude, it is greater for stiffer systems, and it is smaller for objects that have larger masses:

$$v_{max} = \sqrt{\frac{k}{m}}X.$$

16.6 Uniform Circular Motion and Simple Harmonic Motion

A projection of uniform circular motion undergoes simple harmonic oscillation.

16.7 Damped Harmonic Motion

- Damped harmonic oscillators have non-conservative forces that dissipate their energy.
- Critical damping returns the system to equilibrium as fast as possible without overshooting.
- An underdamped system will oscillate through the equilibrium position.
- An overdamped system moves more slowly toward equilibrium than one that is critically damped.

16.8 Forced Oscillations and Resonance

- A system's natural frequency is the frequency at which the system will oscillate if not affected by driving or damping forces.
- A periodic force driving a harmonic oscillator at its natural frequency produces resonance. The system is said to resonate.
- The less damping a system has, the higher the amplitude of the forced oscillations near resonance. The more damping a system has, the broader response it has to varying driving frequencies.

16.9 Waves

- A wave is a disturbance that moves from the point of creation with a wave velocity v_w.
- A wave has a wavelength λ, which is the distance between adjacent identical parts of the wave.
- Wave velocity and wavelength are related to the wave's frequency and period by $v_w = \frac{\lambda}{T}$ or $v_w = f\lambda.$
- A transverse wave has a disturbance perpendicular to its direction of propagation, whereas a longitudinal wave has a disturbance parallel to its direction of propagation.

16.10 Superposition and Interference

- Superposition is the combination of two waves at the same location.

- Constructive interference occurs when two identical waves are superimposed in phase.
- Destructive interference occurs when two identical waves are superimposed exactly out of phase.
- A standing wave is one in which two waves superimpose to produce a wave that varies in amplitude but does not propagate.
- Nodes are points of no motion in standing waves.
- An antinode is the location of maximum amplitude of a standing wave.
- Waves on a string are resonant standing waves with a fundamental frequency and can occur at higher multiples of the fundamental, called overtones or harmonics.
- Beats occur when waves of similar frequencies f_1 and f_2 are superimposed. The resulting amplitude oscillates with a beat frequency given by

$$f_B = \mid f_1 - f_2 \mid .$$

16.11 Energy in Waves: Intensity

Intensity is defined to be the power per unit area:

$I = \frac{P}{A}$ and has units of W/m^2.

Conceptual Questions

16.1 Hooke's Law: Stress and Strain Revisited

1. Describe a system in which elastic potential energy is stored.

16.3 Simple Harmonic Motion: A Special Periodic Motion

2. What conditions must be met to produce simple harmonic motion?

3. (a) If frequency is not constant for some oscillation, can the oscillation be simple harmonic motion?

(b) Can you think of any examples of harmonic motion where the frequency may depend on the amplitude?

4. Give an example of a simple harmonic oscillator, specifically noting how its frequency is independent of amplitude.

5. Explain why you expect an object made of a stiff material to vibrate at a higher frequency than a similar object made of a spongy material.

6. As you pass a freight truck with a trailer on a highway, you notice that its trailer is bouncing up and down slowly. Is it more likely that the trailer is heavily loaded or nearly empty? Explain your answer.

7. Some people modify cars to be much closer to the ground than when manufactured. Should they install stiffer springs? Explain your answer.

16.4 The Simple Pendulum

8. Pendulum clocks are made to run at the correct rate by adjusting the pendulum's length. Suppose you move from one city to another where the acceleration due to gravity is slightly greater, taking your pendulum clock with you, will you have to lengthen or shorten the pendulum to keep the correct time, other factors remaining constant? Explain your answer.

16.5 Energy and the Simple Harmonic Oscillator

9. Explain in terms of energy how dissipative forces such as friction reduce the amplitude of a harmonic oscillator. Also explain how a driving mechanism can compensate. (A pendulum clock is such a system.)

16.7 Damped Harmonic Motion

10. Give an example of a damped harmonic oscillator. (They are more common than undamped or simple harmonic oscillators.)

11. How would a car bounce after a bump under each of these conditions?
- overdamping
- underdamping
- critical damping

12. Most harmonic oscillators are damped and, if undriven, eventually come to a stop. How is this observation related to the second law of thermodynamics?

16.8 Forced Oscillations and Resonance

13. Why are soldiers in general ordered to "route step" (walk out of step) across a bridge?

16.9 Waves

14. Give one example of a transverse wave and another of a longitudinal wave, being careful to note the relative directions of the disturbance and wave propagation in each.

15. What is the difference between propagation speed and the frequency of a wave? Does one or both affect wavelength? If so, how?

16.10 Superposition and Interference

16. Speakers in stereo systems have two color-coded terminals to indicate how to hook up the wires. If the wires are reversed, the speaker moves in a direction opposite that of a properly connected speaker. Explain why it is important to have both speakers connected the same way.

16.11 Energy in Waves: Intensity

17. Two identical waves undergo pure constructive interference. Is the resultant intensity twice that of the individual waves? Explain your answer.

18. Circular water waves decrease in amplitude as they move away from where a rock is dropped. Explain why.

Problems & Exercises

16.1 Hooke's Law: Stress and Strain Revisited

19. Fish are hung on a spring scale to determine their mass (most fishermen feel no obligation to truthfully report the mass).

(a) What is the force constant of the spring in such a scale if it the spring stretches 8.00 cm for a 10.0 kg load?

(b) What is the mass of a fish that stretches the spring 5.50 cm?

(c) How far apart are the half-kilogram marks on the scale?

20. It is weigh-in time for the local under-85-kg rugby team. The bathroom scale used to assess eligibility can be described by Hooke's law and is depressed 0.75 cm by its maximum load of 120 kg. (a) What is the spring's effective spring constant? (b) A player stands on the scales and depresses it by 0.48 cm. Is he eligible to play on this under-85 kg team?

21. One type of BB gun uses a spring-driven plunger to blow the BB from its barrel. (a) Calculate the force constant of its plunger's spring if you must compress it 0.150 m to drive the 0.0500-kg plunger to a top speed of 20.0 m/s. (b) What force must be exerted to compress the spring?

22. (a) The springs of a pickup truck act like a single spring with a force constant of $1.30\times10^{5}\ \text{N/m}$. By how much will the truck be depressed by its maximum load of 1000 kg?

(b) If the pickup truck has four identical springs, what is the force constant of each?

23. When an 80.0-kg man stands on a pogo stick, the spring is compressed 0.120 m.

(a) What is the force constant of the spring? (b) Will the spring be compressed more when he hops down the road?

24. A spring has a length of 0.200 m when a 0.300-kg mass hangs from it, and a length of 0.750 m when a 1.95-kg mass hangs from it. (a) What is the force constant of the spring? (b) What is the unloaded length of the spring?

16.2 Period and Frequency in Oscillations

25. What is the period of $60.0\ \text{Hz}$ electrical power?

26. If your heart rate is 150 beats per minute during strenuous exercise, what is the time per beat in units of seconds?

27. Find the frequency of a tuning fork that takes $2.50\times10^{-3}\ \text{s}$ to complete one oscillation.

28. A stroboscope is set to flash every $8.00\times10^{-5}\ \text{s}$. What is the frequency of the flashes?

29. A tire has a tread pattern with a crevice every 2.00 cm. Each crevice makes a single vibration as the tire moves. What is the frequency of these vibrations if the car moves at 30.0 m/s?

30. Engineering Application

Each piston of an engine makes a sharp sound every other revolution of the engine. (a) How fast is a race car going if its eight-cylinder engine emits a sound of frequency 750 Hz, given that the engine makes 2000 revolutions per kilometer? (b) At how many revolutions per minute is the engine rotating?

16.3 Simple Harmonic Motion: A Special Periodic Motion

31. A type of cuckoo clock keeps time by having a mass bouncing on a spring, usually something cute like a cherub in a chair. What force constant is needed to produce a period of 0.500 s for a 0.0150-kg mass?

32. If the spring constant of a simple harmonic oscillator is doubled, by what factor will the mass of the system need to change in order for the frequency of the motion to remain the same?

33. A 0.500-kg mass suspended from a spring oscillates with a period of 1.50 s. How much mass must be added to the object to change the period to 2.00 s?

34. By how much leeway (both percentage and mass) would you have in the selection of the mass of the object in the previous problem if you did not wish the new period to be greater than 2.01 s or less than 1.99 s?

35. Suppose you attach the object with mass m to a vertical spring originally at rest, and let it bounce up and down. You release the object from rest at the spring's original rest length. (a) Show that the spring exerts an upward force of $2.00\ mg$ on the object at its lowest point. (b) If the spring has a force constant of $10.0\ \text{N/m}$ and a 0.25-kg-mass object is set in motion as described, find the amplitude of the oscillations. (c) Find the maximum velocity.

36. A diver on a diving board is undergoing simple harmonic motion. Her mass is 55.0 kg and the period of her motion is 0.800 s. The next diver is a male whose period of simple harmonic oscillation is 1.05 s. What is his mass if the mass of the board is negligible?

37. Suppose a diving board with no one on it bounces up and down in a simple harmonic motion with a frequency of 4.00 Hz. The board has an effective mass of 10.0 kg. What is the frequency of the simple harmonic motion of a 75.0-kg diver on the board?

38.

Figure 16.46 This child's toy relies on springs to keep infants entertained. (credit: By Humboldthead, Flickr)

The device pictured in **Figure 16.46** entertains infants while keeping them from wandering. The child bounces in a harness suspended from a door frame by a spring constant.

(a) If the spring stretches 0.250 m while supporting an 8.0-kg child, what is its spring constant?

(b) What is the time for one complete bounce of this child? (c) What is the child's maximum velocity if the amplitude of her bounce is 0.200 m?

39. A 90.0-kg skydiver hanging from a parachute bounces up and down with a period of 1.50 s. What is the new period of oscillation when a second skydiver, whose mass is 60.0 kg, hangs from the legs of the first, as seen in **Figure 16.47**.

Figure 16.47 The oscillations of one skydiver are about to be affected by a second skydiver. (credit: U.S. Army, www.army.mil)

16.4 The Simple Pendulum

As usual, the acceleration due to gravity in these problems is taken to be $g = 9.80 \text{ m}/\text{s}^2$ **, unless otherwise specified.**

40. What is the length of a pendulum that has a period of 0.500 s?

41. Some people think a pendulum with a period of 1.00 s can be driven with "mental energy" or psycho kinetically, because its period is the same as an average heartbeat. True or not, what is the length of such a pendulum?

42. What is the period of a 1.00-m-long pendulum?

43. How long does it take a child on a swing to complete one swing if her center of gravity is 4.00 m below the pivot?

44. The pendulum on a cuckoo clock is 5.00 cm long. What is its frequency?

45. Two parakeets sit on a swing with their combined center of mass 10.0 cm below the pivot. At what frequency do they swing?

46. (a) A pendulum that has a period of 3.00000 s and that is located where the acceleration due to gravity is 9.79 m/s^2 is moved to a location where it the acceleration due to gravity is 9.82 m/s^2 . What is its new period? (b) Explain why so many digits are needed in the value for the period, based on the relation between the period and the acceleration due to gravity.

47. A pendulum with a period of 2.00000 s in one location $\left(g = 9.80 \text{ m/s}^2\right)$ is moved to a new location where the period is now 1.99796 s. What is the acceleration due to gravity at its new location?

48. (a) What is the effect on the period of a pendulum if you double its length?

(b) What is the effect on the period of a pendulum if you decrease its length by 5.00%?

49. Find the ratio of the new/old periods of a pendulum if the pendulum were transported from Earth to the Moon, where the acceleration due to gravity is 1.63 m/s^2 .

50. At what rate will a pendulum clock run on the Moon, where the acceleration due to gravity is 1.63 m/s^2 , if it keeps time accurately on Earth? That is, find the time (in hours) it takes the clock's hour hand to make one revolution on the Moon.

51. Suppose the length of a clock's pendulum is changed by 1.000%, exactly at noon one day. What time will it read 24.00 hours later, assuming it the pendulum has kept perfect time before the change? Note that there are two answers, and perform the calculation to four-digit precision.

52. If a pendulum-driven clock gains 5.00 s/day, what fractional change in pendulum length must be made for it to keep perfect time?

16.5 Energy and the Simple Harmonic Oscillator

53. The length of nylon rope from which a mountain climber is suspended has a force constant of $1.40\times10^4 \text{ N/m}$.

(a) What is the frequency at which he bounces, given his mass plus and the mass of his equipment are 90.0 kg?

(b) How much would this rope stretch to break the climber's fall if he free-falls 2.00 m before the rope runs out of slack? Hint: Use conservation of energy.

(c) Repeat both parts of this problem in the situation where twice this length of nylon rope is used.

54. Engineering Application

Near the top of the Citigroup Center building in New York City, there is an object with mass of $4.00\times10^5 \text{ kg}$ on springs that have adjustable force constants. Its function is to dampen wind-driven oscillations of the building by oscillating at the same frequency as the building is being driven—the driving force is transferred to the object, which oscillates instead of the entire building. (a) What effective force constant should the springs have to make the object oscillate with a period of 2.00 s? (b) What energy is stored in the springs for a 2.00-m displacement from equilibrium?

16.6 Uniform Circular Motion and Simple Harmonic Motion

55. (a)What is the maximum velocity of an 85.0-kg person bouncing on a bathroom scale having a force constant of $1.50\times10^6 \text{ N/m}$, if the amplitude of the bounce is 0.200 cm? (b)What is the maximum energy stored in the spring?

56. A novelty clock has a 0.0100-kg mass object bouncing on a spring that has a force constant of 1.25 N/m. What is the maximum velocity of the object if the object bounces 3.00 cm above and below its equilibrium position? (b) How many joules of kinetic energy does the object have at its maximum velocity?

57. At what positions is the speed of a simple harmonic oscillator half its maximum? That is, what values of x/X give $v = \pm v_{\text{max}}/2$, where X is the amplitude of the motion?

58. A ladybug sits 12.0 cm from the center of a Beatles music album spinning at 33.33 rpm. What is the maximum velocity of its shadow on the wall behind the turntable, if illuminated parallel to the record by the parallel rays of the setting Sun?

16.7 Damped Harmonic Motion

59. The amplitude of a lightly damped oscillator decreases by 3.0% during each cycle. What percentage of the mechanical energy of the oscillator is lost in each cycle?

16.8 Forced Oscillations and Resonance

60. How much energy must the shock absorbers of a 1200-kg car dissipate in order to damp a bounce that initially has a velocity of 0.800 m/s at the equilibrium position? Assume the car returns to its original vertical position.

61. If a car has a suspension system with a force constant of $5.00\times10^4 \text{ N/m}$, how much energy must the car's shocks remove to dampen an oscillation starting with a maximum displacement of 0.0750 m?

62. (a) How much will a spring that has a force constant of 40.0 N/m be stretched by an object with a mass of 0.500 kg when hung motionless from the spring? (b) Calculate the decrease in gravitational potential energy of the 0.500-kg object when it descends this distance. (c) Part of this gravitational energy goes into the spring. Calculate the energy stored in the spring by this stretch, and compare it with the gravitational potential energy. Explain where the rest of the energy might go.

63. Suppose you have a 0.750-kg object on a horizontal surface connected to a spring that has a force constant of 150 N/m. There is simple friction between the object and surface with a static coefficient of friction $\mu_s = 0.100$. (a) How far can the spring be stretched without moving the mass? (b) If the object is set into oscillation with an amplitude twice the distance found in part (a), and the kinetic coefficient of friction is $\mu_k = 0.0850$, what total distance does it travel before stopping? Assume it starts at the maximum amplitude.

64. Engineering Application: A suspension bridge oscillates with an effective force constant of $1.00 \times 10^8 \text{ N/m}$. (a) How much energy is needed to make it oscillate with an amplitude of 0.100 m? (b) If soldiers march across the bridge with a cadence equal to the bridge's natural frequency and impart $1.00 \times 10^4 \text{ J}$ of energy each second, how long does it take for the bridge's oscillations to go from 0.100 m to 0.500 m amplitude?

16.9 Waves

65. Storms in the South Pacific can create waves that travel all the way to the California coast, which are 12,000 km away. How long does it take them if they travel at 15.0 m/s?

66. Waves on a swimming pool propagate at 0.750 m/s. You splash the water at one end of the pool and observe the wave go to the opposite end, reflect, and return in 30.0 s. How far away is the other end of the pool?

67. Wind gusts create ripples on the ocean that have a wavelength of 5.00 cm and propagate at 2.00 m/s. What is their frequency?

68. How many times a minute does a boat bob up and down on ocean waves that have a wavelength of 40.0 m and a propagation speed of 5.00 m/s?

69. Scouts at a camp shake the rope bridge they have just crossed and observe the wave crests to be 8.00 m apart. If they shake it the bridge twice per second, what is the propagation speed of the waves?

70. What is the wavelength of the waves you create in a swimming pool if you splash your hand at a rate of 2.00 Hz and the waves propagate at 0.800 m/s?

71. What is the wavelength of an earthquake that shakes you with a frequency of 10.0 Hz and gets to another city 84.0 km away in 12.0 s?

72. Radio waves transmitted through space at $3.00 \times 10^8 \text{ m/s}$ by the Voyager spacecraft have a wavelength of 0.120 m. What is their frequency?

73. Your ear is capable of differentiating sounds that arrive at the ear just 1.00 ms apart. What is the minimum distance between two speakers that produce sounds that arrive at noticeably different times on a day when the speed of sound is 340 m/s?

74. (a) Seismographs measure the arrival times of earthquakes with a precision of 0.100 s. To get the distance to the epicenter of the quake, they compare the arrival times of S- and P-waves, which travel at different speeds. Figure 16.48) If S- and P-waves travel at 4.00 and 7.20 km/s, respectively, in the region considered, how precisely can the distance to the source of the earthquake be determined? (b) Seismic waves from underground detonations of nuclear bombs can be used to locate the test site and detect violations of test bans. Discuss whether your answer to (a) implies a serious limit to such detection. (Note also that the uncertainty is greater if there is an uncertainty in the propagation speeds of the S- and P-waves.)

Figure 16.48 A seismograph as described in above problem.(credit: Oleg Alexandrov)

16.10 Superposition and Interference

75. A car has two horns, one emitting a frequency of 199 Hz and the other emitting a frequency of 203 Hz. What beat frequency do they produce?

76. The middle-C hammer of a piano hits two strings, producing beats of 1.50 Hz. One of the strings is tuned to 260.00 Hz. What frequencies could the other string have?

77. Two tuning forks having frequencies of 460 and 464 Hz are struck simultaneously. What average frequency will you hear, and what will the beat frequency be?

78. Twin jet engines on an airplane are producing an average sound frequency of 4100 Hz with a beat frequency of 0.500 Hz. What are their individual frequencies?

79. A wave traveling on a Slinky® that is stretched to 4 m takes 2.4 s to travel the length of the Slinky and back again. (a) What is the speed of the wave? (b) Using the same Slinky stretched to the same length, a standing wave is created which consists of three antinodes and four nodes. At what frequency must the Slinky be oscillating?

80. Three adjacent keys on a piano (F, F-sharp, and G) are struck simultaneously, producing frequencies of 349, 370, and 392 Hz. What beat frequencies are produced by this discordant combination?

16.11 Energy in Waves: Intensity

81. Medical Application

Ultrasound of intensity $1.50 \times 10^2 \text{ W/m}^2$ is produced by the rectangular head of a medical imaging device measuring 3.00 by 5.00 cm. What is its power output?

82. The low-frequency speaker of a stereo set has a surface area of 0.05 m^2 and produces 1W of acoustical power. What is the intensity at the speaker? If the speaker projects sound uniformly in all directions, at what distance from the speaker is the intensity 0.1 W/m^2 ?

83. To increase intensity of a wave by a factor of 50, by what factor should the amplitude be increased?

84. Engineering Application

A device called an insolation meter is used to measure the intensity of sunlight has an area of 100 cm^2 and registers 6.50 W. What is the intensity in W/m^2 ?

85. Astronomy Application

Energy from the Sun arrives at the top of the Earth's atmosphere with an intensity of $1.30 \text{ kW/m}^2.$ How long does it take for $1.8 \times 10^9 \text{ J}$ to arrive on an area of 1.00 m^2 ?

86. Suppose you have a device that extracts energy from ocean breakers in direct proportion to their intensity. If the device produces 10.0 kW of power on a day when the breakers are 1.20 m high, how much will it produce when they are 0.600 m high?

87. Engineering Application

(a) A photovoltaic array of (solar cells) is 10.0% efficient in gathering solar energy and converting it to electricity. If the average intensity of sunlight on one day is $700\text{ W/m}^2,$ what area should your array have to gather energy at the rate of 100 W? (b) What is the maximum cost of the array if it must pay for itself in two years of operation averaging 10.0 hours per day? Assume that it earns money at the rate of 9.00 ¢ per kilowatt-hour.

88. A microphone receiving a pure sound tone feeds an oscilloscope, producing a wave on its screen. If the sound intensity is originally $2.00\times10^{-5}\text{ W/m}^2,$ but is turned up until the amplitude increases by 30.0%, what is the new intensity?

89. Medical Application

(a) What is the intensity in W/m^2 of a laser beam used to burn away cancerous tissue that, when 90.0% absorbed, puts 500 J of energy into a circular spot 2.00 mm in diameter in 4.00 s? (b) Discuss how this intensity compares to the average intensity of sunlight (about 700 W/m^2) and the implications that would have if the laser beam entered your eye. Note how your answer depends on the time duration of the exposure.

17 PHYSICS OF HEARING

Figure 17.1 This tree fell some time ago. When it fell, atoms in the air were disturbed. Physicists would call this disturbance sound whether someone was around to hear it or not. (credit: B.A. Bowen Photography)

Learning Objectives

17.1. Sound
17.2. Speed of Sound, Frequency, and Wavelength
17.3. Sound Intensity and Sound Level
17.4. Doppler Effect and Sonic Booms
17.5. Sound Interference and Resonance: Standing Waves in Air Columns
17.6. Hearing
17.7. Ultrasound

Introduction to the Physics of Hearing

If a tree falls in the forest and no one is there to hear it, does it make a sound? The answer to this old philosophical question depends on how you define sound. If sound only exists when someone is around to perceive it, then there was no sound. However, if we define sound in terms of physics; that is, a disturbance of the atoms in matter transmitted from its origin outward (in other words, a wave), then there *was* a sound, even if nobody was around to hear it.

Such a wave is the physical phenomenon we call *sound.* Its perception is hearing. Both the physical phenomenon and its perception are interesting and will be considered in this text. We shall explore both sound and hearing; they are related, but are not the same thing. We will also explore the many practical uses of sound waves, such as in medical imaging.

17.1 Sound

Figure 17.2 This glass has been shattered by a high-intensity sound wave of the same frequency as the resonant frequency of the glass. While the sound is not visible, the effects of the sound prove its existence. (credit: ||read||, Flickr)

Sound can be used as a familiar illustration of waves. Because hearing is one of our most important senses, it is interesting to see how the physical properties of sound correspond to our perceptions of it. **Hearing** is the perception of sound, just as vision is the perception of visible light. But sound has important applications beyond hearing. Ultrasound, for example, is not heard but can be employed to form medical images and is also used in treatment.

The physical phenomenon of **sound** is defined to be a disturbance of matter that is transmitted from its source outward. Sound is a wave. On the atomic scale, it is a disturbance of atoms that is far more ordered than their thermal motions. In many instances, sound is a periodic wave, and the atoms undergo simple harmonic motion. In this text, we shall explore such periodic sound waves.

A vibrating string produces a sound wave as illustrated in Figure 17.3, Figure 17.4, and Figure 17.5. As the string oscillates back and forth, it transfers energy to the air, mostly as thermal energy created by turbulence. But a small part of the string's energy goes into compressing and expanding the surrounding air, creating slightly higher and lower local pressures. These compressions (high pressure regions) and rarefactions (low pressure regions) move out as longitudinal pressure waves having the same frequency as the string—they are the disturbance that is a sound wave. (Sound waves in air and most fluids are longitudinal, because fluids have almost no shear strength. In solids, sound waves can be both transverse and longitudinal.) Figure 17.5 shows a graph of gauge pressure versus distance from the vibrating string.

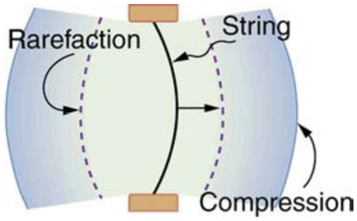


Figure 17.3 A vibrating string moving to the right compresses the air in front of it and expands the air behind it.

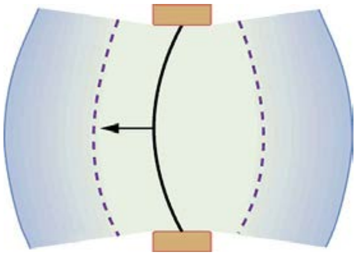


Figure 17.4 As the string moves to the left, it creates another compression and rarefaction as the ones on the right move away from the string.

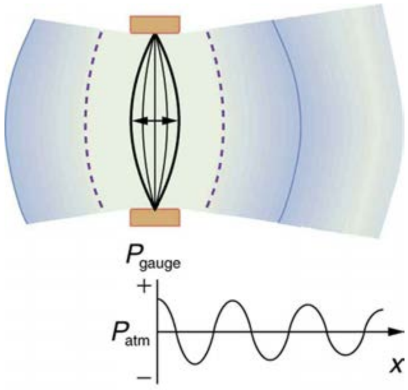


Figure 17.5 After many vibrations, there are a series of compressions and rarefactions moving out from the string as a sound wave. The graph shows gauge pressure versus distance from the source. Pressures vary only slightly from atmospheric for ordinary sounds.

The amplitude of a sound wave decreases with distance from its source, because the energy of the wave is spread over a larger and larger area. But it is also absorbed by objects, such as the eardrum in Figure 17.6, and converted to thermal energy by the viscosity of air. In addition, during each compression a little heat transfers to the air and during each rarefaction even less heat transfers from the air, so that the heat transfer reduces the organized disturbance into random thermal motions. (These processes can be viewed as a manifestation of the second law of thermodynamics presented in **Introduction to the Second Law of Thermodynamics: Heat Engines and Their Efficiency**.) Whether the heat transfer from compression to rarefaction is significant depends on how far apart they are—that is, it depends on wavelength. Wavelength, frequency, amplitude, and speed of propagation are important for sound, as they are for all waves.

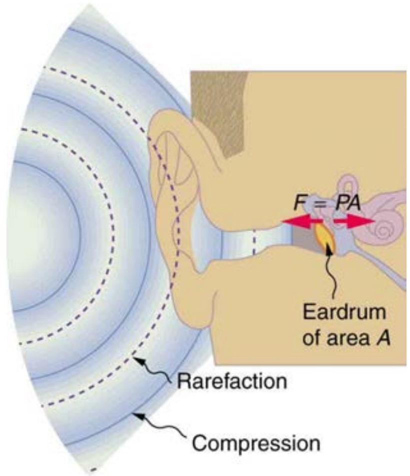


Figure 17.6 Sound wave compressions and rarefactions travel up the ear canal and force the eardrum to vibrate. There is a net force on the eardrum, since the sound wave pressures differ from the atmospheric pressure found behind the eardrum. A complicated mechanism converts the vibrations to nerve impulses, which are perceived by the person.

PhET Explorations: Wave Interference

Make waves with a dripping faucet, audio speaker, or laser! Add a second source or a pair of slits to create an interference pattern.

PhET Interactive Simulation

Figure 17.7 Wave Interference (http://cnx.org/content/m42255/1.3/wave-interference_en.jar)

17.2 Speed of Sound, Frequency, and Wavelength

Figure 17.8 When a firework explodes, the light energy is perceived before the sound energy. Sound travels more slowly than light does. (credit: Dominic Alves, Flickr)

Sound, like all waves, travels at a certain speed and has the properties of frequency and wavelength. You can observe direct evidence of the speed of sound while watching a fireworks display. The flash of an explosion is seen well before its sound is heard, implying both that sound travels at a finite speed and that it is much slower than light. You can also directly sense the frequency of a sound. Perception of frequency is called **pitch**. The wavelength of sound is not directly sensed, but indirect evidence is found in the correlation of the size of musical instruments with their pitch. Small instruments, such as a piccolo, typically make high-pitch sounds, while large instruments, such as a tuba, typically make low-pitch sounds. High pitch means small wavelength, and the size of a musical instrument is directly related to the wavelengths of sound it produces. So a small instrument creates short-wavelength sounds. Similar arguments hold that a large instrument creates long-wavelength sounds.

The relationship of the speed of sound, its frequency, and wavelength is the same as for all waves:

$$v_{\mathrm{w}} = f\lambda, \tag{17.1}$$

where v_{w} is the speed of sound, f is its frequency, and λ is its wavelength. The wavelength of a sound is the distance between adjacent identical parts of a wave—for example, between adjacent compressions as illustrated in **Figure 17.9**. The frequency is the same as that of the source and is the number of waves that pass a point per unit time.

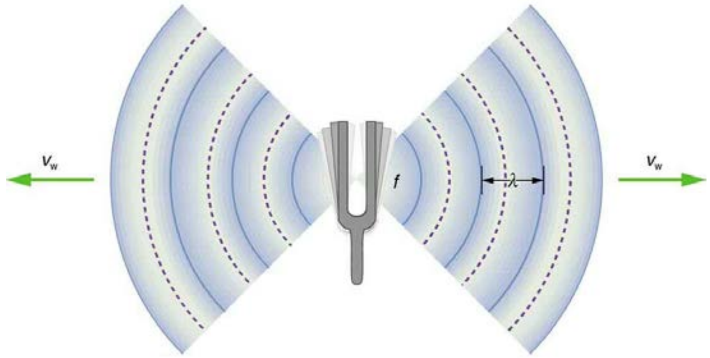


Figure 17.9 A sound wave emanates from a source vibrating at a frequency f, propagates at v_w, and has a wavelength λ.

Table 17.1 makes it apparent that the speed of sound varies greatly in different media. The speed of sound in a medium is determined by a combination of the medium's rigidity (or compressibility in gases) and its density. The more rigid (or less compressible) the medium, the faster the speed of sound. This observation is analogous to the fact that the frequency of a simple harmonic motion is directly proportional to the stiffness of the oscillating object. The greater the density of a medium, the slower the speed of sound. This observation is analogous to the fact that the frequency of a simple harmonic motion is inversely proportional to the mass of the oscillating object. The speed of sound in air is low, because air is compressible. Because liquids and solids are relatively rigid and very difficult to compress, the speed of sound in such media is generally greater than in gases.

Table 17.1 Speed of Sound in Various Media

Medium	v_w(m/s)
Gases at $0^\text{o}C$	
Air	331
Carbon dioxide	259
Oxygen	316
Helium	965
Hydrogen	1290
Liquids at $20^\text{o}C$	
Ethanol	1160
Mercury	1450
Water, fresh	1480
Sea water	1540
Human tissue	1540
Solids (longitudinal or bulk)	
Vulcanized rubber	54
Polyethylene	920
Marble	3810
Glass, Pyrex	5640
Lead	1960
Aluminum	5120
Steel	5960

Earthquakes, essentially sound waves in Earth's crust, are an interesting example of how the speed of sound depends on the rigidity of the medium. Earthquakes have both longitudinal and transverse components, and these travel at different speeds. The bulk modulus of granite is greater than its shear modulus. For that reason, the speed of longitudinal or pressure waves (P-waves) in earthquakes in granite is significantly higher than the speed of transverse or shear waves (S-waves). Both components of earthquakes travel slower in less rigid material, such as sediments. P-waves have speeds of 4 to 7 km/s, and S-waves correspondingly range in speed from 2 to 5 km/s, both being faster in more rigid material. The P-wave gets progressively farther ahead of the S-wave as they travel through Earth's crust. The time between the P- and S-waves is routinely used to determine the distance to their source, the epicenter of the earthquake.

The speed of sound is affected by temperature in a given medium. For air at sea level, the speed of sound is given by

$$v_\text{w} = (331\text{ m/s})\sqrt{\frac{T}{273\text{ K}}}, \tag{17.2}$$

where the temperature (denoted as T) is in units of kelvin. The speed of sound in gases is related to the average speed of particles in the gas, v_{rms} , and that

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}}, \tag{17.3}$$

where k is the Boltzmann constant (1.38×10^{-23} J/K) and m is the mass of each (identical) particle in the gas. So, it is reasonable that the speed of sound in air and other gases should depend on the square root of temperature. While not negligible, this is not a strong dependence. At 0°C , the speed of sound is 331 m/s, whereas at 20.0°C it is 343 m/s, less than a 4% increase. **Figure 17.10** shows a use of the speed of sound by a bat to sense distances. Echoes are also used in medical imaging.

Figure 17.10 A bat uses sound echoes to find its way about and to catch prey. The time for the echo to return is directly proportional to the distance.

One of the more important properties of sound is that its speed is nearly independent of frequency. This independence is certainly true in open air for sounds in the audible range of 20 to 20,000 Hz. If this independence were not true, you would certainly notice it for music played by a marching band in a football stadium, for example. Suppose that high-frequency sounds traveled faster—then the farther you were from the band, the more the sound from the low-pitch instruments would lag that from the high-pitch ones. But the music from all instruments arrives in cadence independent of distance, and so all frequencies must travel at nearly the same speed. Recall that

$$v_{\text{w}} = f\lambda. \tag{17.4}$$

In a given medium under fixed conditions, v_{w} is constant, so that there is a relationship between f and λ ; the higher the frequency, the smaller the wavelength. See **Figure 17.11** and consider the following example.

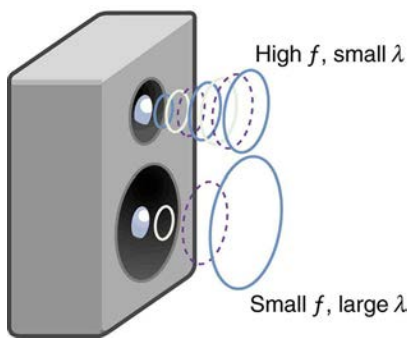


Figure 17.11 Because they travel at the same speed in a given medium, low-frequency sounds must have a greater wavelength than high-frequency sounds. Here, the lower-frequency sounds are emitted by the large speaker, called a woofer, while the higher-frequency sounds are emitted by the small speaker, called a tweeter.

Example 17.1 Calculating Wavelengths: What Are the Wavelengths of Audible Sounds?

Calculate the wavelengths of sounds at the extremes of the audible range, 20 and 20,000 Hz, in 30.0°C air. (Assume that the frequency values are accurate to two significant figures.)

Strategy

To find wavelength from frequency, we can use $v_{\text{w}} = f\lambda$.

Solution

1. Identify knowns. The value for v_{w} , is given by

$$v_{\text{w}} = (331\text{ m/s})\sqrt{\frac{T}{273\text{ K}}}. \tag{17.5}$$

2. Convert the temperature into kelvin and then enter the temperature into the equation

$$v_{\text{w}} = (331\text{ m/s})\sqrt{\frac{303\text{ K}}{273\text{ K}}} = 348.7\text{ m/s}. \tag{17.6}$$

3. Solve the relationship between speed and wavelength for λ :

$$\lambda = \frac{v_{\text{w}}}{f}. \tag{17.7}$$

4. Enter the speed and the minimum frequency to give the maximum wavelength:

$$\lambda_{\text{max}} = \frac{348.7\text{ m/s}}{20\text{ Hz}} = 17\text{ m.} \tag{17.8}$$

5. Enter the speed and the maximum frequency to give the minimum wavelength:

$$\lambda_{\text{min}} = \frac{348.7\text{ m/s}}{20{,}000\text{ Hz}} = 0.017\text{ m} = 1.7\text{ cm.} \tag{17.9}$$

Discussion

Because the product of f multiplied by λ equals a constant, the smaller f is, the larger λ must be, and vice versa.

The speed of sound can change when sound travels from one medium to another. However, the frequency usually remains the same because it is like a driven oscillation and has the frequency of the original source. If v_{w} changes and f remains the same, then the wavelength λ must change. That is, because $v_{\text{w}} = f\lambda$, the higher the speed of a sound, the greater its wavelength for a given frequency.

Making Connections: Take-Home Investigation—Voice as a Sound Wave

Suspend a sheet of paper so that the top edge of the paper is fixed and the bottom edge is free to move. You could tape the top edge of the paper to the edge of a table. Gently blow near the edge of the bottom of the sheet and note how the sheet moves. Speak softly and then louder such that the sounds hit the edge of the bottom of the paper, and note how the sheet moves. Explain the effects.

Check Your Understanding

Imagine you observe two fireworks explode. You hear the explosion of one as soon as you see it. However, you see the other firework for several milliseconds before you hear the explosion. Explain why this is so.

Solution

Sound and light both travel at definite speeds. The speed of sound is slower than the speed of light. The first firework is probably very close by, so the speed difference is not noticeable. The second firework is farther away, so the light arrives at your eyes noticeably sooner than the sound wave arrives at your ears.

Check Your Understanding

You observe two musical instruments that you cannot identify. One plays high-pitch sounds and the other plays low-pitch sounds. How could you determine which is which without hearing either of them play?

Solution

Compare their sizes. High-pitch instruments are generally smaller than low-pitch instruments because they generate a smaller wavelength.

17.3 Sound Intensity and Sound Level

Figure 17.12 Noise on crowded roadways like this one in Delhi makes it hard to hear others unless they shout. (credit: Lingaraj G J, Flickr)

In a quiet forest, you can sometimes hear a single leaf fall to the ground. After settling into bed, you may hear your blood pulsing through your ears. But when a passing motorist has his stereo turned up, you cannot even hear what the person next to you in your car is saying. We are all very familiar with the loudness of sounds and aware that they are related to how energetically the source is vibrating. In cartoons depicting a screaming person (or an animal making a loud noise), the cartoonist often shows an open mouth with a vibrating uvula, the hanging tissue at the back of the mouth, to suggest a loud sound coming from the throat Figure 17.13. High noise exposure is hazardous to hearing, and it is common for musicians to have hearing losses that are sufficiently severe that they interfere with the musicians' abilities to perform. The relevant physical quantity is sound intensity, a concept that is valid for all sounds whether or not they are in the audible range.

Intensity is defined to be the power per unit area carried by a wave. Power is the rate at which energy is transferred by the wave. In equation form, **intensity** I is

$$I = \frac{P}{A}, \tag{17.10}$$

where P is the power through an area A. The SI unit for I is $\mathrm{W/m^2}$. The intensity of a sound wave is related to its amplitude squared by the following relationship:

$$I = \frac{(\Delta p)^2}{2\rho v_{\mathrm{w}}}. \tag{17.11}$$

Here Δp is the pressure variation or pressure amplitude (half the difference between the maximum and minimum pressure in the sound wave) in units of pascals (Pa) or $\mathrm{N/m^2}$. (We are using a lower case p for pressure to distinguish it from power, denoted by P above.) The energy (as kinetic energy $\frac{mv^2}{2}$) of an oscillating element of air due to a traveling sound wave is proportional to its amplitude squared. In this equation, ρ is the density of the material in which the sound wave travels, in units of $\mathrm{kg/m^3}$, and v_{w} is the speed of sound in the medium, in units of m/s. The pressure variation is proportional to the amplitude of the oscillation, and so I varies as $(\Delta p)^2$ (**Figure 17.13**). This relationship is consistent with the fact that the sound wave is produced by some vibration; the greater its pressure amplitude, the more the air is compressed in the sound it creates.

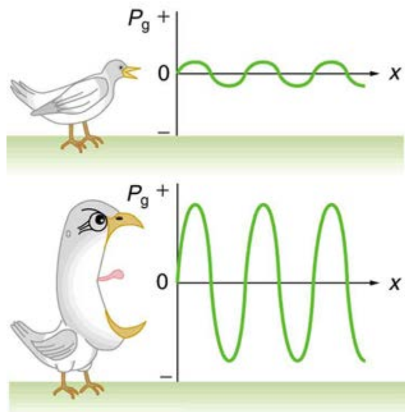


Figure 17.13 Graphs of the gauge pressures in two sound waves of different intensities. The more intense sound is produced by a source that has larger-amplitude oscillations and has greater pressure maxima and minima. Because pressures are higher in the greater-intensity sound, it can exert larger forces on the objects it encounters.

Sound intensity levels are quoted in decibels (dB) much more often than sound intensities in watts per meter squared. Decibels are the unit of choice in the scientific literature as well as in the popular media. The reasons for this choice of units are related to how we perceive sounds. How our ears perceive sound can be more accurately described by the logarithm of the intensity rather than directly to the intensity. The **sound intensity level** β in decibels of a sound having an intensity I in watts per meter squared is defined to be

$$\beta\,(\mathrm{dB}) = 10\log_{10}\left(\frac{I}{I_0}\right), \tag{17.12}$$

where $I_0 = 10^{-12}\ \mathrm{W/m^2}$ is a reference intensity. In particular, I_0 is the lowest or threshold intensity of sound a person with normal hearing can perceive at a frequency of 1000 Hz. Sound intensity level is not the same as intensity. Because β is defined in terms of a ratio, it is a unitless quantity telling you the *level* of the sound relative to a fixed standard ($10^{-12}\ \mathrm{W/m^2}$, in this case). The units of decibels (dB) are used to indicate this ratio is multiplied by 10 in its definition. The bel, upon which the decibel is based, is named for Alexander Graham Bell, the inventor of the telephone.

Table 17.2 Sound Intensity Levels and Intensities

Sound intensity level β (dB)	Intensity I(W/m^2)	Example/effect
0	1×10^{-12}	Threshold of hearing at 1000 Hz
10	1×10^{-11}	Rustle of leaves
20	1×10^{-10}	Whisper at 1 m distance
30	1×10^{-9}	Quiet home
40	1×10^{-8}	Average home
50	1×10^{-7}	Average office, soft music
60	1×10^{-6}	Normal conversation
70	1×10^{-5}	Noisy office, busy traffic
80	1×10^{-4}	Loud radio, classroom lecture
90	1×10^{-3}	Inside a heavy truck; damage from prolonged exposure[1]
100	1×10^{-2}	Noisy factory, siren at 30 m; damage from 8 h per day exposure
110	1×10^{-1}	Damage from 30 min per day exposure
120	1	Loud rock concert, pneumatic chipper at 2 m; threshold of pain
140	1×10^{2}	Jet airplane at 30 m; severe pain, damage in seconds
160	1×10^{4}	Bursting of eardrums

The decibel level of a sound having the threshold intensity of $10^{-12}\ \text{W/m}^2$ is $\beta = 0\ \text{dB}$, because $\log_{10} 1 = 0$. That is, the threshold of hearing is 0 decibels. **Table 17.2** gives levels in decibels and intensities in watts per meter squared for some familiar sounds.

One of the more striking things about the intensities in **Table 17.2** is that the intensity in watts per meter squared is quite small for most sounds. The ear is sensitive to as little as a trillionth of a watt per meter squared—even more impressive when you realize that the area of the eardrum is only about $1\ \text{cm}^2$, so that only 10^{-16} W falls on it at the threshold of hearing! Air molecules in a sound wave of this intensity vibrate over a distance of less than one molecular diameter, and the gauge pressures involved are less than 10^{-9} atm.

Another impressive feature of the sounds in **Table 17.2** is their numerical range. Sound intensity varies by a factor of 10^{12} from threshold to a sound that causes damage in seconds. You are unaware of this tremendous range in sound intensity because how your ears respond can be described approximately as the logarithm of intensity. Thus, sound intensity levels in decibels fit your experience better than intensities in watts per meter squared. The decibel scale is also easier to relate to because most people are more accustomed to dealing with numbers such as 0, 53, or 120 than numbers such as 1.00×10^{-11}.

One more observation readily verified by examining **Table 17.2** or using $I = \frac{(\Delta p)^2}{2\rho v_\text{w}}$ is that each factor of 10 in intensity corresponds to 10 dB. For example, a 90 dB sound compared with a 60 dB sound is 30 dB greater, or three factors of 10 (that is, 10^3 times) as intense. Another example is that if one sound is 10^7 as intense as another, it is 70 dB higher. See **Table 17.3**.

Table 17.3 Ratios of Intensities and Corresponding Differences in Sound Intensity Levels

I_2 / I_1	$\beta_2 - \beta_1$
2.0	3.0 dB
5.0	7.0 dB
10.0	10.0 dB

1. Several government agencies and health-related professional associations recommend that 85 dB not be exceeded for 8-hour daily exposures in the absence of hearing protection.

Example 17.2 Calculating Sound Intensity Levels: Sound Waves

Calculate the sound intensity level in decibels for a sound wave traveling in air at 0^oC and having a pressure amplitude of 0.656 Pa.

Strategy

We are given Δp, so we can calculate I using the equation $I = (\Delta p)^2 / (2 p v_\text{w})^2$. Using I, we can calculate β straight from its definition in $\beta\ (\text{dB}) = 10\ \log_{10}(I/I_0)$.

Solution

(1) Identify knowns:

Sound travels at 331 m/s in air at 0^oC.

Air has a density of $1.29\ \text{kg/m}^3$ at atmospheric pressure and 0^oC.

(2) Enter these values and the pressure amplitude into $I = (\Delta p)^2 / (2\rho v_\text{w})$:

$$I = \frac{(\Delta p)^2}{2\rho v_\text{w}} = \frac{(0.656\ \text{Pa})^2}{2\left(1.29\ \text{kg/m}^3\right)(331\ \text{m/s})} = 5.04\times10^{-4}\ \text{W/m}^2. \tag{17.13}$$

(3) Enter the value for I and the known value for I_0 into $\beta\ (\text{dB}) = 10\ \log_{10}(I/I_0)$. Calculate to find the sound intensity level in decibels:

$$10\ \log_{10}\left(5.04\times10^8\right) = 10\ \left(8.70\right)\ \text{dB} = 87\ \text{dB}. \tag{17.14}$$

Discussion

This 87 dB sound has an intensity five times as great as an 80 dB sound. So a factor of five in intensity corresponds to a difference of 7 dB in sound intensity level. This value is true for any intensities differing by a factor of five.

Example 17.3 Change Intensity Levels of a Sound: What Happens to the Decibel Level?

Show that if one sound is twice as intense as another, it has a sound level about 3 dB higher.

Strategy

You are given that the ratio of two intensities is 2 to 1, and are then asked to find the difference in their sound levels in decibels. You can solve this problem using of the properties of logarithms.

Solution

(1) Identify knowns:

The ratio of the two intensities is 2 to 1, or:

$$\frac{I_2}{I_1} = 2.00. \tag{17.15}$$

We wish to show that the difference in sound levels is about 3 dB. That is, we want to show:

$$\beta_2 - \beta_1 = 3\ \text{dB}. \tag{17.16}$$

Note that:

$$\log_{10} b - \log_{10} a = \log_{10}\left(\frac{b}{a}\right). \tag{17.17}$$

(2) Use the definition of β to get:

$$\beta_2 - \beta_1 = 10\ \log_{10}\left(\frac{I_2}{I_1}\right) = 10\ \log_{10} 2.00 = 10\ (0.301)\ \text{dB}. \tag{17.18}$$

Thus,

$$\beta_2 - \beta_1 = 3.01\ \text{dB}. \tag{17.19}$$

Discussion

This means that the two sound intensity levels differ by 3.01 dB, or about 3 dB, as advertised. Note that because only the ratio I_2/I_1 is given (and not the actual intensities), this result is true for any intensities that differ by a factor of two. For example, a 56.0 dB sound is twice as intense as a 53.0 dB sound, a 97.0 dB sound is half as intense as a 100 dB sound, and so on.

It should be noted at this point that there is another decibel scale in use, called the **sound pressure level**, based on the ratio of the pressure amplitude to a reference pressure. This scale is used particularly in applications where sound travels in water. It is beyond the scope of most introductory texts to treat this scale because it is not commonly used for sounds in air, but it is important to note that very different decibel levels may be encountered when sound pressure levels are quoted. For example, ocean noise pollution produced by ships may be as great as 200 dB expressed in the sound pressure level, where the more familiar sound intensity level we use here would be something under 140 dB for the same sound.

Take-Home Investigation: Feeling Sound

Find a CD player and a CD that has rock music. Place the player on a light table, insert the CD into the player, and start playing the CD. Place your hand gently on the table next to the speakers. Increase the volume and note the level when the table just begins to vibrate as the rock music plays. Increase the reading on the volume control until it doubles. What has happened to the vibrations?

Check Your Understanding

Describe how amplitude is related to the loudness of a sound.

Solution
Amplitude is directly proportional to the experience of loudness. As amplitude increases, loudness increases.

Check Your Understanding

Identify common sounds at the levels of 10 dB, 50 dB, and 100 dB.

Solution

10 dB: Running fingers through your hair.

50 dB: Inside a quiet home with no television or radio.

100 dB: Take-off of a jet plane.

17.4 Doppler Effect and Sonic Booms

The characteristic sound of a motorcycle buzzing by is an example of the **Doppler effect**. The high-pitch scream shifts dramatically to a lower-pitch roar as the motorcycle passes by a stationary observer. The closer the motorcycle brushes by, the more abrupt the shift. The faster the motorcycle moves, the greater the shift. We also hear this characteristic shift in frequency for passing race cars, airplanes, and trains. It is so familiar that it is used to imply motion and children often mimic it in play.

The Doppler effect is an alteration in the observed frequency of a sound due to motion of either the source or the observer. Although less familiar, this effect is easily noticed for a stationary source and moving observer. For example, if you ride a train past a stationary warning bell, you will hear the bell's frequency shift from high to low as you pass by. The actual change in frequency due to relative motion of source and observer is called a **Doppler shift**. The Doppler effect and Doppler shift are named for the Austrian physicist and mathematician Christian Johann Doppler (1803–1853), who did experiments with both moving sources and moving observers. Doppler, for example, had musicians play on a moving open train car and also play standing next to the train tracks as a train passed by. Their music was observed both on and off the train, and changes in frequency were measured.

What causes the Doppler shift? **Figure 17.14**, **Figure 17.15**, and **Figure 17.16** compare sound waves emitted by stationary and moving sources in a stationary air mass. Each disturbance spreads out spherically from the point where the sound was emitted. If the source is stationary, then all of the spheres representing the air compressions in the sound wave centered on the same point, and the stationary observers on either side see the same wavelength and frequency as emitted by the source, as in **Figure 17.14**. If the source is moving, as in **Figure 17.15**, then the situation is different. Each compression of the air moves out in a sphere from the point where it was emitted, but the point of emission moves. This moving emission point causes the air compressions to be closer together on one side and farther apart on the other. Thus, the wavelength is shorter in the direction the source is moving (on the right in **Figure 17.15**), and longer in the opposite direction (on the left in **Figure 17.15**). Finally, if the observers move, as in **Figure 17.16**, the frequency at which they receive the compressions changes. The observer moving toward the source receives them at a higher frequency, and the person moving away from the source receives them at a lower frequency.

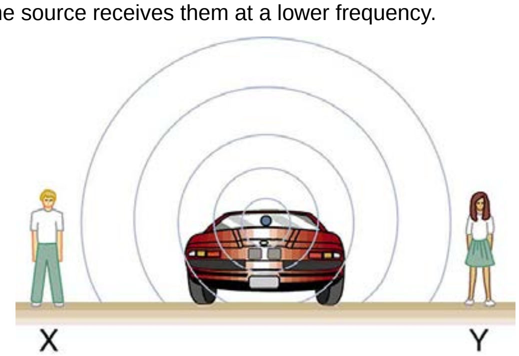


Figure 17.14 Sounds emitted by a source spread out in spherical waves. Because the source, observers, and air are stationary, the wavelength and frequency are the same in all directions and to all observers.

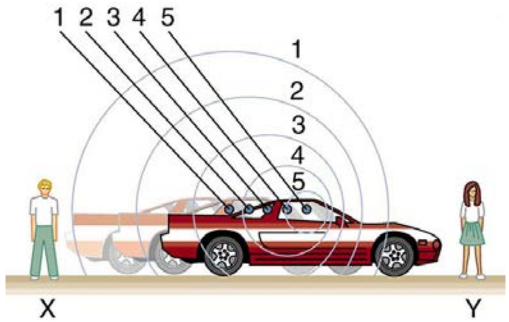


Figure 17.15 Sounds emitted by a source moving to the right spread out from the points at which they were emitted. The wavelength is reduced and, consequently, the frequency is increased in the direction of motion, so that the observer on the right hears a higher-pitch sound. The opposite is true for the observer on the left, where the wavelength is increased and the frequency is reduced.

Figure 17.16 The same effect is produced when the observers move relative to the source. Motion toward the source increases frequency as the observer on the right passes through more wave crests than she would if stationary. Motion away from the source decreases frequency as the observer on the left passes through fewer wave crests than he would if stationary.

We know that wavelength and frequency are related by $v_{\text{w}} = f\lambda$, where v_{w} is the fixed speed of sound. The sound moves in a medium and has the same speed v_{w} in that medium whether the source is moving or not. Thus f multiplied by λ is a constant. Because the observer on the right in **Figure 17.15** receives a shorter wavelength, the frequency she receives must be higher. Similarly, the observer on the left receives a longer wavelength, and hence he hears a lower frequency. The same thing happens in **Figure 17.16**. A higher frequency is received by the observer moving toward the source, and a lower frequency is received by an observer moving away from the source. In general, then, relative motion of source and observer toward one another increases the received frequency. Relative motion apart decreases frequency. The greater the relative speed is, the greater the effect.

The Doppler Effect

The Doppler effect occurs not only for sound but for any wave when there is relative motion between the observer and the source. There are Doppler shifts in the frequency of sound, light, and water waves, for example. Doppler shifts can be used to determine velocity, such as when ultrasound is reflected from blood in a medical diagnostic. The recession of galaxies is determined by the shift in the frequencies of light received from them and has implied much about the origins of the universe. Modern physics has been profoundly affected by observations of Doppler shifts.

For a stationary observer and a moving source, the frequency f_{obs} received by the observer can be shown to be

$$f_{\text{obs}} = f_{\text{s}}\left(\frac{v_{\text{w}}}{v_{\text{w}} \pm v_{\text{s}}}\right), \tag{17.20}$$

where f_{s} is the frequency of the source, v_{s} is the speed of the source along a line joining the source and observer, and v_{w} is the speed of sound. The minus sign is used for motion toward the observer and the plus sign for motion away from the observer, producing the appropriate shifts up and down in frequency. Note that the greater the speed of the source, the greater the effect. Similarly, for a stationary source and moving observer, the frequency received by the observer f_{obs} is given by

$$f_{\text{obs}} = f_{\text{s}}\left(\frac{v_{\text{w}} \pm v_{\text{obs}}}{v_{\text{w}}}\right), \tag{17.21}$$

where v_{obs} is the speed of the observer along a line joining the source and observer. Here the plus sign is for motion toward the source, and the minus is for motion away from the source.

Example 17.4 Calculate Doppler Shift: A Train Horn

Suppose a train that has a 150-Hz horn is moving at 35.0 m/s in still air on a day when the speed of sound is 340 m/s.

(a) What frequencies are observed by a stationary person at the side of the tracks as the train approaches and after it passes?

(b) What frequency is observed by the train's engineer traveling on the train?

Strategy

To find the observed frequency in (a), $f_{\text{obs}} = f_{\text{s}}\left(\frac{v_{\text{w}}}{v_{\text{w}} \pm v_{\text{s}}}\right),$ must be used because the source is moving. The minus sign is used for the approaching train, and the plus sign for the receding train. In (b), there are two Doppler shifts—one for a moving source and the other for a moving observer.

Solution for (a)

(1) Enter known values into $f_{\text{obs}} = f_{\text{s}}\left(\frac{v_{\text{w}}}{v_{\text{w}} - v_{\text{s}}}\right).$

$$f_{\text{obs}} = f_{\text{s}}\left(\frac{v_{\text{w}}}{v_{\text{w}} - v_{\text{s}}}\right) = (150\text{ Hz})\left(\frac{340\text{ m/s}}{340\text{ m/s} - 35.0\text{ m/s}}\right) \tag{17.22}$$

(2) Calculate the frequency observed by a stationary person as the train approaches.

$$f_{\text{obs}} = (150\text{ Hz})(1.11) = 167\text{ Hz} \tag{17.23}$$

(3) Use the same equation with the plus sign to find the frequency heard by a stationary person as the train recedes.

$$f_{\text{obs}} = f_{\text{s}}\left(\frac{v_{\text{w}}}{v_{\text{w}} + v_{\text{s}}}\right) = (150\text{ Hz})\left(\frac{340\text{ m/s}}{340\text{ m/s} + 35.0\text{ m/s}}\right) \tag{17.24}$$

(4) Calculate the second frequency.

$$f_{\text{obs}} = (150\text{ Hz})(0.907) = 136\text{ Hz} \tag{17.25}$$

Discussion on (a)

The numbers calculated are valid when the train is far enough away that the motion is nearly along the line joining train and observer. In both cases, the shift is significant and easily noticed. Note that the shift is 17.0 Hz for motion toward and 14.0 Hz for motion away. The shifts are not symmetric.

Solution for (b)

(1) Identify knowns:

- It seems reasonable that the engineer would receive the same frequency as emitted by the horn, because the relative velocity between them is zero.
- Relative to the medium (air), the speeds are $v_{\text{s}} = v_{\text{obs}} = 35.0\text{ m/s}.$
- The first Doppler shift is for the moving observer; the second is for the moving source.

(2) Use the following equation:

$$f_{\text{obs}} = \left[f_{\text{s}}\left(\frac{v_{\text{w}} \pm v_{\text{obs}}}{v_{\text{w}}}\right)\right]\left(\frac{v_{\text{w}}}{v_{\text{w}} \pm v_{\text{s}}}\right). \tag{17.26}$$

The quantity in the square brackets is the Doppler-shifted frequency due to a moving observer. The factor on the right is the effect of the moving source.

(3) Because the train engineer is moving in the direction toward the horn, we must use the plus sign for $v_{\text{obs}};$ however, because the horn is also moving in the direction away from the engineer, we also use the plus sign for v_{s}. But the train is carrying both the engineer and the horn at the same velocity, so $v_{\text{s}} = v_{\text{obs}}$. As a result, everything but f_{s} cancels, yielding

$$f_{\text{obs}} = f_{\text{s}}. \tag{17.27}$$

Discussion for (b)

We may expect that there is no change in frequency when source and observer move together because it fits your experience. For example, there is no Doppler shift in the frequency of conversations between driver and passenger on a motorcycle. People talking when a wind moves the air between them also observe no Doppler shift in their conversation. The crucial point is that source and observer are not moving relative to each other.

Sonic Booms to Bow Wakes

What happens to the sound produced by a moving source, such as a jet airplane, that approaches or even exceeds the speed of sound? The answer to this question applies not only to sound but to all other waves as well.

Suppose a jet airplane is coming nearly straight at you, emitting a sound of frequency f_{s}. The greater the plane's speed v_{s}, the greater the Doppler shift and the greater the value observed for f_{obs}. Now, as v_{s} approaches the speed of sound, f_{obs} approaches infinity, because the denominator in $f_{\text{obs}} = f_{\text{s}}\left(\frac{v_{\text{w}}}{v_{\text{w}} \pm v_{\text{s}}}\right)$ approaches zero. At the speed of sound, this result means that in front of the source, each successive wave is superimposed on the previous one because the source moves forward at the speed of sound. The observer gets them all at the same instant, and so the frequency is infinite. (Before airplanes exceeded the speed of sound, some people argued it would be impossible because such constructive superposition would produce pressures great enough to destroy the airplane.) If the source exceeds the speed of sound, no sound is received by the observer until the source has passed, so that the sounds from the approaching source are mixed with those from it when receding. This mixing appears messy, but something interesting happens—a sonic boom is created. (See **Figure 17.17**.)

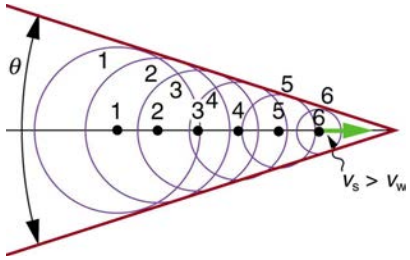


Figure 17.17 Sound waves from a source that moves faster than the speed of sound spread spherically from the point where they are emitted, but the source moves ahead of each. Constructive interference along the lines shown (actually a cone in three dimensions) creates a shock wave called a sonic boom. The faster the speed of the source, the smaller the angle θ.

There is constructive interference along the lines shown (a cone in three dimensions) from similar sound waves arriving there simultaneously. This superposition forms a disturbance called a **sonic boom**, a constructive interference of sound created by an object moving faster than sound. Inside the cone, the interference is mostly destructive, and so the sound intensity there is much less than on the shock wave. An aircraft creates two sonic booms, one from its nose and one from its tail. (See **Figure 17.18**.) During television coverage of space shuttle landings, two distinct booms could often be heard. These were separated by exactly the time it would take the shuttle to pass by a point. Observers on the ground often do not see the aircraft creating the sonic boom, because it has passed by before the shock wave reaches them, as seen in **Figure 17.18**. If the aircraft flies close by at low altitude, pressures in the sonic boom can be destructive and break windows as well as rattle nerves. Because of how destructive sonic booms can be, supersonic flights are banned over populated areas of the United States.

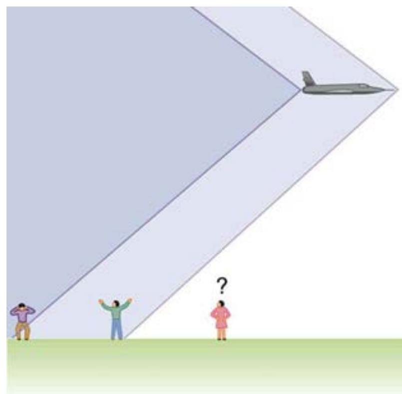


Figure 17.18 Two sonic booms, created by the nose and tail of an aircraft, are observed on the ground after the plane has passed by.

Sonic booms are one example of a broader phenomenon called bow wakes. A **bow wake**, such as the one in **Figure 17.19**, is created when the wave source moves faster than the wave propagation speed. Water waves spread out in circles from the point where created, and the bow wake is the familiar V-shaped wake trailing the source. A more exotic bow wake is created when a subatomic particle travels through a medium faster than the speed of light travels in that medium. (In a vacuum, the maximum speed of light will be $c = 3.00 \times 10^8 \text{ m/s}$; in the medium of water, the speed of light is closer to $0.75c$. If the particle creates light in its passage, that light spreads on a cone with an angle indicative of the speed of the particle, as illustrated in **Figure 17.20**. Such a bow wake is called Cerenkov radiation and is commonly observed in particle physics.

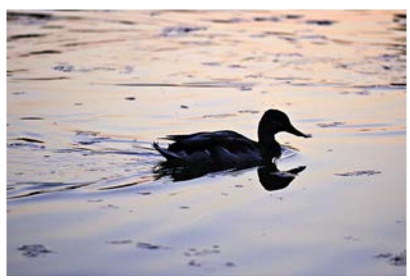


Figure 17.19 Bow wake created by a duck. Constructive interference produces the rather structured wake, while there is relatively little wave action inside the wake, where interference is mostly destructive. (credit: Horia Varlan, Flickr)

Figure 17.20 The blue glow in this research reactor pool is Cerenkov radiation caused by subatomic particles traveling faster than the speed of light in water. (credit: U.S. Nuclear Regulatory Commission)

Doppler shifts and sonic booms are interesting sound phenomena that occur in all types of waves. They can be of considerable use. For example, the Doppler shift in ultrasound can be used to measure blood velocity, while police use the Doppler shift in radar (a microwave) to measure car velocities. In meteorology, the Doppler shift is used to track the motion of storm clouds; such “Doppler Radar” can give velocity and direction and rain or snow potential of imposing weather fronts. In astronomy, we can examine the light emitted from distant galaxies and determine their speed relative to ours. As galaxies move away from us, their light is shifted to a lower frequency, and so to a longer wavelength—the so-called red shift. Such information from galaxies far, far away has allowed us to estimate the age of the universe (from the Big Bang) as about 14 billion years.

Check Your Understanding

Why did scientist Christian Doppler observe musicians both on a moving train and also from a stationary point not on the train?

Solution
Doppler needed to compare the perception of sound when the observer is stationary and the sound source moves, as well as when the sound source and the observer are both in motion.

Check Your Understanding

Describe a situation in your life when you might rely on the Doppler shift to help you either while driving a car or walking near traffic.

Solution
If I am driving and I hear Doppler shift in an ambulance siren, I would be able to tell when it was getting closer and also if it has passed by. This would help me to know whether I needed to pull over and let the ambulance through.

17.5 Sound Interference and Resonance: Standing Waves in Air Columns

Figure 17.21 Some types of headphones use the phenomena of constructive and destructive interference to cancel out outside noises. (credit: JVC America, Flickr)

Interference is the hallmark of waves, all of which exhibit constructive and destructive interference exactly analogous to that seen for water waves. In fact, one way to prove something “is a wave” is to observe interference effects. So, sound being a wave, we expect it to exhibit interference; we have already mentioned a few such effects, such as the beats from two similar notes played simultaneously.

Figure 17.22 shows a clever use of sound interference to cancel noise. Larger-scale applications of active noise reduction by destructive interference are contemplated for entire passenger compartments in commercial aircraft. To obtain destructive interference, a fast electronic analysis is performed, and a second sound is introduced with its maxima and minima exactly reversed from the incoming noise. Sound waves in fluids are pressure waves and consistent with Pascal’s principle; pressures from two different sources add and subtract like simple numbers; that is, positive and negative gauge pressures add to a much smaller pressure, producing a lower-intensity sound. Although completely destructive interference is possible only under the simplest conditions, it is possible to reduce noise levels by 30 dB or more using this technique.

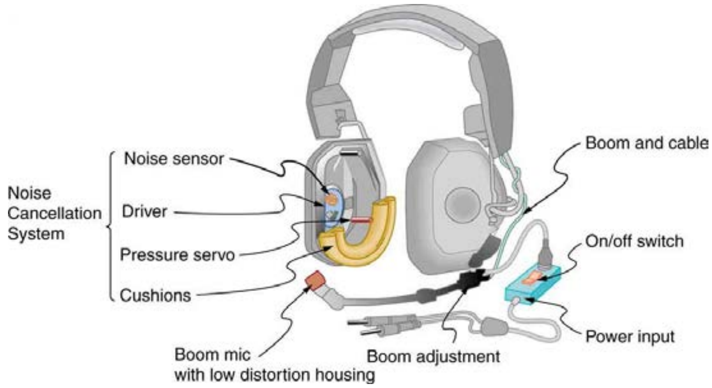


Figure 17.22 Headphones designed to cancel noise with destructive interference create a sound wave exactly opposite to the incoming sound. These headphones can be more effective than the simple passive attenuation used in most ear protection. Such headphones were used on the record-setting, around the world nonstop flight of the Voyager aircraft to protect the pilots' hearing from engine noise.

Where else can we observe sound interference? All sound resonances, such as in musical instruments, are due to constructive and destructive interference. Only the resonant frequencies interfere constructively to form standing waves, while others interfere destructively and are absent. From the toot made by blowing over a bottle, to the characteristic flavor of a violin's sounding box, to the recognizability of a great singer's voice, resonance and standing waves play a vital role.

Interference

Interference is such a fundamental aspect of waves that observing interference is proof that something is a wave. The wave nature of light was established by experiments showing interference. Similarly, when electrons scattered from crystals exhibited interference, their wave nature was confirmed to be exactly as predicted by symmetry with certain wave characteristics of light.

Suppose we hold a tuning fork near the end of a tube that is closed at the other end, as shown in **Figure 17.23**, **Figure 17.24**, **Figure 17.25**, and **Figure 17.26**. If the tuning fork has just the right frequency, the air column in the tube resonates loudly, but at most frequencies it vibrates very little. This observation just means that the air column has only certain natural frequencies. The figures show how a resonance at the lowest of these natural frequencies is formed. A disturbance travels down the tube at the speed of sound and bounces off the closed end. If the tube is just the right length, the reflected sound arrives back at the tuning fork exactly half a cycle later, and it interferes constructively with the continuing sound produced by the tuning fork. The incoming and reflected sounds form a standing wave in the tube as shown.

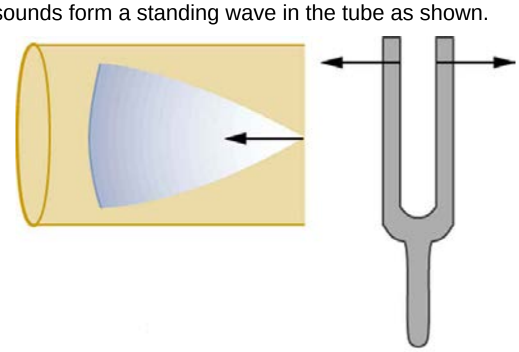


Figure 17.23 Resonance of air in a tube closed at one end, caused by a tuning fork. A disturbance moves down the tube.

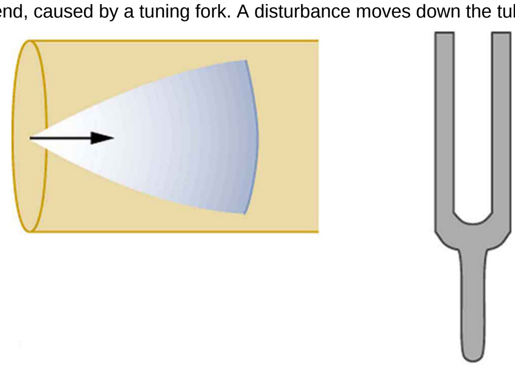


Figure 17.24 Resonance of air in a tube closed at one end, caused by a tuning fork. The disturbance reflects from the closed end of the tube.

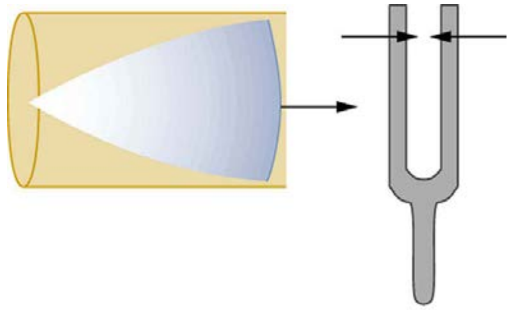


Figure 17.25 Resonance of air in a tube closed at one end, caused by a tuning fork. If the length of the tube L is just right, the disturbance gets back to the tuning fork half a cycle later and interferes constructively with the continuing sound from the tuning fork. This interference forms a standing wave, and the air column resonates.

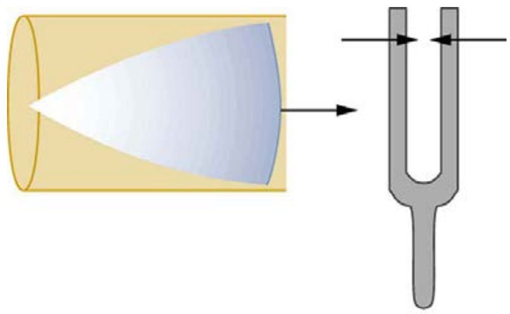


Figure 17.26 Resonance of air in a tube closed at one end, caused by a tuning fork. A graph of air displacement along the length of the tube shows none at the closed end, where the motion is constrained, and a maximum at the open end. This standing wave has one-fourth of its wavelength in the tube, so that $\lambda = 4L$.

The standing wave formed in the tube has its maximum air displacement (an **antinode**) at the open end, where motion is unconstrained, and no displacement (a **node**) at the closed end, where air movement is halted. The distance from a node to an antinode is one-fourth of a wavelength, and this equals the length of the tube; thus, $\lambda = 4L$. This same resonance can be produced by a vibration introduced at or near the closed end of the tube, as shown in **Figure 17.27**. It is best to consider this a natural vibration of the air column independently of how it is induced.

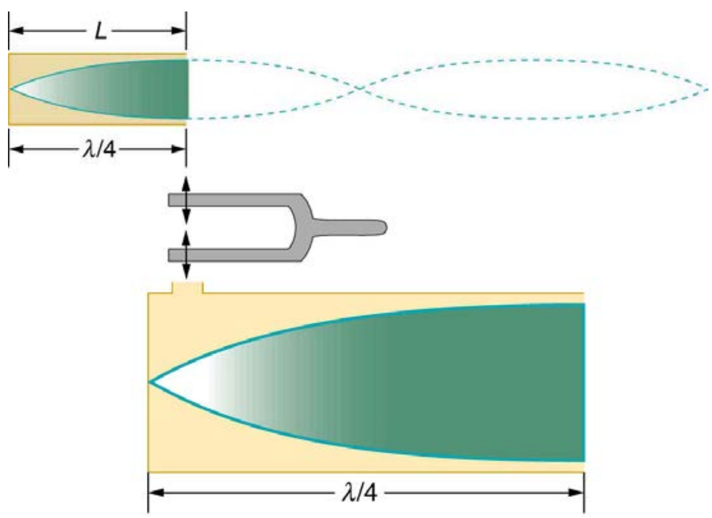


Figure 17.27 The same standing wave is created in the tube by a vibration introduced near its closed end.

Given that maximum air displacements are possible at the open end and none at the closed end, there are other, shorter wavelengths that can resonate in the tube, such as the one shown in **Figure 17.28**. Here the standing wave has three-fourths of its wavelength in the tube, or $L = (3/4)\lambda'$, so that $\lambda' = 4L/3$. Continuing this process reveals a whole series of shorter-wavelength and higher-frequency sounds that resonate in the tube. We use specific terms for the resonances in any system. The lowest resonant frequency is called the **fundamental**, while all higher resonant frequencies are called **overtones**. All resonant frequencies are integral multiples of the fundamental, and they are collectively called **harmonics**. The fundamental is the first harmonic, the first overtone is the second harmonic, and so on. **Figure 17.29** shows the fundamental and the first three overtones (the first four harmonics) in a tube closed at one end.

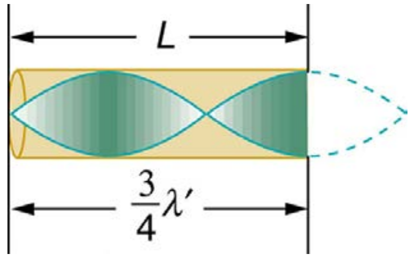


Figure 17.28 Another resonance for a tube closed at one end. This has maximum air displacements at the open end, and none at the closed end. The wavelength is shorter, with three-fourths λ' equaling the length of the tube, so that $\lambda' = 4L/3$. This higher-frequency vibration is the first overtone.

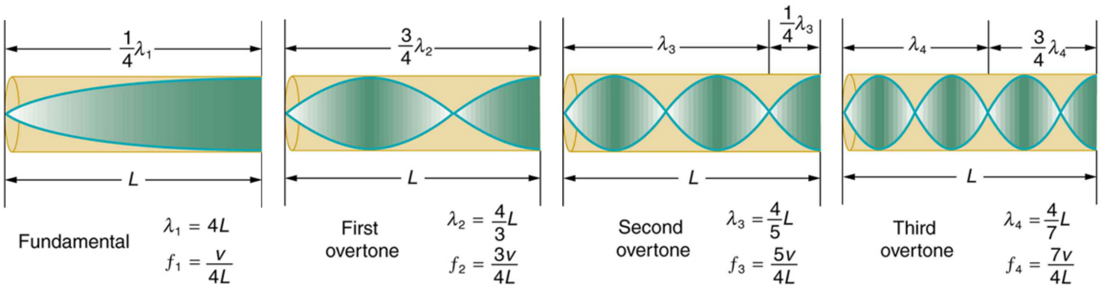


Figure 17.29 The fundamental and three lowest overtones for a tube closed at one end. All have maximum air displacements at the open end and none at the closed end.

The fundamental and overtones can be present simultaneously in a variety of combinations. For example, middle C on a trumpet has a sound distinctively different from middle C on a clarinet, both instruments being modified versions of a tube closed at one end. The fundamental frequency is the same (and usually the most intense), but the overtones and their mix of intensities are different and subject to shading by the musician. This mix is what gives various musical instruments (and human voices) their distinctive characteristics, whether they have air columns, strings, sounding boxes, or drumheads. In fact, much of our speech is determined by shaping the cavity formed by the throat and mouth and positioning the tongue to adjust the fundamental and combination of overtones. Simple resonant cavities can be made to resonate with the sound of the vowels, for example. (See **Figure 17.30**.) In boys, at puberty, the larynx grows and the shape of the resonant cavity changes giving rise to the difference in predominant frequencies in speech between men and women.

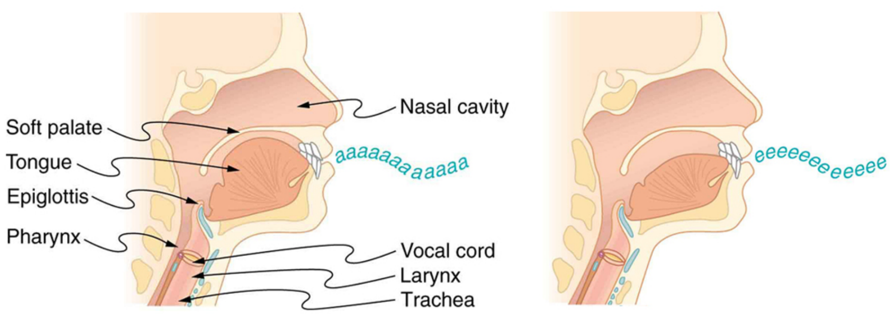


Figure 17.30 The throat and mouth form an air column closed at one end that resonates in response to vibrations in the voice box. The spectrum of overtones and their intensities vary with mouth shaping and tongue position to form different sounds. The voice box can be replaced with a mechanical vibrator, and understandable speech is still possible. Variations in basic shapes make different voices recognizable.

Now let us look for a pattern in the resonant frequencies for a simple tube that is closed at one end. The fundamental has $\lambda = 4L$, and frequency is related to wavelength and the speed of sound as given by:

$$v_\text{w} = f\lambda. \tag{17.28}$$

Solving for f in this equation gives

$$f = \frac{v_\text{w}}{\lambda} = \frac{v_\text{w}}{4L}, \tag{17.29}$$

where v_w is the speed of sound in air. Similarly, the first overtone has $\lambda' = 4L/3$ (see **Figure 17.29**), so that

$$f' = 3\frac{v_\text{w}}{4L} = 3f. \tag{17.30}$$

Because $f' = 3f$, we call the first overtone the third harmonic. Continuing this process, we see a pattern that can be generalized in a single expression. The resonant frequencies of a tube closed at one end are

$$f_n = n\frac{v_\text{w}}{4L}, n = 1,3,5, \tag{17.31}$$

where f_1 is the fundamental, f_3 is the first overtone, and so on. It is interesting that the resonant frequencies depend on the speed of sound and, hence, on temperature. This dependence poses a noticeable problem for organs in old unheated cathedrals, and it is also the reason why musicians commonly bring their wind instruments to room temperature before playing them.

Example 17.5 Find the Length of a Tube with a 128 Hz Fundamental

(a) What length should a tube closed at one end have on a day when the air temperature, is 22.0°C , if its fundamental frequency is to be 128 Hz (C below middle C)?

(b) What is the frequency of its fourth overtone?

Strategy

The length L can be found from the relationship in $f_n = n\frac{v_\text{w}}{4L}$, but we will first need to find the speed of sound v_w .

Solution for (a)

(1) Identify knowns:

- the fundamental frequency is 128 Hz
- the air temperature is 22.0°C

(2) Use $f_n = n\frac{v_\text{w}}{4L}$ to find the fundamental frequency ($n = 1$).

$$f_1 = \frac{v_\text{w}}{4L} \tag{17.32}$$

(3) Solve this equation for length.

$$L = \frac{v_\text{w}}{4f_1} \tag{17.33}$$

(4) Find the speed of sound using $v_\text{w} = (331\ \text{m/s})\sqrt{\frac{T}{273\ \text{K}}}$.

$$v_\text{w} = (331\ \text{m/s})\sqrt{\frac{295\ \text{K}}{273\ \text{K}}} = 344\ \text{m/s} \tag{17.34}$$

(5) Enter the values of the speed of sound and frequency into the expression for L .

$$L = \frac{v_\text{w}}{4f_1} = \frac{344\ \text{m/s}}{4(128\ \text{Hz})} = 0.672\ \text{m} \tag{17.35}$$

Discussion on (a)

Many wind instruments are modified tubes that have finger holes, valves, and other devices for changing the length of the resonating air column and hence, the frequency of the note played. Horns producing very low frequencies, such as tubas, require tubes so long that they are coiled into loops.

Solution for (b)

(1) Identify knowns:

- the first overtone has $n = 3$
- the second overtone has $n = 5$
- the third overtone has $n = 7$
- the fourth overtone has $n = 9$

(2) Enter the value for the fourth overtone into $f_n = n\frac{v_\text{w}}{4L}$.

$$f_9 = 9\frac{v_\text{w}}{4L} = 9f_1 = 1.15\ \text{kHz} \tag{17.36}$$

Discussion on (b)

Whether this overtone occurs in a simple tube or a musical instrument depends on how it is stimulated to vibrate and the details of its shape. The trombone, for example, does not produce its fundamental frequency and only makes overtones.

Another type of tube is one that is *open* at both ends. Examples are some organ pipes, flutes, and oboes. The resonances of tubes open at both ends can be analyzed in a very similar fashion to those for tubes closed at one end. The air columns in tubes open at both ends have maximum air displacements at both ends, as illustrated in **Figure 17.31**. Standing waves form as shown.

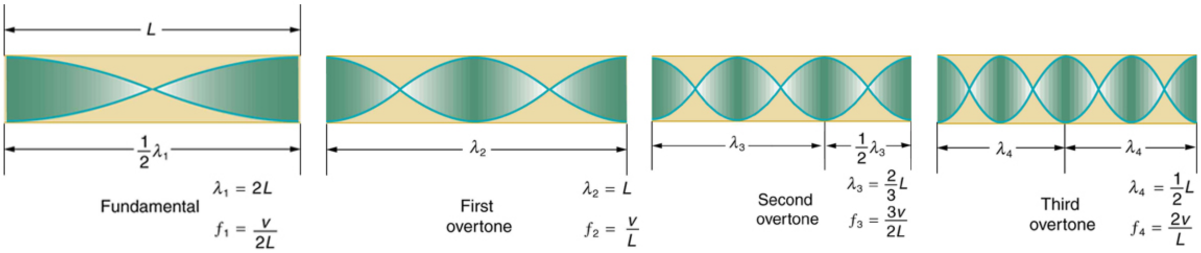


Figure 17.31 The resonant frequencies of a tube open at both ends are shown, including the fundamental and the first three overtones. In all cases the maximum air displacements occur at both ends of the tube, giving it different natural frequencies than a tube closed at one end.

Based on the fact that a tube open at both ends has maximum air displacements at both ends, and using **Figure 17.31** as a guide, we can see that the resonant frequencies of a tube open at both ends are:

$$f_n = n\frac{v_\mathrm{w}}{2L},\ \ n = 1, 2, 3..., \tag{17.37}$$

where f_1 is the fundamental, f_2 is the first overtone, f_3 is the second overtone, and so on. Note that a tube open at both ends has a fundamental frequency twice what it would have if closed at one end. It also has a different spectrum of overtones than a tube closed at one end. So if you had two tubes with the same fundamental frequency but one was open at both ends and the other was closed at one end, they would sound different when played because they have different overtones. Middle C, for example, would sound richer played on an open tube, because it has even multiples of the fundamental as well as odd. A closed tube has only odd multiples.

Real-World Applications: Resonance in Everyday Systems

Resonance occurs in many different systems, including strings, air columns, and atoms. Resonance is the driven or forced oscillation of a system at its natural frequency. At resonance, energy is transferred rapidly to the oscillating system, and the amplitude of its oscillations grows until the system can no longer be described by Hooke’s law. An example of this is the distorted sound intentionally produced in certain types of rock music.

Wind instruments use resonance in air columns to amplify tones made by lips or vibrating reeds. Other instruments also use air resonance in clever ways to amplify sound. **Figure 17.32** shows a violin and a guitar, both of which have sounding boxes but with different shapes, resulting in different overtone structures. The vibrating string creates a sound that resonates in the sounding box, greatly amplifying the sound and creating overtones that give the instrument its characteristic flavor. The more complex the shape of the sounding box, the greater its ability to resonate over a wide range of frequencies. The marimba, like the one shown in **Figure 17.33** uses pots or gourds below the wooden slats to amplify their tones. The resonance of the pot can be adjusted by adding water.

Figure 17.32 String instruments such as violins and guitars use resonance in their sounding boxes to amplify and enrich the sound created by their vibrating strings. The bridge and supports couple the string vibrations to the sounding boxes and air within. (credits: guitar, Feliciano Guimares, Fotopedia; violin, Steve Snodgrass, Flickr)

Figure 17.33 Resonance has been used in musical instruments since prehistoric times. This marimba uses gourds as resonance chambers to amplify its sound. (credit: APC Events, Flickr)

We have emphasized sound applications in our discussions of resonance and standing waves, but these ideas apply to any system that has wave characteristics. Vibrating strings, for example, are actually resonating and have fundamentals and overtones similar to those for air columns. More subtle are the resonances in atoms due to the wave character of their electrons. Their orbitals can be viewed as standing waves, which have a fundamental (ground state) and overtones (excited states). It is fascinating that wave characteristics apply to such a wide range of physical systems.

Check Your Understanding

Describe how noise-canceling headphones differ from standard headphones used to block outside sounds.

Solution
Regular headphones only block sound waves with a physical barrier. Noise-canceling headphones use destructive interference to reduce the loudness of outside sounds.

Check Your Understanding

How is it possible to use a standing wave's node and antinode to determine the length of a closed-end tube?

Solution
When the tube resonates at its natural frequency, the wave's node is located at the closed end of the tube, and the antinode is located at the open end. The length of the tube is equal to one-fourth of the wavelength of this wave. Thus, if we know the wavelength of the wave, we can determine the length of the tube.

PhET Explorations: Sound

This simulation lets you see sound waves. Adjust the frequency or volume and you can see and hear how the wave changes. Move the listener around and hear what she hears.

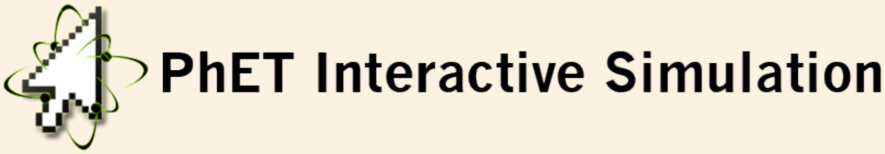


Figure 17.34 Sound (http://cnx.org/content/m42296/1.4/sound_en.jar)

17.6 Hearing

Figure 17.35 Hearing allows this vocalist, his band, and his fans to enjoy music. (credit: West Point Public Affairs, Flickr)

The human ear has a tremendous range and sensitivity. It can give us a wealth of simple information—such as pitch, loudness, and direction. And from its input we can detect musical quality and nuances of voiced emotion. How is our hearing related to the physical qualities of sound, and how does the hearing mechanism work?

Hearing is the perception of sound. (Perception is commonly defined to be awareness through the senses, a typically circular definition of higher-level processes in living organisms.) Normal human hearing encompasses frequencies from 20 to 20,000 Hz, an impressive range. Sounds below 20 Hz are called **infrasound**, whereas those above 20,000 Hz are **ultrasound**. Neither is perceived by the ear, although infrasound can sometimes be felt as vibrations. When we do hear low-frequency vibrations, such as the sounds of a diving board, we hear the individual vibrations only because there are higher-frequency sounds in each. Other animals have hearing ranges different from that of humans. Dogs can hear sounds as high as 30,000 Hz, whereas bats and dolphins can hear up to 100,000-Hz sounds. You may have noticed that dogs respond to the sound of a dog whistle which produces sound out of the range of human hearing. Elephants are known to respond to frequencies below 20 Hz.

The perception of frequency is called **pitch**. Most of us have excellent relative pitch, which means that we can tell whether one sound has a different frequency from another. Typically, we can discriminate between two sounds if their frequencies differ by 0.3% or more. For example, 500.0 and 501.5 Hz are noticeably different. Pitch perception is directly related to frequency and is not greatly affected by other physical quantities such as intensity. Musical **notes** are particular sounds that can be produced by most instruments and in Western music have particular names. Combinations of notes constitute music. Some people can identify musical notes, such as A-sharp, C, or E-flat, just by listening to them. This uncommon ability is called perfect pitch.

The ear is remarkably sensitive to low-intensity sounds. The lowest audible intensity or threshold is about $10^{-12}\ \text{W/m}^2$ or 0 dB. Sounds as much as 10^{12} more intense can be briefly tolerated. Very few measuring devices are capable of observations over a range of a trillion. The perception of intensity is called **loudness**. At a given frequency, it is possible to discern differences of about 1 dB, and a change of 3 dB is easily noticed. But loudness is not related to intensity alone. Frequency has a major effect on how loud a sound seems. The ear has its maximum sensitivity to frequencies in the range of 2000 to 5000 Hz, so that sounds in this range are perceived as being louder than, say, those at 500 or 10,000 Hz, even when they all have the same intensity. Sounds near the high- and low-frequency extremes of the hearing range seem even less loud, because the ear is even less sensitive at those frequencies. Table 17.4 gives the dependence of certain human hearing perceptions on physical quantities.

Table 17.4 Sound Perceptions

Perception	Physical quantity
Pitch	Frequency
Loudness	Intensity and Frequency
Timbre	Number and relative intensity of multiple frequencies. Subtle craftsmanship leads to non-linear effects and more detail.
Note	Basic unit of music with specific names, combined to generate tunes
Tone	Number and relative intensity of multiple frequencies.

When a violin plays middle C, there is no mistaking it for a piano playing the same note. The reason is that each instrument produces a distinctive set of frequencies and intensities. We call our perception of these combinations of frequencies and intensities **tone** quality, or more commonly the **timbre** of the sound. It is more difficult to correlate timbre perception to physical quantities than it is for loudness or pitch perception. Timbre is more subjective. Terms such as dull, brilliant, warm, cold, pure, and rich are employed to describe the timbre of a sound. So the consideration of timbre takes us into the realm of perceptual psychology, where higher-level processes in the brain are dominant. This is true for other perceptions of sound, such as music and noise. We shall not delve further into them; rather, we will concentrate on the question of loudness perception.

A unit called a **phon** is used to express loudness numerically. Phons differ from decibels because the phon is a unit of loudness perception, whereas the decibel is a unit of physical intensity. Figure 17.36 shows the relationship of loudness to intensity (or intensity level) and frequency for persons with normal hearing. The curved lines are equal-loudness curves. Each curve is labeled with its loudness in phons. Any sound along a given curve will be perceived as equally loud by the average person. The curves were determined by having large numbers of people compare the loudness of sounds at different frequencies and sound intensity levels. At a frequency of 1000 Hz, phons are taken to be numerically equal to decibels. The following example helps illustrate how to use the graph:

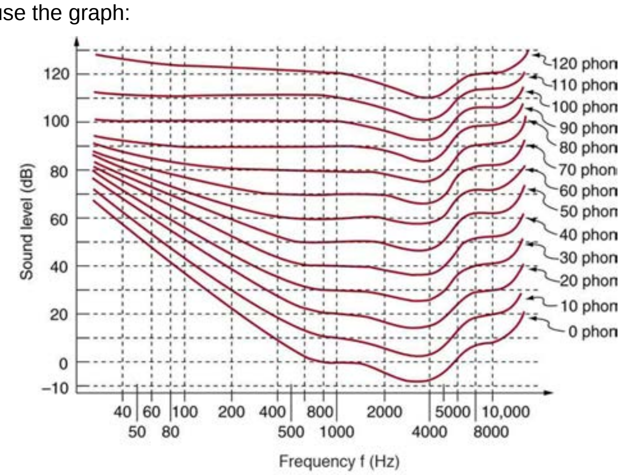


Figure 17.36 The relationship of loudness in phons to intensity level (in decibels) and intensity (in watts per meter squared) for persons with normal hearing. The curved lines are equal-loudness curves—all sounds on a given curve are perceived as equally loud. Phons and decibels are defined to be the same at 1000 Hz.

Example 17.6 Measuring Loudness: Loudness Versus Intensity Level and Frequency

(a) What is the loudness in phons of a 100-Hz sound that has an intensity level of 80 dB? (b) What is the intensity level in decibels of a 4000-Hz sound having a loudness of 70 phons? (c) At what intensity level will an 8000-Hz sound have the same loudness as a 200-Hz sound at 60 dB?

Strategy for (a)

The graph in **Figure 17.36** should be referenced in order to solve this example. To find the loudness of a given sound, you must know its frequency and intensity level and locate that point on the square grid, then interpolate between loudness curves to get the loudness in phons.

Solution for (a)

(1) Identify knowns:

- The square grid of the graph relating phons and decibels is a plot of intensity level versus frequency—both physical quantities.
- 100 Hz at 80 dB lies halfway between the curves marked 70 and 80 phons.

(2) Find the loudness: 75 phons.

Strategy for (b)

The graph in **Figure 17.36** should be referenced in order to solve this example. To find the intensity level of a sound, you must have its frequency and loudness. Once that point is located, the intensity level can be determined from the vertical axis.

Solution for (b)

(1) Identify knowns:

- Values are given to be 4000 Hz at 70 phons.

(2) Follow the 70-phon curve until it reaches 4000 Hz. At that point, it is below the 70 dB line at about 67 dB.

(3) Find the intensity level:

67 dB

Strategy for (c)

The graph in **Figure 17.36** should be referenced in order to solve this example.

Solution for (c)

(1) Locate the point for a 200 Hz and 60 dB sound.

(2) Find the loudness: This point lies just slightly above the 50-phon curve, and so its loudness is 51 phons.

(3) Look for the 51-phon level is at 8000 Hz: 63 dB.

Discussion

These answers, like all information extracted from **Figure 17.36**, have uncertainties of several phons or several decibels, partly due to difficulties in interpolation, but mostly related to uncertainties in the equal-loudness curves.

Further examination of the graph in **Figure 17.36** reveals some interesting facts about human hearing. First, sounds below the 0-phon curve are not perceived by most people. So, for example, a 60 Hz sound at 40 dB is inaudible. The 0-phon curve represents the threshold of normal hearing. We can hear some sounds at intensity levels below 0 dB. For example, a 3-dB, 5000-Hz sound is audible, because it lies above the 0-phon curve. The loudness curves all have dips in them between about 2000 and 5000 Hz. These dips mean the ear is most sensitive to frequencies in that range. For example, a 15-dB sound at 4000 Hz has a loudness of 20 phons, the same as a 20-dB sound at 1000 Hz. The curves rise at both extremes of the frequency range, indicating that a greater-intensity level sound is needed at those frequencies to be perceived to be as loud as at middle frequencies. For example, a sound at 10,000 Hz must have an intensity level of 30 dB to seem as loud as a 20 dB sound at 1000 Hz. Sounds above 120 phons are painful as well as damaging.

We do not often utilize our full range of hearing. This is particularly true for frequencies above 8000 Hz, which are rare in the environment and are unnecessary for understanding conversation or appreciating music. In fact, people who have lost the ability to hear such high frequencies are usually unaware of their loss until tested. The shaded region in **Figure 17.37** is the frequency and intensity region where most conversational sounds fall. The curved lines indicate what effect hearing losses of 40 and 60 phons will have. A 40-phon hearing loss at all frequencies still allows a person to understand conversation, although it will seem very quiet. A person with a 60-phon loss at all frequencies will hear only the lowest frequencies and will not be able to understand speech unless it is much louder than normal. Even so, speech may seem indistinct, because higher frequencies are not as well perceived. The conversational speech region also has a gender component, in that female voices are usually characterized by higher frequencies. So the person with a 60-phon hearing impediment might have difficulty understanding the normal conversation of a woman.

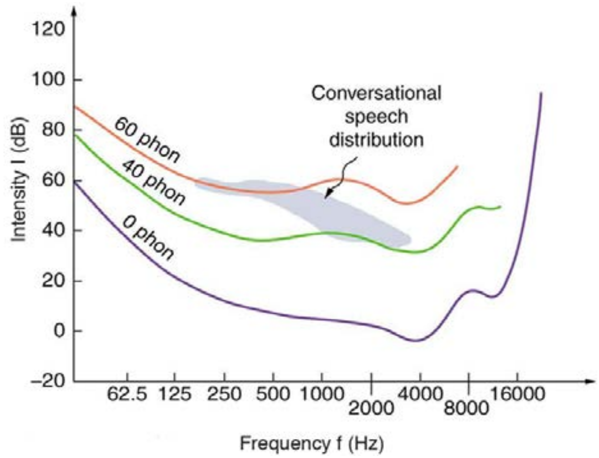


Figure 17.37 The shaded region represents frequencies and intensity levels found in normal conversational speech. The 0-phon line represents the normal hearing threshold, while those at 40 and 60 represent thresholds for people with 40- and 60-phon hearing losses, respectively.

Hearing tests are performed over a range of frequencies, usually from 250 to 8000 Hz, and can be displayed graphically in an audiogram like that in **Figure 17.38**. The hearing threshold is measured in dB *relative to the normal threshold*, so that normal hearing registers as 0 dB at all frequencies. Hearing loss caused by noise typically shows a dip near the 4000 Hz frequency, irrespective of the frequency that caused the loss and often affects both ears. The most common form of hearing loss comes with age and is called *presbycusis*—literally elder ear. Such loss is increasingly severe at higher frequencies, and interferes with music appreciation and speech recognition.

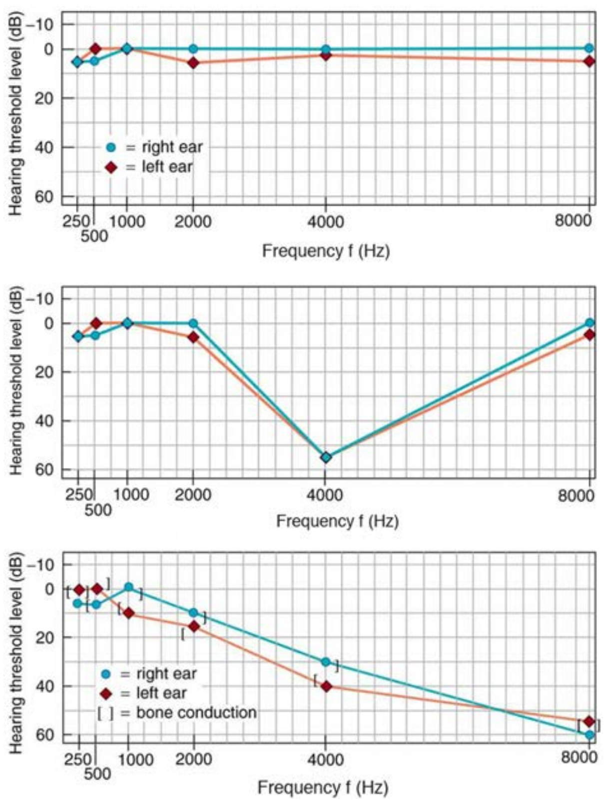


Figure 17.38 Audiograms showing the threshold in intensity level versus frequency for three different individuals. Intensity level is measured relative to the normal threshold. The top left graph is that of a person with normal hearing. The graph to its right has a dip at 4000 Hz and is that of a child who suffered hearing loss due to a cap gun. The third graph is typical of presbycusis, the progressive loss of higher frequency hearing with age. Tests performed by bone conduction (brackets) can distinguish nerve damage from middle ear damage.

The Hearing Mechanism

The hearing mechanism involves some interesting physics. The sound wave that impinges upon our ear is a pressure wave. The ear is a transducer that converts sound waves into electrical nerve impulses in a manner much more sophisticated than, but analogous to, a microphone. **Figure 17.39** shows the gross anatomy of the ear with its division into three parts: the outer ear or ear canal; the middle ear, which runs from the eardrum to the cochlea; and the inner ear, which is the cochlea itself. The body part normally referred to as the ear is technically called the pinna.

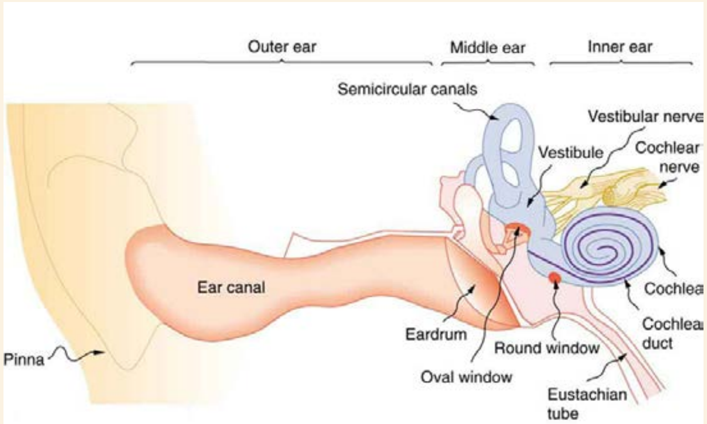


Figure 17.39 The illustration shows the gross anatomy of the human ear.

The outer ear, or ear canal, carries sound to the recessed protected eardrum. The air column in the ear canal resonates and is partially responsible for the sensitivity of the ear to sounds in the 2000 to 5000 Hz range. The middle ear converts sound into mechanical vibrations and applies these vibrations to the cochlea. The lever system of the middle ear takes the force exerted on the eardrum by sound pressure variations, amplifies it and transmits it to the inner ear via the oval window, creating pressure waves in the cochlea approximately 40 times greater than those impinging on the eardrum. (See **Figure 17.40**.) Two muscles in the middle ear (not shown) protect the inner ear from very intense sounds. They react to intense sound in a few milliseconds and reduce the force transmitted to the cochlea. This protective reaction can also be triggered by your own voice, so that humming while shooting a gun, for example, can reduce noise damage.

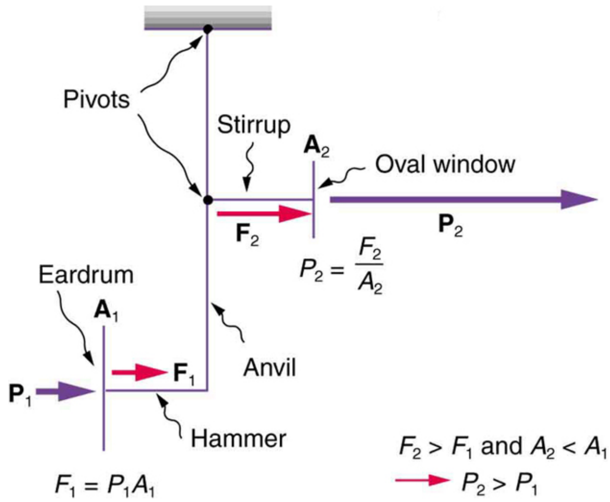


Figure 17.40 This schematic shows the middle ear's system for converting sound pressure into force, increasing that force through a lever system, and applying the increased force to a small area of the cochlea, thereby creating a pressure about 40 times that in the original sound wave. A protective muscle reaction to intense sounds greatly reduces the mechanical advantage of the lever system.

Figure 17.41 shows the middle and inner ear in greater detail. Pressure waves moving through the cochlea cause the tectorial membrane to vibrate, rubbing cilia (called hair cells), which stimulate nerves that send electrical signals to the brain. The membrane resonates at different positions for different frequencies, with high frequencies stimulating nerves at the near end and low frequencies at the far end. The complete operation of the cochlea is still not understood, but several mechanisms for sending information to the brain are known to be involved. For sounds below about 1000 Hz, the nerves send signals at the same frequency as the sound. For frequencies greater than about 1000 Hz, the nerves signal frequency by position. There is a structure to the cilia, and there are connections between nerve cells that perform signal processing before information is sent to the brain. Intensity information is partly indicated by the number of nerve signals and by volleys of signals. The brain processes the cochlear nerve signals to provide additional information such as source direction (based on time and intensity comparisons of sounds from both ears). Higher-level processing produces many nuances, such as music appreciation.

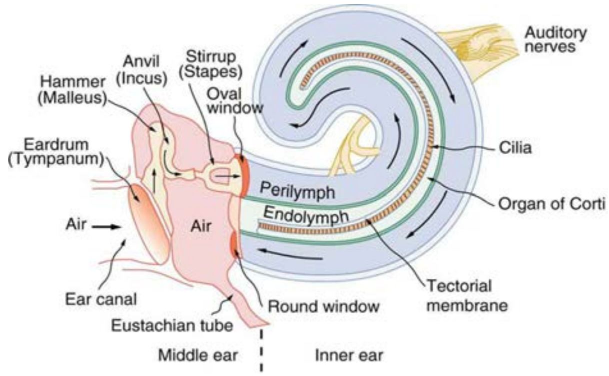


Figure 17.41 The inner ear, or cochlea, is a coiled tube about 3 mm in diameter and 3 cm in length if uncoiled. When the oval window is forced inward, as shown, a pressure wave travels through the perilymph in the direction of the arrows, stimulating nerves at the base of cilia in the organ of Corti.

Hearing losses can occur because of problems in the middle or inner ear. Conductive losses in the middle ear can be partially overcome by sending sound vibrations to the cochlea through the skull. Hearing aids for this purpose usually press against the bone behind the ear, rather than simply amplifying the sound sent into the ear canal as many hearing aids do. Damage to the nerves in the cochlea is not repairable, but amplification can partially compensate. There is a risk that amplification will produce further damage. Another common failure in the cochlea is damage or loss of the cilia but with nerves remaining functional. Cochlear implants that stimulate the nerves directly are now available and widely accepted. Over 100,000 implants are in use, in about equal numbers of adults and children.

The cochlear implant was pioneered in Melbourne, Australia, by Graeme Clark in the 1970s for his deaf father. The implant consists of three external components and two internal components. The external components are a microphone for picking up sound and converting it into an electrical signal, a speech processor to select certain frequencies and a transmitter to transfer the signal to the internal components through electromagnetic induction. The internal components consist of a receiver/transmitter secured in the bone beneath the skin, which converts the signals into electric impulses and sends them through an internal cable to the cochlea and an array of about 24 electrodes wound through the cochlea. These electrodes in turn send the impulses directly into the brain. The electrodes basically emulate the cilia.

Check Your Understanding

Are ultrasound and infrasound imperceptible to all hearing organisms? Explain your answer.

Solution
No, the range of perceptible sound is based in the range of human hearing. Many other organisms perceive either infrasound or ultrasound.

17.7 Ultrasound

Figure 17.42 Ultrasound is used in medicine to painlessly and noninvasively monitor patient health and diagnose a wide range of disorders. (credit: abbybatchelder, Flickr)

Any sound with a frequency above 20,000 Hz (or 20 kHz)—that is, above the highest audible frequency—is defined to be ultrasound. In practice, it is possible to create ultrasound frequencies up to more than a gigahertz. (Higher frequencies are difficult to create; furthermore, they propagate poorly because they are very strongly absorbed.) Ultrasound has a tremendous number of applications, which range from burglar alarms to use in cleaning delicate objects to the guidance systems of bats. We begin our discussion of ultrasound with some of its applications in medicine, in which it is used extensively both for diagnosis and for therapy.

Characteristics of Ultrasound

The characteristics of ultrasound, such as frequency and intensity, are wave properties common to all types of waves. Ultrasound also has a wavelength that limits the fineness of detail it can detect. This characteristic is true of all waves. We can never observe details significantly smaller than the wavelength of our probe; for example, we will never see individual atoms with visible light, because the atoms are so small compared with the wavelength of light.

Ultrasound in Medical Therapy

Ultrasound, like any wave, carries energy that can be absorbed by the medium carrying it, producing effects that vary with intensity. When focused to intensities of 10^3 to 10^5 W/m^2, ultrasound can be used to shatter gallstones or pulverize cancerous tissue in surgical procedures. (See **Figure 17.43**.) Intensities this great can damage individual cells, variously causing their protoplasm to stream inside them, altering their permeability, or rupturing their walls through *cavitation*. Cavitation is the creation of vapor cavities in a fluid—the longitudinal vibrations in ultrasound alternatively compress and expand the medium, and at sufficient amplitudes the expansion separates molecules. Most cavitation damage is done when the cavities collapse, producing even greater shock pressures.

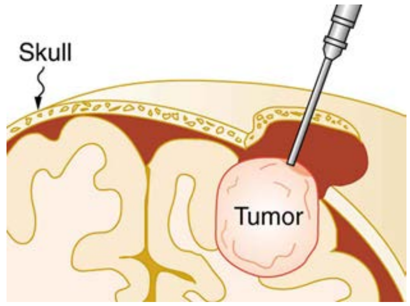


Figure 17.43 The tip of this small probe oscillates at 23 kHz with such a large amplitude that it pulverizes tissue on contact. The debris is then aspirated. The speed of the tip may exceed the speed of sound in tissue, thus creating shock waves and cavitation, rather than a smooth simple harmonic oscillator–type wave.

Most of the energy carried by high-intensity ultrasound in tissue is converted to thermal energy. In fact, intensities of 10^3 to 10^4 W/m^2 are commonly used for deep-heat treatments called ultrasound diathermy. Frequencies of 0.8 to 1 MHz are typical. In both athletics and physical therapy, ultrasound diathermy is most often applied to injured or overworked muscles to relieve pain and improve flexibility. Skill is needed by the therapist to avoid "bone burns" and other tissue damage caused by overheating and cavitation, sometimes made worse by reflection and focusing of the ultrasound by joint and bone tissue.

In some instances, you may encounter a different decibel scale, called the sound *pressure* level, when ultrasound travels in water or in human and other biological tissues. We shall not use the scale here, but it is notable that numbers for sound pressure levels range 60 to 70 dB higher than you would quote for β, the sound intensity level used in this text. Should you encounter a sound pressure level of 220 decibels, then, it is not an astronomically high intensity, but equivalent to about 155 dB—high enough to destroy tissue, but not as unreasonably high as it might seem at first.

Ultrasound in Medical Diagnostics

When used for imaging, ultrasonic waves are emitted from a transducer, a crystal exhibiting the piezoelectric effect (the expansion and contraction of a substance when a voltage is applied across it, causing a vibration of the crystal). These high-frequency vibrations are transmitted into any tissue in contact with the transducer. Similarly, if a pressure is applied to the crystal (in the form of a wave reflected off tissue layers), a voltage is produced which can be recorded. The crystal therefore acts as both a transmitter and a receiver of sound. Ultrasound is also partially absorbed by tissue on its path, both on its journey away from the transducer and on its return journey. From the time between when the original signal is sent and when the reflections from various boundaries between media are received, (as well as a measure of the intensity loss of the signal), the nature and position of each boundary between tissues and organs may be deduced.

Reflections at boundaries between two different media occur because of differences in a characteristic known as the **acoustic impedance** Z of each substance. Impedance is defined as

$$Z = \rho v, \tag{17.38}$$

where ρ is the density of the medium (in kg/m^3) and v is the speed of sound through the medium (in m/s). The units for Z are therefore $\text{kg/(m}^2 \cdot \text{s)}$.

Table 17.5 shows the density and speed of sound through various media (including various soft tissues) and the associated acoustic impedances. Note that the acoustic impedances for soft tissue do not vary much but that there is a big difference between the acoustic impedance of soft tissue and air and also between soft tissue and bone.

Table 17.5 The Ultrasound Properties of Various Media, Including Soft Tissue Found in the Body

Medium	Density (kg/m^3)	Speed of Ultrasound (m/s)	Acoustic Impedance $\left(kg/\left(m^2 \cdot s\right)\right)$
Air	1.3	330	429
Water	1000	1500	1.5×10^6
Blood	1060	1570	1.66×10^6
Fat	925	1450	1.34×10^6
Muscle (average)	1075	1590	1.70×10^6
Bone (varies)	1400–1900	4080	5.7×10^6 to 7.8×10^6
Barium titanate (transducer material)	5600	5500	30.8×10^6

At the boundary between media of different acoustic impedances, some of the wave energy is reflected and some is transmitted. The greater the *difference* in acoustic impedance between the two media, the greater the reflection and the smaller the transmission.

The **intensity reflection coefficient** a is defined as the ratio of the intensity of the reflected wave relative to the incident (transmitted) wave. This statement can be written mathematically as

$$a = \frac{(Z_2 - Z_1)^2}{(Z_1 + Z_2)^2}, \tag{17.39}$$

where Z_1 and Z_2 are the acoustic impedances of the two media making up the boundary. A reflection coefficient of zero (corresponding to total transmission and no reflection) occurs when the acoustic impedances of the two media are the same. An impedance "match" (no reflection) provides an efficient coupling of sound energy from one medium to another. The image formed in an ultrasound is made by tracking reflections (as shown in **Figure 17.44**) and mapping the intensity of the reflected sound waves in a two-dimensional plane.

Example 17.7 Calculate Acoustic Impedance and Intensity Reflection Coefficient: Ultrasound and Fat Tissue

(a) Using the values for density and the speed of ultrasound given in **Table 17.5**, show that the acoustic impedance of fat tissue is indeed $1.34\times10^6\ \text{kg/(m}^2\cdot\text{s)}$.

(b) Calculate the intensity reflection coefficient of ultrasound when going from fat to muscle tissue.

Strategy for (a)

The acoustic impedance can be calculated using $Z = \rho v$ and the values for ρ and v found in **Table 17.5**.

Solution for (a)

(1) Substitute known values from **Table 17.5** into $Z = \rho v$.

$$Z = \rho v = \left(925\ \text{kg/m}^3\right)(1450\ \text{m/s}) \tag{17.40}$$

(2) Calculate to find the acoustic impedance of fat tissue.

$$1.34\times10^6\ \text{kg/(m}^2\cdot\text{s)} \tag{17.41}$$

This value is the same as the value given for the acoustic impedance of fat tissue.

Strategy for (b)

The intensity reflection coefficient for any boundary between two media is given by $a = \frac{(Z_2 - Z_1)^2}{(Z_1\ +\ Z_2)^2}$, and the acoustic impedance of muscle is given in **Table 17.5**.

Solution for (b)

Substitute known values into $a = \frac{(Z_2 - Z_1)^2}{(Z_1\ +\ Z_2)^2}$ to find the intensity reflection coefficient:

$$a = \frac{(Z_2 - Z_1)^2}{(Z_1 + Z_2)^2} = \frac{\left(1.34\times10^6\ \text{kg/(m}^2\cdot\text{s)} - 1.70\times10^6\ \text{kg/(m}^2\cdot\text{s)}\right)^2}{\left(1.70\times10^6\ \text{kg/(m}^2\cdot\text{s)} + 1.34\times10^6\ \text{kg/(m}^2\cdot\text{s)}\right)^2} = 0.014 \tag{17.42}$$

Discussion

This result means that only 1.4% of the incident intensity is reflected, with the remaining being transmitted.

The applications of ultrasound in medical diagnostics have produced untold benefits with no known risks. Diagnostic intensities are too low (about $10^{-2}\ \text{W/m}^2$) to cause thermal damage. More significantly, ultrasound has been in use for several decades and detailed follow-up studies do not show evidence of ill effects, quite unlike the case for x-rays.

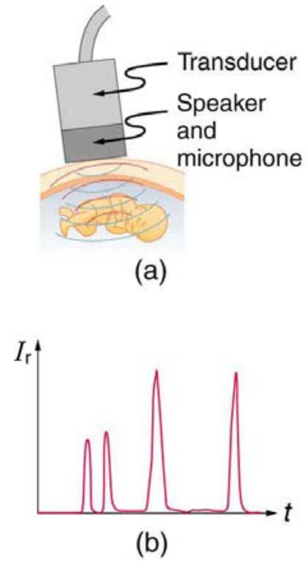


Figure 17.44 (a) An ultrasound speaker doubles as a microphone. Brief bleeps are broadcast, and echoes are recorded from various depths. (b) Graph of echo intensity versus time. The time for echoes to return is directly proportional to the distance of the reflector, yielding this information noninvasively.

The most common ultrasound applications produce an image like that shown in **Figure 17.45**. The speaker-microphone broadcasts a directional beam, sweeping the beam across the area of interest. This is accomplished by having multiple ultrasound sources in the probe's head, which are phased to interfere constructively in a given, adjustable direction. Echoes are measured as a function of position as well as depth. A computer constructs an image that reveals the shape and density of internal structures.

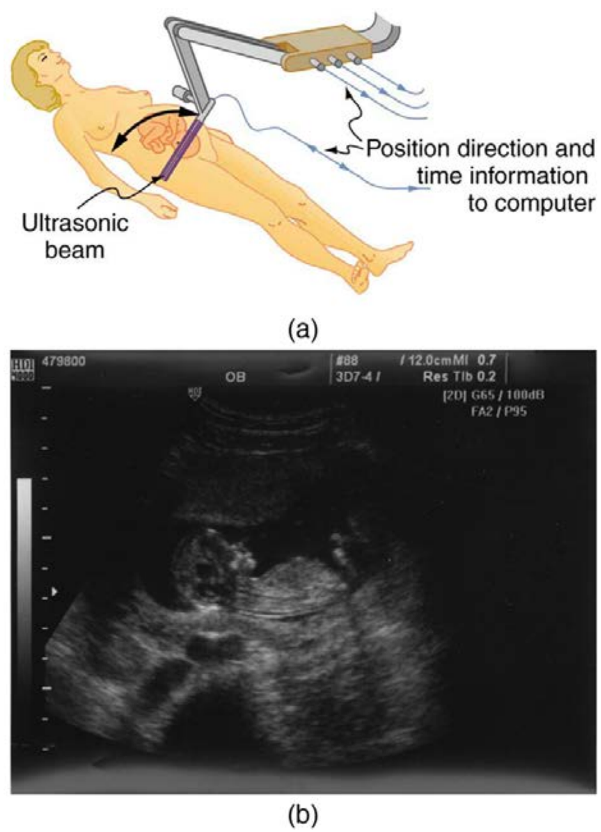


Figure 17.45 (a) An ultrasonic image is produced by sweeping the ultrasonic beam across the area of interest, in this case the woman's abdomen. Data are recorded and analyzed in a computer, providing a two-dimensional image. (b) Ultrasound image of 12-week-old fetus. (credit: Margaret W. Carruthers, Flickr)

How much detail can ultrasound reveal? The image in **Figure 17.45** is typical of low-cost systems, but that in **Figure 17.46** shows the remarkable detail possible with more advanced systems, including 3D imaging. Ultrasound today is commonly used in prenatal care. Such imaging can be used to see if the fetus is developing at a normal rate, and help in the determination of serious problems early in the pregnancy. Ultrasound is also in wide use to image the chambers of the heart and the flow of blood within the beating heart, using the Doppler effect (echocardiology).

Whenever a wave is used as a probe, it is very difficult to detect details smaller than its wavelength λ. Indeed, current technology cannot do quite this well. Abdominal scans may use a 7-MHz frequency, and the speed of sound in tissue is about 1540 m/s—so the wavelength limit to detail would be $\lambda = \frac{v_w}{f} = \frac{1540 \text{ m/s}}{7 \times 10^6 \text{ Hz}} = 0.22 \text{ mm}$. In practice, 1-mm detail is attainable, which is sufficient for many purposes. Higher-frequency ultrasound would allow greater detail, but it does not penetrate as well as lower frequencies do. The accepted rule of thumb is that you can effectively scan to a depth of about 500λ into tissue. For 7 MHz, this penetration limit is $500 \times 0.22 \text{ mm}$, which is 0.11 m. Higher frequencies may be employed in smaller organs, such as the eye, but are not practical for looking deep into the body.

Figure 17.46 A 3D ultrasound image of a fetus. As well as for the detection of any abnormalities, such scans have also been shown to be useful for strengthening the emotional bonding between parents and their unborn child. (credit: Jennie Cu, Wikimedia Commons)

In addition to shape information, ultrasonic scans can produce density information superior to that found in X-rays, because the intensity of a reflected sound is related to changes in density. Sound is most strongly reflected at places where density changes are greatest.

Another major use of ultrasound in medical diagnostics is to detect motion and determine velocity through the Doppler shift of an echo, known as **Doppler-shifted ultrasound**. This technique is used to monitor fetal heartbeat, measure blood velocity, and detect occlusions in blood vessels, for example. (See **Figure 17.47**.) The magnitude of the Doppler shift in an echo is directly proportional to the velocity of whatever reflects the sound. Because an echo is involved, there is actually a double shift. The first occurs because the reflector (say a fetal heart) is a moving observer and receives a Doppler-shifted frequency. The reflector then acts as a moving source, producing a second Doppler shift.

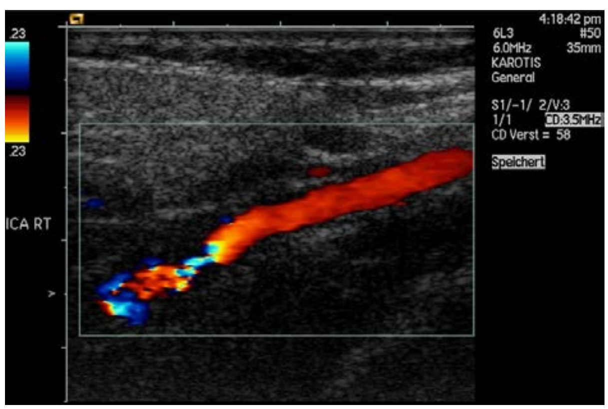


Figure 17.47 This Doppler-shifted ultrasonic image of a partially occluded artery uses color to indicate velocity. The highest velocities are in red, while the lowest are blue. The blood must move faster through the constriction to carry the same flow. (credit: Arning C, Grzyska U, Wikimedia Commons)

A clever technique is used to measure the Doppler shift in an echo. The frequency of the echoed sound is superimposed on the broadcast frequency, producing beats. The beat frequency is $F_B = \mid f_1 - f_2 \mid$, and so it is directly proportional to the Doppler shift ($f_1 - f_2$) and hence, the reflector's velocity. The advantage in this technique is that the Doppler shift is small (because the reflector's velocity is small), so that great accuracy would be needed to measure the shift directly. But measuring the beat frequency is easy, and it is not affected if the broadcast frequency varies somewhat. Furthermore, the beat frequency is in the audible range and can be amplified for audio feedback to the medical observer.

Uses for Doppler-Shifted Radar

Doppler-shifted radar echoes are used to measure wind velocities in storms as well as aircraft and automobile speeds. The principle is the same as for Doppler-shifted ultrasound. There is evidence that bats and dolphins may also sense the velocity of an object (such as prey) reflecting their ultrasound signals by observing its Doppler shift.

Example 17.8 Calculate Velocity of Blood: Doppler-Shifted Ultrasound

Ultrasound that has a frequency of 2.50 MHz is sent toward blood in an artery that is moving toward the source at 20.0 cm/s, as illustrated in Figure 17.48. Use the speed of sound in human tissue as 1540 m/s. (Assume that the frequency of 2.50 MHz is accurate to seven significant figures.)

a. What frequency does the blood receive?
b. What frequency returns to the source?
c. What beat frequency is produced if the source and returning frequencies are mixed?

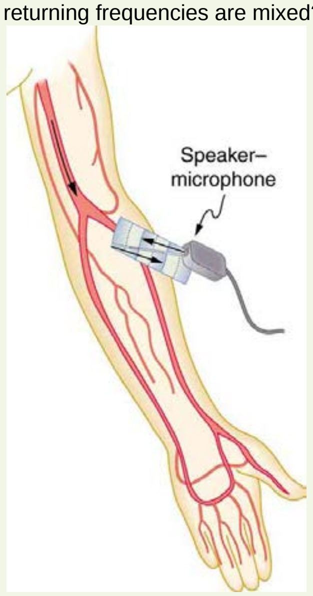


Figure 17.48 Ultrasound is partly reflected by blood cells and plasma back toward the speaker-microphone. Because the cells are moving, two Doppler shifts are produced—one for blood as a moving observer, and the other for the reflected sound coming from a moving source. The magnitude of the shift is directly proportional to blood velocity.

Strategy

The first two questions can be answered using $f_{\text{obs}} = f_{\text{s}}\left(\frac{v_{\text{w}}}{v_{\text{w}} \pm v_{\text{s}}}\right)$ and $f_{\text{obs}} = f_{\text{s}}\left(\frac{v_{\text{w}} \pm v_{\text{obs}}}{v_{\text{w}}}\right)$ for the Doppler shift. The last question asks for beat frequency, which is the difference between the original and returning frequencies.

Solution for (a)

(1) Identify knowns:

- The blood is a moving observer, and so the frequency it receives is given by

$$f_{\text{obs}} = f_{\text{s}}\left(\frac{v_{\text{w}} \pm v_{\text{obs}}}{v_{\text{w}}}\right). \tag{17.43}$$

- v_{b} is the blood velocity (v_{obs} here) and the plus sign is chosen because the motion is toward the source.

(2) Enter the given values into the equation.

$$f_{\text{obs}} = (2{,}500{,}000\ \text{Hz})\left(\frac{1540\ \text{m/s}+0.2\ \text{m/s}}{1540\ \text{m/s}}\right) \tag{17.44}$$

(3) Calculate to find the frequency: 20,500,325 Hz.

Solution for (b)

(1) Identify knowns:

- The blood acts as a moving source.
- The microphone acts as a stationary observer.
- The frequency leaving the blood is 2,500,325 Hz, but it is shifted upward as given by

$$f_{\text{obs}} = f_{\text{s}}\left(\frac{v_{\text{w}}}{v_{\text{w}} - v_{\text{b}}}\right). \tag{17.45}$$

  f_{obs} is the frequency received by the speaker-microphone.

- The source velocity is v_{b} .
- The minus sign is used because the motion is toward the observer.

The minus sign is used because the motion is toward the observer.

(2) Enter the given values into the equation:

$$f_{\text{obs}} = (2{,}500{,}325\ \text{Hz})\left(\frac{1540\ \text{m/s}}{1540\ \text{m/s} - 0.200\ \text{m/s}}\right) \tag{17.46}$$

(3) Calculate to find the frequency returning to the source: 2,500,649 Hz.

Solution for (c)

(1) Identify knowns:

- The beat frequency is simply the absolute value of the difference between f_s and f_{obs}, as stated in:

$$f_B = \mid f_{obs} - f_s \mid . \tag{17.47}$$

(2) Substitute known values:

$$\mid 2{,}500{,}649 \text{ Hz} - 2{,}500{,}000 \text{ Hz} \mid \tag{17.48}$$

(3) Calculate to find the beat frequency: 649 Hz.

Discussion

The Doppler shifts are quite small compared with the original frequency of 2.50 MHz. It is far easier to measure the beat frequency than it is to measure the echo frequency with an accuracy great enough to see shifts of a few hundred hertz out of a couple of megahertz. Furthermore, variations in the source frequency do not greatly affect the beat frequency, because both f_s and f_{obs} would increase or decrease. Those changes subtract out in $f_B = \mid f_{obs} - f_s \mid$.

Industrial and Other Applications of Ultrasound

Industrial, retail, and research applications of ultrasound are common. A few are discussed here. Ultrasonic cleaners have many uses. Jewelry, machined parts, and other objects that have odd shapes and crevices are immersed in a cleaning fluid that is agitated with ultrasound typically about 40 kHz in frequency. The intensity is great enough to cause cavitation, which is responsible for most of the cleansing action. Because cavitation-produced shock pressures are large and well transmitted in a fluid, they reach into small crevices where even a low-surface-tension cleaning fluid might not penetrate.

Sonar is a familiar application of ultrasound. Sonar typically employs ultrasonic frequencies in the range from 30.0 to 100 kHz. Bats, dolphins, submarines, and even some birds use ultrasonic sonar. Echoes are analyzed to give distance and size information both for guidance and finding prey. In most sonar applications, the sound reflects quite well because the objects of interest have significantly different density than the medium in which they travel. When the Doppler shift is observed, velocity information can also be obtained. Submarine sonar can be used to obtain such information, and there is evidence that some bats also sense velocity from their echoes.

Similarly, there are a range of relatively inexpensive devices that measure distance by timing ultrasonic echoes. Many cameras, for example, use such information to focus automatically. Some doors open when their ultrasonic ranging devices detect a nearby object, and certain home security lights turn on when their ultrasonic rangers observe motion. Ultrasonic “measuring tapes” also exist to measure such things as room dimensions. Sinks in public restrooms are sometimes automated with ultrasound devices to turn faucets on and off when people wash their hands. These devices reduce the spread of germs and can conserve water.

Ultrasound is used for nondestructive testing in industry and by the military. Because ultrasound reflects well from any large change in density, it can reveal cracks and voids in solids, such as aircraft wings, that are too small to be seen with x-rays. For similar reasons, ultrasound is also good for measuring the thickness of coatings, particularly where there are several layers involved.

Basic research in solid state physics employs ultrasound. Its attenuation is related to a number of physical characteristics, making it a useful probe. Among these characteristics are structural changes such as those found in liquid crystals, the transition of a material to a superconducting phase, as well as density and other properties.

These examples of the uses of ultrasound are meant to whet the appetites of the curious, as well as to illustrate the underlying physics of ultrasound. There are many more applications, as you can easily discover for yourself.

Check Your Understanding

Why is it possible to use ultrasound both to observe a fetus in the womb and also to destroy cancerous tumors in the body?

Solution

Ultrasound can be used medically at different intensities. Lower intensities do not cause damage and are used for medical imaging. Higher intensities can pulverize and destroy targeted substances in the body, such as tumors.

Glossary

acoustic impedance: property of medium that makes the propagation of sound waves more difficult

antinode: point of maximum displacement

bow wake: V-shaped disturbance created when the wave source moves faster than the wave propagation speed

Doppler effect: an alteration in the observed frequency of a sound due to motion of either the source or the observer

Doppler shift: the actual change in frequency due to relative motion of source and observer

Doppler-shifted ultrasound: a medical technique to detect motion and determine velocity through the Doppler shift of an echo

fundamental: the lowest-frequency resonance

harmonics: the term used to refer collectively to the fundamental and its overtones

hearing: the perception of sound

infrasound: sounds below 20 Hz

intensity reflection coefficient: a measure of the ratio of the intensity of the wave reflected off a boundary between two media relative to the intensity of the incident wave

intensity: the power per unit area carried by a wave

loudness: the perception of sound intensity

node: point of zero displacement

note: basic unit of music with specific names, combined to generate tunes

overtones: all resonant frequencies higher than the fundamental

phon: the numerical unit of loudness

pitch: the perception of the frequency of a sound

sonic boom: a constructive interference of sound created by an object moving faster than sound

sound intensity level: a unitless quantity telling you the level of the sound relative to a fixed standard

sound pressure level: the ratio of the pressure amplitude to a reference pressure

sound: a disturbance of matter that is transmitted from its source outward

timbre: number and relative intensity of multiple sound frequencies

tone: number and relative intensity of multiple sound frequencies

ultrasound: sounds above 20,000 Hz

Section Summary

17.1 Sound

- Sound is a disturbance of matter that is transmitted from its source outward.
- Sound is one type of wave.
- Hearing is the perception of sound.

17.2 Speed of Sound, Frequency, and Wavelength

The relationship of the speed of sound v_w, its frequency f, and its wavelength λ is given by

$$v_\text{w} = f\lambda,$$

which is the same relationship given for all waves.

In air, the speed of sound is related to air temperature T by

$$v_\text{w} = (331\text{ m/s})\sqrt{\frac{T}{273\text{ K}}}.$$

v_w is the same for all frequencies and wavelengths.

17.3 Sound Intensity and Sound Level

- Intensity is the same for a sound wave as was defined for all waves; it is

  $$I = \frac{P}{A},$$

  where P is the power crossing area A. The SI unit for I is watts per meter squared. The intensity of a sound wave is also related to the pressure amplitude Δp

  $$I = \frac{(\Delta p)^2}{2\rho v_\text{w}},$$

  where ρ is the density of the medium in which the sound wave travels and v_w is the speed of sound in the medium.
- Sound intensity level in units of decibels (dB) is

  $$\beta\text{ (dB)} = 10\log_{10}\left(\frac{I}{I_0}\right),$$

  where $I_0 = 10^{-12}\text{ W/m}^2$ is the threshold intensity of hearing.

17.4 Doppler Effect and Sonic Booms

- The Doppler effect is an alteration in the observed frequency of a sound due to motion of either the source or the observer.
- The actual change in frequency is called the Doppler shift.
- A sonic boom is constructive interference of sound created by an object moving faster than sound.
- A sonic boom is a type of bow wake created when any wave source moves faster than the wave propagation speed.
- For a stationary observer and a moving source, the observed frequency f_{obs} is:

$$f_{\text{obs}} = f_{\text{s}}\left(\frac{v_{\text{w}}}{v_{\text{w}} \pm v_{\text{s}}}\right),$$

  where f_{s} is the frequency of the source, v_{s} is the speed of the source, and v_{w} is the speed of sound. The minus sign is used for motion toward the observer and the plus sign for motion away.
- For a stationary source and moving observer, the observed frequency is:

$$f_{\text{obs}} = f_{\text{s}}\left(\frac{v_{\text{w}} \pm v_{\text{obs}}}{v_{\text{w}}}\right),$$

  where v_{obs} is the speed of the observer.

17.5 Sound Interference and Resonance: Standing Waves in Air Columns

- Sound interference and resonance have the same properties as defined for all waves.
- In air columns, the lowest-frequency resonance is called the fundamental, whereas all higher resonant frequencies are called overtones. Collectively, they are called harmonics.
- The resonant frequencies of a tube closed at one end are:

$$f_n = n\frac{v_{\text{w}}}{4L},\ n = 1, 3, 5...,$$

  f_1 is the fundamental and L is the length of the tube.
- The resonant frequencies of a tube open at both ends are:

$$f_n = n\frac{v_{\text{w}}}{2L},\ n = 1, 2, 3...$$

17.6 Hearing

- The range of audible frequencies is 20 to 20,000 Hz.
- Those sounds above 20,000 Hz are ultrasound, whereas those below 20 Hz are infrasound.
- The perception of frequency is pitch.
- The perception of intensity is loudness.
- Loudness has units of phons.

17.7 Ultrasound

- The acoustic impedance is defined as:

$$Z = \rho v,$$

  ρ is the density of a medium through which the sound travels and v is the speed of sound through that medium.
- The intensity reflection coefficient a , a measure of the ratio of the intensity of the wave reflected off a boundary between two media relative to the intensity of the incident wave, is given by

$$a = \frac{(Z_2 - Z_1)^2}{(Z_1 + Z_2)^2}.$$

- The intensity reflection coefficient is a unitless quantity.

Conceptual Questions

17.2 Speed of Sound, Frequency, and Wavelength

1. How do sound vibrations of atoms differ from thermal motion?

2. When sound passes from one medium to another where its propagation speed is different, does its frequency or wavelength change? Explain your answer briefly.

17.3 Sound Intensity and Sound Level

3. Six members of a synchronized swim team wear earplugs to protect themselves against water pressure at depths, but they can still hear the music and perform the combinations in the water perfectly. One day, they were asked to leave the pool so the dive team could practice a few dives, and they tried to practice on a mat, but seemed to have a lot more difficulty. Why might this be?

4. A community is concerned about a plan to bring train service to their downtown from the town's outskirts. The current sound intensity level, even though the rail yard is blocks away, is 70 dB downtown. The mayor assures the public that there will be a difference of only 30 dB in sound in the downtown area. Should the townspeople be concerned? Why?

17.4 Doppler Effect and Sonic Booms

5. Is the Doppler shift real or just a sensory illusion?

6. Due to efficiency considerations related to its bow wake, the supersonic transport aircraft must maintain a cruising speed that is a constant ratio to the speed of sound (a constant Mach number). If the aircraft flies from warm air into colder air, should it increase or decrease its speed? Explain your answer.

7. When you hear a sonic boom, you often cannot see the plane that made it. Why is that?

17.5 Sound Interference and Resonance: Standing Waves in Air Columns

8. How does an unamplified guitar produce sounds so much more intense than those of a plucked string held taut by a simple stick?

9. You are given two wind instruments of identical length. One is open at both ends, whereas the other is closed at one end. Which is able to produce the lowest frequency?

10. What is the difference between an overtone and a harmonic? Are all harmonics overtones? Are all overtones harmonics?

17.6 Hearing

11. Why can a hearing test show that your threshold of hearing is 0 dB at 250 Hz, when **Figure 17.37** implies that no one can hear such a frequency at less than 20 dB?

17.7 Ultrasound

12. If audible sound follows a rule of thumb similar to that for ultrasound, in terms of its absorption, would you expect the high or low frequencies from your neighbor’s stereo to penetrate into your house? How does this expectation compare with your experience?

13. Elephants and whales are known to use infrasound to communicate over very large distances. What are the advantages of infrasound for long distance communication?

14. It is more difficult to obtain a high-resolution ultrasound image in the abdominal region of someone who is overweight than for someone who has a slight build. Explain why this statement is accurate.

15. Suppose you read that 210-dB ultrasound is being used to pulverize cancerous tumors. You calculate the intensity in watts per centimeter squared and find it is unreasonably high ($10^5\ \text{W/cm}^2$). What is a possible explanation?

Problems & Exercises

17.2 Speed of Sound, Frequency, and Wavelength

16. When poked by a spear, an operatic soprano lets out a 1200-Hz shriek. What is its wavelength if the speed of sound is 345 m/s?

17. What frequency sound has a 0.10-m wavelength when the speed of sound is 340 m/s?

18. Calculate the speed of sound on a day when a 1500 Hz frequency has a wavelength of 0.221 m.

19. (a) What is the speed of sound in a medium where a 100-kHz frequency produces a 5.96-cm wavelength? (b) Which substance in **Table 17.1** is this likely to be?

20. Show that the speed of sound in 20.0°C air is 343 m/s, as claimed in the text.

21. Air temperature in the Sahara Desert can reach 56.0°C (about 134°F). What is the speed of sound in air at that temperature?

22. Dolphins make sounds in air and water. What is the ratio of the wavelength of a sound in air to its wavelength in seawater? Assume air temperature is 20.0°C.

23. A sonar echo returns to a submarine 1.20 s after being emitted. What is the distance to the object creating the echo? (Assume that the submarine is in the ocean, not in fresh water.)

24. (a) If a submarine's sonar can measure echo times with a precision of 0.0100 s, what is the smallest difference in distances it can detect? (Assume that the submarine is in the ocean, not in fresh water.)

(b) Discuss the limits this time resolution imposes on the ability of the sonar system to detect the size and shape of the object creating the echo.

25. A physicist at a fireworks display times the lag between seeing an explosion and hearing its sound, and finds it to be 0.400 s. (a) How far away is the explosion if air temperature is 24.0°C and if you neglect the time taken for light to reach the physicist? (b) Calculate the distance to the explosion taking the speed of light into account. Note that this distance is negligibly greater.

26. Suppose a bat uses sound echoes to locate its insect prey, 3.00 m away. (See **Figure 17.10**.) (a) Calculate the echo times for temperatures of 5.00°C and 35.0°C. (b) What percent uncertainty does this cause for the bat in locating the insect? (c) Discuss the significance of this uncertainty and whether it could cause difficulties for the bat. (In practice, the bat continues to use sound as it closes in, eliminating most of any difficulties imposed by this and other effects, such as motion of the prey.)

17.3 Sound Intensity and Sound Level

27. What is the intensity in watts per meter squared of 85.0-dB sound?

28. The warning tag on a lawn mower states that it produces noise at a level of 91.0 dB. What is this in watts per meter squared?

29. A sound wave traveling in 20°C air has a pressure amplitude of 0.5 Pa. What is the intensity of the wave?

30. What intensity level does the sound in the preceding problem correspond to?

31. What sound intensity level in dB is produced by earphones that create an intensity of $4.00\times10^{-2}\ \text{W/m}^2$?

32. Show that an intensity of $10^{-12}\ \text{W/m}^2$ is the same as $10^{-16}\ \text{W/cm}^2$.

33. (a) What is the decibel level of a sound that is twice as intense as a 90.0-dB sound? (b) What is the decibel level of a sound that is one-fifth as intense as a 90.0-dB sound?

34. (a) What is the intensity of a sound that has a level 7.00 dB lower than a $4.00\times10^{-9}\ \text{W/m}^2$ sound? (b) What is the intensity of a sound that is 3.00 dB higher than a $4.00\times10^{-9}\ \text{W/m}^2$ sound?

35. (a) How much more intense is a sound that has a level 17.0 dB higher than another? (b) If one sound has a level 23.0 dB less than another, what is the ratio of their intensities?

36. People with good hearing can perceive sounds as low in level as $-8.00\ \text{dB}$ at a frequency of 3000 Hz. What is the intensity of this sound in watts per meter squared?

37. If a large housefly 3.0 m away from you makes a noise of 40.0 dB, what is the noise level of 1000 flies at that distance, assuming interference has a negligible effect?

38. Ten cars in a circle at a boom box competition produce a 120-dB sound intensity level at the center of the circle. What is the average sound intensity level produced there by each stereo, assuming interference effects can be neglected?

39. The amplitude of a sound wave is measured in terms of its maximum gauge pressure. By what factor does the amplitude of a sound wave increase if the sound intensity level goes up by 40.0 dB?

40. If a sound intensity level of 0 dB at 1000 Hz corresponds to a maximum gauge pressure (sound amplitude) of $10^{-9}\ \text{atm}$, what is the maximum gauge pressure in a 60-dB sound? What is the maximum gauge pressure in a 120-dB sound?

41. An 8-hour exposure to a sound intensity level of 90.0 dB may cause hearing damage. What energy in joules falls on a 0.800-cm-diameter eardrum so exposed?

42. (a) Ear trumpets were never very common, but they did aid people with hearing losses by gathering sound over a large area and concentrating it on the smaller area of the eardrum. What decibel increase does an ear trumpet produce if its sound gathering area is $900\ \text{cm}^2$ and the area of the eardrum is $0.500\ \text{cm}^2$, but the trumpet only has an efficiency of 5.00% in transmitting the sound to the eardrum? (b) Comment on the usefulness of the decibel increase found in part (a).

43. Sound is more effectively transmitted into a stethoscope by direct contact than through the air, and it is further intensified by being concentrated on the smaller area of the eardrum. It is reasonable to assume that sound is transmitted into a stethoscope 100 times as effectively compared with transmission though the air. What, then, is the gain in decibels produced by a stethoscope that has a sound gathering area of $15.0\ \text{cm}^2$, and concentrates the sound onto two eardrums with a total area of $0.900\ \text{cm}^2$ with an efficiency of 40.0%?

44. Loudspeakers can produce intense sounds with surprisingly small energy input in spite of their low efficiencies. Calculate the power input needed to produce a 90.0-dB sound intensity level for a 12.0-cm-diameter speaker that has an efficiency of 1.00%. (This value is the sound intensity level right at the speaker.)

17.4 Doppler Effect and Sonic Booms

45. (a) What frequency is received by a person watching an oncoming ambulance moving at 110 km/h and emitting a steady 800-Hz sound from its siren? The speed of sound on this day is 345 m/s. (b) What frequency does she receive after the ambulance has passed?

46. (a) At an air show a jet flies directly toward the stands at a speed of 1200 km/h, emitting a frequency of 3500 Hz, on a day when the speed of sound is 342 m/s. What frequency is received by the observers? (b) What frequency do they receive as the plane flies directly away from them?

47. What frequency is received by a mouse just before being dispatched by a hawk flying at it at 25.0 m/s and emitting a screech of frequency 3500 Hz? Take the speed of sound to be 331 m/s.

48. A spectator at a parade receives an 888-Hz tone from an oncoming trumpeter who is playing an 880-Hz note. At what speed is the musician approaching if the speed of sound is 338 m/s?

49. A commuter train blows its 200-Hz horn as it approaches a crossing. The speed of sound is 335 m/s. (a) An observer waiting at the crossing receives a frequency of 208 Hz. What is the speed of the train? (b) What frequency does the observer receive as the train moves away?

50. Can you perceive the shift in frequency produced when you pull a tuning fork toward you at 10.0 m/s on a day when the speed of sound is 344 m/s? To answer this question, calculate the factor by which the frequency shifts and see if it is greater than 0.300%.

51. Two eagles fly directly toward one another, the first at 15.0 m/s and the second at 20.0 m/s. Both screech, the first one emitting a frequency of 3200 Hz and the second one emitting a frequency of 3800 Hz. What frequencies do they receive if the speed of sound is 330 m/s?

52. What is the minimum speed at which a source must travel toward you for you to be able to hear that its frequency is Doppler shifted? That is, what speed produces a shift of 0.300% on a day when the speed of sound is 331 m/s?

17.5 Sound Interference and Resonance: Standing Waves in Air Columns

53. A "showy" custom-built car has two brass horns that are supposed to produce the same frequency but actually emit 263.8 and 264.5 Hz. What beat frequency is produced?

54. What beat frequencies will be present: (a) If the musical notes A and C are played together (frequencies of 220 and 264 Hz)? (b) If D and F are played together (frequencies of 297 and 352 Hz)? (c) If all four are played together?

55. What beat frequencies result if a piano hammer hits three strings that emit frequencies of 127.8, 128.1, and 128.3 Hz?

56. A piano tuner hears a beat every 2.00 s when listening to a 264.0-Hz tuning fork and a single piano string. What are the two possible frequencies of the string?

57. (a) What is the fundamental frequency of a 0.672-m-long tube, open at both ends, on a day when the speed of sound is 344 m/s? (b) What is the frequency of its second harmonic?

58. If a wind instrument, such as a tuba, has a fundamental frequency of 32.0 Hz, what are its first three overtones? It is closed at one end. (The overtones of a real tuba are more complex than this example, because it is a tapered tube.)

59. What are the first three overtones of a bassoon that has a fundamental frequency of 90.0 Hz? It is open at both ends. (The overtones of a real bassoon are more complex than this example, because its double reed makes it act more like a tube closed at one end.)

60. How long must a flute be in order to have a fundamental frequency of 262 Hz (this frequency corresponds to middle C on the evenly tempered chromatic scale) on a day when air temperature is 20.0°C ? It is open at both ends.

61. What length should an oboe have to produce a fundamental frequency of 110 Hz on a day when the speed of sound is 343 m/s? It is open at both ends.

62. What is the length of a tube that has a fundamental frequency of 176 Hz and a first overtone of 352 Hz if the speed of sound is 343 m/s?

63. (a) Find the length of an organ pipe closed at one end that produces a fundamental frequency of 256 Hz when air temperature is 18.0°C . (b) What is its fundamental frequency at 25.0°C ?

64. By what fraction will the frequencies produced by a wind instrument change when air temperature goes from 10.0°C to 30.0°C ? That is, find the ratio of the frequencies at those temperatures.

65. The ear canal resonates like a tube closed at one end. (See **Figure 17.39**.) If ear canals range in length from 1.80 to 2.60 cm in an average population, what is the range of fundamental resonant frequencies? Take air temperature to be 37.0°C , which is the same as body temperature. How does this result correlate with the intensity versus frequency graph (**Figure 17.37** of the human ear?

66. Calculate the first overtone in an ear canal, which resonates like a 2.40-cm-long tube closed at one end, by taking air temperature to be 37.0°C . Is the ear particularly sensitive to such a frequency? (The resonances of the ear canal are complicated by its nonuniform shape, which we shall ignore.)

67. A crude approximation of voice production is to consider the breathing passages and mouth to be a resonating tube closed at one end. (See **Figure 17.30**.) (a) What is the fundamental frequency if the tube is 0.240-m long, by taking air temperature to be 37.0°C ? (b) What would this frequency become if the person replaced the air with helium? Assume the same temperature dependence for helium as for air.

68. (a) Students in a physics lab are asked to find the length of an air column in a tube closed at one end that has a fundamental frequency of 256 Hz. They hold the tube vertically and fill it with water to the top, then lower the water while a 256-Hz tuning fork is rung and listen for the first resonance. What is the air temperature if the resonance occurs for a length of 0.336 m? (b) At what length will they observe the second resonance (first overtone)?

69. What frequencies will a 1.80-m-long tube produce in the audible range at 20.0°C if: (a) The tube is closed at one end? (b) It is open at both ends?

17.6 Hearing

70. The factor of 10^{-12} in the range of intensities to which the ear can respond, from threshold to that causing damage after brief exposure, is truly remarkable. If you could measure distances over the same range with a single instrument and the smallest distance you could measure was 1 mm, what would the largest be?

71. The frequencies to which the ear responds vary by a factor of 10^3 . Suppose the speedometer on your car measured speeds differing by the same factor of 10^3 , and the greatest speed it reads is 90.0 mi/h. What would be the slowest nonzero speed it could read?

72. What are the closest frequencies to 500 Hz that an average person can clearly distinguish as being different in frequency from 500 Hz? The sounds are not present simultaneously.

73. Can the average person tell that a 2002-Hz sound has a different frequency than a 1999-Hz sound without playing them simultaneously?

74. If your radio is producing an average sound intensity level of 85 dB, what is the next lowest sound intensity level that is clearly less intense?

75. Can you tell that your roommate turned up the sound on the TV if its average sound intensity level goes from 70 to 73 dB?

76. Based on the graph in **Figure 17.36**, what is the threshold of hearing in decibels for frequencies of 60, 400, 1000, 4000, and 15,000 Hz? Note that many AC electrical appliances produce 60 Hz, music is commonly 400 Hz, a reference frequency is 1000 Hz, your maximum sensitivity is near 4000 Hz, and many older TVs produce a 15,750 Hz whine.

77. What sound intensity levels must sounds of frequencies 60, 3000, and 8000 Hz have in order to have the same loudness as a 40-dB sound of frequency 1000 Hz (that is, to have a loudness of 40 phons)?

78. What is the approximate sound intensity level in decibels of a 600-Hz tone if it has a loudness of 20 phons? If it has a loudness of 70 phons?

79. (a) What are the loudnesses in phons of sounds having frequencies of 200, 1000, 5000, and 10,000 Hz, if they are all at the same 60.0-dB sound intensity level? (b) If they are all at 110 dB? (c) If they are all at 20.0 dB?

80. Suppose a person has a 50-dB hearing loss at all frequencies. By how many factors of 10 will low-intensity sounds need to be amplified to seem normal to this person? Note that smaller amplification is appropriate for more intense sounds to avoid further hearing damage.

81. If a woman needs an amplification of 5.0×10^{12} times the threshold intensity to enable her to hear at all frequencies, what is her overall hearing loss in dB? Note that smaller amplification is appropriate for more intense sounds to avoid further damage to her hearing from levels above 90 dB.

82. (a) What is the intensity in watts per meter squared of a just barely audible 200-Hz sound? (b) What is the intensity in watts per meter squared of a barely audible 4000-Hz sound?

83. (a) Find the intensity in watts per meter squared of a 60.0-Hz sound having a loudness of 60 phons. (b) Find the intensity in watts per meter squared of a 10,000-Hz sound having a loudness of 60 phons.

84. A person has a hearing threshold 10 dB above normal at 100 Hz and 50 dB above normal at 4000 Hz. How much more intense must a 100-Hz tone be than a 4000-Hz tone if they are both barely audible to this person?

85. A child has a hearing loss of 60 dB near 5000 Hz, due to noise exposure, and normal hearing elsewhere. How much more intense is a 5000-Hz tone than a 400-Hz tone if they are both barely audible to the child?

86. What is the ratio of intensities of two sounds of identical frequency if the first is just barely discernible as louder to a person than the second?

17.7 Ultrasound

Unless otherwise indicated, for problems in this section, assume that the speed of sound through human tissues is 1540 m/s.

87. What is the sound intensity level in decibels of ultrasound of intensity $10^5\ \text{W/m}^2$, used to pulverize tissue during surgery?

88. Is 155-dB ultrasound in the range of intensities used for deep heating? Calculate the intensity of this ultrasound and compare this intensity with values quoted in the text.

89. Find the sound intensity level in decibels of $2.00\times10^{-2}\ \text{W/m}^2$ ultrasound used in medical diagnostics.

90. The time delay between transmission and the arrival of the reflected wave of a signal using ultrasound traveling through a piece of fat tissue was 0.13 ms. At what depth did this reflection occur?

91. In the clinical use of ultrasound, transducers are always coupled to the skin by a thin layer of gel or oil, replacing the air that would otherwise exist between the transducer and the skin. (a) Using the values of acoustic impedance given in **Table 17.5** calculate the intensity reflection coefficient between transducer material and air. (b) Calculate the intensity reflection coefficient between transducer material and gel (assuming for this problem that its acoustic impedance is identical to that of water). (c) Based on the results of your calculations, explain why the gel is used.

92. (a) Calculate the minimum frequency of ultrasound that will allow you to see details as small as 0.250 mm in human tissue. (b) What is the effective depth to which this sound is effective as a diagnostic probe?

93. (a) Find the size of the smallest detail observable in human tissue with 20.0-MHz ultrasound. (b) Is its effective penetration depth great enough to examine the entire eye (about 3.00 cm is needed)? (c) What is the wavelength of such ultrasound in 0°C air?

94. (a) Echo times are measured by diagnostic ultrasound scanners to determine distances to reflecting surfaces in a patient. What is the difference in echo times for tissues that are 3.50 and 3.60 cm beneath the surface? (This difference is the minimum resolving time for the scanner to see details as small as 0.100 cm, or 1.00 mm. Discrimination of smaller time differences is needed to see smaller details.) (b) Discuss whether the period T of this ultrasound must be smaller than the minimum time resolution. If so, what is the minimum frequency of the ultrasound and is that out of the normal range for diagnostic ultrasound?

95. (a) How far apart are two layers of tissue that produce echoes having round-trip times (used to measure distances) that differ by $0.750\ \mu\text{s}$?

(b) What minimum frequency must the ultrasound have to see detail this small?

96. (a) A bat uses ultrasound to find its way among trees. If this bat can detect echoes 1.00 ms apart, what minimum distance between objects can it detect? (b) Could this distance explain the difficulty that bats have finding an open door when they accidentally get into a house?

97. A dolphin is able to tell in the dark that the ultrasound echoes received from two sharks come from two different objects only if the sharks are separated by 3.50 m, one being that much farther away than the other. (a) If the ultrasound has a frequency of 100 kHz, show this ability is not limited by its wavelength. (b) If this ability is due to the dolphin's ability to detect the arrival times of echoes, what is the minimum time difference the dolphin can perceive?

98. A diagnostic ultrasound echo is reflected from moving blood and returns with a frequency 500 Hz higher than its original 2.00 MHz. What is the velocity of the blood? (Assume that the frequency of 2.00 MHz is accurate to seven significant figures and 500 Hz is accurate to three significant figures.)

99. Ultrasound reflected from an oncoming bloodstream that is moving at 30.0 cm/s is mixed with the original frequency of 2.50 MHz to produce beats. What is the beat frequency? (Assume that the frequency of 2.50 MHz is accurate to seven significant figures.)

18 ELECTRIC CHARGE AND ELECTRIC FIELD

Figure 18.1 Static electricity from this plastic slide causes the child's hair to stand on end. The sliding motion stripped electrons away from the child's body, leaving an excess of positive charges, which repel each other along each strand of hair. (credit: Ken Bosma/Wikimedia Commons)

Learning Objectives

Introduction to Electric Charge and Electric Field

The image of American politician and scientist Benjamin Franklin (1706–1790) flying a kite in a thunderstorm is familiar to every schoolchild. (See **Figure 18.2**.) In this experiment, Franklin demonstrated a connection between lightning and **static electricity**. Sparks were drawn from a key hung on a kite string during an electrical storm. These sparks were like those produced by static electricity, such as the spark that jumps from your finger to a metal doorknob after you walk across a wool carpet. What Franklin demonstrated in his dangerous experiment was a connection between phenomena on two different scales: one the grand power of an electrical storm, the other an effect of more human proportions. Connections like this one reveal the underlying unity of the laws of nature, an aspect we humans find particularly appealing.

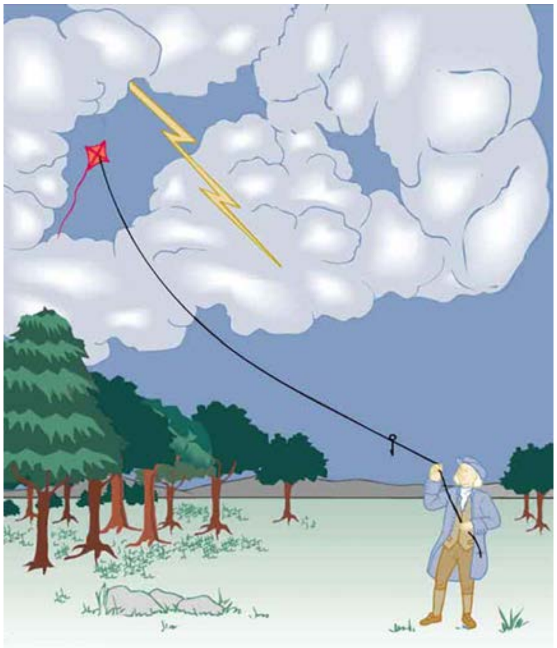


Figure 18.2 When Benjamin Franklin demonstrated that lightning was related to static electricity, he made a connection that is now part of the evidence that all directly experienced forces except the gravitational force are manifestations of the electromagnetic force.

Much has been written about Franklin. His experiments were only part of the life of a man who was a scientist, inventor, revolutionary, statesman, and writer. Franklin's experiments were not performed in isolation, nor were they the only ones to reveal connections.

For example, the Italian scientist Luigi Galvani (1737–1798) performed a series of experiments in which static electricity was used to stimulate contractions of leg muscles of dead frogs, an effect already known in humans subjected to static discharges. But Galvani also found that if he joined two metal wires (say copper and zinc) end to end and touched the other ends to muscles, he produced the same effect in frogs as static discharge. Alessandro Volta (1745–1827), partly inspired by Galvani's work, experimented with various combinations of metals and developed the battery.

During the same era, other scientists made progress in discovering fundamental connections. The periodic table was developed as the systematic properties of the elements were discovered. This influenced the development and refinement of the concept of atoms as the basis of matter. Such submicroscopic descriptions of matter also help explain a great deal more.

Atomic and molecular interactions, such as the forces of friction, cohesion, and adhesion, are now known to be manifestations of the **electromagnetic force**. Static electricity is just one aspect of the electromagnetic force, which also includes moving electricity and magnetism.

All the macroscopic forces that we experience directly, such as the sensations of touch and the tension in a rope, are due to the electromagnetic force, one of the four fundamental forces in nature. The gravitational force, another fundamental force, is actually sensed through the electromagnetic interaction of molecules, such as between those in our feet and those on the top of a bathroom scale. (The other two fundamental forces, the strong nuclear force and the weak nuclear force, cannot be sensed on the human scale.)

This chapter begins the study of electromagnetic phenomena at a fundamental level. The next several chapters will cover static electricity, moving electricity, and magnetism—collectively known as electromagnetism. In this chapter, we begin with the study of electric phenomena due to charges that are at least temporarily stationary, called electrostatics, or static electricity.

18.1 Static Electricity and Charge: Conservation of Charge

Figure 18.3 Borneo amber was mined in Sabah, Malaysia, from shale-sandstone-mudstone veins. When a piece of amber is rubbed with a piece of silk, the amber gains more electrons, giving it a net negative charge. At the same time, the silk, having lost electrons, becomes positively charged. (credit: Sebakoamber, Wikimedia Commons)

What makes plastic wrap cling? Static electricity. Not only are applications of static electricity common these days, its existence has been known since ancient times. The first record of its effects dates to ancient Greeks who noted more than 500 years B.C. that polishing amber temporarily enabled it to attract bits of straw (see **Figure 18.3**). The very word *electric* derives from the Greek word for amber (*electron*).

Many of the characteristics of static electricity can be explored by rubbing things together. Rubbing creates the spark you get from walking across a wool carpet, for example. Static cling generated in a clothes dryer and the attraction of straw to recently polished amber also result from rubbing. Similarly, lightning results from air movements under certain weather conditions. You can also rub a balloon on your hair, and the static electricity created can then make the balloon cling to a wall. We also have to be cautious of static electricity, especially in dry climates. When we pump gasoline, we are warned to discharge ourselves (after sliding across the seat) on a metal surface before grabbing the gas nozzle. Attendants in hospital operating rooms must wear booties with aluminum foil on the bottoms to avoid creating sparks which may ignite the oxygen being used.

Some of the most basic characteristics of static electricity include:

- The effects of static electricity are explained by a physical quantity not previously introduced, called electric charge.
- There are only two types of charge, one called positive and the other called negative.
- Like charges repel, whereas unlike charges attract.
- The force between charges decreases with distance.

How do we know there are two types of **electric charge**? When various materials are rubbed together in controlled ways, certain combinations of materials always produce one type of charge on one material and the opposite type on the other. By convention, we call one type of charge “positive”, and the other type “negative.” For example, when glass is rubbed with silk, the glass becomes positively charged and the silk negatively charged. Since the glass and silk have opposite charges, they attract one another like clothes that have rubbed together in a dryer. Two glass rods rubbed with silk in this manner will repel one another, since each rod has positive charge on it. Similarly, two silk cloths so rubbed will repel, since both cloths have negative charge. **Figure 18.4** shows how these simple materials can be used to explore the nature of the force between charges.

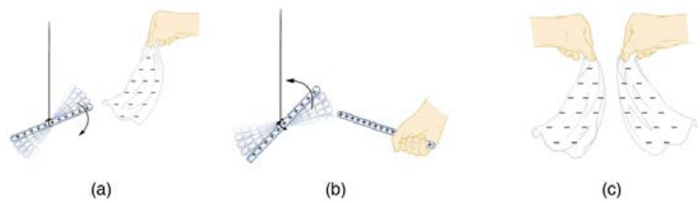


Figure 18.4 A glass rod becomes positively charged when rubbed with silk, while the silk becomes negatively charged. (a) The glass rod is attracted to the silk because their charges are opposite. (b) Two similarly charged glass rods repel. (c) Two similarly charged silk cloths repel.

More sophisticated questions arise. Where do these charges come from? Can you create or destroy charge? Is there a smallest unit of charge? Exactly how does the force depend on the amount of charge and the distance between charges? Such questions obviously occurred to Benjamin Franklin and other early researchers, and they interest us even today.

Charge Carried by Electrons and Protons

Franklin wrote in his letters and books that he could see the effects of electric charge but did not understand what caused the phenomenon. Today we have the advantage of knowing that normal matter is made of atoms, and that atoms contain positive and negative charges, usually in equal amounts.

Figure 18.5 shows a simple model of an atom with negative **electrons** orbiting its positive nucleus. The nucleus is positive due to the presence of positively charged **protons**. Nearly all charge in nature is due to electrons and protons, which are two of the three building blocks of most matter. (The third is the neutron, which is neutral, carrying no charge.) Other charge-carrying particles are observed in cosmic rays and nuclear decay, and are created in particle accelerators. All but the electron and proton survive only a short time and are quite rare by comparison.

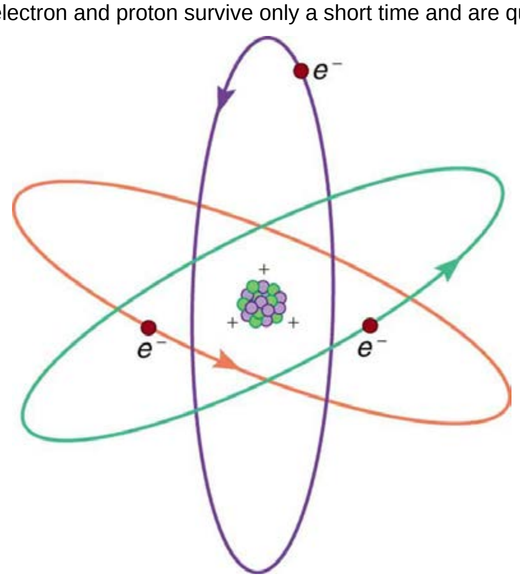


Figure 18.5 This simplified (and not to scale) view of an atom is called the planetary model of the atom. Negative electrons orbit a much heavier positive nucleus, as the planets orbit the much heavier sun. There the similarity ends, because forces in the atom are electromagnetic, whereas those in the planetary system are gravitational. Normal macroscopic amounts of matter contain immense numbers of atoms and molecules and, hence, even greater numbers of individual negative and positive charges.

The charges of electrons and protons are identical in magnitude but opposite in sign. Furthermore, all charged objects in nature are integral multiples of this basic quantity of charge, meaning that all charges are made of combinations of a basic unit of charge. Usually, charges are formed by combinations of electrons and protons. The magnitude of this basic charge is

$$|q_e| = 1.60 \times 10^{-19}\ \text{C}. \tag{18.1}$$

The symbol q is commonly used for charge and the subscript e indicates the charge of a single electron (or proton).

The SI unit of charge is the coulomb (C). The number of protons needed to make a charge of 1.00 C is

$$1.00\ \text{C} \times \frac{1\ \text{proton}}{1.60 \times 10^{-19}\ \text{C}} = 6.25 \times 10^{18}\ \text{protons}. \tag{18.2}$$

Similarly, 6.25×10^{18} electrons have a combined charge of −1.00 coulomb. Just as there is a smallest bit of an element (an atom), there is a smallest bit of charge. There is no directly observed charge smaller than $|q_e|$ (see **Things Great and Small: The Submicroscopic Origin of Charge**), and all observed charges are integral multiples of $|q_e|$.

Things Great and Small: The Submicroscopic Origin of Charge

With the exception of exotic, short-lived particles, all charge in nature is carried by electrons and protons. Electrons carry the charge we have named negative. Protons carry an equal-magnitude charge that we call positive. (See **Figure 18.6**.) Electron and proton charges are considered fundamental building blocks, since all other charges are integral multiples of those carried by electrons and protons. Electrons and protons are also two of the three fundamental building blocks of ordinary matter. The neutron is the third and has zero total charge.

Figure 18.6 shows a person touching a Van de Graaff generator and receiving excess positive charge. The expanded view of a hair shows the existence of both types of charges but an excess of positive. The repulsion of these positive like charges causes the strands of hair to repel other strands of hair and to stand up. The further blowup shows an artist's conception of an electron and a proton perhaps found in an atom in a strand of hair.

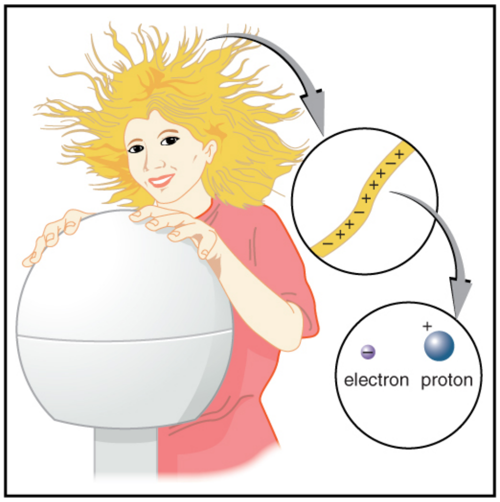


Figure 18.6 When this person touches a Van de Graaff generator, she receives an excess of positive charge, causing her hair to stand on end. The charges in one hair are shown. An artist's conception of an electron and a proton illustrate the particles carrying the negative and positive charges. We cannot really see these particles with visible light because they are so small (the electron seems to be an infinitesimal point), but we know a great deal about their measurable properties, such as the charges they carry.

The electron seems to have no substructure; in contrast, when the substructure of protons is explored by scattering extremely energetic electrons from them, it appears that there are point-like particles inside the proton. These sub-particles, named quarks, have never been directly observed, but they are believed to carry fractional charges as seen in **Figure 18.7**. Charges on electrons and protons and all other directly observable particles are unitary, but these quark substructures carry charges of either $-\frac{1}{3}$ or $+\frac{2}{3}$. There are continuing attempts to observe fractional charge directly and to learn of the properties of quarks, which are perhaps the ultimate substructure of matter.

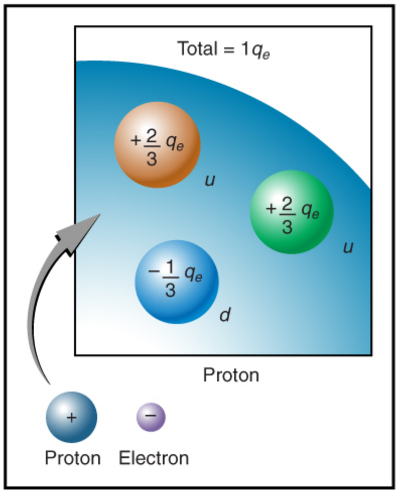


Figure 18.7 Artist's conception of fractional quark charges inside a proton. A group of three quark charges add up to the single positive charge on the proton: $-\frac{1}{3}q_e + \frac{2}{3}q_e + \frac{2}{3}q_e = +1q_e$.

Separation of Charge in Atoms

Charges in atoms and molecules can be separated—for example, by rubbing materials together. Some atoms and molecules have a greater affinity for electrons than others and will become negatively charged by close contact in rubbing, leaving the other material positively charged. (See **Figure 18.8**.) Positive charge can similarly be induced by rubbing. Methods other than rubbing can also separate charges. Batteries, for example, use combinations of substances that interact in such a way as to separate charges. Chemical interactions may transfer negative charge from one substance to the other, making one battery terminal negative and leaving the first one positive.

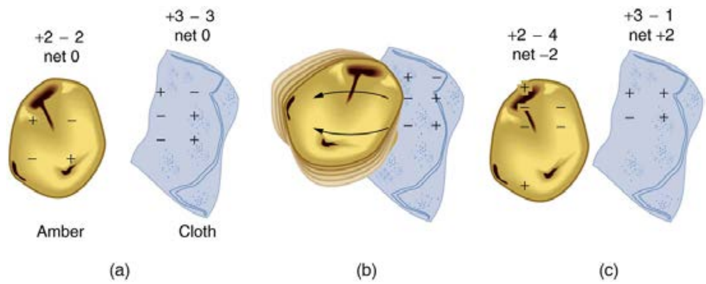


Figure 18.8 When materials are rubbed together, charges can be separated, particularly if one material has a greater affinity for electrons than another. (a) Both the amber and cloth are originally neutral, with equal positive and negative charges. Only a tiny fraction of the charges are involved, and only a few of them are shown here. (b) When rubbed together, some negative charge is transferred to the amber, leaving the cloth with a net positive charge. (c) When separated, the amber and cloth now have net charges, but the absolute value of the net positive and negative charges will be equal.

No charge is actually created or destroyed when charges are separated as we have been discussing. Rather, existing charges are moved about. In fact, in all situations the total amount of charge is always constant. This universally obeyed law of nature is called the **law of conservation of charge**.

Law of Conservation of Charge

Total charge is constant in any process.

In more exotic situations, such as in particle accelerators, mass, Δm, can be created from energy in the amount $\Delta m = \frac{E}{c^2}$. Sometimes, the created mass is charged, such as when an electron is created. Whenever a charged particle is created, another having an opposite charge is always created along with it, so that the total charge created is zero. Usually, the two particles are "matter-antimatter" counterparts. For example, an antielectron would usually be created at the same time as an electron. The antielectron has a positive charge (it is called a positron), and so the total charge created is zero. (See **Figure 18.9**.) All particles have antimatter counterparts with opposite signs. When matter and antimatter counterparts are brought together, they completely annihilate one another. By annihilate, we mean that the mass of the two particles is converted to energy E, again obeying the relationship $\Delta m = \frac{E}{c^2}$. Since the two particles have equal and opposite charge, the total charge is zero before and after the annihilation; thus, total charge is conserved.

Making Connections: Conservation Laws

Only a limited number of physical quantities are universally conserved. Charge is one—energy, momentum, and angular momentum are others. Because they are conserved, these physical quantities are used to explain more phenomena and form more connections than other, less basic quantities. We find that conserved quantities give us great insight into the rules followed by nature and hints to the organization of nature.

Discoveries of conservation laws have led to further discoveries, such as the weak nuclear force and the quark substructure of protons and other particles.

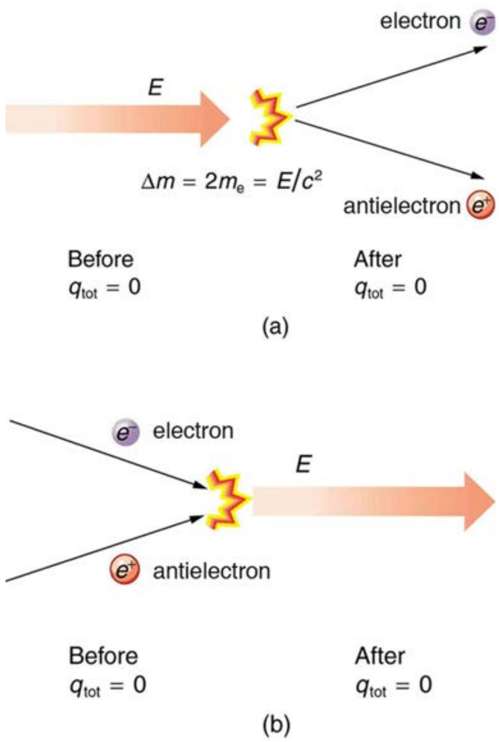


Figure 18.9 (a) When enough energy is present, it can be converted into matter. Here the matter created is an electron–antielectron pair. (m_e is the electron's mass.) The total charge before and after this event is zero. (b) When matter and antimatter collide, they annihilate each other; the total charge is conserved at zero before and after the annihilation.

The law of conservation of charge is absolute—it has never been observed to be violated. Charge, then, is a special physical quantity, joining a very short list of other quantities in nature that are always conserved. Other conserved quantities include energy, momentum, and angular momentum.

PhET Explorations: Balloons and Static Electricity

Why does a balloon stick to your sweater? Rub a balloon on a sweater, then let go of the balloon and it flies over and sticks to the sweater. View the charges in the sweater, balloons, and the wall.

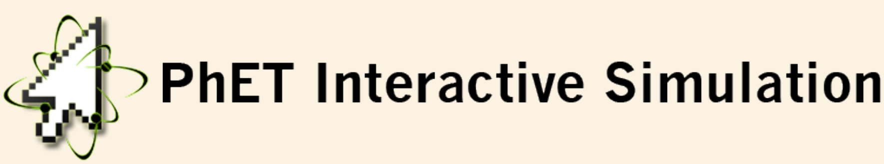


Figure 18.10 Balloons and Static Electricity (http://cnx.org/content/m42300/1.5/balloons_en.jar)

18.2 Conductors and Insulators

Figure 18.11 This power adapter uses metal wires and connectors to conduct electricity from the wall socket to a laptop computer. The conducting wires allow electrons to move freely through the cables, which are shielded by rubber and plastic. These materials act as insulators that don't allow electric charge to escape outward. (credit: Evan-Amos, Wikimedia Commons)

Some substances, such as metals and salty water, allow charges to move through them with relative ease. Some of the electrons in metals and similar conductors are not bound to individual atoms or sites in the material. These **free electrons** can move through the material much as air moves through loose sand. Any substance that has free electrons and allows charge to move relatively freely through it is called a **conductor**. The moving electrons may collide with fixed atoms and molecules, losing some energy, but they can move in a conductor. Superconductors allow the movement of charge without any loss of energy. Salty water and other similar conducting materials contain free ions that can move through them. An ion is an atom or molecule having a positive or negative (nonzero) total charge. In other words, the total number of electrons is not equal to the total number of protons.

Other substances, such as glass, do not allow charges to move through them. These are called **insulators**. Electrons and ions in insulators are bound in the structure and cannot move easily—as much as 10^{23} times more slowly than in conductors. Pure water and dry table salt are insulators, for example, whereas molten salt and salty water are conductors.

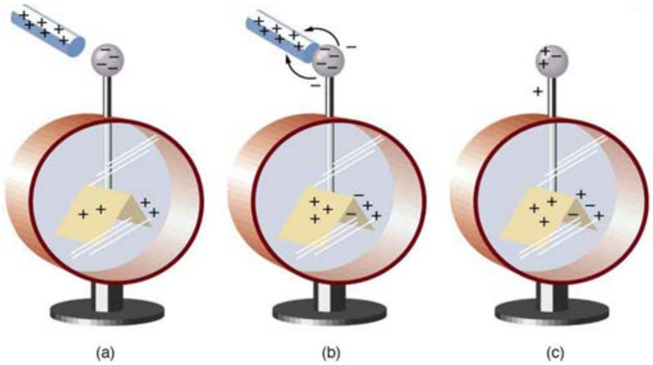


Figure 18.12 An electroscope is a favorite instrument in physics demonstrations and student laboratories. It is typically made with gold foil leaves hung from a (conducting) metal stem and is insulated from the room air in a glass-walled container. (a) A positively charged glass rod is brought near the tip of the electroscope, attracting electrons to the top and leaving a net positive charge on the leaves. Like charges in the light flexible gold leaves repel, separating them. (b) When the rod is touched against the ball, electrons are attracted and transferred, reducing the net charge on the glass rod but leaving the electroscope positively charged. (c) The excess charges are evenly distributed in the stem and leaves of the electroscope once the glass rod is removed.

Charging by Contact

Figure 18.12 shows an electroscope being charged by touching it with a positively charged glass rod. Because the glass rod is an insulator, it must actually touch the electroscope to transfer charge to or from it. (Note that the extra positive charges reside on the surface of the glass rod as a result of rubbing it with silk before starting the experiment.) Since only electrons move in metals, we see that they are attracted to the top of the electroscope. There, some are transferred to the positive rod by touch, leaving the electroscope with a net positive charge.

Electrostatic repulsion in the leaves of the charged electroscope separates them. The electrostatic force has a horizontal component that results in the leaves moving apart as well as a vertical component that is balanced by the gravitational force. Similarly, the electroscope can be negatively charged by contact with a negatively charged object.

Charging by Induction

It is not necessary to transfer excess charge directly to an object in order to charge it. **Figure 18.13** shows a method of **induction** wherein a charge is created in a nearby object, without direct contact. Here we see two neutral metal spheres in contact with one another but insulated from the rest of the world. A positively charged rod is brought near one of them, attracting negative charge to that side, leaving the other sphere positively charged.

This is an example of induced **polarization** of neutral objects. Polarization is the separation of charges in an object that remains neutral. If the spheres are now separated (before the rod is pulled away), each sphere will have a net charge. Note that the object closest to the charged rod receives an opposite charge when charged by induction. Note also that no charge is removed from the charged rod, so that this process can be repeated without depleting the supply of excess charge.

Another method of charging by induction is shown in **Figure 18.14**. The neutral metal sphere is polarized when a charged rod is brought near it. The sphere is then grounded, meaning that a conducting wire is run from the sphere to the ground. Since the earth is large and most ground is a good conductor, it can supply or accept excess charge easily. In this case, electrons are attracted to the sphere through a wire called the ground wire, because it supplies a conducting path to the ground. The ground connection is broken before the charged rod is removed, leaving the sphere with an excess charge opposite to that of the rod. Again, an opposite charge is achieved when charging by induction and the charged rod loses none of its excess charge.

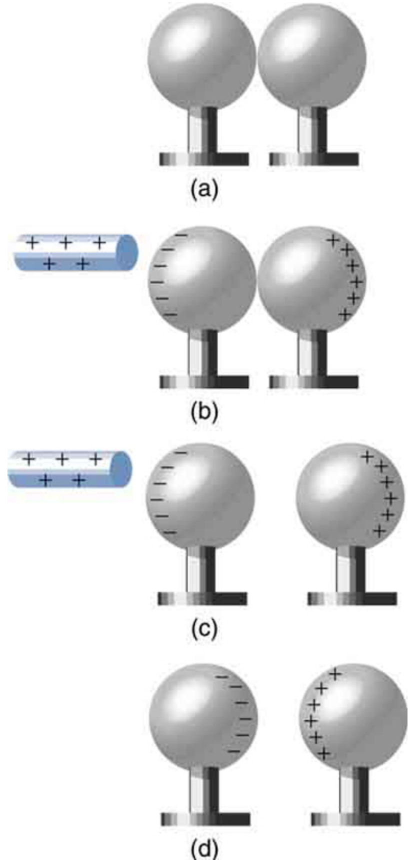


Figure 18.13 Charging by induction. (a) Two uncharged or neutral metal spheres are in contact with each other but insulated from the rest of the world. (b) A positively charged glass rod is brought near the sphere on the left, attracting negative charge and leaving the other sphere positively charged. (c) The spheres are separated before the rod is removed, thus separating negative and positive charge. (d) The spheres retain net charges after the inducing rod is removed—without ever having been touched by a charged object.

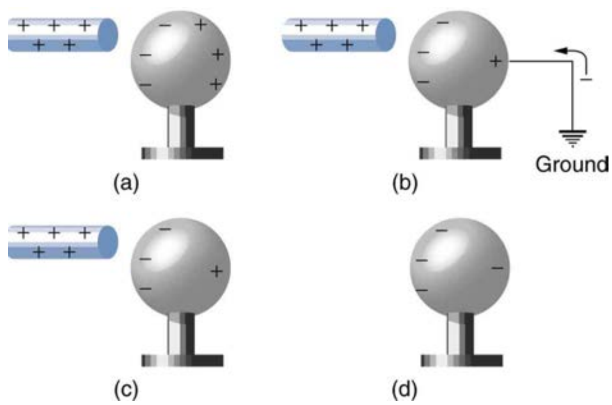


Figure 18.14 Charging by induction, using a ground connection. (a) A positively charged rod is brought near a neutral metal sphere, polarizing it. (b) The sphere is grounded, allowing electrons to be attracted from the earth's ample supply. (c) The ground connection is broken. (d) The positive rod is removed, leaving the sphere with an induced negative charge.

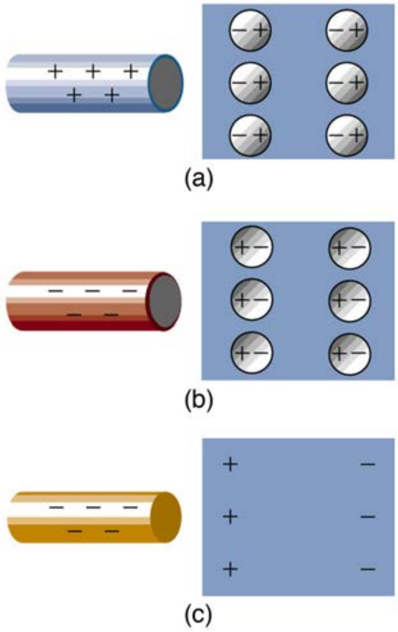


Figure 18.15 Both positive and negative objects attract a neutral object by polarizing its molecules. (a) A positive object brought near a neutral insulator polarizes its molecules. There is a slight shift in the distribution of the electrons orbiting the molecule, with unlike charges being brought nearer and like charges moved away. Since the electrostatic force decreases with distance, there is a net attraction. (b) A negative object produces the opposite polarization, but again attracts the neutral object. (c) The same effect occurs for a conductor; since the unlike charges are closer, there is a net attraction.

Neutral objects can be attracted to any charged object. The pieces of straw attracted to polished amber are neutral, for example. If you run a plastic comb through your hair, the charged comb can pick up neutral pieces of paper. **Figure 18.15** shows how the polarization of atoms and molecules in neutral objects results in their attraction to a charged object.

When a charged rod is brought near a neutral substance, an insulator in this case, the distribution of charge in atoms and molecules is shifted slightly. Opposite charge is attracted nearer the external charged rod, while like charge is repelled. Since the electrostatic force decreases with distance, the repulsion of like charges is weaker than the attraction of unlike charges, and so there is a net attraction. Thus a positively charged glass rod attracts neutral pieces of paper, as will a negatively charged rubber rod. Some molecules, like water, are polar molecules. Polar molecules have a natural or inherent separation of charge, although they are neutral overall. Polar molecules are particularly affected by other charged objects and show greater polarization effects than molecules with naturally uniform charge distributions.

Check Your Understanding

Can you explain the attraction of water to the charged rod in the figure below?

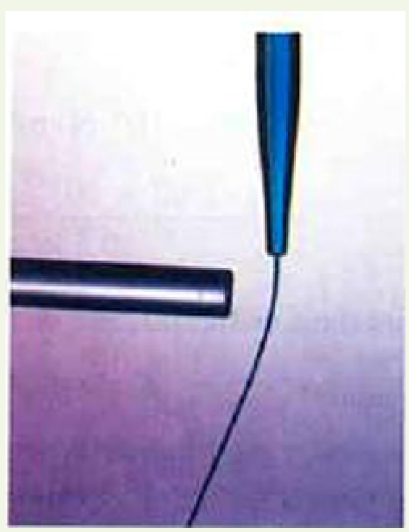


Figure 18.16

Solution

Water molecules are polarized, giving them slightly positive and slightly negative sides. This makes water even more susceptible to a charged rod’s attraction. As the water flows downward, due to the force of gravity, the charged conductor exerts a net attraction to the opposite charges in the stream of water, pulling it closer.

PhET Explorations: John Travoltage

Make sparks fly with John Travoltage. Wiggle Johnnie's foot and he picks up charges from the carpet. Bring his hand close to the door knob and get rid of the excess charge.

PhET Interactive Simulation

Figure 18.17 **John Travoltage (http://cnx.org/content/m42306/1.4/travoltage_en.jar)**

18.3 Coulomb's Law

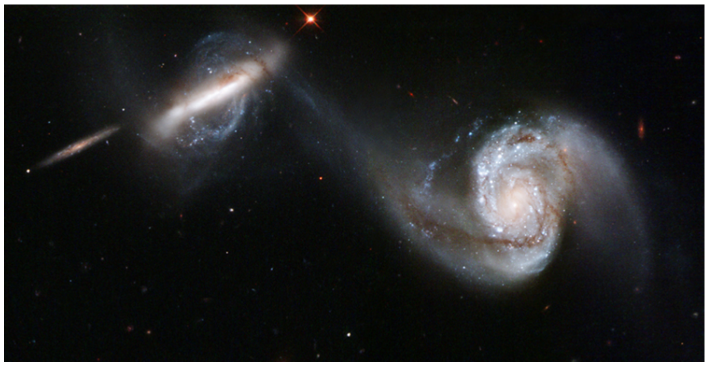


Figure 18.18 This NASA image of Arp 87 shows the result of a strong gravitational attraction between two galaxies. In contrast, at the subatomic level, the electrostatic attraction between two objects, such as an electron and a proton, is far greater than their mutual attraction due to gravity. (credit: NASA/HST)

Through the work of scientists in the late 18th century, the main features of the **electrostatic force**—the existence of two types of charge, the observation that like charges repel, unlike charges attract, and the decrease of force with distance—were eventually refined, and expressed as a mathematical formula. The mathematical formula for the electrostatic force is called **Coulomb's law** after the French physicist Charles Coulomb (1736–1806), who performed experiments and first proposed a formula to calculate it.

Coulomb's Law

$$F = k\frac{|q_1 q_2|}{r^2}. \tag{18.3}$$

Coulomb's law calculates the magnitude of the force F between two point charges, q_1 and q_2, separated by a distance r. In SI units, the constant k is equal to

$$k = 8.988\times10^9\frac{\text{N}\cdot\text{m}^2}{\text{C}^2} \approx 8.99\times10^9\frac{\text{N}\cdot\text{m}^2}{\text{C}^2}. \tag{18.4}$$

The electrostatic force is a vector quantity and is expressed in units of newtons. The force is understood to be along the line joining the two charges. (See **Figure 18.19**.)

Although the formula for Coulomb's law is simple, it was no mean task to prove it. The experiments Coulomb did, with the primitive equipment then available, were difficult. Modern experiments have verified Coulomb's law to great precision. For example, it has been shown that the force is inversely proportional to distance between two objects squared $\left(F \propto 1/r^2\right)$ to an accuracy of 1 part in 10^{16}. No exceptions have ever been found, even at the small distances within the atom.

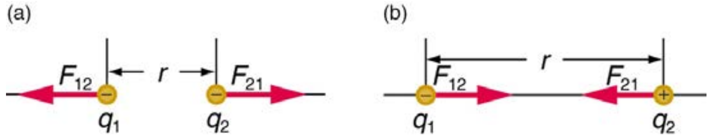


Figure 18.19 The magnitude of the electrostatic force F between point charges q_1 and q_2 separated by a distance r is given by Coulomb's law. Note that Newton's third law (every force exerted creates an equal and opposite force) applies as usual—the force on q_1 is equal in magnitude and opposite in direction to the force it exerts on q_2. (a) Like charges. (b) Unlike charges.

Example 18.1 How Strong is the Coulomb Force Relative to the Gravitational Force?

Compare the electrostatic force between an electron and proton separated by 0.530×10^{-10} m with the gravitational force between them. This distance is their average separation in a hydrogen atom.

Strategy

To compare the two forces, we first compute the electrostatic force using Coulomb's law, $F = k\frac{|q_1 q_2|}{r^2}$. We then calculate the gravitational force using Newton's universal law of gravitation. Finally, we take a ratio to see how the forces compare in magnitude.

Solution

Entering the given and known information about the charges and separation of the electron and proton into the expression of Coulomb's law yields

$$F = k\frac{|q_1 q_2|}{r^2} \quad (18.5)$$

$$= \left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\times\frac{(1.60\times10^{-19}\ \text{C})(1.60\times10^{-19}\ \text{C})}{(0.530\times10^{-10}\ \text{m})^2} \quad (18.6)$$

Thus the Coulomb force is

$$F = 8.19\times10^{-8}\ \text{N}. \quad (18.7)$$

The charges are opposite in sign, so this is an attractive force. This is a very large force for an electron—it would cause an acceleration of $8.99\times10^{22}\ \text{m}/\text{s}^2$ (verification is left as an end-of-section problem).The gravitational force is given by Newton's law of gravitation as:

$$F_G = G\frac{mM}{r^2}, \quad (18.8)$$

where $G = 6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$. Here m and M represent the electron and proton masses, which can be found in the appendices. Entering values for the knowns yields

$$F_G = (6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\times\frac{(9.11\times10^{-31}\ \text{kg})(1.67\times10^{-27}\ \text{kg})}{(0.530\times10^{-10}\ \text{m})^2} = 3.61\times10^{-47}\ \text{N} \quad (18.9)$$

This is also an attractive force, although it is traditionally shown as positive since gravitational force is always attractive. The ratio of the magnitude of the electrostatic force to gravitational force in this case is, thus,

$$\frac{F}{F_G} = 2.27\times10^{39}. \quad (18.10)$$

Discussion

This is a remarkably large ratio! Note that this will be the ratio of electrostatic force to gravitational force for an electron and a proton at any distance (taking the ratio before entering numerical values shows that the distance cancels). This ratio gives some indication of just how much larger the Coulomb force is than the gravitational force between two of the most common particles in nature.

As the example implies, gravitational force is completely negligible on a small scale, where the interactions of individual charged particles are important. On a large scale, such as between the Earth and a person, the reverse is true. Most objects are nearly electrically neutral, and so attractive and repulsive **Coulomb forces** nearly cancel. Gravitational force on a large scale dominates interactions between large objects because it is always attractive, while Coulomb forces tend to cancel.

18.4 Electric Field: Concept of a Field Revisited

Contact forces, such as between a baseball and a bat, are explained on the small scale by the interaction of the charges in atoms and molecules in close proximity. They interact through forces that include the **Coulomb force**. Action at a distance is a force between objects that are not close enough for their atoms to “touch.” That is, they are separated by more than a few atomic diameters.

For example, a charged rubber comb attracts neutral bits of paper from a distance via the Coulomb force. It is very useful to think of an object being surrounded in space by a **force field**. The force field carries the force to another object (called a test object) some distance away.

Concept of a Field

A field is a way of conceptualizing and mapping the force that surrounds any object and acts on another object at a distance without apparent physical connection. For example, the gravitational field surrounding the earth (and all other masses) represents the gravitational force that would be experienced if another mass were placed at a given point within the field.

In the same way, the Coulomb force field surrounding any charge extends throughout space. Using Coulomb's law, $F = k|q_1 q_2| / r^2$, its magnitude is given by the equation $F = k|qQ| / r^2$, for a **point charge** (a particle having a charge Q) acting on a **test charge** q at a distance r (see **Figure 18.20**). Both the magnitude and direction of the Coulomb force field depend on Q and the test charge q.

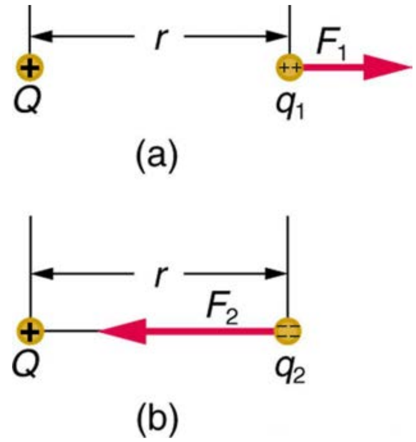


Figure 18.20 The Coulomb force field due to a positive charge Q is shown acting on two different charges. Both charges are the same distance from Q. (a) Since q_1 is positive, the force F_1 acting on it is repulsive. (b) The charge q_2 is negative and greater in magnitude than q_1, and so the force F_2 acting on it is attractive and stronger than F_1. The Coulomb force field is thus not unique at any point in space, because it depends on the test charges q_1 and q_2 as well as the charge Q.

To simplify things, we would prefer to have a field that depends only on Q and not on the test charge q. The electric field is defined in such a manner that it represents only the charge creating it and is unique at every point in space. Specifically, the electric field E is defined to be the ratio of the Coulomb force to the test charge:

$$\mathbf{E} = \frac{\mathbf{F}}{q}, \tag{18.11}$$

where $\mathbf{F}$ is the electrostatic force (or Coulomb force) exerted on a positive test charge q. It is understood that $\mathbf{E}$ is in the same direction as $\mathbf{F}$. It is also assumed that q is so small that it does not alter the charge distribution creating the electric field. The units of electric field are newtons per coulomb (N/C). If the electric field is known, then the electrostatic force on any charge q is simply obtained by multiplying charge times electric field, or $\mathbf{F} = q\mathbf{E}$. Consider the electric field due to a point charge Q. According to Coulomb's law, the force it exerts on a test charge q is $F = k|qQ| / r^2$. Thus the magnitude of the electric field, E, for a point charge is

$$E = \left|\frac{F}{q}\right| = k\left|\frac{qQ}{qr^2}\right| = k\frac{|Q|}{r^2}. \tag{18.12}$$

Since the test charge cancels, we see that

$$E = k\frac{|Q|}{r^2}. \tag{18.13}$$

The electric field is thus seen to depend only on the charge Q and the distance r; it is completely independent of the test charge q.

Example 18.2 Calculating the Electric Field of a Point Charge

Calculate the strength and direction of the electric field E due to a point charge of 2.00 nC (nano-Coulombs) at a distance of 5.00 mm from the charge.

Strategy

We can find the electric field created by a point charge by using the equation $E = kQ / r^2$.

Solution

Here $Q = 2.00\times10^{-9}$ C and $r = 5.00\times10^{-3}$ m. Entering those values into the above equation gives

$$\begin{aligned} E &= k\frac{Q}{r^2} \\ &= (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\times\frac{(2.00\times10^{-9}\ \text{C})}{(5.00\times10^{-3}\ \text{m})^2} \\ &= 7.19\times10^5\ \text{N/C}. \end{aligned} \tag{18.14}$$

Discussion

This **electric field strength** is the same at any point 5.00 mm away from the charge Q that creates the field. It is positive, meaning that it has a direction pointing away from the charge Q.

Example 18.3 Calculating the Force Exerted on a Point Charge by an Electric Field

What force does the electric field found in the previous example exert on a point charge of $-0.250\ \mu\text{C}$?

Strategy

Since we know the electric field strength and the charge in the field, the force on that charge can be calculated using the definition of electric field $\mathbf{E} = \mathbf{F} / q$ rearranged to $\mathbf{F} = q\mathbf{E}$.

Solution

The magnitude of the force on a charge $q = -0.250\ \mu\text{C}$ exerted by a field of strength $E = 7.20\times10^5$ N/C is thus,

$$\begin{aligned} F &= -qE \\ &= (0.250\times10^{-6}\ \text{C})(7.20\times10^5\ \text{N/C}) \\ &= 0.180\ \text{N}. \end{aligned} \tag{18.15}$$

Because q is negative, the force is directed opposite to the direction of the field.

Discussion

The force is attractive, as expected for unlike charges. (The field was created by a positive charge and here acts on a negative charge.) The charges in this example are typical of common static electricity, and the modest attractive force obtained is similar to forces experienced in static cling and similar situations.

PhET Explorations: Electric Field of Dreams

Play ball! Add charges to the Field of Dreams and see how they react to the electric field. Turn on a background electric field and adjust the direction and magnitude.

PhET Interactive Simulation

Figure 18.21 Electric Field of Dreams (http://cnx.org/content/m42310/1.6/efield_en.jar)

18.5 Electric Field Lines: Multiple Charges

Drawings using lines to represent **electric fields** around charged objects are very useful in visualizing field strength and direction. Since the electric field has both magnitude and direction, it is a vector. Like all **vectors**, the electric field can be represented by an arrow that has length proportional to its magnitude and that points in the correct direction. (We have used arrows extensively to represent force vectors, for example.)

Figure 18.22 shows two pictorial representations of the same electric field created by a positive point charge Q. **Figure 18.22** (b) shows the standard representation using continuous lines. **Figure 18.22** (b) shows numerous individual arrows with each arrow representing the force on a test charge q. Field lines are essentially a map of infinitesimal force vectors.

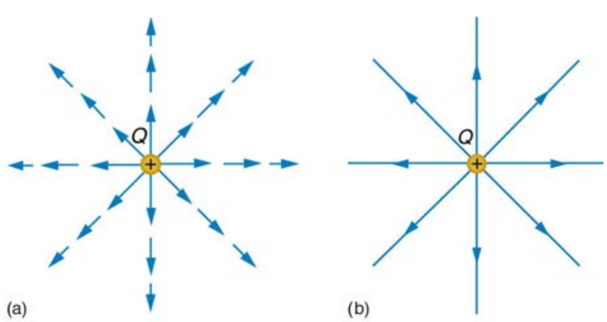


Figure 18.22 Two equivalent representations of the electric field due to a positive charge Q. (a) Arrows representing the electric field's magnitude and direction. (b) In the standard representation, the arrows are replaced by continuous field lines having the same direction at any point as the electric field. The closeness of the lines is directly related to the strength of the electric field. A test charge placed anywhere will feel a force in the direction of the field line; this force will have a strength proportional to the density of the lines (being greater near the charge, for example).

Note that the electric field is defined for a positive test charge q , so that the field lines point away from a positive charge and toward a negative charge. (See **Figure 18.23**.) The electric field strength is exactly proportional to the number of field lines per unit area, since the magnitude of the electric field for a point charge is $E = k|Q| / r^2$ and area is proportional to r^2 . This pictorial representation, in which field lines represent the direction and their closeness (that is, their areal density or the number of lines crossing a unit area) represents strength, is used for all fields: electrostatic, gravitational, magnetic, and others.

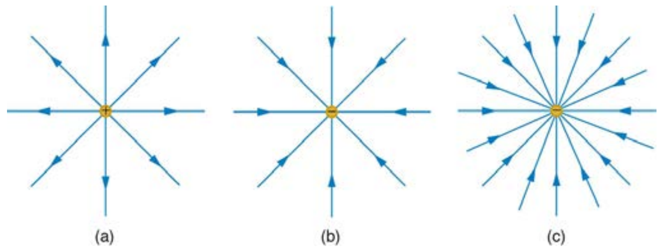


Figure 18.23 The electric field surrounding three different point charges. (a) A positive charge. (b) A negative charge of equal magnitude. (c) A larger negative charge.

In many situations, there are multiple charges. The total electric field created by multiple charges is the vector sum of the individual fields created by each charge. The following example shows how to add electric field vectors.

Example 18.4 Adding Electric Fields

Find the magnitude and direction of the total electric field due to the two point charges, q_1 and q_2 , at the origin of the coordinate system as shown in **Figure 18.24**.

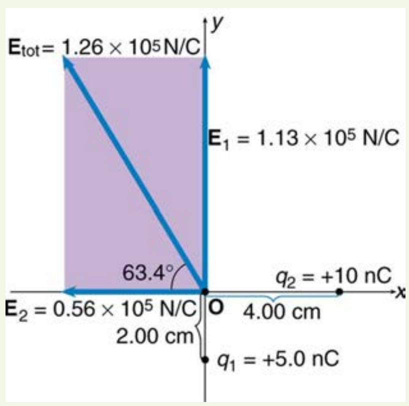


Figure 18.24 The electric fields $\mathbf{E}_1$ and $\mathbf{E}_2$ at the origin O add to $\mathbf{E}_{\text{tot}}$.

Strategy

Since the electric field is a vector (having magnitude and direction), we add electric fields with the same vector techniques used for other types of vectors. We first must find the electric field due to each charge at the point of interest, which is the origin of the coordinate system (O) in this instance. We pretend that there is a positive test charge, q , at point O, which allows us to determine the direction of the fields $\mathbf{E}_1$ and $\mathbf{E}_2$.

Once those fields are found, the total field can be determined using **vector addition**.

Solution

The electric field strength at the origin due to q_1 is labeled E_1 and is calculated:

$$E_1 = k\frac{q_1}{r_1^2} = \left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{\left(5.00\times10^{-9}\ \text{C}\right)}{\left(2.00\times10^{-2}\ \text{m}\right)^2} \tag{18.16}$$

$$E_1 = 1.124\times10^5\ \text{N/C}.$$

Similarly, E_2 is

$$E_2 = k\frac{q_2}{r_2^2} = \left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{\left(10.0\times10^{-9}\ \text{C}\right)}{\left(4.00\times10^{-2}\ \text{m}\right)^2} \tag{18.17}$$

$$E_2 = 0.5619\times10^5\ \text{N/C}.$$

Four digits have been retained in this solution to illustrate that E_1 is exactly twice the magnitude of E_2 . Now arrows are drawn to represent the magnitudes and directions of $\mathbf{E}_1$ and $\mathbf{E}_2$. (See **Figure 18.24**.) The direction of the electric field is that of the force on a positive charge so both arrows point directly away from the positive charges that create them. The arrow for $\mathbf{E}_1$ is exactly twice the length of that for $\mathbf{E}_2$. The arrows form a right triangle in this case and can be added using the Pythagorean theorem. The magnitude of the total field E_{tot} is

$$\begin{aligned} E_{\text{tot}} &= (E_1^2 + E_2^2)^{1/2} \\ &= \{(1.124\times10^5\text{ N/C})^2 + (0.5619\times10^5\text{ N/C})^2\}^{1/2} \\ &= 1.26\times10^5\text{ N/C}. \end{aligned} \tag{18.18}$$

The direction is

$$\begin{aligned} \theta &= \tan^{-1}\left(\frac{E_1}{E_2}\right) \\ &= \tan^{-1}\left(\frac{1.124\times10^5\text{ N/C}}{0.5619\times10^5\text{ N/C}}\right) \\ &= 63.4^\circ, \end{aligned} \tag{18.19}$$

or 63.4° above the *x*-axis.

Discussion

In cases where the electric field vectors to be added are not perpendicular, vector components or graphical techniques can be used. The total electric field found in this example is the total electric field at only one point in space. To find the total electric field due to these two charges over an entire region, the same technique must be repeated for each point in the region. This impossibly lengthy task (there are an infinite number of points in space) can be avoided by calculating the total field at representative points and using some of the unifying features noted next.

Figure 18.25 shows how the electric field from two point charges can be drawn by finding the total field at representative points and drawing electric field lines consistent with those points. While the electric fields from multiple charges are more complex than those of single charges, some simple features are easily noticed.

For example, the field is weaker between like charges, as shown by the lines being farther apart in that region. (This is because the fields from each charge exert opposing forces on any charge placed between them.) (See **Figure 18.25** and **Figure 18.26**(a).) Furthermore, at a great distance from two like charges, the field becomes identical to the field from a single, larger charge.

Figure 18.26(b) shows the electric field of two unlike charges. The field is stronger between the charges. In that region, the fields from each charge are in the same direction, and so their strengths add. The field of two unlike charges is weak at large distances, because the fields of the individual charges are in opposite directions and so their strengths subtract. At very large distances, the field of two unlike charges looks like that of a smaller single charge.

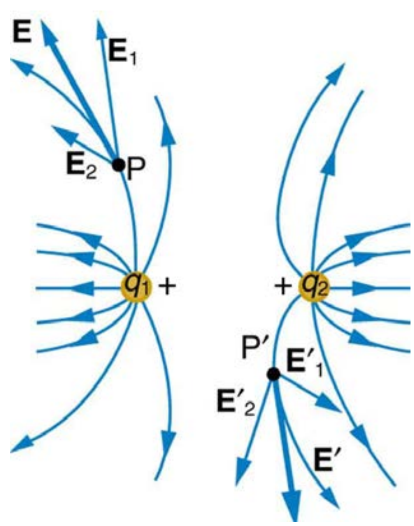


Figure 18.25 Two positive point charges q_1 and q_2 produce the resultant electric field shown. The field is calculated at representative points and then smooth field lines drawn following the rules outlined in the text.

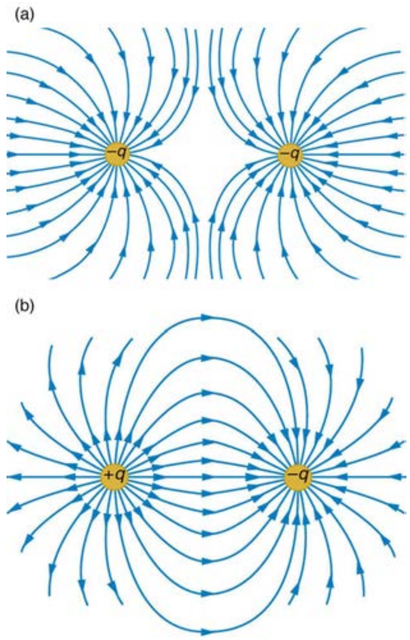


Figure 18.26 (a) Two negative charges produce the fields shown. It is very similar to the field produced by two positive charges, except that the directions are reversed. The field is clearly weaker between the charges. The individual forces on a test charge in that region are in opposite directions. (b) Two opposite charges produce the field shown, which is stronger in the region between the charges.

We use electric field lines to visualize and analyze electric fields (the lines are a pictorial tool, not a physical entity in themselves). The properties of electric field lines for any charge distribution can be summarized as follows:

1. Field lines must begin on positive charges and terminate on negative charges, or at infinity in the hypothetical case of isolated charges.
2. The number of field lines leaving a positive charge or entering a negative charge is proportional to the magnitude of the charge.
3. The strength of the field is proportional to the closeness of the field lines—more precisely, it is proportional to the number of lines per unit area perpendicular to the lines.
4. The direction of the electric field is tangent to the field line at any point in space.
5. Field lines can never cross.

The last property means that the field is unique at any point. The field line represents the direction of the field; so if they crossed, the field would have two directions at that location (an impossibility if the field is unique).

PhET Explorations: Charges and Fields

Move point charges around on the playing field and then view the electric field, voltages, equipotential lines, and more. It's colorful, it's dynamic, it's free.

Figure 18.27 Charges and Fields (http://cnx.org/content/m42312/1.7/charges-and-fields_en.jar)

18.6 Electric Forces in Biology

Classical electrostatics has an important role to play in modern molecular biology. Large molecules such as proteins, nucleic acids, and so on—so important to life—are usually electrically charged. DNA itself is highly charged; it is the electrostatic force that not only holds the molecule together but gives the molecule structure and strength. **Figure 18.28** is a schematic of the DNA double helix.

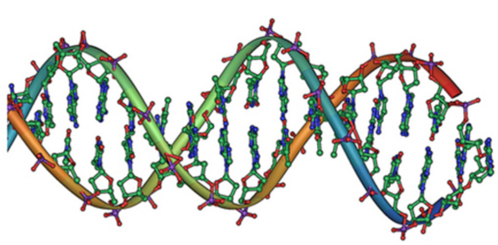


Figure 18.28 DNA is a highly charged molecule. The DNA double helix shows the two coiled strands each containing a row of nitrogenous bases, which “code” the genetic information needed by a living organism. The strands are connected by bonds between pairs of bases. While pairing combinations between certain bases are fixed (C-G and A-T), the sequence of nucleotides in the strand varies. (credit: Jerome Walker)

The four nucleotide bases are given the symbols A (adenine), C (cytosine), G (guanine), and T (thymine). The order of the four bases varies in each strand, but the pairing between bases is always the same. C and G are always paired and A and T are always paired, which helps to preserve the order of bases in cell division (mitosis) so as to pass on the correct genetic information. Since the Coulomb force drops with distance ($F \propto 1/r^2$), the distances between the base pairs must be small enough that the electrostatic force is sufficient to hold them together.

DNA is a highly charged molecule, with about $2q_e$ (fundamental charge) per 0.3×10^{-9} m. The distance separating the two strands that make up the DNA structure is about 1 nm, while the distance separating the individual atoms within each base is about 0.3 nm.

One might wonder why electrostatic forces do not play a larger role in biology than they do if we have so many charged molecules. The reason is that the electrostatic force is “diluted” due to **screening** between molecules. This is due to the presence of other charges in the cell.

Polarity of Water Molecules

The best example of this charge screening is the water molecule, represented as H_2O . Water is a strongly **polar molecule**. Its 10 electrons (8 from the oxygen atom and 2 from the two hydrogen atoms) tend to remain closer to the oxygen nucleus than the hydrogen nuclei. This creates two centers of equal and opposite charges—what is called a **dipole**, as illustrated in Figure 18.29. The magnitude of the dipole is called the dipole moment.

These two centers of charge will terminate some of the electric field lines coming from a free charge, as on a DNA molecule. This results in a reduction in the strength of the **Coulomb interaction**. One might say that screening makes the Coulomb force a short range force rather than long range.

Other ions of importance in biology that can reduce or screen Coulomb interactions are Na^+, and K^+, and Cl^-. These ions are located both inside and outside of living cells. The movement of these ions through cell membranes is crucial to the motion of nerve impulses through nerve axons.

Recent studies of electrostatics in biology seem to show that electric fields in cells can be extended over larger distances, in spite of screening, by “microtubules” within the cell. These microtubules are hollow tubes composed of proteins that guide the movement of chromosomes when cells divide, the motion of other organisms within the cell, and provide mechanisms for motion of some cells (as motors).

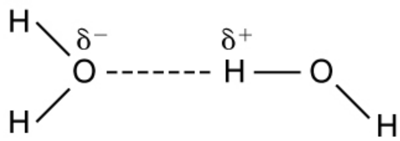


Figure 18.29 This schematic shows water (H_2O) as a polar molecule. Unequal sharing of electrons between the oxygen (O) and hydrogen (H) atoms leads to a net separation of positive and negative charge—forming a dipole. The symbols δ^- and δ^+ indicate that the oxygen side of the H_2O molecule tends to be more negative, while the hydrogen ends tend to be more positive. This leads to an attraction of opposite charges between molecules.

18.7 Conductors and Electric Fields in Static Equilibrium

Conductors contain **free charges** that move easily. When excess charge is placed on a conductor or the conductor is put into a static electric field, charges in the conductor quickly respond to reach a steady state called **electrostatic equilibrium**.

Figure 18.30 shows the effect of an electric field on free charges in a conductor. The free charges move until the field is perpendicular to the conductor’s surface. There can be no component of the field parallel to the surface in electrostatic equilibrium, since, if there were, it would produce further movement of charge. A positive free charge is shown, but free charges can be either positive or negative and are, in fact, negative in metals. The motion of a positive charge is equivalent to the motion of a negative charge in the opposite direction.

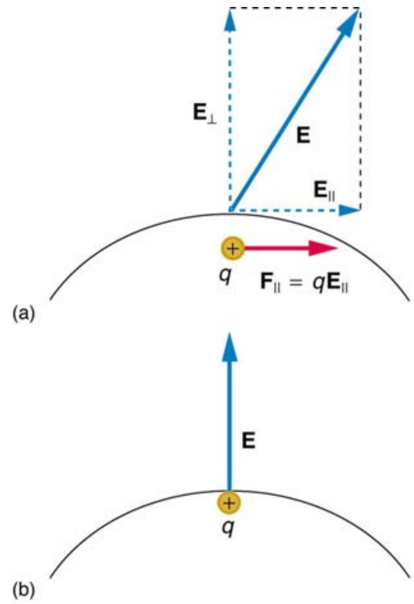


Figure 18.30 When an electric field $\mathbf{E}$ is applied to a conductor, free charges inside the conductor move until the field is perpendicular to the surface. (a) The electric field is a vector quantity, with both parallel and perpendicular components. The parallel component ($\mathbf{E}_{||}$) exerts a force ($\mathbf{F}_{||}$) on the free charge q , which moves the charge until $\mathbf{F}_{||} = 0$. (b) The resulting field is perpendicular to the surface. The free charge has been brought to the conductor's surface, leaving electrostatic forces in equilibrium.

A conductor placed in an **electric field** will be **polarized**. **Figure 18.31** shows the result of placing a neutral conductor in an originally uniform electric field. The field becomes stronger near the conductor but entirely disappears inside it.

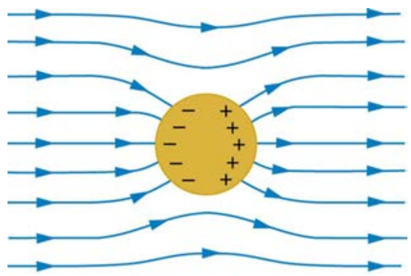


Figure 18.31 This illustration shows a spherical conductor in static equilibrium with an originally uniform electric field. Free charges move within the conductor, polarizing it, until the electric field lines are perpendicular to the surface. The field lines end on excess negative charge on one section of the surface and begin again on excess positive charge on the opposite side. No electric field exists inside the conductor, since free charges in the conductor would continue moving in response to any field until it was neutralized.

Misconception Alert: Electric Field inside a Conductor

Excess charges placed on a spherical conductor repel and move until they are evenly distributed, as shown in **Figure 18.32**. Excess charge is forced to the surface until the field inside the conductor is zero. Outside the conductor, the field is exactly the same as if the conductor were replaced by a point charge at its center equal to the excess charge.

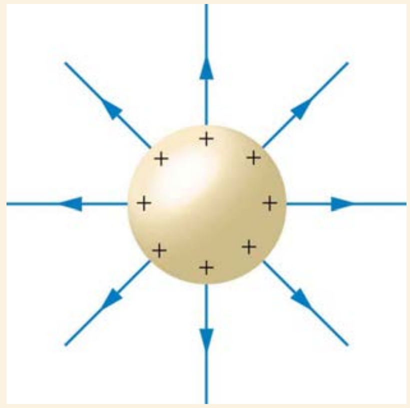


Figure 18.32 The mutual repulsion of excess positive charges on a spherical conductor distributes them uniformly on its surface. The resulting electric field is perpendicular to the surface and zero inside. Outside the conductor, the field is identical to that of a point charge at the center equal to the excess charge.

Properties of a Conductor in Electrostatic Equilibrium

1. The electric field is zero inside a conductor.

2. Just outside a conductor, the electric field lines are perpendicular to its surface, ending or beginning on charges on the surface.
3. Any excess charge resides entirely on the surface or surfaces of a conductor.

The properties of a conductor are consistent with the situations already discussed and can be used to analyze any conductor in electrostatic equilibrium. This can lead to some interesting new insights, such as described below.

How can a very uniform electric field be created? Consider a system of two metal plates with opposite charges on them, as shown in **Figure 18.33**. The properties of conductors in electrostatic equilibrium indicate that the electric field between the plates will be uniform in strength and direction. Except near the edges, the excess charges distribute themselves uniformly, producing field lines that are uniformly spaced (hence uniform in strength) and perpendicular to the surfaces (hence uniform in direction, since the plates are flat). The edge effects are less important when the plates are close together.

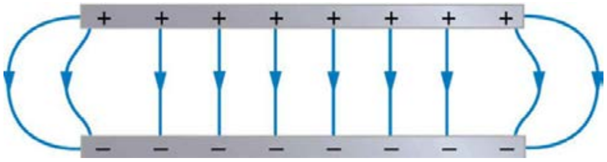


Figure 18.33 Two metal plates with equal, but opposite, excess charges. The field between them is uniform in strength and direction except near the edges. One use of such a field is to produce uniform acceleration of charges between the plates, such as in the electron gun of a TV tube.

Earth's Electric Field

A near uniform electric field of approximately 150 N/C, directed downward, surrounds Earth, with the magnitude increasing slightly as we get closer to the surface. What causes the electric field? At around 100 km above the surface of Earth we have a layer of charged particles, called the **ionosphere**. The ionosphere is responsible for a range of phenomena including the electric field surrounding Earth. In fair weather the ionosphere is positive and the Earth largely negative, maintaining the electric field (**Figure 18.34**(a)).

In storm conditions clouds form and localized electric fields can be larger and reversed in direction (**Figure 18.34**(b)). The exact charge distributions depend on the local conditions, and variations of **Figure 18.34**(b) are possible.

If the electric field is sufficiently large, the insulating properties of the surrounding material break down and it becomes conducting. For air this occurs at around 3×10^6 N/C. Air ionizes ions and electrons recombine, and we get discharge in the form of lightning sparks and corona discharge.

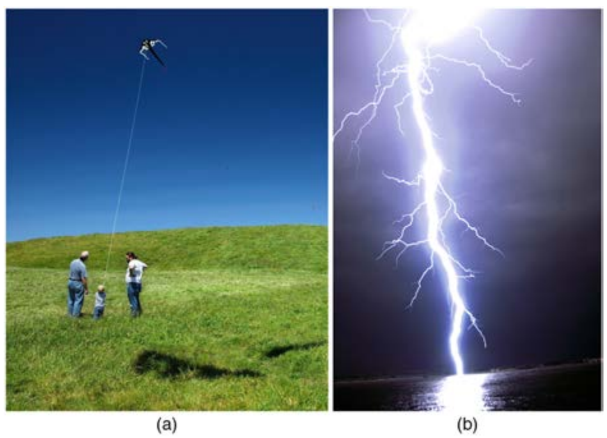


Figure 18.34 Earth's electric field. (a) Fair weather field. Earth and the ionosphere (a layer of charged particles) are both conductors. They produce a uniform electric field of about 150 N/C. (credit: D. H. Parks) (b) Storm fields. In the presence of storm clouds, the local electric fields can be larger. At very high fields, the insulating properties of the air break down and lightning can occur. (credit: Jan-Joost Verhoef)

Electric Fields on Uneven Surfaces

So far we have considered excess charges on a smooth, symmetrical conductor surface. What happens if a conductor has sharp corners or is pointed? Excess charges on a nonuniform conductor become concentrated at the sharpest points. Additionally, excess charge may move on or off the conductor at the sharpest points.

To see how and why this happens, consider the charged conductor in **Figure 18.35**. The electrostatic repulsion of like charges is most effective in moving them apart on the flattest surface, and so they become least concentrated there. This is because the forces between identical pairs of charges at either end of the conductor are identical, but the components of the forces parallel to the surfaces are different. The component parallel to the surface is greatest on the flattest surface and, hence, more effective in moving the charge.

The same effect is produced on a conductor by an externally applied electric field, as seen in **Figure 18.35** (c). Since the field lines must be perpendicular to the surface, more of them are concentrated on the most curved parts.

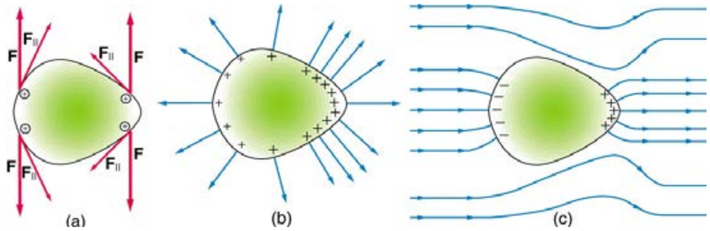


Figure 18.35 Excess charge on a nonuniform conductor becomes most concentrated at the location of greatest curvature. (a) The forces between identical pairs of charges at either end of the conductor are identical, but the components of the forces parallel to the surface are different. It is $\mathbf{F}_{||}$ that moves the charges apart once they have reached the surface. (b) $\mathbf{F}_{||}$ is smallest at the more pointed end, the charges are left closer together, producing the electric field shown. (c) An uncharged conductor in an originally uniform electric field is polarized, with the most concentrated charge at its most pointed end.

Applications of Conductors

On a very sharply curved surface, such as shown in **Figure 18.36**, the charges are so concentrated at the point that the resulting electric field can be great enough to remove them from the surface. This can be useful.

Lightning rods work best when they are most pointed. The large charges created in storm clouds induce an opposite charge on a building that can result in a lightning bolt hitting the building. The induced charge is bled away continually by a lightning rod, preventing the more dramatic lightning strike.

Of course, we sometimes wish to prevent the transfer of charge rather than to facilitate it. In that case, the conductor should be very smooth and have as large a radius of curvature as possible. (See **Figure 18.37**.) Smooth surfaces are used on high-voltage transmission lines, for example, to avoid leakage of charge into the air.

Another device that makes use of some of these principles is a **Faraday cage**. This is a metal shield that encloses a volume. All electrical charges will reside on the outside surface of this shield, and there will be no electrical field inside. A Faraday cage is used to prohibit stray electrical fields in the environment from interfering with sensitive measurements, such as the electrical signals inside a nerve cell.

During electrical storms if you are driving a car, it is best to stay inside the car as its metal body acts as a Faraday cage with zero electrical field inside. If in the vicinity of a lightning strike, its effect is felt on the outside of the car and the inside is unaffected, provided you remain totally inside. This is also true if an active (“hot”) electrical wire was broken (in a storm or an accident) and fell on your car.

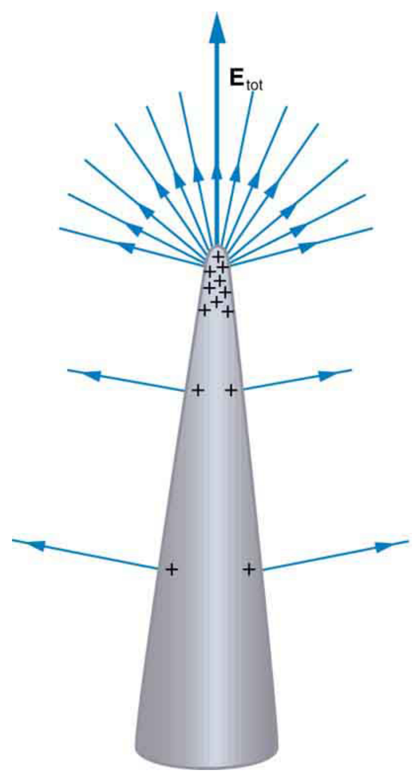


Figure 18.36 A very pointed conductor has a large charge concentration at the point. The electric field is very strong at the point and can exert a force large enough to transfer charge on or off the conductor. Lightning rods are used to prevent the buildup of large excess charges on structures and, thus, are pointed.

Figure 18.37 (a) A lightning rod is pointed to facilitate the transfer of charge. (credit: Romaine, Wikimedia Commons) (b) This Van de Graaff generator has a smooth surface with a large radius of curvature to prevent the transfer of charge and allow a large voltage to be generated. The mutual repulsion of like charges is evident in the person's hair while touching the metal sphere. (credit: Jon 'ShakataGaNai' Davis/Wikimedia Commons).

18.8 Applications of Electrostatics

The study of **electrostatics** has proven useful in many areas. This module covers just a few of the many applications of electrostatics.

The Van de Graaff Generator

Van de Graaff generators (or Van de Graaffs) are not only spectacular devices used to demonstrate high voltage due to static electricity—they are also used for serious research. The first was built by Robert Van de Graaff in 1931 (based on original suggestions by Lord Kelvin) for use in nuclear physics research. Figure 18.38 shows a schematic of a large research version. Van de Graaffs utilize both smooth and pointed surfaces, and conductors and insulators to generate large static charges and, hence, large voltages.

A very large excess charge can be deposited on the sphere, because it moves quickly to the outer surface. Practical limits arise because the large electric fields polarize and eventually ionize surrounding materials, creating free charges that neutralize excess charge or allow it to escape. Nevertheless, voltages of 15 million volts are well within practical limits.

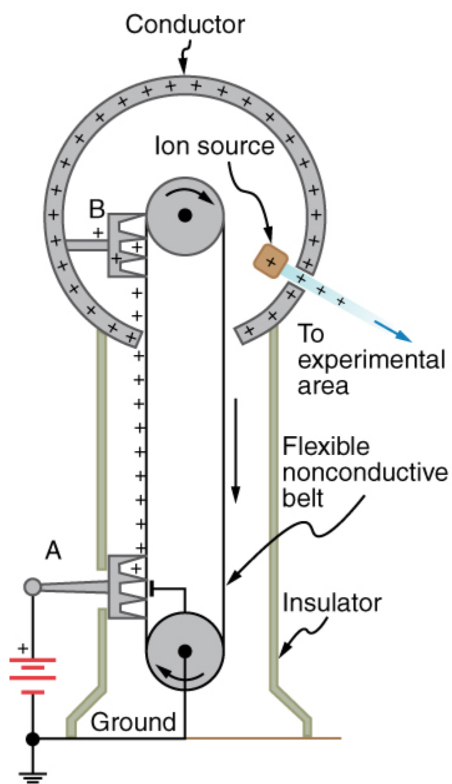


Figure 18.38 Schematic of Van de Graaff generator. A battery (A) supplies excess positive charge to a pointed conductor, the points of which spray the charge onto a moving insulating belt near the bottom. The pointed conductor (B) on top in the large sphere picks up the charge. (The induced electric field at the points is so large that it removes the charge from the belt.) This can be done because the charge does not remain inside the conducting sphere but moves to its outside surface. An ion source inside the sphere produces positive ions, which are accelerated away from the positive sphere to high velocities.

Take-Home Experiment: Electrostatics and Humidity

Rub a comb through your hair and use it to lift pieces of paper. It may help to tear the pieces of paper rather than cut them neatly. Repeat the exercise in your bathroom after you have had a long shower and the air in the bathroom is moist. Is it easier to get electrostatic effects in dry or moist air? Why would torn paper be more attractive to the comb than cut paper? Explain your observations.

Xerography

Most copy machines use an electrostatic process called **xerography**—a word coined from the Greek words *xeros* for dry and *graphos* for writing. The heart of the process is shown in simplified form in **Figure 18.39**.

A selenium-coated aluminum drum is sprayed with positive charge from points on a device called a corotron. Selenium is a substance with an interesting property—it is a **photoconductor**. That is, selenium is an insulator when in the dark and a conductor when exposed to light.

In the first stage of the xerography process, the conducting aluminum drum is **grounded** so that a negative charge is induced under the thin layer of uniformly positively charged selenium. In the second stage, the surface of the drum is exposed to the image of whatever is to be copied. Where the image is light, the selenium becomes conducting, and the positive charge is neutralized. In dark areas, the positive charge remains, and so the image has been transferred to the drum.

The third stage takes a dry black powder, called toner, and sprays it with a negative charge so that it will be attracted to the positive regions of the drum. Next, a blank piece of paper is given a greater positive charge than on the drum so that it will pull the toner from the drum. Finally, the paper and electrostatically held toner are passed through heated pressure rollers, which melt and permanently adhere the toner within the fibers of the paper.

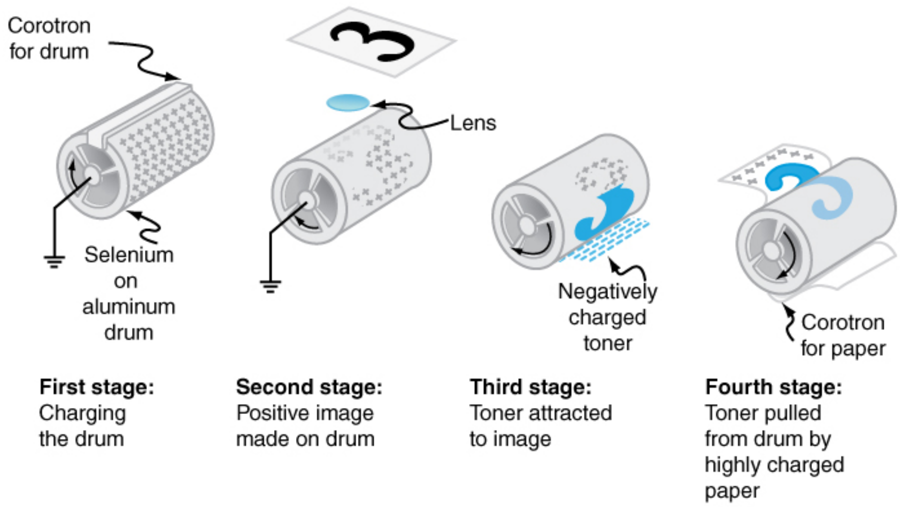


Figure 18.39 Xerography is a dry copying process based on electrostatics. The major steps in the process are the charging of the photoconducting drum, transfer of an image creating a positive charge duplicate, attraction of toner to the charged parts of the drum, and transfer of toner to the paper. Not shown are heat treatment of the paper and cleansing of the drum for the next copy.

Laser Printers

Laser printers use the xerographic process to make high-quality images on paper, employing a laser to produce an image on the photoconducting drum as shown in **Figure 18.40**. In its most common application, the laser printer receives output from a computer, and it can achieve high-quality output because of the precision with which laser light can be controlled. Many laser printers do significant information processing, such as making sophisticated letters or fonts, and may contain a computer more powerful than the one giving them the raw data to be printed.

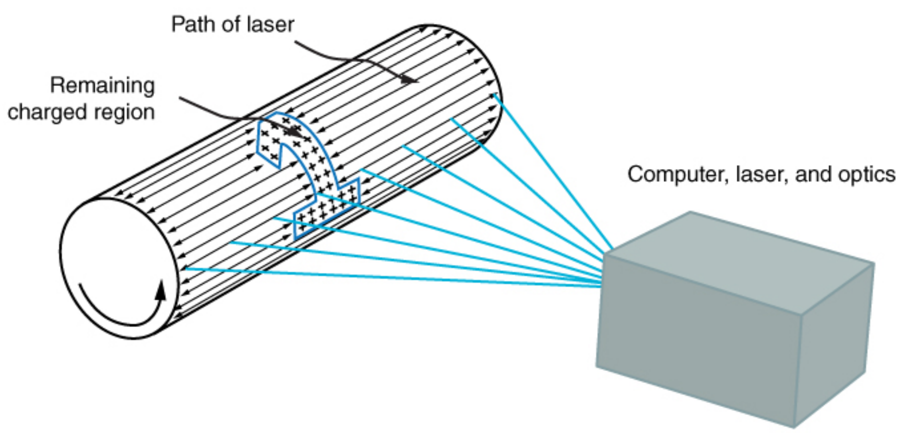


Figure 18.40 In a laser printer, a laser beam is scanned across a photoconducting drum, leaving a positive charge image. The other steps for charging the drum and transferring the image to paper are the same as in xerography. Laser light can be very precisely controlled, enabling laser printers to produce high-quality images.

Ink Jet Printers and Electrostatic Painting

The **ink jet printer**, commonly used to print computer-generated text and graphics, also employs electrostatics. A nozzle makes a fine spray of tiny ink droplets, which are then given an electrostatic charge. (See **Figure 18.41**.)

Once charged, the droplets can be directed, using pairs of charged plates, with great precision to form letters and images on paper. Ink jet printers can produce color images by using a black jet and three other jets with primary colors, usually cyan, magenta, and yellow, much as a color television produces color. (This is more difficult with xerography, requiring multiple drums and toners.)

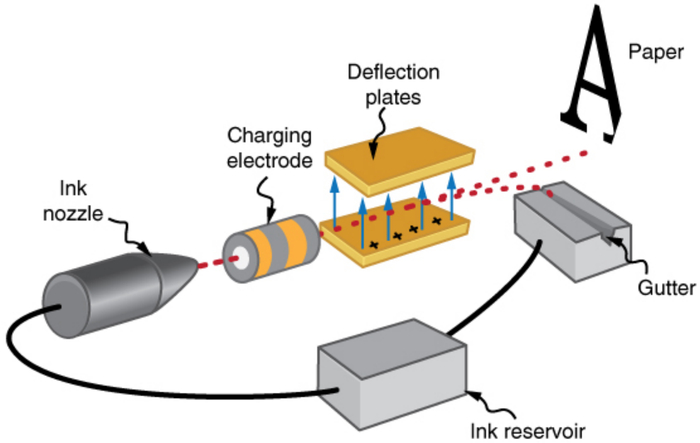


Figure 18.41 The nozzle of an ink-jet printer produces small ink droplets, which are sprayed with electrostatic charge. Various computer-driven devices are then used to direct the droplets to the correct positions on a page.

Electrostatic painting employs electrostatic charge to spray paint onto odd-shaped surfaces. Mutual repulsion of like charges causes the paint to fly away from its source. Surface tension forms drops, which are then attracted by unlike charges to the surface to be painted. Electrostatic painting can reach those hard-to-get at places, applying an even coat in a controlled manner. If the object is a conductor, the electric field is perpendicular to the surface, tending to bring the drops in perpendicularly. Corners and points on conductors will receive extra paint. Felt can similarly be applied.

Smoke Precipitators and Electrostatic Air Cleaning

Another important application of electrostatics is found in air cleaners, both large and small. The electrostatic part of the process places excess (usually positive) charge on smoke, dust, pollen, and other particles in the air and then passes the air through an oppositely charged grid that attracts and retains the charged particles. (See **Figure 18.42**.)

Large **electrostatic precipitators** are used industrially to remove over 99% of the particles from stack gas emissions associated with the burning of coal and oil. Home precipitators, often in conjunction with the home heating and air conditioning system, are very effective in removing polluting particles, irritants, and allergens.

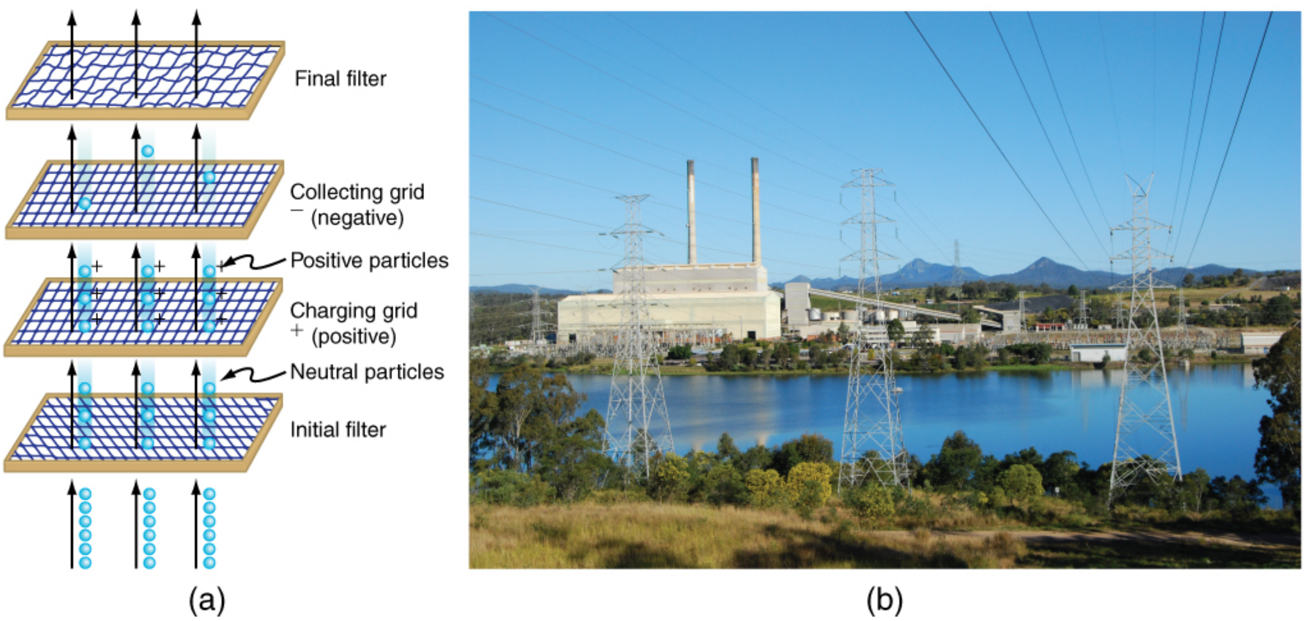


Figure 18.42 (a) Schematic of an electrostatic precipitator. Air is passed through grids of opposite charge. The first grid charges airborne particles, while the second attracts and collects them. (b) The dramatic effect of electrostatic precipitators is seen by the absence of smoke from this power plant. (credit: Cmdalgleish, Wikimedia Commons)

Problem-Solving Strategies for Electrostatics

1. Examine the situation to determine if static electricity is involved. This may concern separated stationary charges, the forces among them, and the electric fields they create.
2. Identify the system of interest. This includes noting the number, locations, and types of charges involved.
3. Identify exactly what needs to be determined in the problem (identify the unknowns). A written list is useful. Determine whether the Coulomb force is to be considered directly—if so, it may be useful to draw a free-body diagram, using electric field lines.
4. Make a list of what is given or can be inferred from the problem as stated (identify the knowns). It is important to distinguish the Coulomb force F from the electric field E, for example.
5. Solve the appropriate equation for the quantity to be determined (the unknown) or draw the field lines as requested.
6. Examine the answer to see if it is reasonable: Does it make sense? Are units correct and the numbers involved reasonable?

Integrated Concepts

The Integrated Concepts exercises for this module involve concepts such as electric charges, electric fields, and several other topics. Physics is most interesting when applied to general situations involving more than a narrow set of physical principles. The electric field exerts force on charges, for example, and hence the relevance of **Dynamics: Force and Newton's Laws of Motion**. The following topics are involved in some or all of the problems labeled "Integrated Concepts":

- **Kinematics**
- **Two-Dimensional Kinematics**
- **Dynamics: Force and Newton's Laws of Motion**
- **Uniform Circular Motion and Gravitation**
- **Statics and Torque**
- **Fluid Statics**

The following worked example illustrates how this strategy is applied to an Integrated Concept problem:

Example 18.5 Acceleration of a Charged Drop of Gasoline

If steps are not taken to ground a gasoline pump, static electricity can be placed on gasoline when filling your car's tank. Suppose a tiny drop of gasoline has a mass of $4.00\times10^{-15}\text{ kg}$ and is given a positive charge of $3.20\times10^{-19}\text{ C}$. (a) Find the weight of the drop. (b) Calculate the electric force on the drop if there is an upward electric field of strength $3.00\times10^{5}\text{ N/C}$ due to other static electricity in the vicinity. (c) Calculate the drop's acceleration.

Strategy

To solve an integrated concept problem, we must first identify the physical principles involved and identify the chapters in which they are found. Part (a) of this example asks for weight. This is a topic of dynamics and is defined in **Dynamics: Force and Newton's Laws of Motion**. Part (b) deals with electric force on a charge, a topic of **Electric Charge and Electric Field**. Part (c) asks for acceleration, knowing forces and mass. These are part of Newton's laws, also found in **Dynamics: Force and Newton's Laws of Motion**.

The following solutions to each part of the example illustrate how the specific problem-solving strategies are applied. These involve identifying knowns and unknowns, checking to see if the answer is reasonable, and so on.

Solution for (a)

Weight is mass times the acceleration due to gravity, as first expressed in

$$w = mg. \tag{18.20}$$

Entering the given mass and the average acceleration due to gravity yields

$$w = (4.00\times10^{-15}\text{ kg})(9.80\text{ m/s}^2) = 3.92\times10^{-14}\text{ N}. \tag{18.21}$$

Discussion for (a)

This is a small weight, consistent with the small mass of the drop.

Solution for (b)

The force an electric field exerts on a charge is given by rearranging the following equation:

$$F = qE. \tag{18.22}$$

Here we are given the charge ($3.20\times10^{-19}\text{ C}$ is twice the fundamental unit of charge) and the electric field strength, and so the electric force is found to be

$$F = (3.20\times10^{-19}\text{ C})(3.00\times10^{5}\text{ N/C}) = 9.60\times10^{-14}\text{ N}. \tag{18.23}$$

Discussion for (b)

While this is a small force, it is greater than the weight of the drop.

Solution for (c)

The acceleration can be found using Newton's second law, provided we can identify all of the external forces acting on the drop. We assume only the drop's weight and the electric force are significant. Since the drop has a positive charge and the electric field is given to be upward, the electric force is upward. We thus have a one-dimensional (vertical direction) problem, and we can state Newton's second law as

$$a = \frac{F_{\text{net}}}{m}. \tag{18.24}$$

where $F_{\text{net}} = F - w$. Entering this and the known values into the expression for Newton's second law yields

$$\begin{aligned} a &= \frac{F - w}{m} \\ &= \frac{9.60\times10^{-14}\text{ N} - 3.92\times10^{-14}\text{ N}}{4.00\times10^{-15}\text{ kg}} \\ &= 14.2\text{ m/s}^2. \end{aligned} \tag{18.25}$$

Discussion for (c)

This is an upward acceleration great enough to carry the drop to places where you might not wish to have gasoline.

This worked example illustrates how to apply problem-solving strategies to situations that include topics in different chapters. The first step is to identify the physical principles involved in the problem. The second step is to solve for the unknown using familiar problem-solving strategies. These are found throughout the text, and many worked examples show how to use them for single topics. In this integrated concepts example, you can see how to apply them across several topics. You will find these techniques useful in applications of physics outside a physics course, such as in your profession, in other science disciplines, and in everyday life. The following problems will build your skills in the broad application of physical principles.

Unreasonable Results

The Unreasonable Results exercises for this module have results that are unreasonable because some premise is unreasonable or because certain of the premises are inconsistent with one another. Physical principles applied correctly then produce unreasonable results. The purpose of these problems is to give practice in assessing whether nature is being accurately described, and if it is not to trace the source of difficulty.

Problem-Solving Strategy

To determine if an answer is reasonable, and to determine the cause if it is not, do the following.

1. Solve the problem using strategies as outlined above. Use the format followed in the worked examples in the text to solve the problem as usual.
2. Check to see if the answer is reasonable. Is it too large or too small, or does it have the wrong sign, improper units, and so on?
3. If the answer is unreasonable, look for what specifically could cause the identified difficulty. Usually, the manner in which the answer is unreasonable is an indication of the difficulty. For example, an extremely large Coulomb force could be due to the assumption of an excessively large separated charge.

Glossary

Coulomb force: another term for the electrostatic force

Coulomb interaction: the interaction between two charged particles generated by the Coulomb forces they exert on one another

Coulomb's law: the mathematical equation calculating the electrostatic force vector between two charged particles

conductor: a material that allows electrons to move separately from their atomic orbits

conductor: an object with properties that allow charges to move about freely within it

dipole: a molecule's lack of symmetrical charge distribution, causing one side to be more positive and another to be more negative

electric charge: a physical property of an object that causes it to be attracted toward or repelled from another charged object; each charged object generates and is influenced by a force called an electromagnetic force

electric field lines: a series of lines drawn from a point charge representing the magnitude and direction of force exerted by that charge

electric field: a three-dimensional map of the electric force extended out into space from a point charge

electromagnetic force: one of the four fundamental forces of nature; the electromagnetic force consists of static electricity, moving electricity and magnetism

electron: a particle orbiting the nucleus of an atom and carrying the smallest unit of negative charge

electrostatic equilibrium: an electrostatically balanced state in which all free electrical charges have stopped moving about

electrostatic force: the amount and direction of attraction or repulsion between two charged bodies

electrostatic precipitators: filters that apply charges to particles in the air, then attract those charges to a filter, removing them from the airstream

electrostatic repulsion: the phenomenon of two objects with like charges repelling each other

electrostatics: the study of electric forces that are static or slow-moving

Faraday cage: a metal shield which prevents electric charge from penetrating its surface

field: a map of the amount and direction of a force acting on other objects, extending out into space

free charge: an electrical charge (either positive or negative) which can move about separately from its base molecule

free electron: an electron that is free to move away from its atomic orbit

grounded: when a conductor is connected to the Earth, allowing charge to freely flow to and from Earth's unlimited reservoir

grounded: connected to the ground with a conductor, so that charge flows freely to and from the Earth to the grounded object

induction: the process by which an electrically charged object brought near a neutral object creates a charge in that object

ink-jet printer: small ink droplets sprayed with an electric charge are controlled by electrostatic plates to create images on paper

insulator: a material that holds electrons securely within their atomic orbits

ionosphere: a layer of charged particles located around 100 km above the surface of Earth, which is responsible for a range of phenomena including the electric field surrounding Earth

laser printer: uses a laser to create a photoconductive image on a drum, which attracts dry ink particles that are then rolled onto a sheet of paper to print a high-quality copy of the image

law of conservation of charge: states that whenever a charge is created, an equal amount of charge with the opposite sign is created simultaneously

photoconductor: a substance that is an insulator until it is exposed to light, when it becomes a conductor

point charge: A charged particle, designated Q, generating an electric field

polar molecule: a molecule with an asymmetrical distribution of positive and negative charge

polarization: slight shifting of positive and negative charges to opposite sides of an atom or molecule

polarized: a state in which the positive and negative charges within an object have collected in separate locations

proton: a particle in the nucleus of an atom and carrying a positive charge equal in magnitude and opposite in sign to the amount of negative charge carried by an electron

screening: the dilution or blocking of an electrostatic force on a charged object by the presence of other charges nearby

static electricity: a buildup of electric charge on the surface of an object

test charge: A particle (designated q) with either a positive or negative charge set down within an electric field generated by a point charge

Van de Graaff generator: a machine that produces a large amount of excess charge, used for experiments with high voltage

vector addition: mathematical combination of two or more vectors, including their magnitudes, directions, and positions

vector: a quantity with both magnitude and direction

xerography: a dry copying process based on electrostatics

Section Summary

18.1 Static Electricity and Charge: Conservation of Charge

- There are only two types of charge, which we call positive and negative.
- Like charges repel, unlike charges attract, and the force between charges decreases with the square of the distance.
- The vast majority of positive charge in nature is carried by protons, while the vast majority of negative charge is carried by electrons.
- The electric charge of one electron is equal in magnitude and opposite in sign to the charge of one proton.
- An ion is an atom or molecule that has nonzero total charge due to having unequal numbers of electrons and protons.
- The SI unit for charge is the coulomb (C), with protons and electrons having charges of opposite sign but equal magnitude; the magnitude of this basic charge $|q_e|$ is

$$|q_e| = 1.60\times10^{-19}\text{ C}.$$

- Whenever charge is created or destroyed, equal amounts of positive and negative are involved.
- Most often, existing charges are separated from neutral objects to obtain some net charge.
- Both positive and negative charges exist in neutral objects and can be separated by rubbing one object with another. For macroscopic objects, negatively charged means an excess of electrons and positively charged means a depletion of electrons.
- The law of conservation of charge ensures that whenever a charge is created, an equal charge of the opposite sign is created at the same time.

18.2 Conductors and Insulators

- Polarization is the separation of positive and negative charges in a neutral object.
- A conductor is a substance that allows charge to flow freely through its atomic structure.
- An insulator holds charge within its atomic structure.
- Objects with like charges repel each other, while those with unlike charges attract each other.
- A conducting object is said to be grounded if it is connected to the Earth through a conductor. Grounding allows transfer of charge to and from the earth's large reservoir.
- Objects can be charged by contact with another charged object and obtain the same sign charge.
- If an object is temporarily grounded, it can be charged by induction, and obtains the opposite sign charge.
- Polarized objects have their positive and negative charges concentrated in different areas, giving them a non-symmetrical charge.
- Polar molecules have an inherent separation of charge.

18.3 Coulomb's Law

- Frenchman Charles Coulomb was the first to publish the mathematical equation that describes the electrostatic force between two objects.
- Coulomb's law gives the magnitude of the force between point charges. It is

$$F = k\frac{|q_1 q_2|}{r^2},$$

where q_1 and q_2 are two point charges separated by a distance r, and $k \approx 8.99 \times 10^9\ \text{N} \cdot \text{m}^2 / \text{C}^2$

- This Coulomb force is extremely basic, since most charges are due to point-like particles. It is responsible for all electrostatic effects and underlies most macroscopic forces.
- The Coulomb force is extraordinarily strong compared with the gravitational force, another basic force—but unlike gravitational force it can cancel, since it can be either attractive or repulsive.
- The electrostatic force between two subatomic particles is far greater than the gravitational force between the same two particles.

18.4 Electric Field: Concept of a Field Revisited

- The electrostatic force field surrounding a charged object extends out into space in all directions.
- The electrostatic force exerted by a point charge on a test charge at a distance r depends on the charge of both charges, as well as the distance between the two.
- The electric field $\mathbf{E}$ is defined to be

$$\mathbf{E} = \frac{\mathbf{F}}{q},$$

where $\mathbf{F}$ is the Coulomb or electrostatic force exerted on a small positive test charge q. $\mathbf{E}$ has units of N/C.

- The magnitude of the electric field $\mathbf{E}$ created by a point charge Q is

$$\mathbf{E} = k\frac{|Q|}{r^2}.$$

where r is the distance from Q. The electric field $\mathbf{E}$ is a vector and fields due to multiple charges add like vectors.

18.5 Electric Field Lines: Multiple Charges

- Drawings of electric field lines are useful visual tools. The properties of electric field lines for any charge distribution are that:
- Field lines must begin on positive charges and terminate on negative charges, or at infinity in the hypothetical case of isolated charges.
- The number of field lines leaving a positive charge or entering a negative charge is proportional to the magnitude of the charge.
- The strength of the field is proportional to the closeness of the field lines—more precisely, it is proportional to the number of lines per unit area perpendicular to the lines.
- The direction of the electric field is tangent to the field line at any point in space.
- Field lines can never cross.

18.6 Electric Forces in Biology

- Many molecules in living organisms, such as DNA, carry a charge.
- An uneven distribution of the positive and negative charges within a polar molecule produces a dipole.
- The effect of a Coulomb field generated by a charged object may be reduced or blocked by other nearby charged objects.
- Biological systems contain water, and because water molecules are polar, they have a strong effect on other molecules in living systems.

18.7 Conductors and Electric Fields in Static Equilibrium

- A conductor allows free charges to move about within it.
- The electrical forces around a conductor will cause free charges to move around inside the conductor until static equilibrium is reached.
- Any excess charge will collect along the surface of a conductor.
- Conductors with sharp corners or points will collect more charge at those points.
- A lightning rod is a conductor with sharply pointed ends that collect excess charge on the building caused by an electrical storm and allow it to dissipate back into the air.
- Electrical storms result when the electrical field of Earth's surface in certain locations becomes more strongly charged, due to changes in the insulating effect of the air.
- A Faraday cage acts like a shield around an object, preventing electric charge from penetrating inside.

18.8 Applications of Electrostatics

- Electrostatics is the study of electric fields in static equilibrium.
- In addition to research using equipment such as a Van de Graaff generator, many practical applications of electrostatics exist, including photocopiers, laser printers, ink-jet printers and electrostatic air filters.

Conceptual Questions

18.1 Static Electricity and Charge: Conservation of Charge

1. There are very large numbers of charged particles in most objects. Why, then, don't most objects exhibit static electricity?

2. Why do most objects tend to contain nearly equal numbers of positive and negative charges?

18.2 Conductors and Insulators

3. An eccentric inventor attempts to levitate by first placing a large negative charge on himself and then putting a large positive charge on the ceiling of his workshop. Instead, while attempting to place a large negative charge on himself, his clothes fly off. Explain.

4. If you have charged an electroscope by contact with a positively charged object, describe how you could use it to determine the charge of other objects. Specifically, what would the leaves of the electroscope do if other charged objects were brought near its knob?

5. When a glass rod is rubbed with silk, it becomes positive and the silk becomes negative—yet both attract dust. Does the dust have a third type of charge that is attracted to both positive and negative? Explain.

6. Why does a car always attract dust right after it is polished? (Note that car wax and car tires are insulators.)

7. Describe how a positively charged object can be used to give another object a negative charge. What is the name of this process?

8. What is grounding? What effect does it have on a charged conductor? On a charged insulator?

18.3 Coulomb's Law

9. Figure 18.43 shows the charge distribution in a water molecule, which is called a polar molecule because it has an inherent separation of charge. Given water's polar character, explain what effect humidity has on removing excess charge from objects.

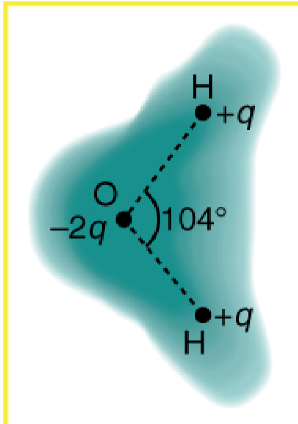


Figure 18.43 Schematic representation of the outer electron cloud of a neutral water molecule. The electrons spend more time near the oxygen than the hydrogens, giving a permanent charge separation as shown. Water is thus a *polar molecule*. It is more easily affected by electrostatic forces than molecules with uniform charge distributions.

10. Using Figure 18.43, explain, in terms of Coulomb's law, why a polar molecule (such as in Figure 18.43) is attracted by both positive and negative charges.

11. Given the polar character of water molecules, explain how ions in the air form nucleation centers for rain droplets.

18.4 Electric Field: Concept of a Field Revisited

12. Why must the test charge q in the definition of the electric field be vanishingly small?

13. Are the direction and magnitude of the Coulomb force unique at a given point in space? What about the electric field?

18.5 Electric Field Lines: Multiple Charges

14. Compare and contrast the Coulomb force field and the electric field. To do this, make a list of five properties for the Coulomb force field analogous to the five properties listed for electric field lines. Compare each item in your list of Coulomb force field properties with those of the electric field—are they the same or different? (For example, electric field lines cannot cross. Is the same true for Coulomb field lines?)

15. Figure 18.44 shows an electric field extending over three regions, labeled I, II, and III. Answer the following questions. (a) Are there any isolated charges? If so, in what region and what are their signs? (b) Where is the field strongest? (c) Where is it weakest? (d) Where is the field the most uniform?

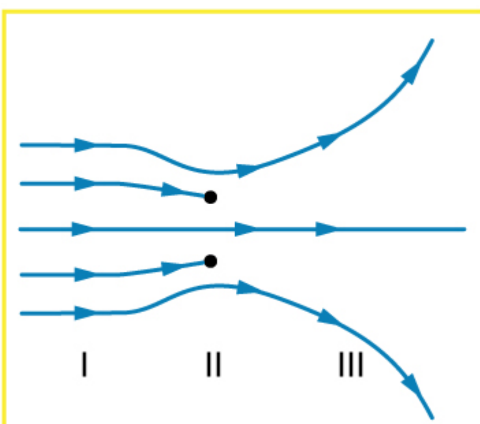


Figure 18.44

18.6 Electric Forces in Biology

16. A cell membrane is a thin layer enveloping a cell. The thickness of the membrane is much less than the size of the cell. In a static situation the membrane has a charge distribution of -2.5×10^{-6} C/m ^{2} on its inner surface and $+2.5\times10^{-6}$ C/m^2 on its outer surface. Draw a diagram of the cell and the surrounding cell membrane. Include on this diagram the charge distribution and the corresponding electric field. Is there any electric field inside the cell? Is there any electric field outside the cell?

18.7 Conductors and Electric Fields in Static Equilibrium

17. Is the object in **Figure 18.45** a conductor or an insulator? Justify your answer.

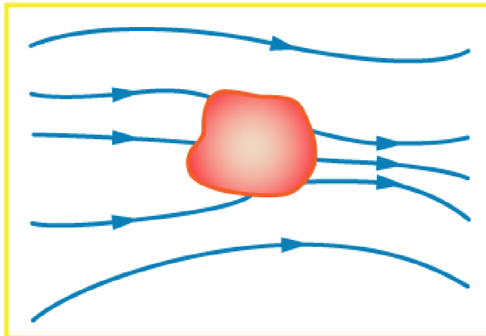


Figure 18.45

18. If the electric field lines in the figure above were perpendicular to the object, would it necessarily be a conductor? Explain.

19. The discussion of the electric field between two parallel conducting plates, in this module states that edge effects are less important if the plates are close together. What does close mean? That is, is the actual plate separation crucial, or is the ratio of plate separation to plate area crucial?

20. Would the self-created electric field at the end of a pointed conductor, such as a lightning rod, remove positive or negative charge from the conductor? Would the same sign charge be removed from a neutral pointed conductor by the application of a similar externally created electric field? (The answers to both questions have implications for charge transfer utilizing points.)

21. Why is a golfer with a metal club over her shoulder vulnerable to lightning in an open fairway? Would she be any safer under a tree?

22. Can the belt of a Van de Graaff accelerator be a conductor? Explain.

23. Are you relatively safe from lightning inside an automobile? Give two reasons.

24. Discuss pros and cons of a lightning rod being grounded versus simply being attached to a building.

25. Using the symmetry of the arrangement, show that the net Coulomb force on the charge q at the center of the square below (**Figure 18.46**) is zero if the charges on the four corners are exactly equal.

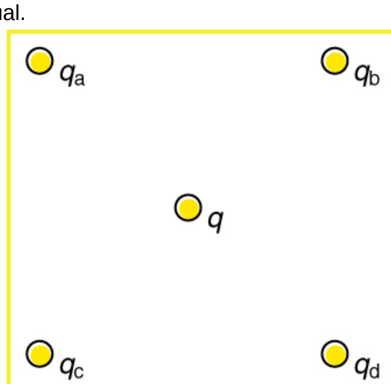


Figure 18.46 Four point charges q_a, q_b, q_c, and q_d lie on the corners of a square and q is located at its center.

26. (a) Using the symmetry of the arrangement, show that the electric field at the center of the square in **Figure 18.46** is zero if the charges on the four corners are exactly equal. (b) Show that this is also true for any combination of charges in which $q_a = q_b$ and $q_b = q_c$

27. (a) What is the direction of the total Coulomb force on q in **Figure 18.46** if q is negative, $q_a = q_c$ and both are negative, and $q_b = q_c$ and both are positive? (b) What is the direction of the electric field at the center of the square in this situation?

28. Considering **Figure 18.46**, suppose that $q_a = q_d$ and $q_b = q_c$. First show that q is in static equilibrium. (You may neglect the gravitational force.) Then discuss whether the equilibrium is stable or unstable, noting that this may depend on the signs of the charges and the direction of displacement of q from the center of the square.

29. If $q_a = 0$ in **Figure 18.46**, under what conditions will there be no net Coulomb force on q?

30. In regions of low humidity, one develops a special “grip” when opening car doors, or touching metal door knobs. This involves placing as much of the hand on the device as possible, not just the ends of one’s fingers. Discuss the induced charge and explain why this is done.

31. Tollbooth stations on roadways and bridges usually have a piece of wire stuck in the pavement before them that will touch a car as it approaches. Why is this done?

32. Suppose a woman carries an excess charge. To maintain her charged status can she be standing on ground wearing just any pair of shoes? How would you discharge her? What are the consequences if she simply walks away?

Problems & Exercises

18.1 Static Electricity and Charge: Conservation of Charge

33. Common static electricity involves charges ranging from nanocoulombs to microcoulombs. (a) How many electrons are needed to form a charge of -2.00 nC (b) How many electrons must be removed from a neutral object to leave a net charge of $0.500\ \mu\text{C}$?

34. If 1.80×10^{20} electrons move through a pocket calculator during a full day's operation, how many coulombs of charge moved through it?

35. To start a car engine, the car battery moves 3.75×10^{21} electrons through the starter motor. How many coulombs of charge were moved?

36. A certain lightning bolt moves 40.0 C of charge. How many fundamental units of charge $|q_e|$ is this?

18.2 Conductors and Insulators

37. Suppose a speck of dust in an electrostatic precipitator has 1.0000×10^{12} protons in it and has a net charge of –5.00 nC (a very large charge for a small speck). How many electrons does it have?

38. An amoeba has 1.00×10^{16} protons and a net charge of 0.300 pC. (a) How many fewer electrons are there than protons? (b) If you paired them up, what fraction of the protons would have no electrons?

39. A 50.0 g ball of copper has a net charge of $2.00\ \mu\text{C}$. What fraction of the copper's electrons has been removed? (Each copper atom has 29 protons, and copper has an atomic mass of 63.5.)

40. What net charge would you place on a 100 g piece of sulfur if you put an extra electron on 1 in 10^{12} of its atoms? (Sulfur has an atomic mass of 32.1.)

41. How many coulombs of positive charge are there in 4.00 kg of plutonium, given its atomic mass is 244 and that each plutonium atom has 94 protons?

18.3 Coulomb's Law

42. What is the repulsive force between two pith balls that are 8.00 cm apart and have equal charges of – 30.0 nC?

43. (a) How strong is the attractive force between a glass rod with a $0.700\ \mu\text{C}$ charge and a silk cloth with a $-0.600\ \mu\text{C}$ charge, which are 12.0 cm apart, using the approximation that they act like point charges? (b) Discuss how the answer to this problem might be affected if the charges are distributed over some area and do not act like point charges.

44. Two point charges exert a 5.00 N force on each other. What will the force become if the distance between them is increased by a factor of three?

45. Two point charges are brought closer together, increasing the force between them by a factor of 25. By what factor was their separation decreased?

46. How far apart must two point charges of 75.0 nC (typical of static electricity) be to have a force of 1.00 N between them?

47. If two equal charges each of 1 C each are separated in air by a distance of 1 km, what is the magnitude of the force acting between them? You will see that even at a distance as large as 1 km, the repulsive force is substantial because 1 C is a very significant amount of charge.

48. A test charge of $+2\ \mu\text{C}$ is placed halfway between a charge of $+6\ \mu\text{C}$ and another of $+4\ \mu\text{C}$ separated by 10 cm. (a) What is the magnitude of the force on the test charge? (b) What is the direction of this force (away from or toward the $+6\ \mu\text{C}$ charge)?

49. Bare free charges do not remain stationary when close together. To illustrate this, calculate the acceleration of two isolated protons separated by 2.00 nm (a typical distance between gas atoms). Explicitly show how you follow the steps in the Problem-Solving Strategy for electrostatics.

50. (a) By what factor must you change the distance between two point charges to change the force between them by a factor of 10? (b) Explain how the distance can either increase or decrease by this factor and still cause a factor of 10 change in the force.

51. Suppose you have a total charge q_{tot} that you can split in any manner. Once split, the separation distance is fixed. How do you split the charge to achieve the greatest force?

52. (a) Common transparent tape becomes charged when pulled from a dispenser. If one piece is placed above another, the repulsive force can be great enough to support the top piece's weight. Assuming equal point charges (only an approximation), calculate the magnitude of the charge if electrostatic force is great enough to support the weight of a 10.0 mg piece of tape held 1.00 cm above another. (b) Discuss whether the magnitude of this charge is consistent with what is typical of static electricity.

53. (a) Find the ratio of the electrostatic to gravitational force between two electrons. (b) What is this ratio for two protons? (c) Why is the ratio different for electrons and protons?

54. At what distance is the electrostatic force between two protons equal to the weight of one proton?

55. A certain five cent coin contains 5.00 g of nickel. What fraction of the nickel atoms' electrons, removed and placed 1.00 m above it, would support the weight of this coin? The atomic mass of nickel is 58.7, and each nickel atom contains 28 electrons and 28 protons.

56. (a) Two point charges totaling $8.00\ \mu\text{C}$ exert a repulsive force of 0.150 N on one another when separated by 0.500 m. What is the charge on each? (b) What is the charge on each if the force is attractive?

57. Point charges of $5.00\ \mu\text{C}$ and $-3.00\ \mu\text{C}$ are placed 0.250 m apart. (a) Where can a third charge be placed so that the net force on it is zero? (b) What if both charges are positive?

58. Two point charges q_1 and q_2 are 3.00 m apart, and their total charge is $20\ \mu\text{C}$. (a) If the force of repulsion between them is 0.075N, what are magnitudes of the two charges? (b) If one charge attracts the other with a force of 0.525N, what are the magnitudes of the two charges? Note that you may need to solve a quadratic equation to reach your answer.

18.4 Electric Field: Concept of a Field Revisited

59. What is the magnitude and direction of an electric field that exerts a $2.00\times10^{-5}\text{ N}$ upward force on a $-1.75\ \mu\text{C}$ charge?

60. What is the magnitude and direction of the force exerted on a $3.50\ \mu\text{C}$ charge by a 250 N/C electric field that points due east?

61. Calculate the magnitude of the electric field 2.00 m from a point charge of 5.00 mC (such as found on the terminal of a Van de Graaff).

62. (a) What magnitude point charge creates a 10,000 N/C electric field at a distance of 0.250 m? (b) How large is the field at 10.0 m?

63. Calculate the initial (from rest) acceleration of a proton in a $5.00\times10^{6}\text{ N/C}$ electric field (such as created by a research Van de Graaff). Explicitly show how you follow the steps in the Problem-Solving Strategy for electrostatics.

64. (a) Find the direction and magnitude of an electric field that exerts a $4.80\times10^{-17}\text{ N}$ westward force on an electron. (b) What magnitude and direction force does this field exert on a proton?

18.5 Electric Field Lines: Multiple Charges

65. (a) Sketch the electric field lines near a point charge $+q$. (b) Do the same for a point charge $-3.00q$.

66. Sketch the electric field lines a long distance from the charge distributions shown in **Figure 18.26** (a) and (b)

67. **Figure 18.47** shows the electric field lines near two charges q_1 and q_2. What is the ratio of their magnitudes? (b) Sketch the electric field lines a long distance from the charges shown in the figure.

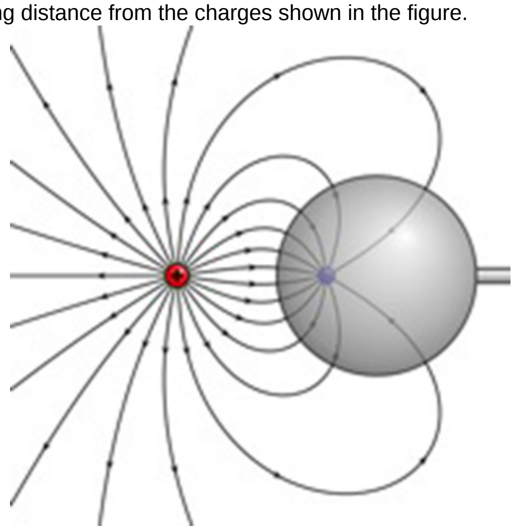


Figure 18.47 The electric field near two charges.

68. Sketch the electric field lines in the vicinity of two opposite charges, where the negative charge is three times greater in magnitude than the positive. (See **Figure 18.47** for a similar situation).

18.7 Conductors and Electric Fields in Static Equilibrium

69. Sketch the electric field lines in the vicinity of the conductor in **Figure 18.48** given the field was originally uniform and parallel to the object's long axis. Is the resulting field small near the long side of the object?

Figure 18.48

70. Sketch the electric field lines in the vicinity of the conductor in **Figure 18.49** given the field was originally uniform and parallel to the object's long axis. Is the resulting field small near the long side of the object?

Figure 18.49

71. Sketch the electric field between the two conducting plates shown in **Figure 18.50**, given the top plate is positive and an equal amount of negative charge is on the bottom plate. Be certain to indicate the distribution of charge on the plates.

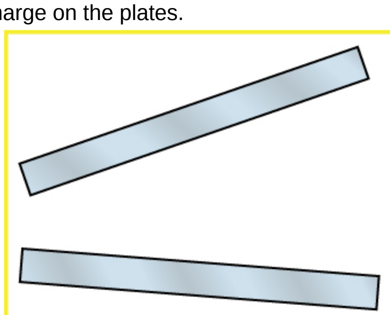


Figure 18.50

72. Sketch the electric field lines in the vicinity of the charged insulator in **Figure 18.51** noting its nonuniform charge distribution.

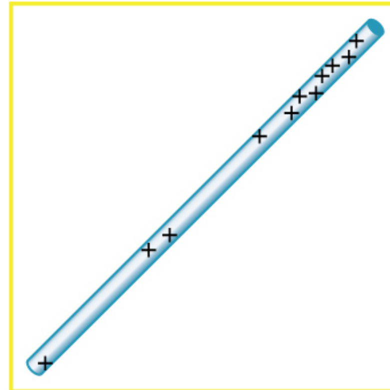


Figure 18.51 A charged insulating rod such as might be used in a classroom demonstration.

73. What is the force on the charge located at $x = 8.00 \text{ cm}$ in **Figure 18.52**(a) given that $q = 1.00 \ \mu\text{C}$?

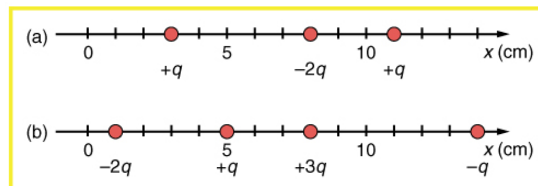


Figure 18.52 (a) Point charges located at 3.00, 8.00, and 11.0 cm along the *x*-axis. (b) Point charges located at 1.00, 5.00, 8.00, and 14.0 cm along the *x*-axis.

74. (a) Find the total electric field at $x = 1.00 \text{ cm}$ in **Figure 18.52**(b) given that $q = 5.00 \text{ nC}$. (b) Find the total electric field at $x = 11.00 \text{ cm}$ in **Figure 18.52**(b). (c) If the charges are allowed to move and eventually be brought to rest by friction, what will the final charge configuration be? (That is, will there be a single charge, double charge, etc., and what will its value(s) be?)

75. (a) Find the electric field at $x = 5.00 \text{ cm}$ in **Figure 18.52**(a), given that $q = 1.00 \ \mu\text{C}$. (b) At what position between 3.00 and 8.00 cm is the total electric field the same as that for $-2q$ alone? (c) Can the electric field be zero anywhere between 0.00 and 8.00 cm? (d) At very large positive or negative values of *x,* the electric field approaches zero in both (a) and (b). In which does it most rapidly approach zero and why? (e) At what position to the right of 11.0 cm is the total electric field zero, other than at infinity? (Hint: A graphing calculator can yield considerable insight in this problem.)

76. (a) Find the total Coulomb force on a charge of 2.00 nC located at $x = 4.00 \text{ cm}$ in **Figure 18.52** (b), given that $q = 1.00 \ \mu\text{C}$. (b) Find the *x*-position at which the electric field is zero in **Figure 18.52** (b).

77. Using the symmetry of the arrangement, determine the direction of the force on q in the figure below, given that $q_a = q_b = +7.50 \ \mu\text{C}$ and $q_c = q_d = -7.50 \ \mu\text{C}$. (b) Calculate the magnitude of the force on the charge q , given that the square is 10.0 cm on a side and $q = 2.00 \ \mu\text{C}$.

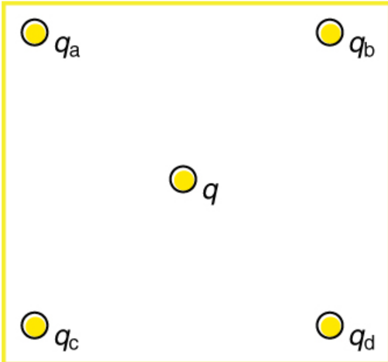


Figure 18.53

78. (a) Using the symmetry of the arrangement, determine the direction of the electric field at the center of the square in Figure 18.53, given that $q_a = q_b = -1.00\ \mu\text{C}$ and $q_c = q_d = +1.00\ \mu\text{C}$. (b) Calculate the magnitude of the electric field at the location of q, given that the square is 5.00 cm on a side.

79. Find the electric field at the location of q_a in Figure 18.53 given that $q_b = q_c = q_d = +2.00\ \text{nC}$, $q = -1.00\ \text{nC}$, and the square is 20.0 cm on a side.

80. Find the total Coulomb force on the charge q in Figure 18.53, given that $q = 1.00\ \mu\text{C}$, $q_a = 2.00\ \mu\text{C}$, $q_b = -3.00\ \mu\text{C}$, $q_c = -4.00\ \mu\text{C}$, and $q_d = +1.00\ \mu\text{C}$. The square is 50.0 cm on a side.

81. (a) Find the electric field at the location of q_a in Figure 18.54, given that $q_\text{b} = +10.00\ \mu\text{C}$ and $q_\text{c} = -5.00\ \mu\text{C}$. (b) What is the force on q_a, given that $q_\text{a} = +1.50\ \text{nC}$?

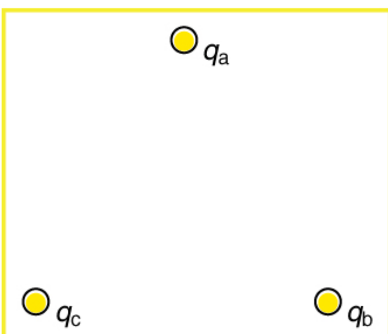


Figure 18.54 Point charges located at the corners of an equilateral triangle 25.0 cm on a side.

82. (a) Find the electric field at the center of the triangular configuration of charges in Figure 18.54, given that $q_a = +2.50\ \text{nC}$, $q_b = -8.00\ \text{nC}$, and $q_c = +1.50\ \text{nC}$. (b) Is there any combination of charges, other than $q_a = q_b = q_c$, that will produce a zero strength electric field at the center of the triangular configuration?

18.8 Applications of Electrostatics

83. (a) What is the electric field 5.00 m from the center of the terminal of a Van de Graaff with a 3.00 mC charge, noting that the field is equivalent to that of a point charge at the center of the terminal? (b) At this distance, what force does the field exert on a $2.00\ \mu\text{C}$ charge on the Van de Graaff's belt?

84. (a) What is the direction and magnitude of an electric field that supports the weight of a free electron near the surface of Earth? (b) Discuss what the small value for this field implies regarding the relative strength of the gravitational and electrostatic forces.

85. A simple and common technique for accelerating electrons is shown in Figure 18.55, where there is a uniform electric field between two plates. Electrons are released, usually from a hot filament, near the negative plate, and there is a small hole in the positive plate that allows the electrons to continue moving. (a) Calculate the acceleration of the electron if the field strength is $2.50\times10^4\ \text{N/C}$. (b) Explain why the electron will not be pulled back to the positive plate once it moves through the hole.

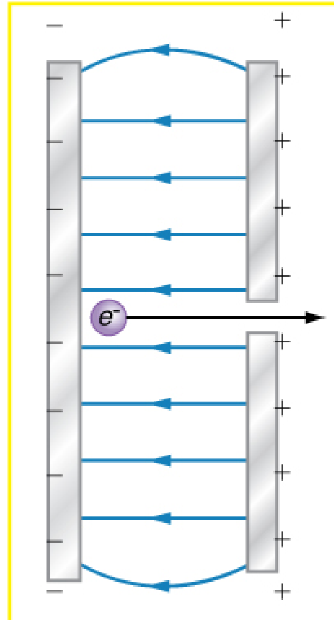


Figure 18.55 Parallel conducting plates with opposite charges on them create a relatively uniform electric field used to accelerate electrons to the right. Those that go through the hole can be used to make a TV or computer screen glow or to produce X-rays.

86. Earth has a net charge that produces an electric field of approximately 150 N/C downward at its surface. (a) What is the magnitude and sign of the excess charge, noting the electric field of a conducting sphere is equivalent to a point charge at its center? (b) What acceleration will the field produce on a free electron near Earth's surface? (c) What mass object with a single extra electron will have its weight supported by this field?

87. Point charges of $25.0\ \mu\text{C}$ and $45.0\ \mu\text{C}$ are placed 0.500 m apart. (a) At what point along the line between them is the electric field zero? (b) What is the electric field halfway between them?

88. What can you say about two charges q_1 and q_2, if the electric field one-fourth of the way from q_1 to q_2 is zero?

89. Integrated Concepts

Calculate the angular velocity ω of an electron orbiting a proton in the hydrogen atom, given the radius of the orbit is $0.530\times10^{-10}\ \text{m}$. You may assume that the proton is stationary and the centripetal force is supplied by Coulomb attraction.

90. Integrated Concepts

An electron has an initial velocity of $5.00\times10^6\ \text{m/s}$ in a uniform $2.00\times10^5\ \text{N/C}$ strength electric field. The field accelerates the electron in the direction opposite to its initial velocity. (a) What is the direction of the electric field? (b) How far does the electron travel before coming to rest? (c) How long does it take the electron to come to rest? (d) What is the electron's velocity when it returns to its starting point?

91. Integrated Concepts

The practical limit to an electric field in air is about $3.00\times10^6\ \text{N/C}$. Above this strength, sparking takes place because air begins to ionize and charges flow, reducing the field. (a) Calculate the distance a free proton must travel in this field to reach 3.00% of the speed of light, starting from rest. (b) Is this practical in air, or must it occur in a vacuum?

92. Integrated Concepts

A 5.00 g charged insulating ball hangs on a 30.0 cm long string in a uniform horizontal electric field as shown in **Figure 18.56**. Given the charge on the ball is $1.00\ \mu C$, find the strength of the field.

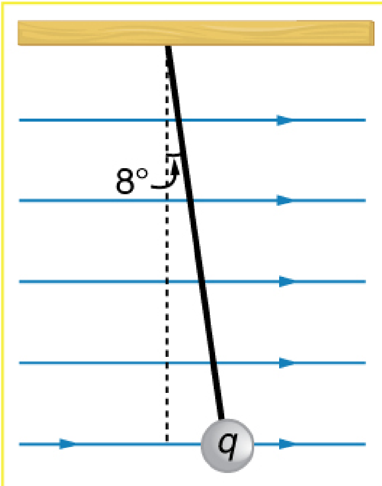


Figure 18.56 A horizontal electric field causes the charged ball to hang at an angle of 8.00^{o}.

93. Integrated Concepts

Figure 18.57 shows an electron passing between two charged metal plates that create an 100 N/C vertical electric field perpendicular to the electron's original horizontal velocity. (These can be used to change the electron's direction, such as in an oscilloscope.) The initial speed of the electron is $3.00\times10^6\ \text{m/s}$, and the horizontal distance it travels in the uniform field is 4.00 cm. (a) What is its vertical deflection? (b) What is the vertical component of its final velocity? (c) At what angle does it exit? Neglect any edge effects.

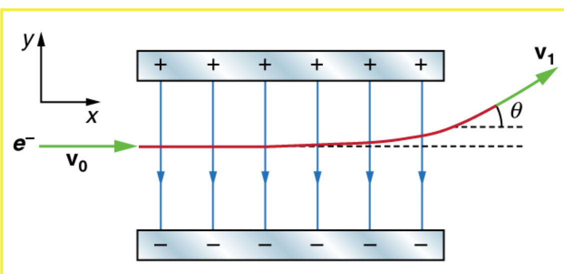


Figure 18.57

94. Integrated Concepts

The classic Millikan oil drop experiment was the first to obtain an accurate measurement of the charge on an electron. In it, oil drops were suspended against the gravitational force by a vertical electric field. (See **Figure 18.58**.) Given the oil drop to be $1.00\ \mu\text{m}$ in radius and have a density of $920\ \text{kg/m}^3$: (a) Find the weight of the drop. (b) If the drop has a single excess electron, find the electric field strength needed to balance its weight.

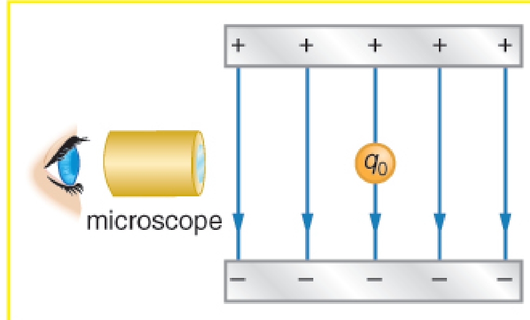


Figure 18.58 In the Millikan oil drop experiment, small drops can be suspended in an electric field by the force exerted on a single excess electron. Classically, this experiment was used to determine the electron charge q_e by measuring the electric field and mass of the drop.

95. Integrated Concepts

(a) In **Figure 18.59**, four equal charges q lie on the corners of a square. A fifth charge Q is on a mass m directly above the center of the square, at a height equal to the length d of one side of the square. Determine the magnitude of q in terms of Q , m , and d , if the Coulomb force is to equal the weight of m . (b) Is this equilibrium stable or unstable? Discuss.

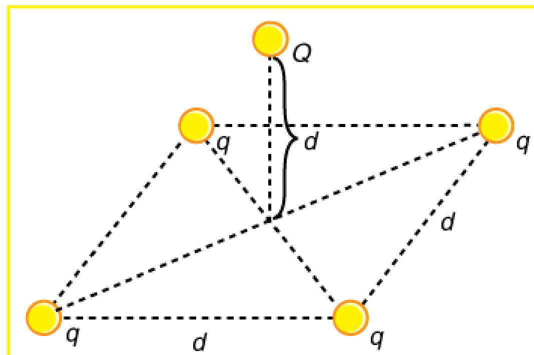


Figure 18.59 Four equal charges on the corners of a horizontal square support the weight of a fifth charge located directly above the center of the square.

96. Unreasonable Results

(a) Calculate the electric field strength near a 10.0 cm diameter conducting sphere that has 1.00 C of excess charge on it. (b) What is unreasonable about this result? (c) Which assumptions are responsible?

97. Unreasonable Results

(a) Two 0.500 g raindrops in a thunderhead are 1.00 cm apart when they each acquire 1.00 mC charges. Find their acceleration. (b) What is unreasonable about this result? (c) Which premise or assumption is responsible?

98. Unreasonable Results

A wrecking yard inventor wants to pick up cars by charging a 0.400 m diameter ball and inducing an equal and opposite charge on the car. If a car has a 1000 kg mass and the ball is to be able to lift it from a distance of 1.00 m: (a) What minimum charge must be used? (b) What is the electric field near the surface of the ball? (c) Why are these results unreasonable? (d) Which premise or assumption is responsible?

99. Construct Your Own Problem

Consider two insulating balls with evenly distributed equal and opposite charges on their surfaces, held with a certain distance between the centers of the balls. Construct a problem in which you calculate the electric field (magnitude and direction) due to the balls at various points along a line running through the centers of the balls and extending to infinity on either side. Choose interesting points and comment on the meaning of the field at those points. For example, at what points might the field be just that due to one ball and where does the field become negligibly small? Among the things to be considered are the magnitudes of the charges and the distance between the centers of the balls. Your instructor may wish for you to consider the electric field off axis or for a more complex array of charges, such as those in a water molecule.

100. Construct Your Own Problem

Consider identical spherical conducting space ships in deep space where gravitational fields from other bodies are negligible compared to the gravitational attraction between the ships. Construct a problem in which you place identical excess charges on the space ships to exactly counter their gravitational attraction. Calculate the amount of excess charge needed. Examine whether that charge depends on the distance between the centers of the ships, the masses of the ships, or any other factors. Discuss whether this would be an easy, difficult, or even impossible thing to do in practice.

19 ELECTRIC POTENTIAL AND ELECTRIC FIELD

Figure 19.1 Automated external defibrillator unit (AED) (credit: U.S. Defense Department photo/Tech. Sgt. Suzanne M. Day)

Learning Objectives

Introduction to Electric Potential and Electric Energy

In **Electric Charge and Electric Field**, we just scratched the surface (or at least rubbed it) of electrical phenomena. Two of the most familiar aspects of electricity are its energy and *voltage*. We know, for example, that great amounts of electrical energy can be stored in batteries, are transmitted cross-country through power lines, and may jump from clouds to explode the sap of trees. In a similar manner, at molecular levels, *ions* cross cell membranes and transfer information. We also know about voltages associated with electricity. Batteries are typically a few volts, the outlets in your home produce 120 volts, and power lines can be as high as hundreds of thousands of volts. But energy and voltage are not the same thing. A motorcycle battery, for example, is small and would not be very successful in replacing the much larger car battery, yet each has the same voltage. In this chapter, we shall examine the relationship between voltage and electrical energy and begin to explore some of the many applications of electricity.

19.1 Electric Potential Energy: Potential Difference

When a free positive charge q is accelerated by an electric field, such as shown in **Figure 19.2**, it is given kinetic energy. The process is analogous to an object being accelerated by a gravitational field. It is as if the charge is going down an electrical hill where its electric potential energy is converted to kinetic energy. Let us explore the work done on a charge q by the electric field in this process, so that we may develop a definition of electric potential energy.

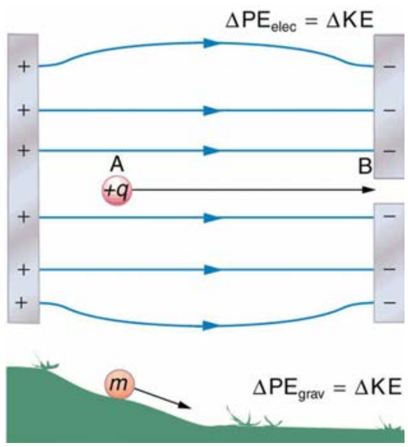


Figure 19.2 A charge accelerated by an electric field is analogous to a mass going down a hill. In both cases potential energy is converted to another form. Work is done by a force, but since this force is conservative, we can write $W = -\Delta\text{PE}$.

The electrostatic or Coulomb force is conservative, which means that the work done on q is independent of the path taken. This is exactly analogous to the gravitational force in the absence of dissipative forces such as friction. When a force is conservative, it is possible to define a potential energy associated with the force, and it is usually easier to deal with the potential energy (because it depends only on position) than to calculate the work directly.

We use the letters PE to denote electric potential energy, which has units of joules (J). The change in potential energy, ΔPE , is crucial, since the work done by a conservative force is the negative of the change in potential energy; that is, $W = -\Delta\text{PE}$. For example, work W done to accelerate a positive charge from rest is positive and results from a loss in PE, or a negative ΔPE . There must be a minus sign in front of ΔPE to make W positive. PE can be found at any point by taking one point as a reference and calculating the work needed to move a charge to the other point.

Potential Energy

$W = -\Delta\text{PE}$. For example, work W done to accelerate a positive charge from rest is positive and results from a loss in PE, or a negative $\Delta\text{PE}.$ There must be a minus sign in front of ΔPE to make W positive. PE can be found at any point by taking one point as a reference and calculating the work needed to move a charge to the other point.

Gravitational potential energy and electric potential energy are quite analogous. Potential energy accounts for work done by a conservative force and gives added insight regarding energy and energy transformation without the necessity of dealing with the force directly. It is much more common, for example, to use the concept of voltage (related to electric potential energy) than to deal with the Coulomb force directly.

Calculating the work directly is generally difficult, since $W = Fd \cos\theta$ and the direction and magnitude of F can be complex for multiple charges, for odd-shaped objects, and along arbitrary paths. But we do know that, since $F = qE$, the work, and hence ΔPE , is proportional to the test charge $q.$ To have a physical quantity that is independent of test charge, we define **electric potential** V (or simply potential, since electric is understood) to be the potential energy per unit charge:

$$V = \frac{\text{PE}}{q}. \tag{19.1}$$

Electric Potential

This is the electric potential energy per unit charge.

$$V = \frac{\text{PE}}{q} \tag{19.2}$$

Since PE is proportional to q , the dependence on q cancels. Thus V does not depend on q . The change in potential energy ΔPE is crucial, and so we are concerned with the difference in potential or potential difference ΔV between two points, where

$$\Delta V = V_\text{B} - V_\text{A} = \frac{\Delta\text{PE}}{q}. \tag{19.3}$$

The **potential difference** between points A and B, $V_\text{B} - V_\text{A}$, is thus defined to be the change in potential energy of a charge q moved from A to B, divided by the charge. Units of potential difference are joules per coulomb, given the name volt (V) after Alessandro Volta.

$$1\text{ V} = 1\,\frac{\text{J}}{\text{C}} \tag{19.4}$$

Potential Difference

The potential difference between points A and B, V_B - V_A, is defined to be the change in potential energy of a charge q moved from A to B, divided by the charge. Units of potential difference are joules per coulomb, given the name volt (V) after Alessandro Volta.

$$1 \text{ V} = 1 \frac{\text{J}}{\text{C}} \tag{19.5}$$

The familiar term **voltage** is the common name for potential difference. Keep in mind that whenever a voltage is quoted, it is understood to be the potential difference between two points. For example, every battery has two terminals, and its voltage is the potential difference between them. More fundamentally, the point you choose to be zero volts is arbitrary. This is analogous to the fact that gravitational potential energy has an arbitrary zero, such as sea level or perhaps a lecture hall floor.

In summary, the relationship between potential difference (or voltage) and electrical potential energy is given by

$$\Delta V = \frac{\Delta\text{PE}}{q} \text{ and } \Delta\text{PE} = q\Delta V. \tag{19.6}$$

Potential Difference and Electrical Potential Energy

The relationship between potential difference (or voltage) and electrical potential energy is given by

$$\Delta V = \frac{\Delta\text{PE}}{q} \text{ and } \Delta\text{PE} = q\Delta V. \tag{19.7}$$

The second equation is equivalent to the first.

Voltage is not the same as energy. Voltage is the energy per unit charge. Thus a motorcycle battery and a car battery can both have the same voltage (more precisely, the same potential difference between battery terminals), yet one stores much more energy than the other since $\Delta\text{PE} = q\Delta V$. The car battery can move more charge than the motorcycle battery, although both are 12 V batteries.

Example 19.1 Calculating Energy

Suppose you have a 12.0 V motorcycle battery that can move 5000 C of charge, and a 12.0 V car battery that can move 60,000 C of charge. How much energy does each deliver? (Assume that the numerical value of each charge is accurate to three significant figures.)

Strategy

To say we have a 12.0 V battery means that its terminals have a 12.0 V potential difference. When such a battery moves charge, it puts the charge through a potential difference of 12.0 V, and the charge is given a change in potential energy equal to $\Delta\text{PE} = q\Delta V$.

So to find the energy output, we multiply the charge moved by the potential difference.

Solution

For the motorcycle battery, $q = 5000 \text{ C}$ and $\Delta V = 12.0 \text{ V}$. The total energy delivered by the motorcycle battery is

$$\begin{aligned} \Delta\text{PE}_{\text{cycle}} &= (5000 \text{ C})(12.0 \text{ V}) \\ &= (5000 \text{ C})(12.0 \text{ J/C}) \\ &= 6.00\times10^4 \text{ J.} \end{aligned} \tag{19.8}$$

Similarly, for the car battery, $q = 60{,}000 \text{ C}$ and

$$\begin{aligned} \Delta\text{PE}_{\text{car}} &= (60{,}000 \text{ C})(12.0 \text{ V}) \\ &= 7.20\times10^5 \text{ J.} \end{aligned} \tag{19.9}$$

Discussion

While voltage and energy are related, they are not the same thing. The voltages of the batteries are identical, but the energy supplied by each is quite different. Note also that as a battery is discharged, some of its energy is used internally and its terminal voltage drops, such as when headlights dim because of a low car battery. The energy supplied by the battery is still calculated as in this example, but not all of the energy is available for external use.

Note that the energies calculated in the previous example are absolute values. The change in potential energy for the battery is negative, since it loses energy. These batteries, like many electrical systems, actually move negative charge—electrons in particular. The batteries repel electrons from their negative terminals (A) through whatever circuitry is involved and attract them to their positive terminals (B) as shown in Figure 19.3. The change in potential is $\Delta V = V_B - V_A = +12 \text{ V}$ and the charge q is negative, so that $\Delta\text{PE} = q\Delta V$ is negative, meaning the potential energy of the battery has decreased when q has moved from A to B.

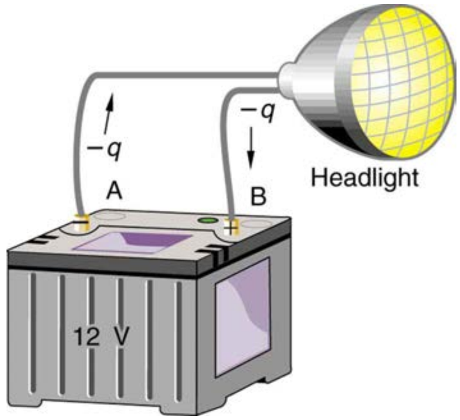


Figure 19.3 A battery moves negative charge from its negative terminal through a headlight to its positive terminal. Appropriate combinations of chemicals in the battery separate charges so that the negative terminal has an excess of negative charge, which is repelled by it and attracted to the excess positive charge on the other terminal. In terms of potential, the positive terminal is at a higher voltage than the negative. Inside the battery, both positive and negative charges move.

Example 19.2 How Many Electrons Move through a Headlight Each Second?

When a 12.0 V car battery runs a single 30.0 W headlight, how many electrons pass through it each second?

Strategy

To find the number of electrons, we must first find the charge that moved in 1.00 s. The charge moved is related to voltage and energy through the equation $\Delta\text{PE} = q\Delta V$. A 30.0 W lamp uses 30.0 joules per second. Since the battery loses energy, we have $\Delta\text{PE} = –30.0\text{ J}$ and, since the electrons are going from the negative terminal to the positive, we see that $\Delta V = +12.0\text{ V}$.

Solution

To find the charge q moved, we solve the equation $\Delta\text{PE} = q\Delta V$:

$$q = \frac{\Delta\text{PE}}{\Delta V}. \tag{19.10}$$

Entering the values for ΔPE and ΔV , we get

$$q = \frac{–30.0\text{ J}}{+12.0\text{ V}} = \frac{–30.0\text{ J}}{+12.0\text{ J/C}} = –2.50\text{ C}. \tag{19.11}$$

The number of electrons n_e is the total charge divided by the charge per electron. That is,

$$n_e = \frac{–2.50\text{ C}}{–1.60\times10^{-19}\text{ C/e}^-} = 1.56\times10^{19}\text{ electrons}. \tag{19.12}$$

Discussion

This is a very large number. It is no wonder that we do not ordinarily observe individual electrons with so many being present in ordinary systems. In fact, electricity had been in use for many decades before it was determined that the moving charges in many circumstances were negative. Positive charge moving in the opposite direction of negative charge often produces identical effects; this makes it difficult to determine which is moving or whether both are moving.

The Electron Volt

The energy per electron is very small in macroscopic situations like that in the previous example—a tiny fraction of a joule. But on a submicroscopic scale, such energy per particle (electron, proton, or ion) can be of great importance. For example, even a tiny fraction of a joule can be great enough for these particles to destroy organic molecules and harm living tissue. The particle may do its damage by direct collision, or it may create harmful x rays, which can also inflict damage. It is useful to have an energy unit related to submicroscopic effects. Figure 19.4 shows a situation related to the definition of such an energy unit. An electron is accelerated between two charged metal plates as it might be in an old-model television tube or oscilloscope. The electron is given kinetic energy that is later converted to another form—light in the television tube, for example. (Note that downhill for the electron is uphill for a positive charge.) Since energy is related to voltage by $\Delta\text{PE} = q\Delta V,$ we can think of the joule as a coulomb-volt.

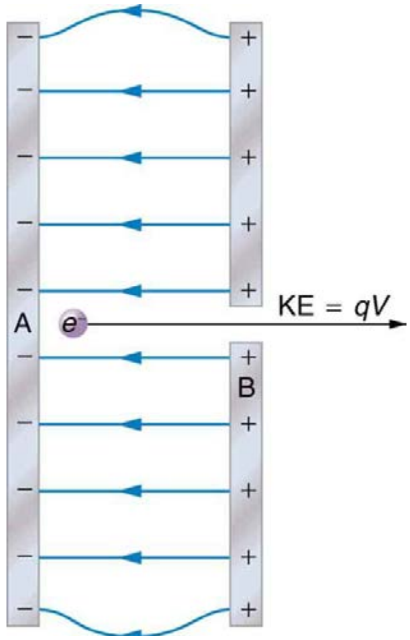


Figure 19.4 A typical electron gun accelerates electrons using a potential difference between two metal plates. The energy of the electron in electron volts is numerically the same as the voltage between the plates. For example, a 5000 V potential difference produces 5000 eV electrons.

On the submicroscopic scale, it is more convenient to define an energy unit called the **electron volt** (eV), which is the energy given to a fundamental charge accelerated through a potential difference of 1 V. In equation form,

$$\begin{aligned} 1\text{ eV} &= \left(1.60\times10^{-19}\text{ C}\right)(1\text{ V}) = \left(1.60\times10^{-19}\text{ C}\right)(1\text{ J/C}) \\ &= 1.60\times10^{-19}\text{ J.} \end{aligned} \tag{19.13}$$

Electron Volt

On the submicroscopic scale, it is more convenient to define an energy unit called the electron volt (eV), which is the energy given to a fundamental charge accelerated through a potential difference of 1 V. In equation form,

$$\begin{aligned} 1\text{ eV} &= \left(1.60\times10^{-19}\text{ C}\right)(1\text{ V}) = \left(1.60\times10^{-19}\text{ C}\right)(1\text{ J/C}) \\ &= 1.60\times10^{-19}\text{ J.} \end{aligned} \tag{19.14}$$

An electron accelerated through a potential difference of 1 V is given an energy of 1 eV. It follows that an electron accelerated through 50 V is given 50 eV. A potential difference of 100,000 V (100 kV) will give an electron an energy of 100,000 eV (100 keV), and so on. Similarly, an ion with a double positive charge accelerated through 100 V will be given 200 eV of energy. These simple relationships between accelerating voltage and particle charges make the electron volt a simple and convenient energy unit in such circumstances.

Connections: Energy Units

The electron volt (eV) is the most common energy unit for submicroscopic processes. This will be particularly noticeable in the chapters on modern physics. Energy is so important to so many subjects that there is a tendency to define a special energy unit for each major topic. There are, for example, calories for food energy, kilowatt-hours for electrical energy, and therms for natural gas energy.

The electron volt is commonly employed in submicroscopic processes—chemical valence energies and molecular and nuclear binding energies are among the quantities often expressed in electron volts. For example, about 5 eV of energy is required to break up certain organic molecules. If a proton is accelerated from rest through a potential difference of 30 kV, it is given an energy of 30 keV (30,000 eV) and it can break up as many as 6000 of these molecules ($30{,}000\text{ eV} \div 5\text{ eV per molecule} = 6000\text{ molecules}$). Nuclear decay energies are on the order of 1 MeV (1,000,000 eV) per event and can, thus, produce significant biological damage.

Conservation of Energy

The total energy of a system is conserved if there is no net addition (or subtraction) of work or heat transfer. For conservative forces, such as the electrostatic force, conservation of energy states that mechanical energy is a constant.

Mechanical energy is the sum of the kinetic energy and potential energy of a system; that is, $\text{KE} + \text{PE} = \text{constant}$. A loss of PE of a charged particle becomes an increase in its KE. Here PE is the electric potential energy. Conservation of energy is stated in equation form as

$$\text{KE} + \text{PE} = \text{constant} \tag{19.15}$$

or

$$\text{KE}_\text{i} + \text{PE}_\text{i} = \text{KE}_\text{f} + \text{PE}_\text{f} , \tag{19.16}$$

where i and f stand for initial and final conditions. As we have found many times before, considering energy can give us insights and facilitate problem solving.

Example 19.3 Electrical Potential Energy Converted to Kinetic Energy

Calculate the final speed of a free electron accelerated from rest through a potential difference of 100 V. (Assume that this numerical value is accurate to three significant figures.)

Strategy

We have a system with only conservative forces. Assuming the electron is accelerated in a vacuum, and neglecting the gravitational force (we will check on this assumption later), all of the electrical potential energy is converted into kinetic energy. We can identify the initial and final forms of energy to be $\text{KE}_\text{i} = 0, \text{KE}_\text{f} = ½mv^2, \text{PE}_\text{i} = qV, \text{and PE}_\text{f} = 0.$

Solution

Conservation of energy states that

$$\text{KE}_\text{i} + \text{PE}_\text{i} = \text{KE}_\text{f} + \text{PE}_\text{f}. \tag{19.17}$$

Entering the forms identified above, we obtain

$$qV = \frac{mv^2}{2}. \tag{19.18}$$

We solve this for v :

$$v = \sqrt{\frac{2qV}{m}}. \tag{19.19}$$

Entering values for $q, V, \text{and } m$ gives

$$\begin{aligned} v &= \sqrt{\frac{2\left(-1.60\times10^{-19}\text{ C}\right)(-100\text{ J/C})}{9.11\times10^{-31}\text{ kg}}} \\ &= 5.93\times10^{6}\text{ m/s.} \end{aligned} \tag{19.20}$$

Discussion

Note that both the charge and the initial voltage are negative, as in **Figure 19.4**. From the discussions in **Electric Charge and Electric Field**, we know that electrostatic forces on small particles are generally very large compared with the gravitational force. The large final speed confirms that the gravitational force is indeed negligible here. The large speed also indicates how easy it is to accelerate electrons with small voltages because of their very small mass. Voltages much higher than the 100 V in this problem are typically used in electron guns. Those higher voltages produce electron speeds so great that relativistic effects must be taken into account. That is why a low voltage is considered (accurately) in this example.

19.2 Electric Potential in a Uniform Electric Field

In the previous section, we explored the relationship between voltage and energy. In this section, we will explore the relationship between voltage and electric field. For example, a uniform electric field $\mathbf{E}$ is produced by placing a potential difference (or voltage) ΔV across two parallel metal plates, labeled A and B. (See **Figure 19.5**.) Examining this will tell us what voltage is needed to produce a certain electric field strength; it will also reveal a more fundamental relationship between electric potential and electric field. From a physicist's point of view, either ΔV or $\mathbf{E}$ can be used to describe any charge distribution. ΔV is most closely tied to energy, whereas $\mathbf{E}$ is most closely related to force. ΔV is a **scalar** quantity and has no direction, while $\mathbf{E}$ is a **vector** quantity, having both magnitude and direction. (Note that the magnitude of the electric field strength, a scalar quantity, is represented by E below.) The relationship between ΔV and $\mathbf{E}$ is revealed by calculating the work done by the force in moving a charge from point A to point B. But, as noted in **Electric Potential Energy: Potential Difference**, this is complex for arbitrary charge distributions, requiring calculus. We therefore look at a uniform electric field as an interesting special case.

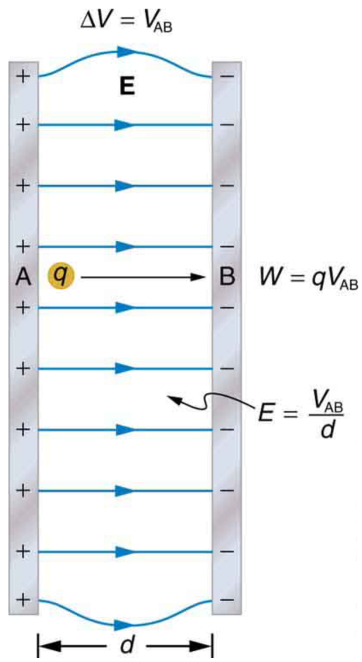


Figure 19.5 The relationship between V and E for parallel conducting plates is $E = V/d$. (Note that $\Delta V = V_{\text{AB}}$ in magnitude. For a charge that is moved from plate A at higher potential to plate B at lower potential, a minus sign needs to be included as follows: $-\Delta V = V_{\text{A}} - V_{\text{B}} = V_{\text{AB}}$. See the text for details.)

The work done by the electric field in **Figure 19.5** to move a positive charge q from A, the positive plate, higher potential, to B, the negative plate, lower potential, is

$$W = -\Delta\text{PE} = -q\Delta V. \tag{19.21}$$

The potential difference between points A and B is

$$-\Delta V = -(V_{\text{B}} - V_{\text{A}}) = V_{\text{A}} - V_{\text{B}} = V_{\text{AB}}. \tag{19.22}$$

Entering this into the expression for work yields

$$W = qV_{\text{AB}}. \tag{19.23}$$

Work is $W = Fd\cos\theta$; here $\cos\theta = 1$, since the path is parallel to the field, and so $W = Fd$. Since $F = qE$, we see that $W = qEd$. Substituting this expression for work into the previous equation gives

$$qEd = qV_{\text{AB}}. \tag{19.24}$$

The charge cancels, and so the voltage between points A and B is seen to be

$$\left.\begin{array}{l} V_{\text{AB}} = Ed \\ E = \dfrac{V_{\text{AB}}}{d} \end{array}\right\}\text{(uniform } E\text{ - field only),} \tag{19.25}$$

where d is the distance from A to B, or the distance between the plates in **Figure 19.5**. Note that the above equation implies the units for electric field are volts per meter. We already know the units for electric field are newtons per coulomb; thus the following relation among units is valid:

$$1\text{ N/C} = 1\text{ V/m}. \tag{19.26}$$

Voltage between Points A and B

$$\left.\begin{array}{l} V_{\text{AB}} = Ed \\ E = \dfrac{V_{\text{AB}}}{d} \end{array}\right\}\text{(uniform } E\text{ - field only),} \tag{19.27}$$

where d is the distance from A to B, or the distance between the plates.

Example 19.4 What Is the Highest Voltage Possible between Two Plates?

Dry air will support a maximum electric field strength of about 3.0×10^{6} V/m. Above that value, the field creates enough ionization in the air to make the air a conductor. This allows a discharge or spark that reduces the field. What, then, is the maximum voltage between two parallel conducting plates separated by 2.5 cm of dry air?

Strategy

We are given the maximum electric field E between the plates and the distance d between them. The equation $V_{\text{AB}} = Ed$ can thus be used to calculate the maximum voltage.

Solution

The potential difference or voltage between the plates is

$$\text{V}_{\text{AB}} = Ed. \tag{19.28}$$

Entering the given values for E and d gives

$$V_{\text{AB}} = (3.0\times10^6\text{ V/m})(0.025\text{ m}) = 7.5\times10^4\text{ V} \tag{19.29}$$

or

$$V_{\text{AB}} = 75\text{ kV}. \tag{19.30}$$

(The answer is quoted to only two digits, since the maximum field strength is approximate.)

Discussion

One of the implications of this result is that it takes about 75 kV to make a spark jump across a 2.5 cm (1 in.) gap, or 150 kV for a 5 cm spark. This limits the voltages that can exist between conductors, perhaps on a power transmission line. A smaller voltage will cause a spark if there are points on the surface, since points create greater fields than smooth surfaces. Humid air breaks down at a lower field strength, meaning that a smaller voltage will make a spark jump through humid air. The largest voltages can be built up, say with static electricity, on dry days.

Figure 19.6 A spark chamber is used to trace the paths of high-energy particles. Ionization created by the particles as they pass through the gas between the plates allows a spark to jump. The sparks are perpendicular to the plates, following electric field lines between them. The potential difference between adjacent plates is not high enough to cause sparks without the ionization produced by particles from accelerator experiments (or cosmic rays). (credit: Daderot, Wikimedia Commons)

Example 19.5 Field and Force inside an Electron Gun

(a) An electron gun has parallel plates separated by 4.00 cm and gives electrons 25.0 keV of energy. What is the electric field strength between the plates? (b) What force would this field exert on a piece of plastic with a $0.500\ \mu\text{C}$ charge that gets between the plates?

Strategy

Since the voltage and plate separation are given, the electric field strength can be calculated directly from the expression $E = \frac{V_{\text{AB}}}{d}$. Once the electric field strength is known, the force on a charge is found using $\mathbf{F} = q\,\mathbf{E}$. Since the electric field is in only one direction, we can write this equation in terms of the magnitudes, $F = q\,E$.

Solution for (a)

The expression for the magnitude of the electric field between two uniform metal plates is

$$E = \frac{V_{\text{AB}}}{d}. \tag{19.31}$$

Since the electron is a single charge and is given 25.0 keV of energy, the potential difference must be 25.0 kV. Entering this value for V_{AB} and the plate separation of 0.0400 m, we obtain

$$E = \frac{25.0\text{ kV}}{0.0400\text{ m}} = 6.25\times10^5\text{ V/m}. \tag{19.32}$$

Solution for (b)

The magnitude of the force on a charge in an electric field is obtained from the equation

$$F = qE. \tag{19.33}$$

Substituting known values gives

$$F = (0.500\times10^{-6}\text{ C})(6.25\times10^5\text{ V/m}) = 0.313\text{ N}. \tag{19.34}$$

Discussion

Note that the units are newtons, since $1\text{ V/m} = 1\text{ N/C}$. The force on the charge is the same no matter where the charge is located between the plates. This is because the electric field is uniform between the plates.

In more general situations, regardless of whether the electric field is uniform, it points in the direction of decreasing potential, because the force on a positive charge is in the direction of $\mathbf{E}$ and also in the direction of lower potential V. Furthermore, the magnitude of $\mathbf{E}$ equals the rate of decrease of V with distance. The faster V decreases over distance, the greater the electric field. In equation form, the general relationship between voltage and electric field is

$$E = -\frac{\Delta V}{\Delta s}, \tag{19.35}$$

where Δs is the distance over which the change in potential, ΔV, takes place. The minus sign tells us that $\mathbf{E}$ points in the direction of decreasing potential. The electric field is said to be the *gradient* (as in grade or slope) of the electric potential.

Relationship between Voltage and Electric Field

In equation form, the general relationship between voltage and electric field is

$$E = -\frac{\Delta V}{\Delta s}, \tag{19.36}$$

where Δs is the distance over which the change in potential, ΔV, takes place. The minus sign tells us that $\mathbf{E}$ points in the direction of decreasing potential. The electric field is said to be the *gradient* (as in grade or slope) of the electric potential.

For continually changing potentials, ΔV and Δs become infinitesimals and differential calculus must be employed to determine the electric field.

19.3 Electrical Potential Due to a Point Charge

Point charges, such as electrons, are among the fundamental building blocks of matter. Furthermore, spherical charge distributions (like on a metal sphere) create external electric fields exactly like a point charge. The electric potential due to a point charge is, thus, a case we need to consider. Using calculus to find the work needed to move a test charge q from a large distance away to a distance of r from a point charge Q, and noting the connection between work and potential $(W = -q\Delta V)$, it can be shown that the *electric potential* V *of a point charge* is

$$V = \frac{kQ}{r}\text{ (Point Charge)}, \tag{19.37}$$

where *k* is a constant equal to $9.0\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2$.

Electric Potential V of a Point Charge

The electric potential V of a point charge is given by

$$V = \frac{kQ}{r}\text{ (Point Charge)}. \tag{19.38}$$

The potential at infinity is chosen to be zero. Thus V for a point charge decreases with distance, whereas $\mathbf{E}$ for a point charge decreases with distance squared:

$$E = \frac{F}{q} = \frac{kQ}{r^2}. \tag{19.39}$$

Recall that the electric potential V is a scalar and has no direction, whereas the electric field $\mathbf{E}$ is a vector. To find the voltage due to a combination of point charges, you add the individual voltages as numbers. To find the total electric field, you must add the individual fields as *vectors*, taking magnitude and direction into account. This is consistent with the fact that V is closely associated with energy, a scalar, whereas $\mathbf{E}$ is closely associated with force, a vector.

Example 19.6 What Voltage Is Produced by a Small Charge on a Metal Sphere?

Charges in static electricity are typically in the nanocoulomb (nC) to microcoulomb (μC) range. What is the voltage 5.00 cm away from the center of a 1-cm diameter metal sphere that has a -3.00 nC static charge?

Strategy

As we have discussed in **Electric Charge and Electric Field**, charge on a metal sphere spreads out uniformly and produces a field like that of a point charge located at its center. Thus we can find the voltage using the equation $V = kQ/r$.

Solution

Entering known values into the expression for the potential of a point charge, we obtain

$$\begin{aligned} V &= k\frac{Q}{r} \\ &= \left(8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(\frac{-3.00\times10^{-9}\text{ C}}{5.00\times10^{-2}\text{ m}}\right) \\ &= -539\text{ V}. \end{aligned} \tag{19.40}$$

Discussion

The negative value for voltage means a positive charge would be attracted from a larger distance, since the potential is lower (more negative) than at larger distances. Conversely, a negative charge would be repelled, as expected.

Example 19.7 What Is the Excess Charge on a Van de Graaff Generator

A demonstration Van de Graaff generator has a 25.0 cm diameter metal sphere that produces a voltage of 100 kV near its surface. (See **Figure 19.7**.) What excess charge resides on the sphere? (Assume that each numerical value here is shown with three significant figures.)

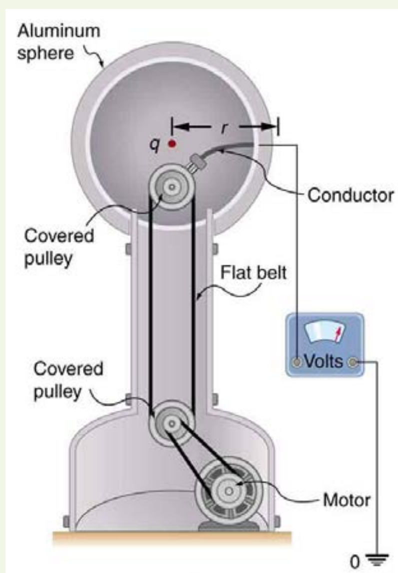


Figure 19.7 The voltage of this demonstration Van de Graaff generator is measured between the charged sphere and ground. Earth's potential is taken to be zero as a reference. The potential of the charged conducting sphere is the same as that of an equal point charge at its center.

Strategy

The potential on the surface will be the same as that of a point charge at the center of the sphere, 12.5 cm away. (The radius of the sphere is 12.5 cm.) We can thus determine the excess charge using the equation

$$V = \frac{kQ}{r}. \tag{19.41}$$

Solution

Solving for Q and entering known values gives

$$\begin{aligned} Q &= \frac{rV}{k} \\ &= \frac{(0.125\text{ m})\left(100\times10^{3}\text{ V}\right)}{8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2} \\ &= 1.39\times10^{-6}\text{ C} = 1.39\ \mu\text{C}. \end{aligned} \tag{19.42}$$

Discussion

This is a relatively small charge, but it produces a rather large voltage. We have another indication here that it is difficult to store isolated charges.

The voltages in both of these examples could be measured with a meter that compares the measured potential with ground potential. Ground potential is often taken to be zero (instead of taking the potential at infinity to be zero). It is the potential difference between two points that is of importance, and very often there is a tacit assumption that some reference point, such as Earth or a very distant point, is at zero potential. As noted in **Electric Potential Energy: Potential Difference**, this is analogous to taking sea level as $h = 0$ when considering gravitational potential energy, $\text{PE}_\text{g} = mgh$.

19.4 Equipotential Lines

We can represent electric potentials (voltages) pictorially, just as we drew pictures to illustrate electric fields. Of course, the two are related. Consider **Figure 19.8**, which shows an isolated positive point charge and its electric field lines. Electric field lines radiate out from a positive charge and terminate on negative charges. While we use blue arrows to represent the magnitude and direction of the electric field, we use green lines to represent places where the electric potential is constant. These are called **equipotential lines** in two dimensions, or *equipotential surfaces* in three dimensions. The term *equipotential* is also used as a noun, referring to an equipotential line or surface. The potential for a point charge is the same anywhere on an imaginary sphere of radius r surrounding the charge. This is true since the potential for a point charge is given by $V = kQ/r$ and, thus, has the same value at any point that is a given distance r from the charge. An equipotential sphere is a circle in the two-dimensional view of **Figure 19.8**. Since the electric field lines point radially away from the charge, they are perpendicular to the equipotential lines.

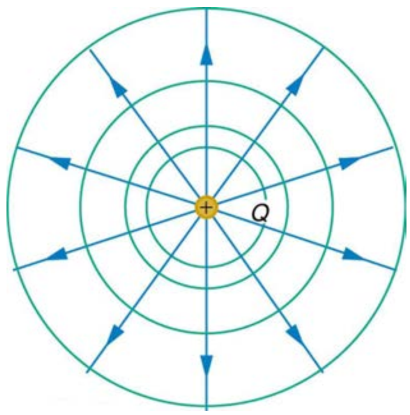


Figure 19.8 An isolated point charge Q with its electric field lines in blue and equipotential lines in green. The potential is the same along each equipotential line, meaning that no work is required to move a charge anywhere along one of those lines. Work is needed to move a charge from one equipotential line to another. Equipotential lines are perpendicular to electric field lines in every case.

It is important to note that *equipotential lines are always perpendicular to electric field lines.* No work is required to move a charge along an equipotential, since $\Delta V = 0$. Thus the work is

$$W = -\Delta\text{PE} = -q\Delta V = 0. \tag{19.43}$$

Work is zero if force is perpendicular to motion. Force is in the same direction as $\mathbf{E}$, so that motion along an equipotential must be perpendicular to $\mathbf{E}$. More precisely, work is related to the electric field by

$$W = Fd\cos\theta = qEd\cos\theta = 0. \tag{19.44}$$

Note that in the above equation, E and F symbolize the magnitudes of the electric field strength and force, respectively. Neither q nor $\mathbf{E}$ nor d is zero, and so $\cos\theta$ must be 0, meaning θ must be 90^o. In other words, motion along an equipotential is perpendicular to $\mathbf{E}$.

One of the rules for static electric fields and conductors is that the electric field must be perpendicular to the surface of any conductor. This implies that a *conductor is an equipotential surface in static situations.* There can be no voltage difference across the surface of a conductor, or charges will flow. One of the uses of this fact is that a conductor can be fixed at zero volts by connecting it to the earth with a good conductor—a process called **grounding**. Grounding can be a useful safety tool. For example, grounding the metal case of an electrical appliance ensures that it is at zero volts relative to the earth.

Grounding

A conductor can be fixed at zero volts by connecting it to the earth with a good conductor—a process called grounding.

Because a conductor is an equipotential, it can replace any equipotential surface. For example, in **Figure 19.8** a charged spherical conductor can replace the point charge, and the electric field and potential surfaces outside of it will be unchanged, confirming the contention that a spherical charge distribution is equivalent to a point charge at its center.

Figure 19.9 shows the electric field and equipotential lines for two equal and opposite charges. Given the electric field lines, the equipotential lines can be drawn simply by making them perpendicular to the electric field lines. Conversely, given the equipotential lines, as in **Figure 19.10**(a), the electric field lines can be drawn by making them perpendicular to the equipotentials, as in **Figure 19.10**(b).

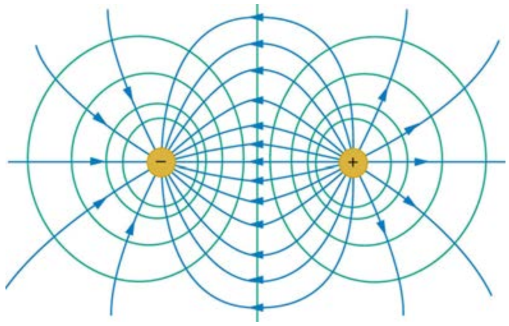


Figure 19.9 The electric field lines and equipotential lines for two equal but opposite charges. The equipotential lines can be drawn by making them perpendicular to the electric field lines, if those are known. Note that the potential is greatest (most positive) near the positive charge and least (most negative) near the negative charge.

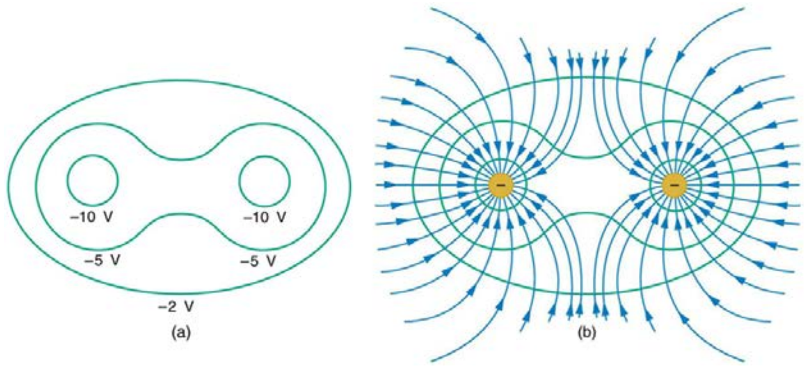


Figure 19.10 (a) These equipotential lines might be measured with a voltmeter in a laboratory experiment. (b) The corresponding electric field lines are found by drawing them perpendicular to the equipotentials. Note that these fields are consistent with two equal negative charges.

One of the most important cases is that of the familiar parallel conducting plates shown in **Figure 19.11**. Between the plates, the equipotentials are evenly spaced and parallel. The same field could be maintained by placing conducting plates at the equipotential lines at the potentials shown.

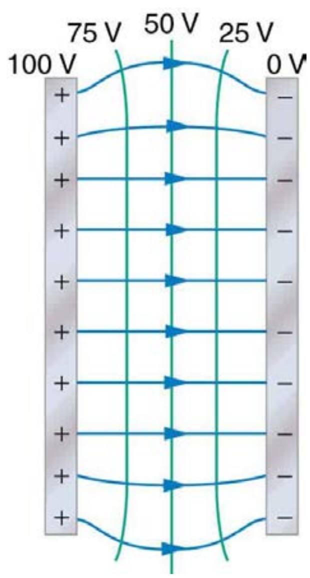


Figure 19.11 The electric field and equipotential lines between two metal plates.

An important application of electric fields and equipotential lines involves the heart. The heart relies on electrical signals to maintain its rhythm. The movement of electrical signals causes the chambers of the heart to contract and relax. When a person has a heart attack, the movement of these electrical signals may be disturbed. An artificial pacemaker and a defibrillator can be used to initiate the rhythm of electrical signals. The equipotential lines around the heart, the thoracic region, and the axis of the heart are useful ways of monitoring the structure and functions of the heart. An electrocardiogram (ECG) measures the small electric signals being generated during the activity of the heart. More about the relationship between electric fields and the heart is discussed in **Energy Stored in Capacitors**.

PhET Explorations: Charges and Fields

Move point charges around on the playing field and then view the electric field, voltages, equipotential lines, and more. It's colorful, it's dynamic, it's free.

PhET Interactive Simulation

Figure 19.12 Charges and Fields (http://cnx.org/content/m42331/1.3/charges-and-fields_en.jar)

19.5 Capacitors and Dielectrics

A **capacitor** is a device used to store electric charge. Capacitors have applications ranging from filtering static out of radio reception to energy storage in heart defibrillators. Typically, commercial capacitors have two conducting parts close to one another, but not touching, such as those in **Figure 19.13**. (Most of the time an insulator is used between the two plates to provide separation—see the discussion on dielectrics below.) When battery terminals are connected to an initially uncharged capacitor, equal amounts of positive and negative charge, $+Q$ and $-Q$, are separated into its two plates. The capacitor remains neutral overall, but we refer to it as storing a charge Q in this circumstance.

Capacitor

A capacitor is a device used to store electric charge.

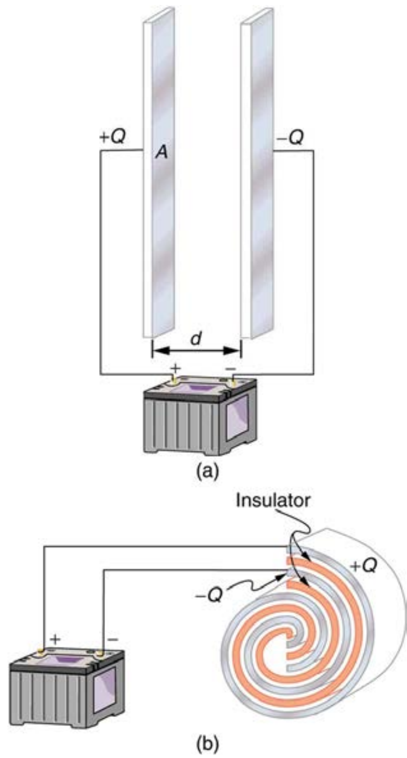


Figure 19.13 Both capacitors shown here were initially uncharged before being connected to a battery. They now have separated charges of $+Q$ and $-Q$ on their two halves. (a) A parallel plate capacitor. (b) A rolled capacitor with an insulating material between its two conducting sheets.

The amount of charge Q a *capacitor* can store depends on two major factors—the voltage applied and the capacitor's physical characteristics, such as its size.

The Amount of Charge Q a Capacitor Can Store

The amount of charge Q a *capacitor* can store depends on two major factors—the voltage applied and the capacitor's physical characteristics, such as its size.

A system composed of two identical, parallel conducting plates separated by a distance, as in **Figure 19.14**, is called a **parallel plate capacitor**. It is easy to see the relationship between the voltage and the stored charge for a parallel plate capacitor, as shown in **Figure 19.14**. Each electric field line starts on an individual positive charge and ends on a negative one, so that there will be more field lines if there is more charge. (Drawing a single field line per charge is a convenience, only. We can draw many field lines for each charge, but the total number is proportional to the number of charges.) The electric field strength is, thus, directly proportional to Q.

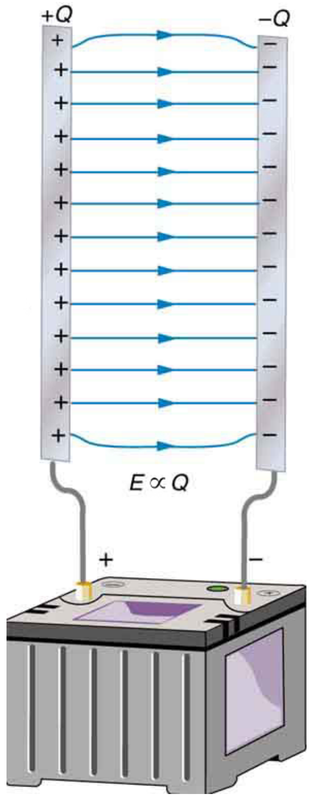


Figure 19.14 Electric field lines in this parallel plate capacitor, as always, start on positive charges and end on negative charges. Since the electric field strength is proportional to the density of field lines, it is also proportional to the amount of charge on the capacitor.

The field is proportional to the charge:

$$E \propto Q, \tag{19.45}$$

where the symbol $\propto$ means "proportional to." From the discussion in **Electric Potential in a Uniform Electric Field**, we know that the voltage across parallel plates is $V = Ed$. Thus,

$$V \propto E. \tag{19.46}$$

It follows, then, that $V \propto Q$, and conversely,

$$Q \propto V. \tag{19.47}$$

This is true in general: The greater the voltage applied to any capacitor, the greater the charge stored in it.

Different capacitors will store different amounts of charge for the same applied voltage, depending on their physical characteristics. We define their **capacitance** C to be such that the charge Q stored in a capacitor is proportional to C. The charge stored in a capacitor is given by

$$Q = CV. \tag{19.48}$$

This equation expresses the two major factors affecting the amount of charge stored. Those factors are the physical characteristics of the capacitor, C, and the voltage, V. Rearranging the equation, we see that *capacitance* C *is the amount of charge stored per volt,* or

$$C = \frac{Q}{V}. \tag{19.49}$$

Capacitance

Capacitance C is the amount of charge stored per volt, or

$$C = \frac{Q}{V}. \tag{19.50}$$

The unit of capacitance is the farad (F), named for Michael Faraday (1791–1867), an English scientist who contributed to the fields of electromagnetism and electrochemistry. Since capacitance is charge per unit voltage, we see that a farad is a coulomb per volt, or

$$1\ \text{F} = \frac{1\ \text{C}}{1\ \text{V}}. \tag{19.51}$$

A 1-farad capacitor would be able to store 1 coulomb (a very large amount of charge) with the application of only 1 volt. One farad is, thus, a very large capacitance. Typical capacitors range from fractions of a picofarad $\left(1\ \text{pF} = 10^{-12}\ \text{F}\right)$ to millifarads $\left(1\ \text{mF} = 10^{-3}\ \text{F}\right)$.

Figure 19.15 shows some common capacitors. Capacitors are primarily made of ceramic, glass, or plastic, depending upon purpose and size. Insulating materials, called dielectrics, are commonly used in their construction, as discussed below.

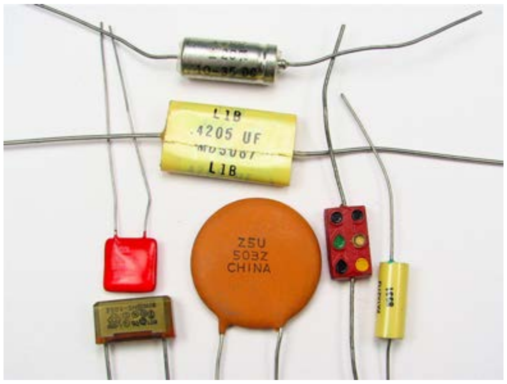


Figure 19.15 Some typical capacitors. Size and value of capacitance are not necessarily related. (credit: Windell Oskay)

Parallel Plate Capacitor

The parallel plate capacitor shown in **Figure 19.16** has two identical conducting plates, each having a surface area A , separated by a distance d (with no material between the plates). When a voltage V is applied to the capacitor, it stores a charge Q , as shown. We can see how its capacitance depends on A and d by considering the characteristics of the Coulomb force. We know that like charges repel, unlike charges attract, and the force between charges decreases with distance. So it seems quite reasonable that the bigger the plates are, the more charge they can store—because the charges can spread out more. Thus C should be greater for larger A . Similarly, the closer the plates are together, the greater the attraction of the opposite charges on them. So C should be greater for smaller d .

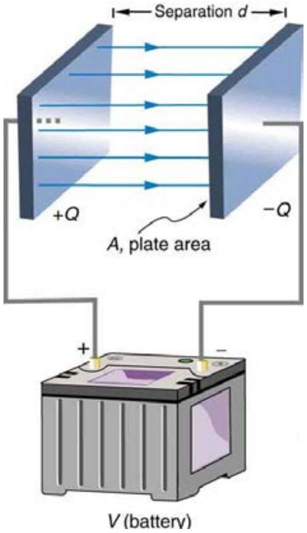


Figure 19.16 Parallel plate capacitor with plates separated by a distance d . Each plate has an area A .

It can be shown that for a parallel plate capacitor there are only two factors (A and d) that affect its capacitance C . The capacitance of a parallel plate capacitor in equation form is given by

$$C = \varepsilon_0 \frac{A}{d}. \tag{19.52}$$

Capacitance of a Parallel Plate Capacitor

$$C = \varepsilon_0 \frac{A}{d} \tag{19.53}$$

A is the area of one plate in square meters, and d is the distance between the plates in meters. The constant ε_0 is the permittivity of free space; its numerical value in SI units is $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m . The units of F/m are equivalent to $\text{C}^2/\text{N} \cdot \text{m}^2$. The small numerical value of ε_0 is related to the large size of the farad. A parallel plate capacitor must have a large area to have a capacitance approaching a farad. (Note that the above equation is valid when the parallel plates are separated by air or free space. When another material is placed between the plates, the equation is modified, as discussed below.)

Example 19.8 Capacitance and Charge Stored in a Parallel Plate Capacitor

(a) What is the capacitance of a parallel plate capacitor with metal plates, each of area $1.00\ \text{m}^2$, separated by 1.00 mm? (b) What charge is stored in this capacitor if a voltage of $3.00\times10^3\ \text{V}$ is applied to it?

Strategy

Finding the capacitance C is a straightforward application of the equation $C = \varepsilon_0 A / d$. Once C is found, the charge stored can be found using the equation $Q = CV$.

Solution for (a)

Entering the given values into the equation for the capacitance of a parallel plate capacitor yields

$$\begin{aligned} C &= \varepsilon_0 \frac{A}{d} = \left(8.85\times10^{-12}\,\frac{\text{F}}{\text{m}}\right)\frac{1.00\ \text{m}^2}{1.00\times10^{-3}\ \text{m}} \\ &= 8.85\times10^{-9}\ \text{F} = 8.85\ \text{nF}. \end{aligned} \tag{19.54}$$

Discussion for (a)

This small value for the capacitance indicates how difficult it is to make a device with a large capacitance. Special techniques help, such as using very large area thin foils placed close together.

Solution for (b)

The charge stored in any capacitor is given by the equation $Q = CV$. Entering the known values into this equation gives

$$\begin{aligned} Q &= CV = \left(8.85\times10^{-9}\ \text{F}\right)\left(3.00\times10^{3}\ \text{V}\right) \\ &= 26.6\ \mu\text{C}. \end{aligned} \tag{19.55}$$

Discussion for (b)

This charge is only slightly greater than those found in typical static electricity. Since air breaks down at about $3.00\times10^6\ \text{V/m}$, more charge cannot be stored on this capacitor by increasing the voltage.

Another interesting biological example dealing with electric potential is found in the cell's plasma membrane. The membrane sets a cell off from its surroundings and also allows ions to selectively pass in and out of the cell. There is a potential difference across the membrane of about $-70\ \text{mV}$. This is due to the mainly negatively charged ions in the cell and the predominance of positively charged sodium (Na^+) ions outside. Things change when a nerve cell is stimulated. Na^+ ions are allowed to pass through the membrane into the cell, producing a positive membrane potential—the nerve signal. The cell membrane is about 7 to 10 nm thick. An approximate value of the electric field across it is given by

$$E = \frac{V}{d} = \frac{-70\times10^{-3}\ \text{V}}{8\times10^{-9}\ \text{m}} = -9\times10^{6}\ \text{V/m}. \tag{19.56}$$

This electric field is enough to cause a breakdown in air.

Dielectric

The previous example highlights the difficulty of storing a large amount of charge in capacitors. If d is made smaller to produce a larger capacitance, then the maximum voltage must be reduced proportionally to avoid breakdown (since $E = V/d$). An important solution to this difficulty is to put an insulating material, called a **dielectric**, between the plates of a capacitor and allow d to be as small as possible. Not only does the smaller d make the capacitance greater, but many insulators can withstand greater electric fields than air before breaking down.

There is another benefit to using a dielectric in a capacitor. Depending on the material used, the capacitance is greater than that given by the equation $C = \varepsilon_0 \frac{A}{d}$ by a factor κ, called the *dielectric constant*. A parallel plate capacitor with a dielectric between its plates has a capacitance given by

$$C = \kappa\varepsilon_0 \frac{A}{d}\ \text{(parallel plate capacitor with dielectric)}. \tag{19.57}$$

Values of the dielectric constant κ for various materials are given in **Table 19.1**. Note that κ for vacuum is exactly 1, and so the above equation is valid in that case, too. If a dielectric is used, perhaps by placing Teflon between the plates of the capacitor in **Example 19.8**, then the capacitance is greater by the factor κ, which for Teflon is 2.1.

Take-Home Experiment: Building a Capacitor

How large a capacitor can you make using a chewing gum wrapper? The plates will be the aluminum foil, and the separation (dielectric) in between will be the paper.

Table 19.1 Dielectric Constants and Dielectric Strengths for Various Materials at 20ºC

Material	Dielectric constant κ	Dielectric strength (V/m)
Vacuum	1.00000	—
Air	1.00059	$3{\times}10^6$
Bakelite	4.9	$24{\times}10^6$
Fused quartz	3.78	$8{\times}10^6$
Neoprene rubber	6.7	$12{\times}10^6$
Nylon	3.4	$14{\times}10^6$
Paper	3.7	$16{\times}10^6$
Polystyrene	2.56	$24{\times}10^6$
Pyrex glass	5.6	$14{\times}10^6$
Silicon oil	2.5	$15{\times}10^6$
Strontium titanate	233	$8{\times}10^6$
Teflon	2.1	$60{\times}10^6$
Water	80	—

Note also that the dielectric constant for air is very close to 1, so that air-filled capacitors act much like those with vacuum between their plates *except* that the air can become conductive if the electric field strength becomes too great. (Recall that $E = V/d$ for a parallel plate capacitor.) Also shown in **Table 19.1** are maximum electric field strengths in V/m, called **dielectric strengths**, for several materials. These are the fields above which the material begins to break down and conduct. The dielectric strength imposes a limit on the voltage that can be applied for a given plate separation. For instance, in **Example 19.8**, the separation is 1.00 mm, and so the voltage limit for air is

$$\begin{aligned} V &= E \cdot d \\ &= (3{\times}10^6\ \text{V/m})(1.00{\times}10^{-3}\ \text{m}) \\ &= 3000\ \text{V}. \end{aligned} \tag{19.58}$$

However, the limit for a 1.00 mm separation filled with Teflon is 60,000 V, since the dielectric strength of Teflon is $60{\times}10^6$ V/m. So the same capacitor filled with Teflon has a greater capacitance and can be subjected to a much greater voltage. Using the capacitance we calculated in the above example for the air-filled parallel plate capacitor, we find that the Teflon-filled capacitor can store a maximum charge of

$$\begin{aligned} Q &= CV \\ &= \kappa C_{\text{air}} V \\ &= (2.1)(8.85\ \text{nF})(6.0{\times}10^4\ \text{V}) \\ &= 1.1\ \text{mC}. \end{aligned} \tag{19.59}$$

This is 42 times the charge of the same air-filled capacitor.

Dielectric Strength

The maximum electric field strength above which an insulating material begins to break down and conduct is called its dielectric strength.

Microscopically, how does a dielectric increase capacitance? Polarization of the insulator is responsible. The more easily it is polarized, the greater its dielectric constant κ. Water, for example, is a **polar molecule** because one end of the molecule has a slight positive charge and the other end has a slight negative charge. The polarity of water causes it to have a relatively large dielectric constant of 80. The effect of polarization can be best explained in terms of the characteristics of the Coulomb force. **Figure 19.17** shows the separation of charge schematically in the molecules of a dielectric material placed between the charged plates of a capacitor. The Coulomb force between the closest ends of the molecules and the charge on the plates is attractive and very strong, since they are very close together. This attracts more charge onto the plates than if the space were empty and the opposite charges were a distance d away.

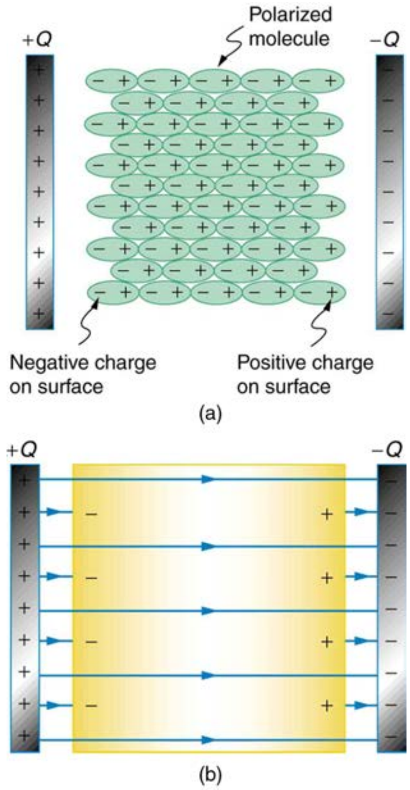


Figure 19.17 (a) The molecules in the insulating material between the plates of a capacitor are polarized by the charged plates. This produces a layer of opposite charge on the surface of the dielectric that attracts more charge onto the plate, increasing its capacitance. (b) The dielectric reduces the electric field strength inside the capacitor, resulting in a smaller voltage between the plates for the same charge. The capacitor stores the same charge for a smaller voltage, implying that it has a larger capacitance because of the dielectric.

Another way to understand how a dielectric increases capacitance is to consider its effect on the electric field inside the capacitor. **Figure 19.17**(b) shows the electric field lines with a dielectric in place. Since the field lines end on charges in the dielectric, there are fewer of them going from one side of the capacitor to the other. So the electric field strength is less than if there were a vacuum between the plates, even though the same charge is on the plates. The voltage between the plates is $V = Ed$, so it too is reduced by the dielectric. Thus there is a smaller voltage V for the same charge Q ; since $C = Q / V$, the capacitance C is greater.

The dielectric constant is generally defined to be $\kappa = E_0 / E$, or the ratio of the electric field in a vacuum to that in the dielectric material, and is intimately related to the polarizability of the material.

Things Great and Small

The Submicroscopic Origin of Polarization

Polarization is a separation of charge within an atom or molecule. As has been noted, the planetary model of the atom pictures it as having a positive nucleus orbited by negative electrons, analogous to the planets orbiting the Sun. Although this model is not completely accurate, it is very helpful in explaining a vast range of phenomena and will be refined elsewhere, such as in **Atomic Physics**. The submicroscopic origin of polarization can be modeled as shown in **Figure 19.18**.

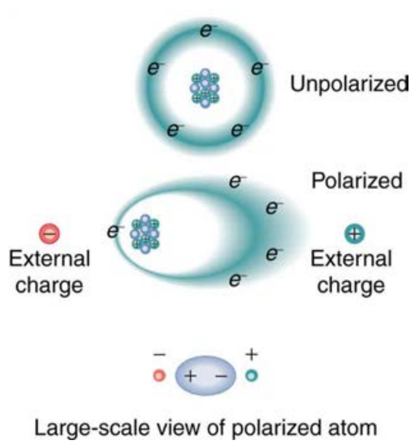


Figure 19.18 Artist's conception of a polarized atom. The orbits of electrons around the nucleus are shifted slightly by the external charges (shown exaggerated). The resulting separation of charge within the atom means that it is polarized. Note that the unlike charge is now closer to the external charges, causing the polarization.

We will find in **Atomic Physics** that the orbits of electrons are more properly viewed as electron clouds with the density of the cloud related to the probability of finding an electron in that location (as opposed to the definite locations and paths of planets in their orbits around the Sun). This cloud is shifted by the Coulomb force so that the atom on average has a separation of charge. Although the atom remains neutral, it can now be the source of a Coulomb force, since a charge brought near the atom will be closer to one type of charge than the other.

Some molecules, such as those of water, have an inherent separation of charge and are thus called polar molecules. **Figure 19.19** illustrates the separation of charge in a water molecule, which has two hydrogen atoms and one oxygen atom (H_2O). The water molecule is not symmetric—the hydrogen atoms are repelled to one side, giving the molecule a boomerang shape. The electrons in a water molecule are more concentrated around the more highly charged oxygen nucleus than around the hydrogen nuclei. This makes the oxygen end of the molecule slightly negative and leaves the hydrogen ends slightly positive. The inherent separation of charge in polar molecules makes it easier to align them with external fields and charges. Polar molecules therefore exhibit greater polarization effects and have greater dielectric constants. Those who study chemistry will find that the polar nature of water has many effects. For example, water molecules gather ions much more effectively because they have an electric field and a separation of charge to attract charges of both signs. Also, as brought out in the previous chapter, polar water provides a shield or screening of the electric fields in the highly charged molecules of interest in biological systems.

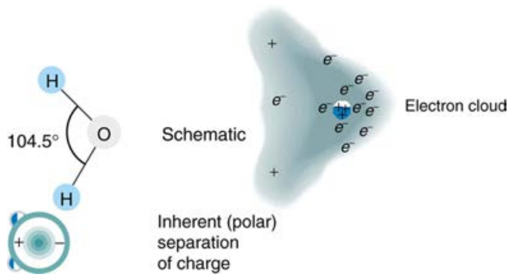


Figure 19.19 Artist's conception of a water molecule. There is an inherent separation of charge, and so water is a polar molecule. Electrons in the molecule are attracted to the oxygen nucleus and leave an excess of positive charge near the two hydrogen nuclei. (Note that the schematic on the right is a rough illustration of the distribution of electrons in the water molecule. It does not show the actual numbers of protons and electrons involved in the structure.)

PhET Explorations: Capacitor Lab

Explore how a capacitor works! Change the size of the plates and add a dielectric to see the effect on capacitance. Change the voltage and see charges built up on the plates. Observe the electric field in the capacitor. Measure the voltage and the electric field.

PhET Interactive Simulation

Figure 19.20 Capacitor Lab (http://cnx.org/content/m42333/1.4/capacitor-lab_en.jar)

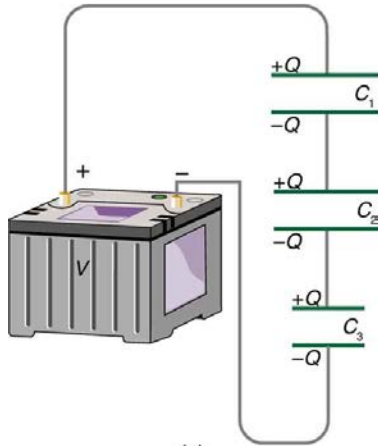
19.6 Capacitors in Series and Parallel

Several capacitors may be connected together in a variety of applications. Multiple connections of capacitors act like a single equivalent capacitor. The total capacitance of this equivalent single capacitor depends both on the individual capacitors and how they are connected. There are two simple and common types of connections, called *series* and *parallel*, for which we can easily calculate the total capacitance. Certain more complicated connections can also be related to combinations of series and parallel.

Capacitance in Series

Figure 19.21(a) shows a series connection of three capacitors with a voltage applied. As for any capacitor, the capacitance of the combination is related to charge and voltage by $C = \frac{Q}{V}$.

Note in **Figure 19.21** that opposite charges of magnitude Q flow to either side of the originally uncharged combination of capacitors when the voltage V is applied. Conservation of charge requires that equal-magnitude charges be created on the plates of the individual capacitors, since charge is only being separated in these originally neutral devices. The end result is that the combination resembles a single capacitor with an effective plate separation greater than that of the individual capacitors alone. (See **Figure 19.21**(b).) Larger plate separation means smaller capacitance. It is a general feature of series connections of capacitors that the total capacitance is less than any of the individual capacitances.



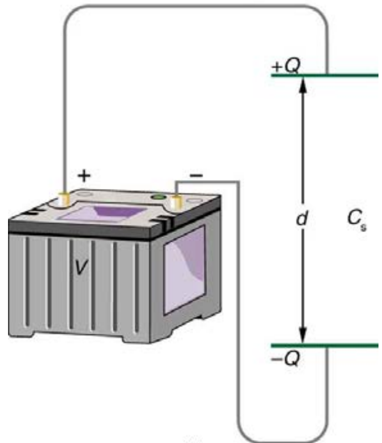


Figure 19.21 (a) Capacitors connected in series. The magnitude of the charge on each plate is Q. (b) An equivalent capacitor has a larger plate separation d. Series connections produce a total capacitance that is less than that of any of the individual capacitors.

We can find an expression for the total capacitance by considering the voltage across the individual capacitors shown in **Figure 19.21**. Solving $C = \frac{Q}{V}$ for V gives $V = \frac{Q}{C}$. The voltages across the individual capacitors are thus $V_1 = \frac{Q}{C_1}$, $V_2 = \frac{Q}{C_2}$, and $V_3 = \frac{Q}{C_3}$. The total voltage is the sum of the individual voltages:

$$V = V_1 + V_2 + V_3. \tag{19.60}$$

Now, calling the total capacitance C_S for series capacitance, consider that

$$V = \frac{Q}{C_\text{S}} = V_1 + V_2 + V_3. \tag{19.61}$$

Entering the expressions for V_1, V_2, and V_3, we get

$$\frac{Q}{C_\text{S}} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}. \tag{19.62}$$

Canceling the Q s, we obtain the equation for the total capacitance in series C_S to be

$$\frac{1}{C_\text{S}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + ..., \tag{19.63}$$

where "..." indicates that the expression is valid for any number of capacitors connected in series. An expression of this form always results in a total capacitance C_S that is less than any of the individual capacitances C_1, C_2, ..., as the next example illustrates.

Total Capacitance in Series, C_s

Total capacitance in series: $\frac{1}{C_\text{S}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + ...$

Example 19.9 What Is the Series Capacitance?

Find the total capacitance for three capacitors connected in series, given their individual capacitances are 1.000, 5.000, and 8.000 μF.

Strategy

With the given information, the total capacitance can be found using the equation for capacitance in series.

Solution

Entering the given capacitances into the expression for $\frac{1}{C_S}$ gives $\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$.

$$\frac{1}{C_S} = \frac{1}{1.000\ \mu F} + \frac{1}{5.000\ \mu F} + \frac{1}{8.000\ \mu F} = \frac{1.325}{\mu F} \quad (19.64)$$

Inverting to find C_S yields $C_S = \frac{\mu F}{1.325} = 0.755\ \mu F$.

Discussion

The total series capacitance C_s is less than the smallest individual capacitance, as promised. In series connections of capacitors, the sum is less than the parts. In fact, it is less than any individual. Note that it is sometimes possible, and more convenient, to solve an equation like the above by finding the least common denominator, which in this case (showing only whole-number calculations) is 40. Thus,

$$\frac{1}{C_S} = \frac{40}{40\ \mu F} + \frac{8}{40\ \mu F} + \frac{5}{40\ \mu F} = \frac{53}{40\ \mu F}, \quad (19.65)$$

so that

$$C_S = \frac{40\ \mu F}{53} = 0.755\ \mu F. \quad (19.66)$$

Capacitors in Parallel

Figure 19.22(a) shows a parallel connection of three capacitors with a voltage applied. Here the total capacitance is easier to find than in the series case. To find the equivalent total capacitance C_p, we first note that the voltage across each capacitor is V, the same as that of the source, since they are connected directly to it through a conductor. (Conductors are equipotentials, and so the voltage across the capacitors is the same as that across the voltage source.) Thus the capacitors have the same charges on them as they would have if connected individually to the voltage source. The total charge Q is the sum of the individual charges:

$$Q = Q_1 + Q_2 + Q_3. \quad (19.67)$$

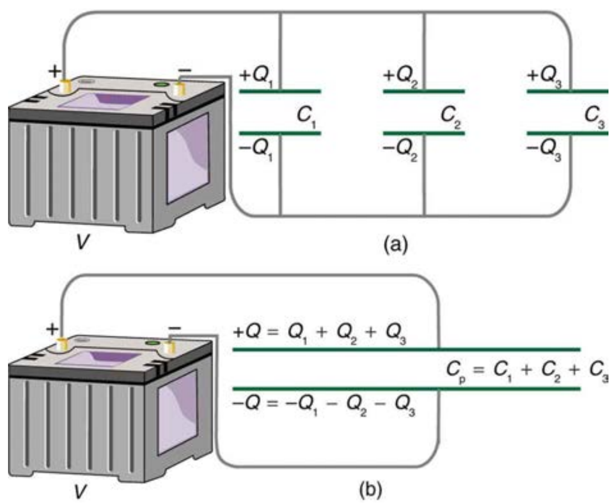


Figure 19.22 (a) Capacitors in parallel. Each is connected directly to the voltage source just as if it were all alone, and so the total capacitance in parallel is just the sum of the individual capacitances. (b) The equivalent capacitor has a larger plate area and can therefore hold more charge than the individual capacitors.

Using the relationship $Q = CV$, we see that the total charge is $Q = C_pV$, and the individual charges are $Q_1 = C_1V$, $Q_2 = C_2V$, and $Q_3 = C_3V$. Entering these into the previous equation gives

$$C_pV = C_1V + C_2V + C_3V. \quad (19.68)$$

Canceling V from the equation, we obtain the equation for the total capacitance in parallel C_p:

$$C_p = C_1 + C_2 + C_3 + ... \quad (19.69)$$

Total capacitance in parallel is simply the sum of the individual capacitances. (Again the "..." indicates the expression is valid for any number of capacitors connected in parallel.) So, for example, if the capacitors in the example above were connected in parallel, their capacitance would be

$$C_\text{p} = 1.000\ \mu\text{F} + 5.000\ \mu\text{F} + 8.000\ \mu\text{F} = 14.000\ \mu\text{F}. \tag{19.70}$$

The equivalent capacitor for a parallel connection has an effectively larger plate area and, thus, a larger capacitance, as illustrated in Figure 19.22(b).

Total Capacitance in Parallel, C_p

Total capacitance in parallel $C_\text{p} = C_1 + C_2 + C_3 + ...$

More complicated connections of capacitors can sometimes be combinations of series and parallel. (See Figure 19.23.) To find the total capacitance of such combinations, we identify series and parallel parts, compute their capacitances, and then find the total.

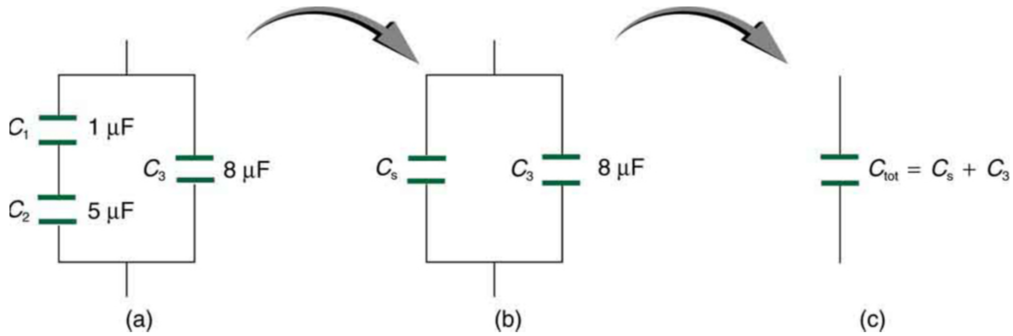


Figure 19.23 (a) This circuit contains both series and parallel connections of capacitors. See Example 19.10 for the calculation of the overall capacitance of the circuit. (b) C_1 and C_2 are in series; their equivalent capacitance C_S is less than either of them. (c) Note that C_S is in parallel with C_3. The total capacitance is, thus, the sum of C_S and C_3.

Example 19.10 A Mixture of Series and Parallel Capacitance

Find the total capacitance of the combination of capacitors shown in Figure 19.23. Assume the capacitances in Figure 19.23 are known to three decimal places ($C_1 = 1.000\ \mu\text{F}$, $C_2 = 3.000\ \mu\text{F}$, and $C_3 = 8.000\ \mu\text{F}$), and round your answer to three decimal places.

Strategy

To find the total capacitance, we first identify which capacitors are in series and which are in parallel. Capacitors C_1 and C_2 are in series. Their combination, labeled C_S in the figure, is in parallel with C_3.

Solution

Since C_1 and C_2 are in series, their total capacitance is given by $\frac{1}{C_\text{S}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$. Entering their values into the equation gives

$$\frac{1}{C_\text{S}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{1.000\ \mu\text{F}} + \frac{1}{5.000\ \mu\text{F}} = \frac{1.200}{\mu\text{F}}. \tag{19.71}$$

Inverting gives

$$C_\text{S} = 0.833\ \mu\text{F}. \tag{19.72}$$

This equivalent series capacitance is in parallel with the third capacitor; thus, the total is the sum

$$\begin{aligned} C_\text{tot} &= C_\text{S} + C_\text{S} \\ &= 0.833\ \mu\text{F} + 8.000\ \mu\text{F} \\ &= 8.833\ \mu\text{F}. \end{aligned} \tag{19.73}$$

Discussion

This technique of analyzing the combinations of capacitors piece by piece until a total is obtained can be applied to larger combinations of capacitors.

19.7 Energy Stored in Capacitors

Most of us have seen dramatizations in which medical personnel use a **defibrillator** to pass an electric current through a patient's heart to get it to beat normally. (Review Figure 19.24.) Often realistic in detail, the person applying the shock directs another person to "make it 400 joules this time." The energy delivered by the defibrillator is stored in a capacitor and can be adjusted to fit the situation. SI units of joules are often employed. Less dramatic is the use of capacitors in microelectronics, such as certain handheld calculators, to supply energy when batteries are charged. (See **Figure 19.24**.) Capacitors are also used to supply energy for flash lamps on cameras.

Figure 19.24 Energy stored in the large capacitor is used to preserve the memory of an electronic calculator when its batteries are charged. (credit: Kucharek, Wikimedia Commons)

Energy stored in a capacitor is electrical potential energy, and it is thus related to the charge Q and voltage V on the capacitor. We must be careful when applying the equation for electrical potential energy $\Delta\text{PE} = q\Delta V$ to a capacitor. Remember that ΔPE is the potential energy of a charge q going through a voltage ΔV. But the capacitor starts with zero voltage and gradually comes up to its full voltage as it is charged. The first charge placed on a capacitor experiences a change in voltage $\Delta V = 0$, since the capacitor has zero voltage when uncharged. The final charge placed on a capacitor experiences $\Delta V = V$, since the capacitor now has its full voltage V on it. The average voltage on the capacitor during the charging process is $V/2$, and so the average voltage experienced by the full charge q is $V/2$. Thus the energy stored in a capacitor, E_{cap}, is

$$E_{\text{cap}} = \frac{QV}{2}, \tag{19.74}$$

where Q is the charge on a capacitor with a voltage V applied. (Note that the energy is not QV, but $QV/2$.) Charge and voltage are related to the capacitance C of a capacitor by $Q = CV$, and so the expression for E_{cap} can be algebraically manipulated into three equivalent expressions:

$$E_{\text{cap}} = \frac{QV}{2} = \frac{CV^2}{2} = \frac{Q^2}{2C}, \tag{19.75}$$

where Q is the charge and V the voltage on a capacitor C. The energy is in joules for a charge in coulombs, voltage in volts, and capacitance in farads.

Energy Stored in Capacitors

The energy stored in a capacitor can be expressed in three ways:

$$E_{\text{cap}} = \frac{QV}{2} = \frac{CV^2}{2} = \frac{Q^2}{2C}, \tag{19.76}$$

where Q is the charge, V is the voltage, and C is the capacitance of the capacitor. The energy is in joules for a charge in coulombs, voltage in volts, and capacitance in farads.

In a defibrillator, the delivery of a large charge in a short burst to a set of paddles across a person's chest can be a lifesaver. The person's heart attack might have arisen from the onset of fast, irregular beating of the heart—cardiac or ventricular fibrillation. The application of a large shock of electrical energy can terminate the arrhythmia and allow the body's pacemaker to resume normal patterns. Today it is common for ambulances to carry a defibrillator, which also uses an electrocardiogram to analyze the patient's heartbeat pattern. Automated external defibrillators (AED) are found in many public places (**Figure 19.25**). These are designed to be used by lay persons. The device automatically diagnoses the patient's heart condition and then applies the shock with appropriate energy and waveform. CPR is recommended in many cases before use of an AED.

Figure 19.25 Automated external defibrillators are found in many public places. These portable units provide verbal instructions for use in the important first few minutes for a person suffering a cardiac attack. (credit: Owain Davies, Wikimedia Commons)

Example 19.11 Capacitance in a Heart Defibrillator

A heart defibrillator delivers $4.00\times10^2\ \text{J}$ of energy by discharging a capacitor initially at $1.00\times10^4\ \text{V}$. What is its capacitance?

Strategy

We are given E_{cap} and V, and we are asked to find the capacitance C. Of the three expressions in the equation for E_{cap}, the most convenient relationship is

$$E_{\text{cap}} = \frac{CV^2}{2}. \tag{19.77}$$

Solution

Solving this expression for C and entering the given values yields

$$\begin{aligned} C &= \frac{2E_{\text{cap}}}{V^2} = \frac{2(4.00\times10^2\ \text{J})}{(1.00\times10^4\ \text{V})^2} = 8.00\times10^{-6}\ \text{F} \\ &= 8.00\ \mu\text{F}. \end{aligned} \tag{19.78}$$

Discussion

This is a fairly large, but manageable, capacitance at $1.00\times10^4\ \text{V}$.

Glossary

capacitance: amount of charge stored per unit volt

capacitor: a device that stores electric charge

defibrillator: a machine used to provide an electrical shock to a heart attack victim's heart in order to restore the heart's normal rhythmic pattern

dielectric strength: the maximum electric field above which an insulating material begins to break down and conduct

dielectric: an insulating material

electric potential: potential energy per unit charge

electron volt: the energy given to a fundamental charge accelerated through a potential difference of one volt

equipotential line: a line along which the electric potential is constant

grounding: fixing a conductor at zero volts by connecting it to the earth or ground

mechanical energy: sum of the kinetic energy and potential energy of a system; this sum is a constant

parallel plate capacitor: two identical conducting plates separated by a distance

polar molecule: a molecule with inherent separation of charge

potential difference (or voltage): change in potential energy of a charge moved from one point to another, divided by the charge; units of potential difference are joules per coulomb, known as volt

scalar: physical quantity with magnitude but no direction

vector: physical quantity with both magnitude and direction

Section Summary

19.1 Electric Potential Energy: Potential Difference

- Electric potential is potential energy per unit charge.
- The potential difference between points A and B, $V_B - V_A$, defined to be the change in potential energy of a charge q moved from A to B, is equal to the change in potential energy divided by the charge, Potential difference is commonly called voltage, represented by the symbol ΔV.
$$\Delta V = \frac{\Delta \text{PE}}{q} \text{ and } \Delta \text{PE} = q\Delta V.$$
- An electron volt is the energy given to a fundamental charge accelerated through a potential difference of 1 V. In equation form,
$$\begin{aligned} 1 \text{ eV} &= \left(1.60\times10^{-19}\text{ C}\right)(1\text{ V}) = \left(1.60\times10^{-19}\text{ C}\right)(1\text{ J/C}) \\ &= 1.60\times10^{-19}\text{ J.} \end{aligned}$$
- Mechanical energy is the sum of the kinetic energy and potential energy of a system, that is, $\text{KE} + \text{PE}.$ This sum is a constant.

19.2 Electric Potential in a Uniform Electric Field

- The voltage between points A and B is
$$\left.\begin{aligned} V_{\text{AB}} &= Ed \\ E &= \frac{V_{\text{AB}}}{d} \end{aligned}\right\}(\text{uniform } E \text{ - field only}),$$
where d is the distance from A to B, or the distance between the plates.
- In equation form, the general relationship between voltage and electric field is
$$E = -\frac{\Delta V}{\Delta s},$$
where Δs is the distance over which the change in potential, ΔV, takes place. The minus sign tells us that $\mathbf{E}$ points in the direction of decreasing potential.) The electric field is said to be the *gradient* (as in grade or slope) of the electric potential.

19.3 Electrical Potential Due to a Point Charge

- Electric potential of a point charge is $V = kQ/r$.
- Electric potential is a scalar, and electric field is a vector. Addition of voltages as numbers gives the voltage due to a combination of point charges, whereas addition of individual fields as vectors gives the total electric field.

19.4 Equipotential Lines

- An equipotential line is a line along which the electric potential is constant.
- An equipotential surface is a three-dimensional version of equipotential lines.
- Equipotential lines are always perpendicular to electric field lines.
- The process by which a conductor can be fixed at zero volts by connecting it to the earth with a good conductor is called grounding.

19.5 Capacitors and Dielectrics

- A capacitor is a device used to store charge.
- The amount of charge Q a capacitor can store depends on two major factors—the voltage applied and the capacitor's physical characteristics, such as its size.
- The capacitance C is the amount of charge stored per volt, or
$$C = \frac{Q}{V}.$$
- The capacitance of a parallel plate capacitor is $C = \varepsilon_0 \frac{A}{d}$, when the plates are separated by air or free space. ε_0 is called the permittivity of free space.
- A parallel plate capacitor with a dielectric between its plates has a capacitance given by
$$C = \kappa\varepsilon_0 \frac{A}{d},$$
where κ is the dielectric constant of the material.
- The maximum electric field strength above which an insulating material begins to break down and conduct is called dielectric strength.

19.6 Capacitors in Series and Parallel

- Total capacitance in series $\frac{1}{C_\text{S}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + ...$
- Total capacitance in parallel $C_\text{p} = C_1 + C_2 + C_3 + ...$
- If a circuit contains a combination of capacitors in series and parallel, identify series and parallel parts, compute their capacitances, and then find the total.

19.7 Energy Stored in Capacitors

- Capacitors are used in a variety of devices, including defibrillators, microelectronics such as calculators, and flash lamps, to supply energy.
- The energy stored in a capacitor can be expressed in three ways:

$$E_\text{cap} = \frac{QV}{2} = \frac{CV^2}{2} = \frac{Q^2}{2C},$$

where Q is the charge, V is the voltage, and C is the capacitance of the capacitor. The energy is in joules when the charge is in coulombs, voltage is in volts, and capacitance is in farads.

Conceptual Questions

19.1 Electric Potential Energy: Potential Difference

1. Voltage is the common word for potential difference. Which term is more descriptive, voltage or potential difference?

2. If the voltage between two points is zero, can a test charge be moved between them with zero net work being done? Can this necessarily be done without exerting a force? Explain.

3. What is the relationship between voltage and energy? More precisely, what is the relationship between potential difference and electric potential energy?

4. Voltages are always measured between two points. Why?

5. How are units of volts and electron volts related? How do they differ?

19.2 Electric Potential in a Uniform Electric Field

6. Discuss how potential difference and electric field strength are related. Give an example.

7. What is the strength of the electric field in a region where the electric potential is constant?

8. Will a negative charge, initially at rest, move toward higher or lower potential? Explain why.

19.3 Electrical Potential Due to a Point Charge

9. In what region of space is the potential due to a uniformly charged sphere the same as that of a point charge? In what region does it differ from that of a point charge?

10. Can the potential of a non-uniformly charged sphere be the same as that of a point charge? Explain.

19.4 Equipotential Lines

11. What is an equipotential line? What is an equipotential surface?

12. Explain in your own words why equipotential lines and surfaces must be perpendicular to electric field lines.

13. Can different equipotential lines cross? Explain.

19.5 Capacitors and Dielectrics

14. Does the capacitance of a device depend on the applied voltage? What about the charge stored in it?

15. Use the characteristics of the Coulomb force to explain why capacitance should be proportional to the plate area of a capacitor. Similarly, explain why capacitance should be inversely proportional to the separation between plates.

16. Give the reason why a dielectric material increases capacitance compared with what it would be with air between the plates of a capacitor. What is the independent reason that a dielectric material also allows a greater voltage to be applied to a capacitor? (The dielectric thus increases C and permits a greater V.)

17. How does the polar character of water molecules help to explain water's relatively large dielectric constant? (**Figure 19.19**)

18. Sparks will occur between the plates of an air-filled capacitor at lower voltage when the air is humid than when dry. Explain why, considering the polar character of water molecules.

19. Water has a large dielectric constant, but it is rarely used in capacitors. Explain why.

20. Membranes in living cells, including those in humans, are characterized by a separation of charge across the membrane. Effectively, the membranes are thus charged capacitors with important functions related to the potential difference across the membrane. Is energy required to separate these charges in living membranes and, if so, is its source the metabolization of food energy or some other source?

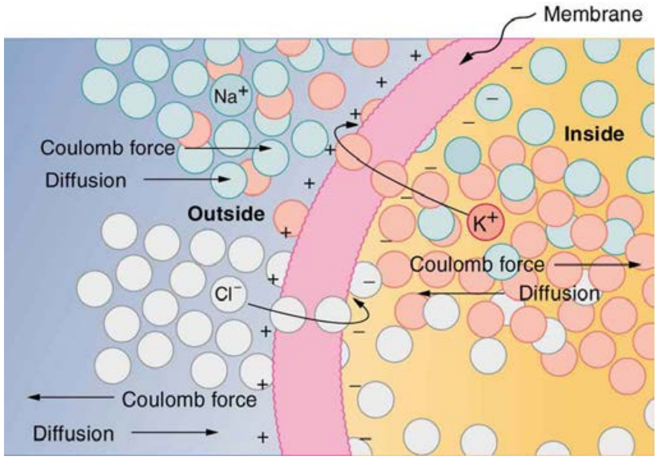


Figure 19.26 The semipermeable membrane of a cell has different concentrations of ions inside and out. Diffusion moves the K^+ (potassium) and Cl^- (chloride) ions in the directions shown, until the Coulomb force halts further transfer. This results in a layer of positive charge on the outside, a layer of negative charge on the inside, and thus a voltage across the cell membrane. The membrane is normally impermeable to Na^+ (sodium ions).

19.6 Capacitors in Series and Parallel

21. If you wish to store a large amount of energy in a capacitor bank, would you connect capacitors in series or parallel? Explain.

19.7 Energy Stored in Capacitors

22. How does the energy contained in a charged capacitor change when a dielectric is inserted, assuming the capacitor is isolated and its charge is constant? Does this imply that work was done?

23. What happens to the energy stored in a capacitor connected to a battery when a dielectric is inserted? Was work done in the process?

Problems & Exercises

19.1 Electric Potential Energy: Potential Difference

24. Find the ratio of speeds of an electron and a negative hydrogen ion (one having an extra electron) accelerated through the same voltage, assuming non-relativistic final speeds. Take the mass of the hydrogen ion to be $1.67\times10^{-27}\ \text{kg.}$

25. An evacuated tube uses an accelerating voltage of 40 kV to accelerate electrons to hit a copper plate and produce x rays. Non-relativistically, what would be the maximum speed of these electrons?

26. A bare helium nucleus has two positive charges and a mass of $6.64\times10^{-27}\ \text{kg.}$ (a) Calculate its kinetic energy in joules at 2.00% of the speed of light. (b) What is this in electron volts? (c) What voltage would be needed to obtain this energy?

27. Integrated Concepts

Singly charged gas ions are accelerated from rest through a voltage of 13.0 V. At what temperature will the average kinetic energy of gas molecules be the same as that given these ions?

28. Integrated Concepts

The temperature near the center of the Sun is thought to be 15 million degrees Celsius $\left(1.5\times10^{7}\ ^\circ\text{C}\right)$. Through what voltage must a singly charged ion be accelerated to have the same energy as the average kinetic energy of ions at this temperature?

29. Integrated Concepts

(a) What is the average power output of a heart defibrillator that dissipates 400 J of energy in 10.0 ms? (b) Considering the high-power output, why doesn't the defibrillator produce serious burns?

30. Integrated Concepts

A lightning bolt strikes a tree, moving 20.0 C of charge through a potential difference of $1.00\times10^{2}\ \text{MV}$. (a) What energy was dissipated? (b) What mass of water could be raised from 15°C to the boiling point and then boiled by this energy? (c) Discuss the damage that could be caused to the tree by the expansion of the boiling steam.

31. Integrated Concepts

A 12.0 V battery-operated bottle warmer heats 50.0 g of glass, $2.50\times10^{2}\ \text{g}$ of baby formula, and $2.00\times10^{2}\ \text{g}$ of aluminum from 20.0°C to 90.0°C . (a) How much charge is moved by the battery? (b) How many electrons per second flow if it takes 5.00 min to warm the formula? (Hint: Assume that the specific heat of baby formula is about the same as the specific heat of water.)

32. Integrated Concepts

A battery-operated car utilizes a 12.0 V system. Find the charge the batteries must be able to move in order to accelerate the 750 kg car from rest to 25.0 m/s, make it climb a $2.00\times10^{2}\ \text{m}$ high hill, and then cause it to travel at a constant 25.0 m/s by exerting a $5.00\times10^{2}\ \text{N}$ force for an hour.

33. Integrated Concepts

Fusion probability is greatly enhanced when appropriate nuclei are brought close together, but mutual Coulomb repulsion must be overcome. This can be done using the kinetic energy of high-temperature gas ions or by accelerating the nuclei toward one another. (a) Calculate the potential energy of two singly charged nuclei separated by $1.00\times10^{-12}\ \text{m}$ by finding the voltage of one at that distance and multiplying by the charge of the other. (b) At what temperature will atoms of a gas have an average kinetic energy equal to this needed electrical potential energy?

34. Unreasonable Results

(a) Find the voltage near a 10.0 cm diameter metal sphere that has 8.00 C of excess positive charge on it. (b) What is unreasonable about this result? (c) Which assumptions are responsible?

35. Construct Your Own Problem

Consider a battery used to supply energy to a cellular phone. Construct a problem in which you determine the energy that must be supplied by the battery, and then calculate the amount of charge it must be able to move in order to supply this energy. Among the things to be considered are the energy needs and battery voltage. You may need to look ahead to interpret manufacturer's battery ratings in ampere-hours as energy in joules.

19.2 Electric Potential in a Uniform Electric Field

36. Show that units of V/m and N/C for electric field strength are indeed equivalent.

37. What is the strength of the electric field between two parallel conducting plates separated by 1.00 cm and having a potential difference (voltage) between them of $1.50\times10^{4}\ \text{V}$?

38. The electric field strength between two parallel conducting plates separated by 4.00 cm is $7.50\times10^{4}\ \text{V/m}$. (a) What is the potential difference between the plates? (b) The plate with the lowest potential is taken to be at zero volts. What is the potential 1.00 cm from that plate (and 3.00 cm from the other)?

39. How far apart are two conducting plates that have an electric field strength of $4.50\times10^{3}\ \text{V/m}$ between them, if their potential difference is 15.0 kV?

40. (a) Will the electric field strength between two parallel conducting plates exceed the breakdown strength for air ($3.0\times10^{6}\ \text{V/m}$) if the plates are separated by 2.00 mm and a potential difference of $5.0\times10^{3}\ \text{V}$ is applied? (b) How close together can the plates be with this applied voltage?

41. The voltage across a membrane forming a cell wall is 80.0 mV and the membrane is 9.00 nm thick. What is the electric field strength? (The value is surprisingly large, but correct. Membranes are discussed in **Capacitors and Dielectrics** and **Nerve Conduction—Electrocardiograms**.) You may assume a uniform electric field.

42. Membrane walls of living cells have surprisingly large electric fields across them due to separation of ions. (Membranes are discussed in some detail in **Nerve Conduction—Electrocardiograms**.) What is the voltage across an 8.00 nm–thick membrane if the electric field strength across it is 5.50 MV/m? You may assume a uniform electric field.

43. Two parallel conducting plates are separated by 10.0 cm, and one of them is taken to be at zero volts. (a) What is the electric field strength between them, if the potential 8.00 cm from the zero volt plate (and 2.00 cm from the other) is 450 V? (b) What is the voltage between the plates?

44. Find the maximum potential difference between two parallel conducting plates separated by 0.500 cm of air, given the maximum sustainable electric field strength in air to be $3.0\times10^{6}\ \text{V/m}$.

45. A doubly charged ion is accelerated to an energy of 32.0 keV by the electric field between two parallel conducting plates separated by 2.00 cm. What is the electric field strength between the plates?

46. An electron is to be accelerated in a uniform electric field having a strength of $2.00\times10^{6}\ \text{V/m}$. (a) What energy in keV is given to the electron if it is accelerated through 0.400 m? (b) Over what distance would it have to be accelerated to increase its energy by 50.0 GeV?

19.3 Electrical Potential Due to a Point Charge

47. A 0.500 cm diameter plastic sphere, used in a static electricity demonstration, has a uniformly distributed 40.0 pC charge on its surface. What is the potential near its surface?

48. What is the potential $0.530\times10^{-10}\ \text{m}$ from a proton (the average distance between the proton and electron in a hydrogen atom)?

49. (a) A sphere has a surface uniformly charged with 1.00 C. At what distance from its center is the potential 5.00 MV? (b) What does your answer imply about the practical aspect of isolating such a large charge?

50. How far from a $1.00\ \mu\text{C}$ point charge will the potential be 100 V? At what distance will it be $2.00\times10^{2}\ \text{V}$?

51. What are the sign and magnitude of a point charge that produces a potential of $-2.00\ \text{V}$ at a distance of 1.00 mm?

52. If the potential due to a point charge is $5.00\times10^{2}\ \text{V}$ at a distance of 15.0 m, what are the sign and magnitude of the charge?

53. In nuclear fission, a nucleus splits roughly in half. (a) What is the potential $2.00\times10^{-14}\ \text{m}$ from a fragment that has 46 protons in it? (b) What is the potential energy in MeV of a similarly charged fragment at this distance?

54. A research Van de Graaff generator has a 2.00-m-diameter metal sphere with a charge of 5.00 mC on it. (a) What is the potential near its surface? (b) At what distance from its center is the potential 1.00 MV? (c) An oxygen atom with three missing electrons is released near the Van de Graaff generator. What is its energy in MeV at this distance?

55. An electrostatic paint sprayer has a 0.200-m-diameter metal sphere at a potential of 25.0 kV that repels paint droplets onto a grounded object. (a) What charge is on the sphere? (b) What charge must a 0.100-mg drop of paint have to arrive at the object with a speed of 10.0 m/s?

56. In one of the classic nuclear physics experiments at the beginning of the 20th century, an alpha particle was accelerated toward a gold nucleus, and its path was substantially deflected by the Coulomb interaction. If the energy of the doubly charged alpha nucleus was 5.00 MeV, how close to the gold nucleus (79 protons) could it come before being deflected?

57. (a) What is the potential between two points situated 10 cm and 20 cm from a $3.0\ \mu\text{C}$ point charge? (b) To what location should the point at 20 cm be moved to increase this potential difference by a factor of two?

58. Unreasonable Results

(a) What is the final speed of an electron accelerated from rest through a voltage of 25.0 MV by a negatively charged Van de Graaff terminal?

(b) What is unreasonable about this result?

(c) Which assumptions are responsible?

19.4 Equipotential Lines

59. (a) Sketch the equipotential lines near a point charge $+q$. Indicate the direction of increasing potential. (b) Do the same for a point charge $-3q$.

60. Sketch the equipotential lines for the two equal positive charges shown in **Figure 19.27**. Indicate the direction of increasing potential.

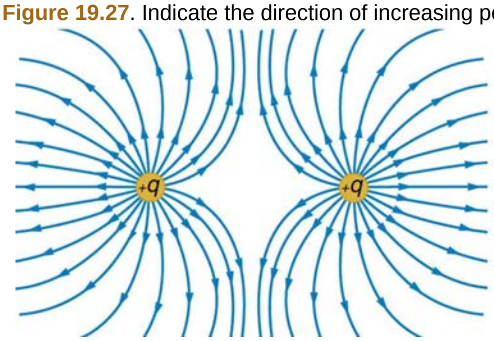


Figure 19.27 The electric field near two equal positive charges is directed away from each of the charges.

61. Figure 19.28 shows the electric field lines near two charges q_1 and q_2, the first having a magnitude four times that of the second. Sketch the equipotential lines for these two charges, and indicate the direction of increasing potential.

62. Sketch the equipotential lines a long distance from the charges shown in **Figure 19.28**. Indicate the direction of increasing potential.

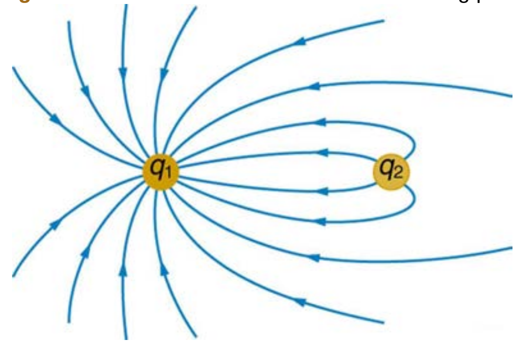


Figure 19.28 The electric field near two charges.

63. Sketch the equipotential lines in the vicinity of two opposite charges, where the negative charge is three times as great in magnitude as the positive. See **Figure 19.28** for a similar situation. Indicate the direction of increasing potential.

64. Sketch the equipotential lines in the vicinity of the negatively charged conductor in **Figure 19.29**. How will these equipotentials look a long distance from the object?

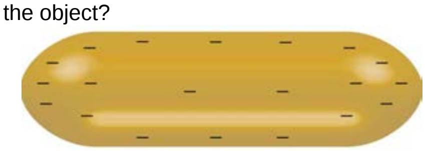


Figure 19.29 A negatively charged conductor.

65. Sketch the equipotential lines surrounding the two conducting plates shown in **Figure 19.30**, given the top plate is positive and the bottom plate has an equal amount of negative charge. Be certain to indicate the distribution of charge on the plates. Is the field strongest where the plates are closest? Why should it be?

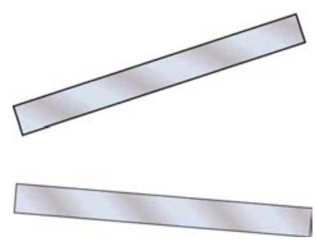


Figure 19.30

66. (a) Sketch the electric field lines in the vicinity of the charged insulator in **Figure 19.31**. Note its non-uniform charge distribution. (b) Sketch equipotential lines surrounding the insulator. Indicate the direction of increasing potential.

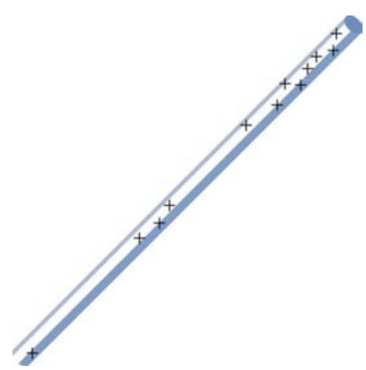


Figure 19.31 A charged insulating rod such as might be used in a classroom demonstration.

67. The naturally occurring charge on the ground on a fine day out in the open country is $-1.00\ \text{nC/m}^2$. (a) What is the electric field relative to ground at a height of 3.00 m? (b) Calculate the electric potential at this height. (c) Sketch electric field and equipotential lines for this scenario.

68. The lesser electric ray (*Narcine bancroftii*) maintains an incredible charge on its head and a charge equal in magnitude but opposite in sign on its tail (**Figure 19.32**). (a) Sketch the equipotential lines surrounding the ray. (b) Sketch the equipotentials when the ray is near a ship with a conducting surface. (c) How could this charge distribution be of use to the ray?

Figure 19.32 Lesser electric ray (*Narcine bancroftii*) (credit: National Oceanic and Atmospheric Administration, NOAA's Fisheries Collection).

19.5 Capacitors and Dielectrics

69. What charge is stored in a $180\ \mu\text{F}$ capacitor when 120 V is applied to it?

70. Find the charge stored when 5.50 V is applied to an 8.00 pF capacitor.

71. What charge is stored in the capacitor in **Example 19.8**?

72. Calculate the voltage applied to a $2.00\ \mu\text{F}$ capacitor when it holds $3.10\ \mu\text{C}$ of charge.

73. What voltage must be applied to an 8.00 nF capacitor to store 0.160 mC of charge?

74. What capacitance is needed to store $3.00\ \mu\text{C}$ of charge at a voltage of 120 V?

75. What is the capacitance of a large Van de Graaff generator's terminal, given that it stores 8.00 mC of charge at a voltage of 12.0 MV?

76. Find the capacitance of a parallel plate capacitor having plates of area $5.00\ \text{m}^2$ that are separated by 0.100 mm of Teflon.

77. (a)What is the capacitance of a parallel plate capacitor having plates of area $1.50\ \text{m}^2$ that are separated by 0.0200 mm of neoprene rubber? (b) What charge does it hold when 9.00 V is applied to it?

78. Integrated Concepts

A prankster applies 450 V to an $80.0\ \mu\text{F}$ capacitor and then tosses it to an unsuspecting victim. The victim's finger is burned by the discharge of the capacitor through 0.200 g of flesh. What is the temperature increase of the flesh? Is it reasonable to assume no phase change?

79. Unreasonable Results

(a) A certain parallel plate capacitor has plates of area $4.00\ \text{m}^2$, separated by 0.0100 mm of nylon, and stores 0.170 C of charge. What is the applied voltage? (b) What is unreasonable about this result? (c) Which assumptions are responsible or inconsistent?

19.6 Capacitors in Series and Parallel

80. Find the total capacitance of the combination of capacitors in **Figure 19.33**.

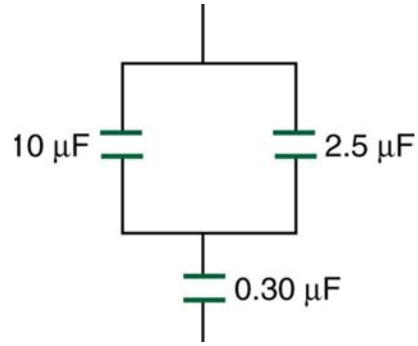


Figure 19.33 A combination of series and parallel connections of capacitors.

81. Suppose you want a capacitor bank with a total capacitance of 0.750 F and you possess numerous 1.50 mF capacitors. What is the smallest number you could hook together to achieve your goal, and how would you connect them?

82. What total capacitances can you make by connecting a $5.00\ \mu\text{F}$ and an $8.00\ \mu\text{F}$ capacitor together?

83. Find the total capacitance of the combination of capacitors shown in **Figure 19.34**.

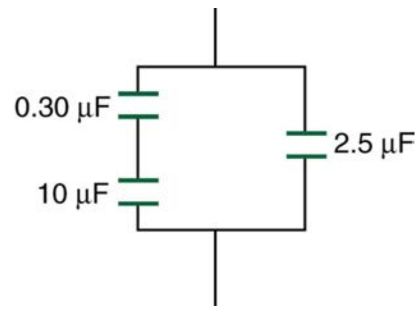


Figure 19.34 A combination of series and parallel connections of capacitors.

84. Find the total capacitance of the combination of capacitors shown in **Figure 19.35**.

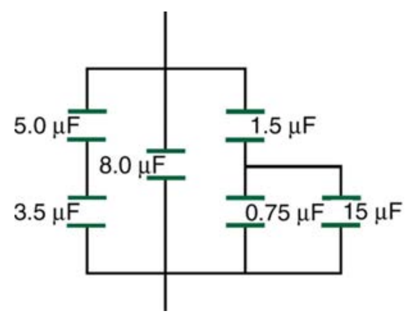


Figure 19.35 A combination of series and parallel connections of capacitors.

85. Unreasonable Results

(a) An $8.00\ \mu\text{F}$ capacitor is connected in parallel to another capacitor, producing a total capacitance of $5.00\ \mu\text{F}$. What is the capacitance of the second capacitor? (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

19.7 Energy Stored in Capacitors

86. (a) What is the energy stored in the $10.0\ \mu\text{F}$ capacitor of a heart defibrillator charged to $9.00\times10^3\ \text{V}$? (b) Find the amount of stored charge.

87. In open heart surgery, a much smaller amount of energy will defibrillate the heart. (a) What voltage is applied to the $8.00\ \mu\text{F}$ capacitor of a heart defibrillator that stores 40.0 J of energy? (b) Find the amount of stored charge.

88. A $165\ \mu F$ capacitor is used in conjunction with a motor. How much energy is stored in it when 119 V is applied?

89. Suppose you have a 9.00 V battery, a $2.00\ \mu F$ capacitor, and a $7.40\ \mu F$ capacitor. (a) Find the charge and energy stored if the capacitors are connected to the battery in series. (b) Do the same for a parallel connection.

90. A nervous physicist worries that the two metal shelves of his wood frame bookcase might obtain a high voltage if charged by static electricity, perhaps produced by friction. (a) What is the capacitance of the empty shelves if they have area $1.00\times10^{2}\ m^{2}$ and are 0.200 m apart? (b) What is the voltage between them if opposite charges of magnitude 2.00 nC are placed on them? (c) To show that this voltage poses a small hazard, calculate the energy stored.

91. Show that for a given dielectric material the maximum energy a parallel plate capacitor can store is directly proportional to the volume of dielectric ($\text{Volume} = A \cdot d$). Note that the applied voltage is limited by the dielectric strength.

92. Construct Your Own Problem

Consider a heart defibrillator similar to that discussed in **Example 19.11**. Construct a problem in which you examine the charge stored in the capacitor of a defibrillator as a function of stored energy. Among the things to be considered are the applied voltage and whether it should vary with energy to be delivered, the range of energies involved, and the capacitance of the defibrillator. You may also wish to consider the much smaller energy needed for defibrillation during open-heart surgery as a variation on this problem.

93. Unreasonable Results

(a) On a particular day, it takes $9.60\times10^{3}\ J$ of electric energy to start a truck's engine. Calculate the capacitance of a capacitor that could store that amount of energy at 12.0 V. (b) What is unreasonable about this result? (c) Which assumptions are responsible?

20 ELECTRIC CURRENT, RESISTANCE, AND OHM'S LAW

Figure 20.1 Electric energy in massive quantities is transmitted from this hydroelectric facility, the Srisailam power station located along the Krishna River in India, by the movement of charge—that is, by electric current. (credit: Chintohere, Wikimedia Commons)

Learning Objectives

Introduction to Electric Current, Resistance, and Ohm's Law

The flicker of numbers on a handheld calculator, nerve impulses carrying signals of vision to the brain, an ultrasound device sending a signal to a computer screen, the brain sending a message for a baby to twitch its toes, an electric train pulling its load over a mountain pass, a hydroelectric plant sending energy to metropolitan and rural users—these and many other examples of electricity involve *electric current, the movement of charge*. Humankind has indeed harnessed electricity, the basis of technology, to improve our quality of life. Whereas the previous two chapters concentrated on static electricity and the fundamental force underlying its behavior, the next few chapters will be devoted to electric and magnetic phenomena involving current. In addition to exploring applications of electricity, we shall gain new insights into nature—in particular, the fact that all magnetism results from electric current.

20.1 Current

Electric Current

Electric current is defined to be the rate at which charge flows. A large current, such as that used to start a truck engine, moves a large amount of charge in a small time, whereas a small current, such as that used to operate a hand-held calculator, moves a small amount of charge over a long period of time. In equation form, **electric current** I is defined to be

$$I = \frac{\Delta Q}{\Delta t}, \tag{20.1}$$

where ΔQ is the amount of charge passing through a given area in time Δt. (As in previous chapters, initial time is often taken to be zero, in which case $\Delta t = t$.) (See **Figure 20.2**.) The SI unit for current is the **ampere** (A), named for the French physicist André-Marie Ampère (1775–1836). Since $I = \Delta Q / \Delta t$, we see that an ampere is one coulomb per second:

$$1\text{ A} = 1\text{ C/s} \tag{20.2}$$

Not only are fuses and circuit breakers rated in amperes (or amps), so are many electrical appliances.

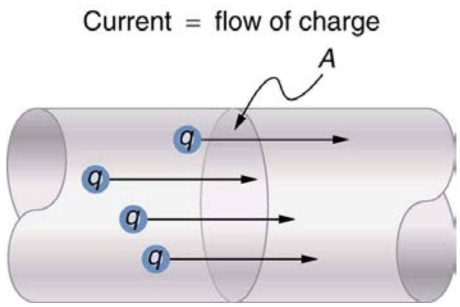


Figure 20.2 The rate of flow of charge is current. An ampere is the flow of one coulomb through an area in one second.

Example 20.1 Calculating Currents: Current in a Truck Battery and a Handheld Calculator

(a) What is the current involved when a truck battery sets in motion 720 C of charge in 4.00 s while starting an engine? (b) How long does it take 1.00 C of charge to flow through a handheld calculator if a 0.300-mA current is flowing?

Strategy

We can use the definition of current in the equation $I = \Delta Q / \Delta t$ to find the current in part (a), since charge and time are given. In part (b), we rearrange the definition of current and use the given values of charge and current to find the time required.

Solution for (a)

Entering the given values for charge and time into the definition of current gives

$$\begin{aligned} I &= \frac{\Delta Q}{\Delta t} = \frac{720\text{ C}}{4.00\text{ s}} = 180\text{ C/s} \\ &= 180\text{ A}. \end{aligned} \tag{20.3}$$

Discussion for (a)

This large value for current illustrates the fact that a large charge is moved in a small amount of time. The currents in these “starter motors” are fairly large because large frictional forces need to be overcome when setting something in motion.

Solution for (b)

Solving the relationship $I = \Delta Q / \Delta t$ for time Δt, and entering the known values for charge and current gives

$$\begin{aligned} \Delta t &= \frac{\Delta Q}{I} = \frac{1.00\text{ C}}{0.300\times10^{-3}\text{ C/s}} \\ &= 3.33\times10^{3}\text{ s}. \end{aligned} \tag{20.4}$$

Discussion for (b)

This time is slightly less than an hour. The small current used by the hand-held calculator takes a much longer time to move a smaller charge than the large current of the truck starter. So why can we operate our calculators only seconds after turning them on? It’s because calculators require very little energy. Such small current and energy demands allow handheld calculators to operate from solar cells or to get many hours of use out of small batteries. Remember, calculators do not have moving parts in the same way that a truck engine has with cylinders and pistons, so the technology requires smaller currents.

Figure 20.3 shows a simple circuit and the standard schematic representation of a battery, conducting path, and load (a resistor). Schematics are very useful in visualizing the main features of a circuit. A single schematic can represent a wide variety of situations. The schematic in **Figure 20.3** (b), for example, can represent anything from a truck battery connected to a headlight lighting the street in front of the truck to a small battery connected to a penlight lighting a keyhole in a door. Such schematics are useful because the analysis is the same for a wide variety of situations. We need to understand a few schematics to apply the concepts and analysis to many more situations.

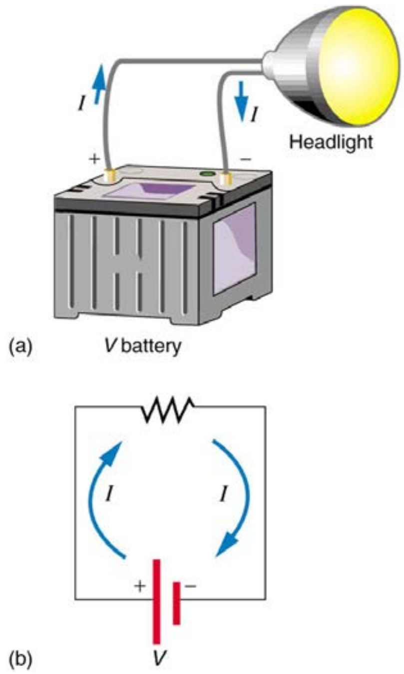


Figure 20.3 (a) A simple electric circuit. A closed path for current to flow through is supplied by conducting wires connecting a load to the terminals of a battery. (b) In this schematic, the battery is represented by the two parallel red lines, conducting wires are shown as straight lines, and the zigzag represents the load. The schematic represents a wide variety of similar circuits.

Note that the direction of current flow in **Figure 20.3** is from positive to negative. *The direction of conventional current is the direction that positive charge would flow*. Depending on the situation, positive charges, negative charges, or both may move. In metal wires, for example, current is carried by electrons—that is, negative charges move. In ionic solutions, such as salt water, both positive and negative charges move. This is also true in nerve cells. A Van de Graaff generator used for nuclear research can produce a current of pure positive charges, such as protons. **Figure 20.4** illustrates the movement of charged particles that compose a current. The fact that conventional current is taken to be in the direction that positive charge would flow can be traced back to American politician and scientist Benjamin Franklin in the 1700s. He named the type of charge associated with electrons negative, long before they were known to carry current in so many situations. Franklin, in fact, was totally unaware of the small-scale structure of electricity.

It is important to realize that there is an electric field in conductors responsible for producing the current, as illustrated in **Figure 20.4**. Unlike static electricity, where a conductor in equilibrium cannot have an electric field in it, conductors carrying a current have an electric field and are not in static equilibrium. An electric field is needed to supply energy to move the charges.

Making Connections: Take-Home Investigation—Electric Current Illustration

Find a straw and little peas that can move freely in the straw. Place the straw flat on a table and fill the straw with peas. When you pop one pea in at one end, a different pea should pop out the other end. This demonstration is an analogy for an electric current. Identify what compares to the electrons and what compares to the supply of energy. What other analogies can you find for an electric current?

Note that the flow of peas is based on the peas physically bumping into each other; electrons flow due to mutually repulsive electrostatic forces.

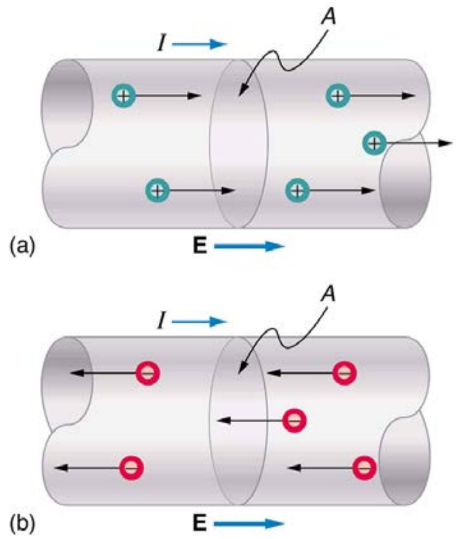


Figure 20.4 Current I is the rate at which charge moves through an area A, such as the cross-section of a wire. Conventional current is defined to move in the direction of the electric field. (a) Positive charges move in the direction of the electric field and the same direction as conventional current. (b) Negative charges move in the direction opposite to the electric field. Conventional current is in the direction opposite to the movement of negative charge. The flow of electrons is sometimes referred to as electronic flow.

Example 20.2 Calculating the Number of Electrons that Move through a Calculator

If the 0.300-mA current through the calculator mentioned in the **Example 20.1** example is carried by electrons, how many electrons per second pass through it?

Strategy

The current calculated in the previous example was defined for the flow of positive charge. For electrons, the magnitude is the same, but the sign is opposite, $I_{\text{electrons}} = -0.300\times10^{-3}\text{ C/s}$ .Since each electron (e^-) has a charge of $-1.60\times10^{-19}\text{ C}$, we can convert the current in coulombs per second to electrons per second.

Solution

Starting with the definition of current, we have

$$I_{\text{electrons}} = \frac{\Delta Q_{\text{electrons}}}{\Delta t} = \frac{-0.300\times10^{-3}\text{ C}}{\text{s}}. \tag{20.5}$$

We divide this by the charge per electron, so that

$$\begin{aligned}\frac{e^-}{\text{s}} &= \frac{-0.300\times10^{-3}\text{ C}}{\text{s}}\times\frac{1\ e^-}{-1.60\times10^{-19}\text{ C}} \\ &= 1.88\times10^{15}\ \frac{e^-}{\text{s}}.\end{aligned} \tag{20.6}$$

Discussion

There are so many charged particles moving, even in small currents, that individual charges are not noticed, just as individual water molecules are not noticed in water flow. Even more amazing is that they do not always keep moving forward like soldiers in a parade. Rather they are like a crowd of people with movement in different directions but a general trend to move forward. There are lots of collisions with atoms in the metal wire and, of course, with other electrons.

Drift Velocity

Electrical signals are known to move very rapidly. Telephone conversations carried by currents in wires cover large distances without noticeable delays. Lights come on as soon as a switch is flicked. Most electrical signals carried by currents travel at speeds on the order of 10^8 m/s, a significant fraction of the speed of light. Interestingly, the individual charges that make up the current move *much* more slowly on average, typically drifting at speeds on the order of 10^{-4} m/s. How do we reconcile these two speeds, and what does it tell us about standard conductors?

The high speed of electrical signals results from the fact that the force between charges acts rapidly at a distance. Thus, when a free charge is forced into a wire, as in **Figure 20.5**, the incoming charge pushes other charges ahead of it, which in turn push on charges farther down the line. The density of charge in a system cannot easily be increased, and so the signal is passed on rapidly. The resulting electrical shock wave moves through the system at nearly the speed of light. To be precise, this rapidly moving signal or shock wave is a rapidly propagating change in electric field.

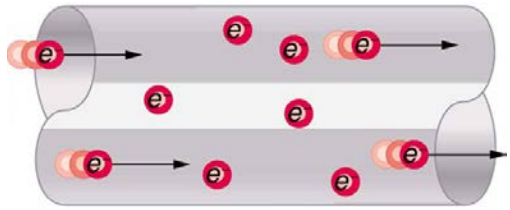


Figure 20.5 When charged particles are forced into this volume of a conductor, an equal number are quickly forced to leave. The repulsion between like charges makes it difficult to increase the number of charges in a volume. Thus, as one charge enters, another leaves almost immediately, carrying the signal rapidly forward.

Good conductors have large numbers of free charges in them. In metals, the free charges are free electrons. Figure 20.6 shows how free electrons move through an ordinary conductor. The distance that an individual electron can move between collisions with atoms or other electrons is quite small. The electron paths thus appear nearly random, like the motion of atoms in a gas. But there is an electric field in the conductor that causes the electrons to drift in the direction shown (opposite to the field, since they are negative). The **drift velocity** v_d is the average velocity of the free charges. Drift velocity is quite small, since there are so many free charges. If we have an estimate of the density of free electrons in a conductor, we can calculate the drift velocity for a given current. The larger the density, the lower the velocity required for a given current.

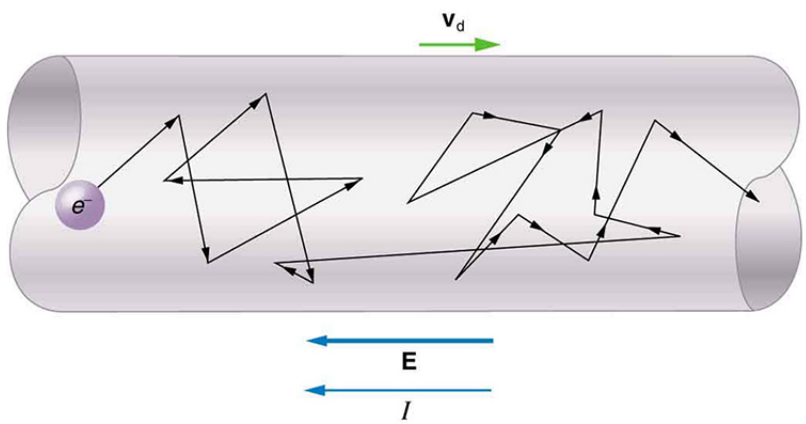


Figure 20.6 Free electrons moving in a conductor make many collisions with other electrons and atoms. The path of one electron is shown. The average velocity of the free charges is called the drift velocity, v_d, and it is in the direction opposite to the electric field for electrons. The collisions normally transfer energy to the conductor, requiring a constant supply of energy to maintain a steady current.

Conduction of Electricity and Heat

Good electrical conductors are often good heat conductors, too. This is because large numbers of free electrons can carry electrical current and can transport thermal energy.

The free-electron collisions transfer energy to the atoms of the conductor. The electric field does work in moving the electrons through a distance, but that work does not increase the kinetic energy (nor speed, therefore) of the electrons. The work is transferred to the conductor's atoms, possibly increasing temperature. Thus a continuous power input is required to keep a current flowing. An exception, of course, is found in superconductors, for reasons we shall explore in a later chapter. Superconductors can have a steady current without a continual supply of energy—a great energy savings. In contrast, the supply of energy can be useful, such as in a lightbulb filament. The supply of energy is necessary to increase the temperature of the tungsten filament, so that the filament glows.

Making Connections: Take-Home Investigation—Filament Observations

Find a lightbulb with a filament. Look carefully at the filament and describe its structure. To what points is the filament connected?

We can obtain an expression for the relationship between current and drift velocity by considering the number of free charges in a segment of wire, as illustrated in Figure 20.7. *The number of free charges per unit volume* is given the symbol n and depends on the material. The shaded segment has a volume Ax, so that the number of free charges in it is nAx. The charge ΔQ in this segment is thus $qnAx$, where q is the amount of charge on each carrier. (Recall that for electrons, q is $-1.60\times10^{-19}\text{ C}$.) Current is charge moved per unit time; thus, if all the original charges move out of this segment in time Δt, the current is

$$I = \frac{\Delta Q}{\Delta t} = \frac{qnAx}{\Delta t}. \tag{20.7}$$

Note that $x/\Delta t$ is the magnitude of the drift velocity, v_d, since the charges move an average distance x in a time Δt. Rearranging terms gives

$$I = nqAv_d, \tag{20.8}$$

where I is the current through a wire of cross-sectional area A made of a material with a free charge density n. The carriers of the current each have charge q and move with a drift velocity of magnitude v_d.

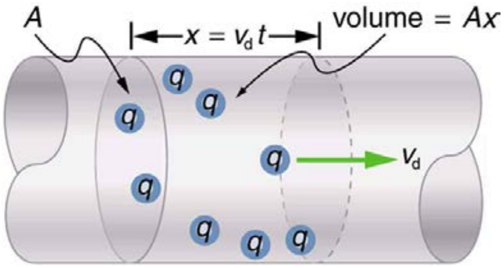


Figure 20.7 All the charges in the shaded volume of this wire move out in a time t, having a drift velocity of magnitude $v_d = x/t$. See text for further discussion.

Note that simple drift velocity is not the entire story. The speed of an electron is much greater than its drift velocity. In addition, not all of the electrons in a conductor can move freely, and those that do might move somewhat faster or slower than the drift velocity. So what do we mean by free electrons? Atoms in a metallic conductor are packed in the form of a lattice structure. Some electrons are far enough away from the atomic nuclei that they do not experience the attraction of the nuclei as much as the inner electrons do. These are the free electrons. They are not bound to a single atom but can instead move freely among the atoms in a "sea" of electrons. These free electrons respond by accelerating when an electric field is applied. Of course as they move they collide with the atoms in the lattice and other electrons, generating thermal energy, and the conductor gets warmer. In an insulator, the organization of the atoms and the structure do not allow for such free electrons.

Example 20.3 Calculating Drift Velocity in a Common Wire

Calculate the drift velocity of electrons in a 12-gauge copper wire (which has a diameter of 2.053 mm) carrying a 20.0-A current, given that there is one free electron per copper atom. (Household wiring often contains 12-gauge copper wire, and the maximum current allowed in such wire is usually 20 A.) The density of copper is $8.80\times10^3\ \text{kg/m}^3$.

Strategy

We can calculate the drift velocity using the equation $I = nqAv_d$. The current $I = 20.0\ \text{A}$ is given, and $q = -1.60\times10^{-19}\ \text{C}$ is the charge of an electron. We can calculate the area of a cross-section of the wire using the formula $A = \pi r^2,$ where r is one-half the given diameter, 2.053 mm. We are given the density of copper, $8.80\times10^3\ \text{kg/m}^3,$ and the periodic table shows that the atomic mass of copper is 63.54 g/mol. We can use these two quantities along with Avogadro's number, $6.02\times10^{23}\ \text{atoms/mol},$ to determine $n,$ the number of free electrons per cubic meter.

Solution

First, calculate the density of free electrons in copper. There is one free electron per copper atom. Therefore, is the same as the number of copper atoms per m^3. We can now find n as follows:

$$\begin{aligned} n &= \frac{1\ e^-}{\text{atom}}\times\frac{6.02\times10^{23}\ \text{atoms}}{\text{mol}}\times\frac{1\ \text{mol}}{63.54\ \text{g}}\times\frac{1000\ \text{g}}{\text{kg}}\times\frac{8.80\times10^3\ \text{kg}}{1\ \text{m}^3} \\ &= 8.342\times10^{28}\ e^-/\text{m}^3. \end{aligned} \tag{20.9}$$

The cross-sectional area of the wire is

$$\begin{aligned} A &= \pi r^2 \\ &= \pi\left(\frac{2.053\times10^{-3}\ \text{m}}{2}\right)^2 \\ &= 3.310\times10^{-6}\ \text{m}^2. \end{aligned} \tag{20.10}$$

Rearranging $I = nqAv_d$ to isolate drift velocity gives

$$\begin{aligned} v_d &= \frac{I}{nqA} \\ &= \frac{20.0\ \text{A}}{(8.342\times10^{28}/\text{m}^3)(-1.60\times10^{-19}\ \text{C})(3.310\times10^{-6}\ \text{m}^2)} \\ &= -4.53\times10^{-4}\ \text{m/s}. \end{aligned} \tag{20.11}$$

Discussion

The minus sign indicates that the negative charges are moving in the direction opposite to conventional current. The small value for drift velocity (on the order of 10^{-4} m/s) confirms that the signal moves on the order of 10^{12} times faster (about 10^8 m/s) than the charges that carry it.

20.2 Ohm’s Law: Resistance and Simple Circuits

What drives current? We can think of various devices—such as batteries, generators, wall outlets, and so on—which are necessary to maintain a current. All such devices create a potential difference and are loosely referred to as voltage sources. When a voltage source is connected to a conductor, it applies a potential difference V that creates an electric field. The electric field in turn exerts force on charges, causing current.

Ohm’s Law

The current that flows through most substances is directly proportional to the voltage V applied to it. The German physicist Georg Simon Ohm (1787–1854) was the first to demonstrate experimentally that the current in a metal wire is *directly proportional to the voltage applied*:

$$I \propto V. \tag{20.12}$$

This important relationship is known as **Ohm’s law**. It can be viewed as a cause-and-effect relationship, with voltage the cause and current the effect. This is an empirical law like that for friction—an experimentally observed phenomenon. Such a linear relationship doesn’t always occur.

Resistance and Simple Circuits

If voltage drives current, what impedes it? The electric property that impedes current (crudely similar to friction and air resistance) is called **resistance** R. Collisions of moving charges with atoms and molecules in a substance transfer energy to the substance and limit current. Resistance is defined as inversely proportional to current, or

$$I \propto \frac{1}{R}. \tag{20.13}$$

Thus, for example, current is cut in half if resistance doubles. Combining the relationships of current to voltage and current to resistance gives

$$I = \frac{V}{R}. \tag{20.14}$$

This relationship is also called Ohm’s law. Ohm’s law in this form really defines resistance for certain materials. Ohm’s law (like Hooke’s law) is not universally valid. The many substances for which Ohm’s law holds are called **ohmic**. These include good conductors like copper and aluminum, and some poor conductors under certain circumstances. Ohmic materials have a resistance R that is independent of voltage V and current I. An object that has simple resistance is called a *resistor*, even if its resistance is small. The unit for resistance is an **ohm** and is given the symbol Ω (upper case Greek omega). Rearranging $I = V/R$ gives $R = V/I$, and so the units of resistance are 1 ohm = 1 volt per ampere:

$$1\ \Omega = 1\frac{V}{A}. \tag{20.15}$$

Figure 20.8 shows the schematic for a simple circuit. A **simple circuit** has a single voltage source and a single resistor. The wires connecting the voltage source to the resistor can be assumed to have negligible resistance, or their resistance can be included in R.

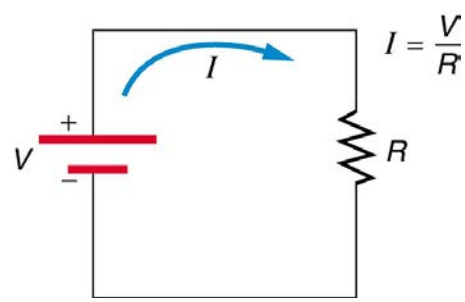


Figure 20.8 A simple electric circuit in which a closed path for current to flow is supplied by conductors (usually metal wires) connecting a load to the terminals of a battery, represented by the red parallel lines. The zigzag symbol represents the single resistor and includes any resistance in the connections to the voltage source.

Example 20.4 Calculating Resistance: An Automobile Headlight

What is the resistance of an automobile headlight through which 2.50 A flows when 12.0 V is applied to it?

Strategy

We can rearrange Ohm’s law as stated by $I = V/R$ and use it to find the resistance.

Solution

Rearranging $I = V/R$ and substituting known values gives

$$R = \frac{V}{I} = \frac{12.0\text{ V}}{2.50\text{ A}} = 4.80\ \Omega. \tag{20.16}$$

Discussion

This is a relatively small resistance, but it is larger than the cold resistance of the headlight. As we shall see in **Resistance and Resistivity**, resistance usually increases with temperature, and so the bulb has a lower resistance when it is first switched on and will draw considerably more current during its brief warm-up period.

Resistances range over many orders of magnitude. Some ceramic insulators, such as those used to support power lines, have resistances of $10^{12}\ \Omega$ or more. A dry person may have a hand-to-foot resistance of $10^{5}\ \Omega$, whereas the resistance of the human heart is about $10^{3}\ \Omega$. A meter-long piece of large-diameter copper wire may have a resistance of $10^{-5}\ \Omega$, and superconductors have no resistance at all (they are non-ohmic). Resistance is related to the shape of an object and the material of which it is composed, as will be seen in **Resistance and Resistivity**.

Additional insight is gained by solving $I = V/R$ for $V,$ yielding

$$V = IR. \tag{20.17}$$

This expression for V can be interpreted as the *voltage drop across a resistor produced by the flow of current* I. The phrase IR *drop* is often used for this voltage. For instance, the headlight in **Example 20.4** has an IR drop of 12.0 V. If voltage is measured at various points in a circuit, it will be seen to increase at the voltage source and decrease at the resistor. Voltage is similar to fluid pressure. The voltage source is like a pump, creating a pressure difference, causing current—the flow of charge. The resistor is like a pipe that reduces pressure and limits flow because of its resistance. Conservation of energy has important consequences here. The voltage source supplies energy (causing an electric field and a current), and the resistor converts it to another form (such as thermal energy). In a simple circuit (one with a single simple resistor), the voltage supplied by the source equals the voltage drop across the resistor, since $\mathrm{PE} = q\Delta V$, and the same q flows through each. Thus the energy supplied by the voltage source and the energy converted by the resistor are equal. (See **Figure 20.9**.)

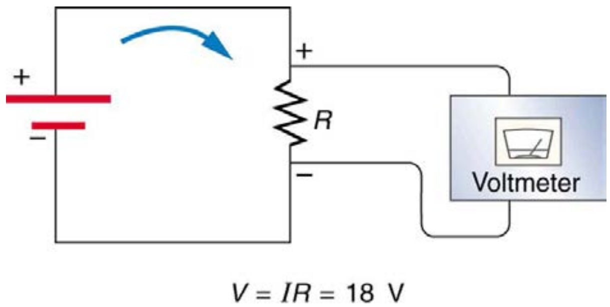


Figure 20.9 The voltage drop across a resistor in a simple circuit equals the voltage output of the battery.

Making Connections: Conservation of Energy

In a simple electrical circuit, the sole resistor converts energy supplied by the source into another form. Conservation of energy is evidenced here by the fact that all of the energy supplied by the source is converted to another form by the resistor alone. We will find that conservation of energy has other important applications in circuits and is a powerful tool in circuit analysis.

PhET Explorations: Ohm's Law

See how the equation form of Ohm's law relates to a simple circuit. Adjust the voltage and resistance, and see the current change according to Ohm's law. The sizes of the symbols in the equation change to match the circuit diagram.

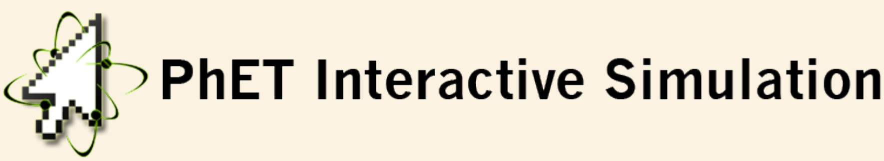


Figure 20.10 Ohm's Law (http://cnx.org/content/m42344/1.4/ohms-law_en.jar)

20.3 Resistance and Resistivity

Material and Shape Dependence of Resistance

The resistance of an object depends on its shape and the material of which it is composed. The cylindrical resistor in **Figure 20.11** is easy to analyze, and, by so doing, we can gain insight into the resistance of more complicated shapes. As you might expect, the cylinder's electric resistance R is directly proportional to its length L, similar to the resistance of a pipe to fluid flow. The longer the cylinder, the more collisions charges will make with its atoms. The greater the diameter of the cylinder, the more current it can carry (again similar to the flow of fluid through a pipe). In fact, R is inversely proportional to the cylinder's cross-sectional area A.

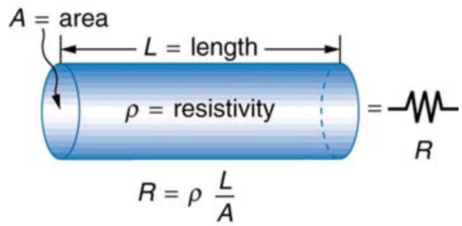


Figure 20.11 A uniform cylinder of length L and cross-sectional area A. Its resistance to the flow of current is similar to the resistance posed by a pipe to fluid flow. The longer the cylinder, the greater its resistance. The larger its cross-sectional area A, the smaller its resistance.

For a given shape, the resistance depends on the material of which the object is composed. Different materials offer different resistance to the flow of charge. We define the **resistivity** ρ of a substance so that the **resistance** R of an object is directly proportional to ρ. Resistivity ρ is an *intrinsic* property of a material, independent of its shape or size. The resistance R of a uniform cylinder of length L, of cross-sectional area A, and made of a material with resistivity ρ, is

$$R = \frac{\rho L}{A}. \tag{20.18}$$

Table 20.1 gives representative values of ρ. The materials listed in the table are separated into categories of conductors, semiconductors, and insulators, based on broad groupings of resistivities. Conductors have the smallest resistivities, and insulators have the largest; semiconductors have intermediate resistivities. Conductors have varying but large free charge densities, whereas most charges in insulators are bound to atoms and are not free to move. Semiconductors are intermediate, having far fewer free charges than conductors, but having properties that make the number of free charges depend strongly on the type and amount of impurities in the semiconductor. These unique properties of semiconductors are put to use in modern electronics, as will be explored in later chapters.

Table 20.1 Resistivities ρ of Various materials at 20°C

Material	Resistivity ρ ($\Omega \cdot \text{m}$)
Conductors	
Silver	1.59×10^{-8}
Copper	1.72×10^{-8}
Gold	2.44×10^{-8}
Aluminum	2.65×10^{-8}
Tungsten	5.6×10^{-8}
Iron	9.71×10^{-8}
Platinum	10.6×10^{-8}
Steel	20×10^{-8}
Lead	22×10^{-8}
Manganin (Cu, Mn, Ni alloy)	44×10^{-8}
Constantan (Cu, Ni alloy)	49×10^{-8}
Mercury	96×10^{-8}
Nichrome (Ni, Fe, Cr alloy)	100×10^{-8}
Semiconductors[1]	
Carbon (pure)	3.5×10^{5}
Carbon	$(3.5 - 60) \times 10^{5}$
Germanium (pure)	600×10^{-3}
Germanium	$(1 - 600) \times 10^{-3}$
Silicon (pure)	2300
Silicon	$0.1–2300$
Insulators	
Amber	5×10^{14}
Glass	$10^{9} - 10^{14}$
Lucite	$>10^{13}$
Mica	$10^{11} - 10^{15}$
Quartz (fused)	75×10^{16}
Rubber (hard)	$10^{13} - 10^{16}$
Sulfur	10^{15}
Teflon	$>10^{13}$
Wood	$10^{8} - 10^{11}$

1. Values depend strongly on amounts and types of impurities

Example 20.5 Calculating Resistor Diameter: A Headlight Filament

A car headlight filament is made of tungsten and has a cold resistance of $0.350\ \Omega$. If the filament is a cylinder 4.00 cm long (it may be coiled to save space), what is its diameter?

Strategy

We can rearrange the equation $R = \frac{\rho L}{A}$ to find the cross-sectional area A of the filament from the given information. Then its diameter can be found by assuming it has a circular cross-section.

Solution

The cross-sectional area, found by rearranging the expression for the resistance of a cylinder given in $R = \frac{\rho L}{A}$, is

$$A = \frac{\rho L}{R}. \tag{20.19}$$

Substituting the given values, and taking ρ from **Table 20.1**, yields

$$\begin{aligned} A &= \frac{(5.6\times10^{-8}\ \Omega\cdot\text{m})(4.00\times10^{-2}\ \text{m})}{1.350\ \Omega} \\ &= 6.40\times10^{-9}\ \text{m}^2. \end{aligned} \tag{20.20}$$

The area of a circle is related to its diameter D by

$$A = \frac{\pi D^2}{4}. \tag{20.21}$$

Solving for the diameter D, and substituting the value found for A, gives

$$\begin{aligned} D &= 2\left(\frac{A}{p}\right)^{\frac{1}{2}} = 2\left(\frac{6.40\times10^{-9}\ \text{m}^2}{3.14}\right)^{\frac{1}{2}} \\ &= 9.0\times10^{-5}\ \text{m}. \end{aligned} \tag{20.22}$$

Discussion

The diameter is just under a tenth of a millimeter. It is quoted to only two digits, because ρ is known to only two digits.

Temperature Variation of Resistance

The resistivity of all materials depends on temperature. Some even become superconductors (zero resistivity) at very low temperatures. (See **Figure 20.12**.) Conversely, the resistivity of conductors increases with increasing temperature. Since the atoms vibrate more rapidly and over larger distances at higher temperatures, the electrons moving through a metal make more collisions, effectively making the resistivity higher. Over relatively small temperature changes (about 100°C or less), resistivity ρ varies with temperature change ΔT as expressed in the following equation

$$\rho = \rho_0(1 + \alpha\Delta T), \tag{20.23}$$

where ρ_0 is the original resistivity and α is the **temperature coefficient of resistivity**. (See the values of α in **Table 20.2** below.) For larger temperature changes, α may vary or a nonlinear equation may be needed to find ρ. Note that α is positive for metals, meaning their resistivity increases with temperature. Some alloys have been developed specifically to have a small temperature dependence. Manganin (which is made of copper, manganese and nickel), for example, has α close to zero (to three digits on the scale in **Table 20.2**), and so its resistivity varies only slightly with temperature. This is useful for making a temperature-independent resistance standard, for example.

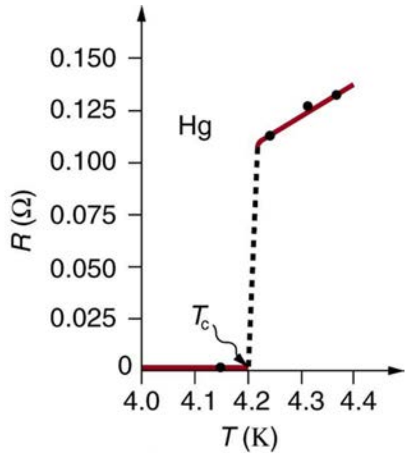


Figure 20.12 The resistance of a sample of mercury is zero at very low temperatures—it is a superconductor up to about 4.2 K. Above that critical temperature, its resistance makes a sudden jump and then increases nearly linearly with temperature.

Table 20.2 Tempature Coefficients of Resistivity α

Material	Coefficient α (1/°C)[2]
Conductors	
Silver	3.8×10^{-3}
Copper	3.9×10^{-3}
Gold	3.4×10^{-3}
Aluminum	3.9×10^{-3}
Tungsten	4.5×10^{-3}
Iron	5.0×10^{-3}
Platinum	3.93×10^{-3}
Lead	3.9×10^{-3}
Manganin (Cu, Mn, Ni alloy)	0.000×10^{-3}
Constantan (Cu, Ni alloy)	0.002×10^{-3}
Mercury	0.89×10^{-3}
Nichrome (Ni, Fe, Cr alloy)	0.4×10^{-3}
Semiconductors	
Carbon (pure)	-0.5×10^{-3}
Germanium (pure)	-50×10^{-3}
Silicon (pure)	-70×10^{-3}

Note also that α is negative for the semiconductors listed in **Table 20.2**, meaning that their resistivity decreases with increasing temperature. They become better conductors at higher temperature, because increased thermal agitation increases the number of free charges available to carry current. This property of decreasing ρ with temperature is also related to the type and amount of impurities present in the semiconductors.

The resistance of an object also depends on temperature, since R_0 is directly proportional to ρ. For a cylinder we know $R = \rho L / A$, and so, if L and A do not change greatly with temperature, R will have the same temperature dependence as ρ. (Examination of the coefficients of linear expansion shows them to be about two orders of magnitude less than typical temperature coefficients of resistivity, and so the effect of temperature on L and A is about two orders of magnitude less than on ρ.) Thus,

$$R = R_0(1 + \alpha\Delta T) \tag{20.24}$$

2.

is the temperature dependence of the resistance of an object, where R_0 is the original resistance and R is the resistance after a temperature change ΔT. Numerous thermometers are based on the effect of temperature on resistance. (See **Figure 20.13**.) One of the most common is the thermistor, a semiconductor crystal with a strong temperature dependence, the resistance of which is measured to obtain its temperature. The device is small, so that it quickly comes into thermal equilibrium with the part of a person it touches.

Figure 20.13 These familiar thermometers are based on the automated measurement of a thermistor's temperature-dependent resistance. (credit: Biol, Wikimedia Commons)

Example 20.6 Calculating Resistance: Hot-Filament Resistance

Although caution must be used in applying $\rho = \rho_0(1 + \alpha\Delta T)$ and $R = R_0(1 + \alpha\Delta T)$ for temperature changes greater than 100°C, for tungsten the equations work reasonably well for very large temperature changes. What, then, is the resistance of the tungsten filament in the previous example if its temperature is increased from room temperature (20°C) to a typical operating temperature of 2850°C ?

Strategy

This is a straightforward application of $R = R_0(1 + \alpha\Delta T)$, since the original resistance of the filament was given to be $R_0 = 0.350\ \Omega$, and the temperature change is $\Delta T = 2830^{\circ}\text{C}$.

Solution

The hot resistance R is obtained by entering known values into the above equation:

$$\begin{aligned} R &= R_0(1 + \alpha\Delta T) \\ &= (0.350\ \Omega)[1 + (4.5\times10^{-3} / {}^{\circ}\text{C})(2830^{\circ}\text{C})] \\ &= 4.8\ \Omega. \end{aligned} \tag{20.25}$$

Discussion

This value is consistent with the headlight resistance example in **Ohm's Law: Resistance and Simple Circuits**.

PhET Explorations: Resistance in a Wire

Learn about the physics of resistance in a wire. Change its resistivity, length, and area to see how they affect the wire's resistance. The sizes of the symbols in the equation change along with the diagram of a wire.

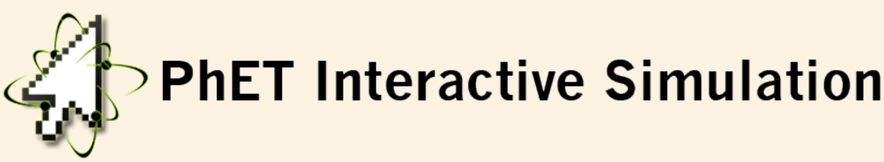
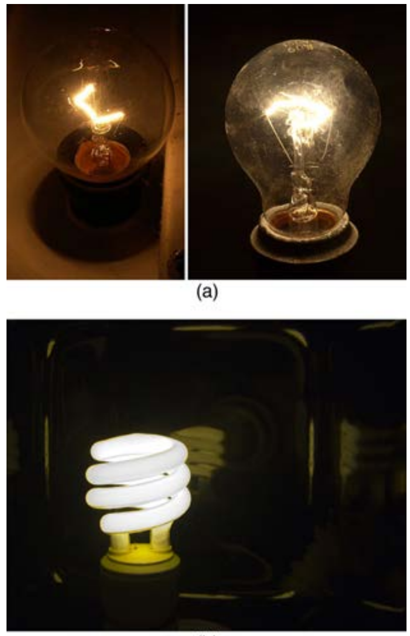


Figure 20.14 Resistance in a Wire (http://cnx.org/content/m42346/1.6/resistance-in-a-wire_en.jar)

20.4 Electric Power and Energy

Power in Electric Circuits

Power is associated by many people with electricity. Knowing that power is the rate of energy use or energy conversion, what is the expression for **electric power**? Power transmission lines might come to mind. We also think of lightbulbs in terms of their power ratings in watts. Let us compare a 25-W bulb with a 60-W bulb. (See **Figure 20.15**(a).) Since both operate on the same voltage, the 60-W bulb must draw more current to have a greater power rating. Thus the 60-W bulb's resistance must be lower than that of a 25-W bulb. If we increase voltage, we also increase power. For example, when a 25-W bulb that is designed to operate on 120 V is connected to 240 V, it briefly glows very brightly and then burns out. Precisely how are voltage, current, and resistance related to electric power?



(a)

(b)

Figure 20.15 (a) Which of these lightbulbs, the 25-W bulb (upper left) or the 60-W bulb (upper right), has the higher resistance? Which draws more current? Which uses the most energy? Can you tell from the color that the 25-W filament is cooler? Is the brighter bulb a different color and if so why? (credits: Dickbauch, Wikimedia Commons; Greg Westfall, Flickr) (b) This compact fluorescent light (CFL) puts out the same intensity of light as the 60-W bulb, but at 1/4 to 1/10 the input power. (credit: dbgg1979, Flickr)

Electric energy depends on both the voltage involved and the charge moved. This is expressed most simply as $\text{PE} = qV$, where q is the charge moved and V is the voltage (or more precisely, the potential difference the charge moves through). Power is the rate at which energy is moved, and so electric power is

$$P = \frac{PE}{t} = \frac{qV}{t}. \tag{20.26}$$

Recognizing that current is $I = q/t$ (note that $\Delta t = t$ here), the expression for power becomes

$$P = IV. \tag{20.27}$$

Electric power (P) is simply the product of current times voltage. Power has familiar units of watts. Since the SI unit for potential energy (PE) is the joule, power has units of joules per second, or watts. Thus, $1\ \text{A} \cdot \text{V} = 1\ \text{W}$. For example, cars often have one or more auxiliary power outlets with which you can charge a cell phone or other electronic devices. These outlets may be rated at 20 A, so that the circuit can deliver a maximum power $P = IV = (20\ \text{A})(12\ \text{V}) = 240\ \text{W}$. In some applications, electric power may be expressed as volt-amperes or even kilovolt-amperes ($1\ \text{kA} \cdot \text{V} = 1\ \text{kW}$).

To see the relationship of power to resistance, we combine Ohm's law with $P = IV$. Substituting $I = V/R$ gives $P = (V/R)V = V^2/R$. Similarly, substituting $V = IR$ gives $P = I(IR) = I^2R$. Three expressions for electric power are listed together here for convenience:

$$P = IV \tag{20.28}$$

$$P = \frac{V^2}{R} \tag{20.29}$$

$$P = I^2R. \tag{20.30}$$

Note that the first equation is always valid, whereas the other two can be used only for resistors. In a simple circuit, with one voltage source and a single resistor, the power supplied by the voltage source and that dissipated by the resistor are identical. (In more complicated circuits, P can be the power dissipated by a single device and not the total power in the circuit.)

Different insights can be gained from the three different expressions for electric power. For example, $P = V^2/R$ implies that the lower the resistance connected to a given voltage source, the greater the power delivered. Furthermore, since voltage is squared in $P = V^2/R$, the effect of applying a higher voltage is perhaps greater than expected. Thus, when the voltage is doubled to a 25-W bulb, its power nearly quadruples to about 100 W, burning it out. If the bulb's resistance remained constant, its power would be exactly 100 W, but at the higher temperature its resistance is higher, too.

Example 20.7 Calculating Power Dissipation and Current: Hot and Cold Power

(a) Consider the examples given in **Ohm's Law: Resistance and Simple Circuits** and **Resistance and Resistivity**. Then find the power dissipated by the car headlight in these examples, both when it is hot and when it is cold. (b) What current does it draw when cold?

Strategy for (a)

For the hot headlight, we know voltage and current, so we can use $P = IV$ to find the power. For the cold headlight, we know the voltage and resistance, so we can use $P = V^2 / R$ to find the power.

Solution for (a)

Entering the known values of current and voltage for the hot headlight, we obtain

$$P = IV = (2.50 \text{ A})(12.0 \text{ V}) = 30.0 \text{ W}. \tag{20.31}$$

The cold resistance was $0.350 \ \Omega$, and so the power it uses when first switched on is

$$P = \frac{V^2}{R} = \frac{(12.0 \text{ V})^2}{0.350 \ \Omega} = 411 \text{ W}. \tag{20.32}$$

Discussion for (a)

The 30 W dissipated by the hot headlight is typical. But the 411 W when cold is surprisingly higher. The initial power quickly decreases as the bulb's temperature increases and its resistance increases.

Strategy and Solution for (b)

The current when the bulb is cold can be found several different ways. We rearrange one of the power equations, $P = I^2 R$, and enter known values, obtaining

$$I = \sqrt{\frac{P}{R}} = \sqrt{\frac{411 \text{ W}}{0.350 \ \Omega}} = 34.3 \text{ A}. \tag{20.33}$$

Discussion for (b)

The cold current is remarkably higher than the steady-state value of 2.50 A, but the current will quickly decline to that value as the bulb's temperature increases. Most fuses and circuit breakers (used to limit the current in a circuit) are designed to tolerate very high currents briefly as a device comes on. In some cases, such as with electric motors, the current remains high for several seconds, necessitating special "slow blow" fuses.

The Cost of Electricity

The more electric appliances you use and the longer they are left on, the higher your electric bill. This familiar fact is based on the relationship between energy and power. You pay for the energy used. Since $P = E / t$, we see that

$$E = Pt \tag{20.34}$$

is the energy used by a device using power P for a time interval t . For example, the more lightbulbs burning, the greater P used; the longer they are on, the greater t is. The energy unit on electric bills is the kilowatt-hour ($\text{kW} \cdot \text{h}$), consistent with the relationship $E = Pt$. It is easy to estimate the cost of operating electric appliances if you have some idea of their power consumption rate in watts or kilowatts, the time they are on in hours, and the cost per kilowatt-hour for your electric utility. Kilowatt-hours, like all other specialized energy units such as food calories, can be converted to joules. You can prove to yourself that $1 \text{ kW} \cdot \text{h} = 3.6{\times}10^6 \text{ J}$.

The electrical energy (E) used can be reduced either by reducing the time of use or by reducing the power consumption of that appliance or fixture. This will not only reduce the cost, but it will also result in a reduced impact on the environment. Improvements to lighting are some of the fastest ways to reduce the electrical energy used in a home or business. About 20% of a home's use of energy goes to lighting, while the number for commercial establishments is closer to 40%. Fluorescent lights are about four times more efficient than incandescent lights—this is true for both the long tubes and the compact fluorescent lights (CFL). (See **Figure 20.15**(b).) Thus, a 60-W incandescent bulb can be replaced by a 15-W CFL, which has the same brightness and color. CFLs have a bent tube inside a globe or a spiral-shaped tube, all connected to a standard screw-in base that fits standard incandescent light sockets. (Original problems with color, flicker, shape, and high initial investment for CFLs have been addressed in recent years.) The heat transfer from these CFLs is less, and they last up to 10 times longer. The significance of an investment in such bulbs is addressed in the next example. New white LED lights (which are clusters of small LED bulbs) are even more efficient (twice that of CFLs) and last 5 times longer than CFLs. However, their cost is still high.

Making Connections: Energy, Power, and Time

The relationship $E = Pt$ is one that you will find useful in many different contexts. The energy your body uses in exercise is related to the power level and duration of your activity, for example. The amount of heating by a power source is related to the power level and time it is applied. Even the radiation dose of an X-ray image is related to the power and time of exposure.

Example 20.8 Calculating the Cost Effectiveness of Compact Fluorescent Lights (CFL)

If the cost of electricity in your area is 12 cents per kWh, what is the total cost (capital plus operation) of using a 60-W incandescent bulb for 1000 hours (the lifetime of that bulb) if the bulb cost 25 cents? (b) If we replace this bulb with a compact fluorescent light that provides the same light output, but at one-quarter the wattage, and which costs $1.50 but lasts 10 times longer (10,000 hours), what will that total cost be?

Strategy

To find the operating cost, we first find the energy used in kilowatt-hours and then multiply by the cost per kilowatt-hour.

Solution for (a)

The energy used in kilowatt-hours is found by entering the power and time into the expression for energy:

$$E = Pt = (60\ \text{W})(1000\ \text{h}) = 60{,}000\ \text{W} \cdot \text{h}. \tag{20.35}$$

In kilowatt-hours, this is

$$E = 60.0\ \text{kW} \cdot \text{h}. \tag{20.36}$$

Now the electricity cost is

$$\text{cost} = (60.0\ \text{kW} \cdot \text{h})(\$0.12/\text{kW} \cdot \text{h}) = \$7.20. \tag{20.37}$$

The total cost will be $7.20 for 1000 hours (about one-half year at 5 hours per day).

Solution for (b)

Since the CFL uses only 15 W and not 60 W, the electricity cost will be $7.20/4 = $1.80. The CFL will last 10 times longer than the incandescent, so that the investment cost will be 1/10 of the bulb cost for that time period of use, or 0.1($1.50) = $0.15. Therefore, the total cost will be $1.95 for 1000 hours.

Discussion

Therefore, it is much cheaper to use the CFLs, even though the initial investment is higher. The increased cost of labor that a business must include for replacing the incandescent bulbs more often has not been figured in here.

Making Connections: Take-Home Experiment—Electrical Energy Use Inventory

1) Make a list of the power ratings on a range of appliances in your home or room. Explain why something like a toaster has a higher rating than a digital clock. Estimate the energy consumed by these appliances in an average day (by estimating their time of use). Some appliances might only state the operating current. If the household voltage is 120 V, then use $P = IV$. 2) Check out the total wattage used in the rest rooms of your school's floor or building. (You might need to assume the long fluorescent lights in use are rated at 32 W.) Suppose that the building was closed all weekend and that these lights were left on from 6 p.m. Friday until 8 a.m. Monday. What would this oversight cost? How about for an entire year of weekends?

20.5 Alternating Current versus Direct Current

Alternating Current

Most of the examples dealt with so far, and particularly those utilizing batteries, have constant voltage sources. Once the current is established, it is thus also a constant. **Direct current** (DC) is the flow of electric charge in only one direction. It is the steady state of a constant-voltage circuit. Most well-known applications, however, use a time-varying voltage source. **Alternating current** (AC) is the flow of electric charge that periodically reverses direction. If the source varies periodically, particularly sinusoidally, the circuit is known as an alternating current circuit. Examples include the commercial and residential power that serves so many of our needs. Figure 20.16 shows graphs of voltage and current versus time for typical DC and AC power. The AC voltages and frequencies commonly used in homes and businesses vary around the world.

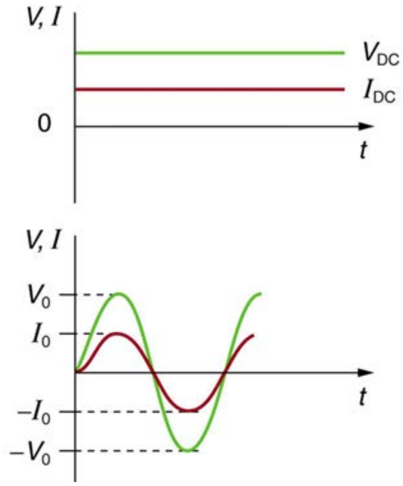


Figure 20.16 (a) DC voltage and current are constant in time, once the current is established. (b) A graph of voltage and current versus time for 60-Hz AC power. The voltage and current are sinusoidal and are in phase for a simple resistance circuit. The frequencies and peak voltages of AC sources differ greatly.

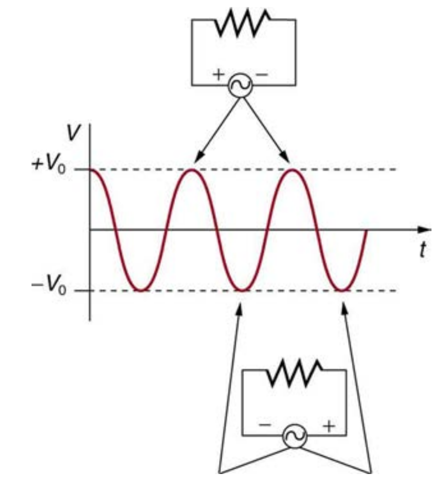


Figure 20.17 The potential difference V between the terminals of an AC voltage source fluctuates as shown. The mathematical expression for V is given by $V = V_0 \sin 2\pi ft$.

Figure 20.17 shows a schematic of a simple circuit with an AC voltage source. The voltage between the terminals fluctuates as shown, with the **AC voltage** given by

$$V = V_0 \sin 2\pi ft, \tag{20.38}$$

where V is the voltage at time t, V_0 is the peak voltage, and f is the frequency in hertz. For this simple resistance circuit, $I = V/R$, and so the **AC current** is

$$I = I_0 \sin 2\pi ft, \tag{20.39}$$

where I is the current at time t, and $I_0 = V_0/R$ is the peak current. For this example, the voltage and current are said to be in phase, as seen in **Figure 20.16**(b).

Current in the resistor alternates back and forth just like the driving voltage, since $I = V/R$. If the resistor is a fluorescent light bulb, for example, it brightens and dims 120 times per second as the current repeatedly goes through zero. A 120-Hz flicker is too rapid for your eyes to detect, but if you wave your hand back and forth between your face and a fluorescent light, you will see a stroboscopic effect evidencing AC. The fact that the light output fluctuates means that the power is fluctuating. The power supplied is $P = IV$. Using the expressions for I and V above, we see that the time dependence of power is $P = I_0 V_0 \sin^2 2\pi ft$, as shown in **Figure 20.18**.

Making Connections: Take-Home Experiment—AC/DC Lights

Wave your hand back and forth between your face and a fluorescent light bulb. Do you observe the same thing with the headlights on your car? Explain what you observe. *Warning: Do not look directly at very bright light*.

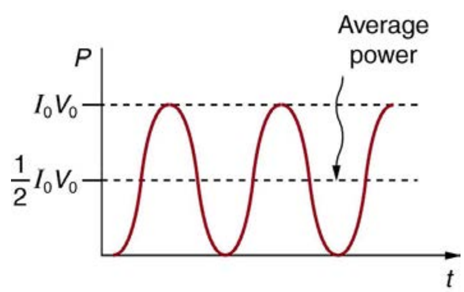


Figure 20.18 AC power as a function of time. Since the voltage and current are in phase here, their product is non-negative and fluctuates between zero and $I_0 V_0$. Average power is $(1/2)I_0 V_0$.

We are most often concerned with average power rather than its fluctuations—that 60-W light bulb in your desk lamp has an average power consumption of 60 W, for example. As illustrated in **Figure 20.18**, the average power P_{ave} is

$$P_{\text{ave}} = \frac{1}{2} I_0 V_0. \tag{20.40}$$

This is evident from the graph, since the areas above and below the $(1/2)I_0 V_0$ line are equal, but it can also be proven using trigonometric identities. Similarly, we define an average or **rms current** I_{rms} and average or **rms voltage** V_{rms} to be, respectively,

$$I_{\text{rms}} = \frac{I_0}{\sqrt{2}} \tag{20.41}$$

and

$$V_{\text{rms}} = \frac{V_0}{\sqrt{2}}. \tag{20.42}$$

where rms stands for root mean square, a particular kind of average. In general, to obtain a root mean square, the particular quantity is squared, its mean (or average) is found, and the square root is taken. This is useful for AC, since the average value is zero. Now,

$$P_{\text{ave}} = I_{\text{rms}} V_{\text{rms}}, \tag{20.43}$$

which gives

$$P_{\text{ave}} = \frac{I_0}{\sqrt{2}} \cdot \frac{V_0}{\sqrt{2}} = \frac{1}{2} I_0 V_0, \tag{20.44}$$

as stated above. It is standard practice to quote I_{rms}, V_{rms}, and P_{ave} rather than the peak values. For example, most household electricity is 120 V AC, which means that V_{rms} is 120 V. The common 10-A circuit breaker will interrupt a sustained I_{rms} greater than 10 A. Your 1.0-kW microwave oven consumes $P_{\text{ave}} = 1.0 \text{ kW}$, and so on. You can think of these rms and average values as the equivalent DC values for a simple resistive circuit.

To summarize, when dealing with AC, Ohm's law and the equations for power are completely analogous to those for DC, but rms and average values are used for AC. Thus, for AC, Ohm's law is written

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{R}. \tag{20.45}$$

The various expressions for AC power P_{ave} are

$$P_{\text{ave}} = I_{\text{rms}} V_{\text{rms}}, \tag{20.46}$$

$$P_{\text{ave}} = \frac{V_{\text{rms}}^2}{R}, \tag{20.47}$$

and

$$P_{\text{ave}} = I_{\text{rms}}^2 R. \tag{20.48}$$

Example 20.9 Peak Voltage and Power for AC

(a) What is the value of the peak voltage for 120-V AC power? (b) What is the peak power consumption rate of a 60.0-W AC light bulb?

Strategy

We are told that V_{rms} is 120 V and P_{ave} is 60.0 W. We can use $V_{\text{rms}} = \frac{V_0}{\sqrt{2}}$ to find the peak voltage, and we can manipulate the definition of power to find the peak power from the given average power.

Solution for (a)

Solving the equation $V_{\text{rms}} = \frac{V_0}{\sqrt{2}}$ for the peak voltage V_0 and substituting the known value for V_{rms} gives

$$V_0 = \sqrt{2} V_{\text{rms}} = 1.414(120 \text{ V}) = 170 \text{ V}. \tag{20.49}$$

Discussion for (a)

This means that the AC voltage swings from 170 V to -170 V and back 60 times every second. An equivalent DC voltage is a constant 120 V.

Solution for (b)

Peak power is peak current times peak voltage. Thus,

$$P_0 = I_0 V_0 = 2\left(\frac{1}{2} I_0 V_0\right) = 2P_{\text{ave}}. \tag{20.50}$$

We know the average power is 60.0 W, and so

$$P_0 = 2(60.0 \text{ W}) = 120 \text{ W}. \tag{20.51}$$

Discussion

So the power swings from zero to 120 W one hundred twenty times per second (twice each cycle), and the power averages 60 W.

Why Use AC for Power Distribution?

Most large power-distribution systems are AC. Moreover, the power is transmitted at much higher voltages than the 120-V AC (240 V in most parts of the world) we use in homes and on the job. Economies of scale make it cheaper to build a few very large electric power-generation plants than to build numerous small ones. This necessitates sending power long distances, and it is obviously important that energy losses en route be minimized. High voltages can be transmitted with much smaller power losses than low voltages, as we shall see. (See **Figure 20.19**.) For safety reasons, the voltage at the user is reduced to familiar values. The crucial factor is that it is much easier to increase and decrease AC voltages than DC, so AC is used in most large power distribution systems.

Figure 20.19 Power is distributed over large distances at high voltage to reduce power loss in the transmission lines. The voltages generated at the power plant are stepped up by passive devices called transformers (see **Transformers**) to 330,000 volts (or more in some places worldwide). At the point of use, the transformers reduce the voltage transmitted for safe residential and commercial use. (Credit: GeorgHH, Wikimedia Commons)

Example 20.10 Power Losses Are Less for High-Voltage Transmission

(a) What current is needed to transmit 100 MW of power at 200 kV? (b) What is the power dissipated by the transmission lines if they have a resistance of $1.00\ \Omega$? (c) What percentage of the power is lost in the transmission lines?

Strategy

We are given $P_{\text{ave}} = 100\text{ MW}$, $V_{\text{rms}} = 200\text{ kV}$, and the resistance of the lines is $R = 1.00\ \Omega$. Using these givens, we can find the current flowing (from $P = IV$) and then the power dissipated in the lines ($P = I^2R$), and we take the ratio to the total power transmitted.

Solution

To find the current, we rearrange the relationship $P_{\text{ave}} = I_{\text{rms}}V_{\text{rms}}$ and substitute known values. This gives

$$I_{\text{rms}} = \frac{P_{\text{ave}}}{V_{\text{rms}}} = \frac{100\times10^6\text{ W}}{200\times10^3\text{ V}} = 500\text{ A}. \tag{20.52}$$

Solution

Knowing the current and given the resistance of the lines, the power dissipated in them is found from $P_{\text{ave}} = I^2_{\text{rms}}R$. Substituting the known values gives

$$P_{\text{ave}} = I^2_{\text{rms}}R = (500\text{ A})^2(1.00\ \Omega) = 250\text{ kW}. \tag{20.53}$$

Solution

The percent loss is the ratio of this lost power to the total or input power, multiplied by 100:

$$\%\text{ loss} = \frac{250\text{ kW}}{100\text{ MW}}\times100 = 0.250\ \%. \tag{20.54}$$

Discussion

One-fourth of a percent is an acceptable loss. Note that if 100 MW of power had been transmitted at 25 kV, then a current of 4000 A would have been needed. This would result in a power loss in the lines of 16.0 MW, or 16.0% rather than 0.250%. The lower the voltage, the more current is needed, and the greater the power loss in the fixed-resistance transmission lines. Of course, lower-resistance lines can be built, but this requires larger and more expensive wires. If superconducting lines could be economically produced, there would be no loss in the transmission lines at all. But, as we shall see in a later chapter, there is a limit to current in superconductors, too. In short, high voltages are more economical for transmitting power, and AC voltage is much easier to raise and lower, so that AC is used in most large-scale power distribution systems.

It is widely recognized that high voltages pose greater hazards than low voltages. But, in fact, some high voltages, such as those associated with common static electricity, can be harmless. So it is not voltage alone that determines a hazard. It is not so widely recognized that AC shocks are often more harmful than similar DC shocks. Thomas Edison thought that AC shocks were more harmful and set up a DC power-distribution system in New York City in the late 1800s. There were bitter fights, in particular between Edison and George Westinghouse and Nikola Tesla, who were advocating the use of AC in early power-distribution systems. AC has prevailed largely due to transformers and lower power losses with high-voltage transmission.

PhET Explorations: Generator

Generate electricity with a bar magnet! Discover the physics behind the phenomena by exploring magnets and how you can use them to make a bulb light.

PhET Interactive Simulation

Figure 20.20 Generator (http://cnx.org/content/m42348/1.5/generator_en.jar)

20.6 Electric Hazards and the Human Body

There are two known hazards of electricity—thermal and shock. A **thermal hazard** is one where excessive electric power causes undesired thermal effects, such as starting a fire in the wall of a house. A **shock hazard** occurs when electric current passes through a person. Shocks range in severity from painful, but otherwise harmless, to heart-stopping lethality. This section considers these hazards and the various factors affecting them in a quantitative manner. **Electrical Safety: Systems and Devices** will consider systems and devices for preventing electrical hazards.

Thermal Hazards

Electric power causes undesired heating effects whenever electric energy is converted to thermal energy at a rate faster than it can be safely dissipated. A classic example of this is the **short circuit**, a low-resistance path between terminals of a voltage source. An example of a short circuit is shown in **Figure 20.21**. Insulation on wires leading to an appliance has worn through, allowing the two wires to come into contact. Such an undesired contact with a high voltage is called a *short*. Since the resistance of the short, r , is very small, the power dissipated in the short, $P = V^2 / r$, is very large. For example, if V is 120 V and r is $0.100\ \Omega$, then the power is 144 kW, *much* greater than that used by a typical household appliance. Thermal energy delivered at this rate will very quickly raise the temperature of surrounding materials, melting or perhaps igniting them.

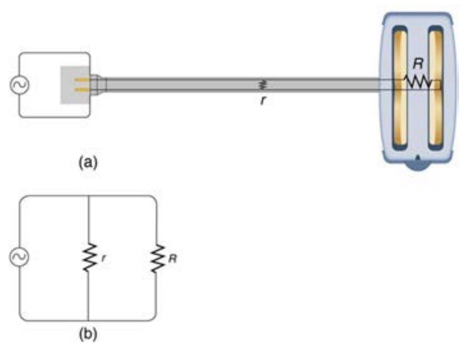


Figure 20.21 A short circuit is an undesired low-resistance path across a voltage source. (a) Worn insulation on the wires of a toaster allow them to come into contact with a low resistance r . Since $P = V^2 / r$, thermal power is created so rapidly that the cord melts or burns. (b) A schematic of the short circuit.

One particularly insidious aspect of a short circuit is that its resistance may actually be decreased due to the increase in temperature. This can happen if the short creates ionization. These charged atoms and molecules are free to move and, thus, lower the resistance r . Since $P = V^2 / r$, the power dissipated in the short rises, possibly causing more ionization, more power, and so on. High voltages, such as the 480-V AC used in some industrial applications, lend themselves to this hazard, because higher voltages create higher initial power production in a short.

Another serious, but less dramatic, thermal hazard occurs when wires supplying power to a user are overloaded with too great a current. As discussed in the previous section, the power dissipated in the supply wires is $P = I^2 R_{\mathrm{w}}$, where R_{w} is the resistance of the wires and I the current flowing through them. If either I or R_{w} is too large, the wires overheat. For example, a worn appliance cord (with some of its braided wires broken) may have $R_{\mathrm{w}} = 2.00\ \Omega$ rather than the $0.100\ \Omega$ it should be. If 10.0 A of current passes through the cord, then $P = I^2 R_{\mathrm{w}} = 200\ \mathrm{W}$ is dissipated in the cord—much more than is safe. Similarly, if a wire with a $0.100\text{-}\Omega$ resistance is meant to carry a few amps, but is instead carrying 100 A, it will severely overheat. The power dissipated in the wire will in that case be $P = 1000\ \mathrm{W}$. Fuses and circuit breakers are used to limit excessive currents. (See **Figure 20.22** and **Figure 20.23**.) Each device opens the circuit automatically when a sustained current exceeds safe limits.

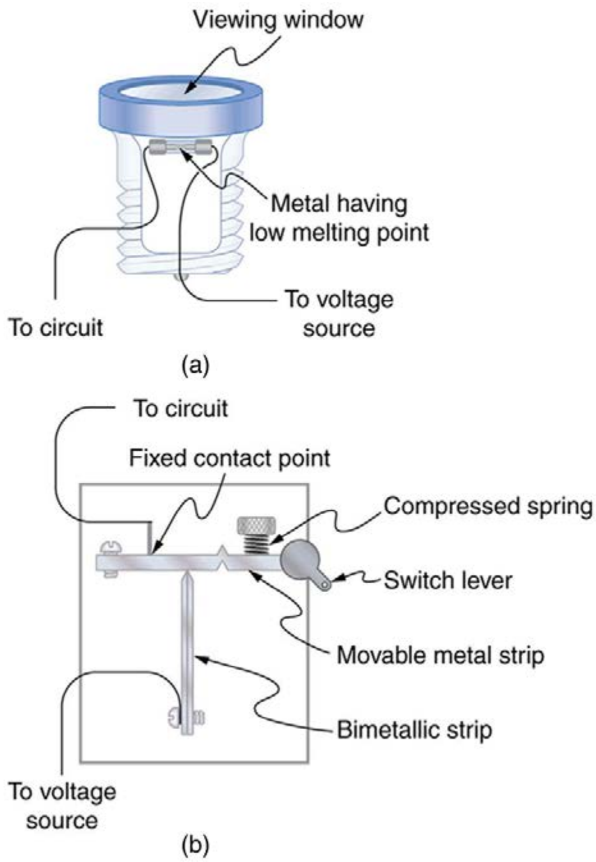


Figure 20.22 (a) A fuse has a metal strip with a low melting point that, when overheated by an excessive current, permanently breaks the connection of a circuit to a voltage source. (b) A circuit breaker is an automatic but restorable electric switch. The one shown here has a bimetallic strip that bends to the right and into the notch if overheated. The spring then forces the metal strip downward, breaking the electrical connection at the points.

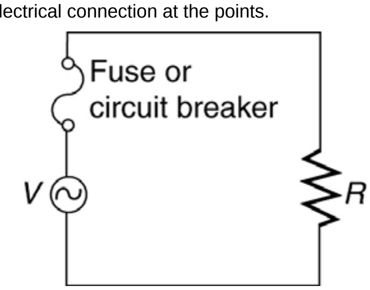


Figure 20.23 Schematic of a circuit with a fuse or circuit breaker in it. Fuses and circuit breakers act like automatic switches that open when sustained current exceeds desired limits.

Fuses and circuit breakers for typical household voltages and currents are relatively simple to produce, but those for large voltages and currents experience special problems. For example, when a circuit breaker tries to interrupt the flow of high-voltage electricity, a spark can jump across its points that ionizes the air in the gap and allows the current to continue flowing. Large circuit breakers found in power-distribution systems employ insulating gas and even use jets of gas to blow out such sparks. Here AC is safer than DC, since AC current goes through zero 120 times per second, giving a quick opportunity to extinguish these arcs.

Shock Hazards

Electrical currents through people produce tremendously varied effects. An electrical current can be used to block back pain. The possibility of using electrical current to stimulate muscle action in paralyzed limbs, perhaps allowing paraplegics to walk, is under study. TV dramatizations in which electrical shocks are used to bring a heart attack victim out of ventricular fibrillation (a massively irregular, often fatal, beating of the heart) are more than common. Yet most electrical shock fatalities occur because a current put the heart into fibrillation. A pacemaker uses electrical shocks to stimulate the heart to beat properly. Some fatal shocks do not produce burns, but warts can be safely burned off with electric current (though freezing using liquid nitrogen is now more common). Of course, there are consistent explanations for these disparate effects. The major factors upon which the effects of electrical shock depend are

1. The amount of current I
2. The path taken by the current
3. The duration of the shock
4. The frequency f of the current ($f = 0$ for DC)

Table 20.3 gives the effects of electrical shocks as a function of current for a typical accidental shock. The effects are for a shock that passes through the trunk of the body, has a duration of 1 s, and is caused by 60-Hz power.

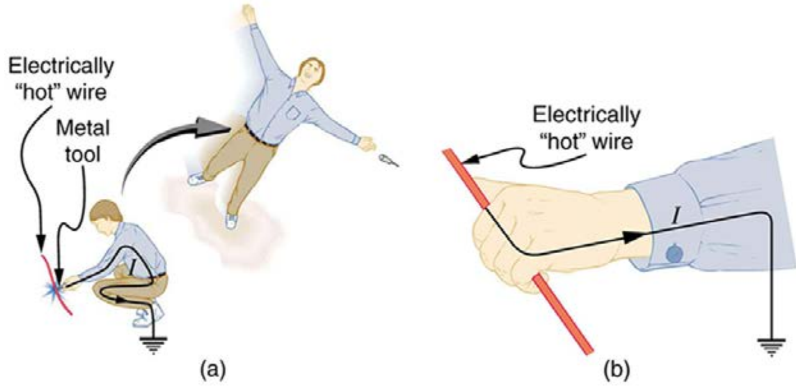


Figure 20.24 An electric current can cause muscular contractions with varying effects. (a) The victim is "thrown" backward by involuntary muscle contractions that extend the legs and torso. (b) The victim can't let go of the wire that is stimulating all the muscles in the hand. Those that close the fingers are stronger than those that open them.

Table 20.3 Effects of Electrical Shock as a Function of Current[3]

Current (mA)	Effect
1	Threshold of sensation
5	Maximum harmless current
10–20	Onset of sustained muscular contraction; cannot let go for duration of shock; contraction of chest muscles may stop breathing during shock
50	Onset of pain
100–300+	Ventricular fibrillation possible; often fatal
300	Onset of burns depending on concentration of current
6000 (6 A)	Onset of sustained ventricular contraction and respiratory paralysis; both cease when shock ends; heartbeat may return to normal; used to defibrillate the heart

Our bodies are relatively good conductors due to the water in our bodies. Given that larger currents will flow through sections with lower resistance (to be further discussed in the next chapter), electric currents preferentially flow through paths in the human body that have a minimum resistance in a direct path to earth. The earth is a natural electron sink. Wearing insulating shoes, a requirement in many professions, prohibits a pathway for electrons by providing a large resistance in that path. Whenever working with high-power tools (drills), or in risky situations, ensure that you do not provide a pathway for current flow (especially through the heart).

Very small currents pass harmlessly and unfelt through the body. This happens to you regularly without your knowledge. The threshold of sensation is only 1 mA and, although unpleasant, shocks are apparently harmless for currents less than 5 mA. A great number of safety rules take the 5-mA value for the maximum allowed shock. At 10 to 20 mA and above, the current can stimulate sustained muscular contractions much as regular nerve impulses do. People sometimes say they were knocked across the room by a shock, but what really happened was that certain muscles contracted, propelling them in a manner not of their own choosing. (See **Figure 20.24**(a).) More frightening, and potentially more dangerous, is the "can't let go" effect illustrated in **Figure 20.24**(b). The muscles that close the fingers are stronger than those that open them, so the hand closes involuntarily on the wire shocking it. This can prolong the shock indefinitely. It can also be a danger to a person trying to rescue the victim, because the rescuer's hand may close about the victim's wrist. Usually the best way to help the victim is to give the fist a hard knock/blow/jar with an insulator or to throw an insulator at the fist. Modern electric fences, used in animal enclosures, are now pulsed on and off to allow people who touch them to get free, rendering them less lethal than in the past.

Greater currents may affect the heart. Its electrical patterns can be disrupted, so that it beats irregularly and ineffectively in a condition called "ventricular fibrillation." This condition often lingers after the shock and is fatal due to a lack of blood circulation. The threshold for ventricular fibrillation is between 100 and 300 mA. At about 300 mA and above, the shock can cause burns, depending on the concentration of current—the more concentrated, the greater the likelihood of burns.

Very large currents cause the heart and diaphragm to contract for the duration of the shock. Both the heart and breathing stop. Interestingly, both often return to normal following the shock. The electrical patterns on the heart are completely erased in a manner that the heart can start afresh with normal beating, as opposed to the permanent disruption caused by smaller currents that can put the heart into ventricular fibrillation. The latter is something like scribbling on a blackboard, whereas the former completely erases it. TV dramatizations of electric shock used to bring a heart attack victim out of ventricular fibrillation also show large paddles. These are used to spread out current passed through the victim to reduce the likelihood of burns.

Current is the major factor determining shock severity (given that other conditions such as path, duration, and frequency are fixed, such as in the table and preceding discussion). A larger voltage is more hazardous, but since $I = V/R$, the severity of the shock depends on the combination of voltage and resistance. For example, a person with dry skin has a resistance of about $200\text{ k}\Omega$. If he comes into contact with 120-V AC, a current $I = (120\text{ V})/(200\text{ k}\Omega) = 0.6\text{ mA}$ passes harmlessly through him. The same person soaking wet may have a resistance of $10.0\text{ k}\Omega$ and the same 120 V will produce a current of 12 mA—above the "can't let go" threshold and potentially dangerous.

Most of the body's resistance is in its dry skin. When wet, salts go into ion form, lowering the resistance significantly. The interior of the body has a much lower resistance than dry skin because of all the ionic solutions and fluids it contains. If skin resistance is bypassed, such as by an intravenous

3. For an average male shocked through trunk of body for 1 s by 60-Hz AC. Values for females are 60–80% of those listed.

infusion, a catheter, or exposed pacemaker leads, a person is rendered **microshock sensitive**. In this condition, currents about 1/1000 those listed in **Table 20.3** produce similar effects. During open-heart surgery, currents as small as $20\ \mu\text{A}$ can be used to still the heart. Stringent electrical safety requirements in hospitals, particularly in surgery and intensive care, are related to the doubly disadvantaged microshock-sensitive patient. The break in the skin has reduced his resistance, and so the same voltage causes a greater current, and a much smaller current has a greater effect.

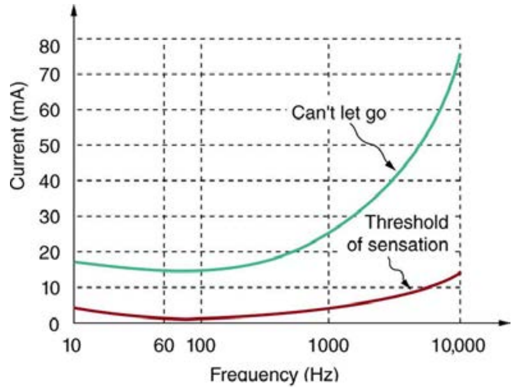


Figure 20.25 Graph of average values for the threshold of sensation and the “can't let go” current as a function of frequency. The lower the value, the more sensitive the body is at that frequency.

Factors other than current that affect the severity of a shock are its path, duration, and AC frequency. Path has obvious consequences. For example, the heart is unaffected by an electric shock through the brain, such as may be used to treat manic depression. And it is a general truth that the longer the duration of a shock, the greater its effects. **Figure 20.25** presents a graph that illustrates the effects of frequency on a shock. The curves show the minimum current for two different effects, as a function of frequency. The lower the current needed, the more sensitive the body is at that frequency. Ironically, the body is most sensitive to frequencies near the 50- or 60-Hz frequencies in common use. The body is slightly less sensitive for DC ($f=0$), mildly confirming Edison's claims that AC presents a greater hazard. At higher and higher frequencies, the body becomes progressively less sensitive to any effects that involve nerves. This is related to the maximum rates at which nerves can fire or be stimulated. At very high frequencies, electrical current travels only on the surface of a person. Thus a wart can be burned off with very high frequency current without causing the heart to stop. (Do not try this at home with 60-Hz AC!) Some of the spectacular demonstrations of electricity, in which high-voltage arcs are passed through the air and over people's bodies, employ high frequencies and low currents. (See **Figure 20.26**.) Electrical safety devices and techniques are discussed in detail in **Electrical Safety: Systems and Devices**.

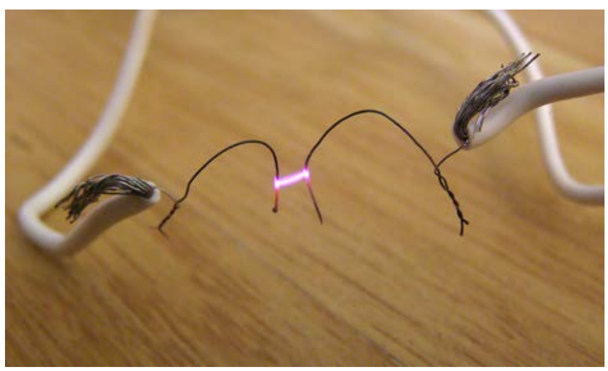


Figure 20.26 Is this electric arc dangerous? The answer depends on the AC frequency and the power involved. (credit: Khimich Alex, Wikimedia Commons)

20.7 Nerve Conduction–Electrocardiograms

Nerve Conduction

Electric currents in the vastly complex system of billions of nerves in our body allow us to sense the world, control parts of our body, and think. These are representative of the three major functions of nerves. First, nerves carry messages from our sensory organs and others to the central nervous system, consisting of the brain and spinal cord. Second, nerves carry messages from the central nervous system to muscles and other organs. Third, nerves transmit and process signals within the central nervous system. The sheer number of nerve cells and the incredibly greater number of connections between them makes this system the subtle wonder that it is. **Nerve conduction** is a general term for electrical signals carried by nerve cells. It is one aspect of **bioelectricity**, or electrical effects in and created by biological systems.

Nerve cells, properly called *neurons*, look different from other cells—they have tendrils, some of them many centimeters long, connecting them with other cells. (See **Figure 20.27**.) Signals arrive at the cell body across *synapses* or through *dendrites*, stimulating the neuron to generate its own signal, sent along its long *axon* to other nerve or muscle cells. Signals may arrive from many other locations and be transmitted to yet others, conditioning the synapses by use, giving the system its complexity and its ability to learn.

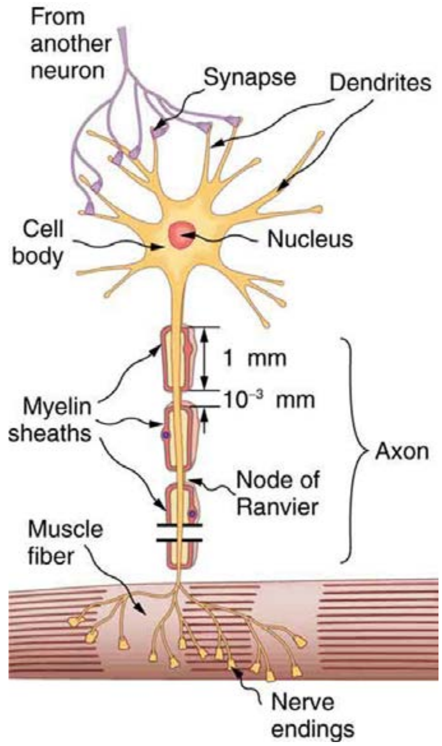


Figure 20.27 A neuron with its dendrites and long axon. Signals in the form of electric currents reach the cell body through dendrites and across synapses, stimulating the neuron to generate its own signal sent down the axon. The number of interconnections can be far greater than shown here.

The method by which these electric currents are generated and transmitted is more complex than the simple movement of free charges in a conductor, but it can be understood with principles already discussed in this text. The most important of these are the Coulomb force and diffusion.

Figure 20.28 illustrates how a voltage (potential difference) is created across the cell membrane of a neuron in its resting state. This thin membrane separates electrically neutral fluids having differing concentrations of ions, the most important varieties being Na^+, K^+, and Cl^- (these are sodium, potassium, and chlorine ions with single plus or minus charges as indicated). As discussed in **Molecular Transport Phenomena: Diffusion, Osmosis, and Related Processes**, free ions will diffuse from a region of high concentration to one of low concentration. But the cell membrane is **semipermeable**, meaning that some ions may cross it while others cannot. In its resting state, the cell membrane is permeable to K^+ and Cl^-, and impermeable to Na^+. Diffusion of K^+ and Cl^- thus creates the layers of positive and negative charge on the outside and inside of the membrane. The Coulomb force prevents the ions from diffusing across in their entirety. Once the charge layer has built up, the repulsion of like charges prevents more from moving across, and the attraction of unlike charges prevents more from leaving either side. The result is two layers of charge right on the membrane, with diffusion being balanced by the Coulomb force. A tiny fraction of the charges move across and the fluids remain neutral (other ions are present), while a separation of charge and a voltage have been created across the membrane.

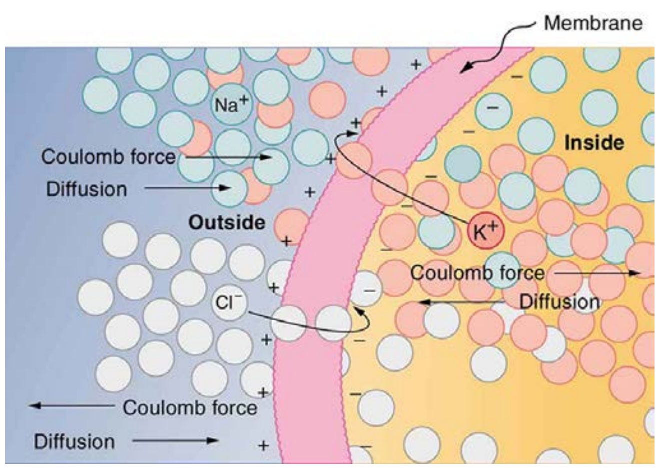


Figure 20.28 The semipermeable membrane of a cell has different concentrations of ions inside and out. Diffusion moves the K^+ and Cl^- ions in the direction shown, until the Coulomb force halts further transfer. This results in a layer of positive charge on the outside, a layer of negative charge on the inside, and thus a voltage across the cell membrane. The membrane is normally impermeable to Na^+.

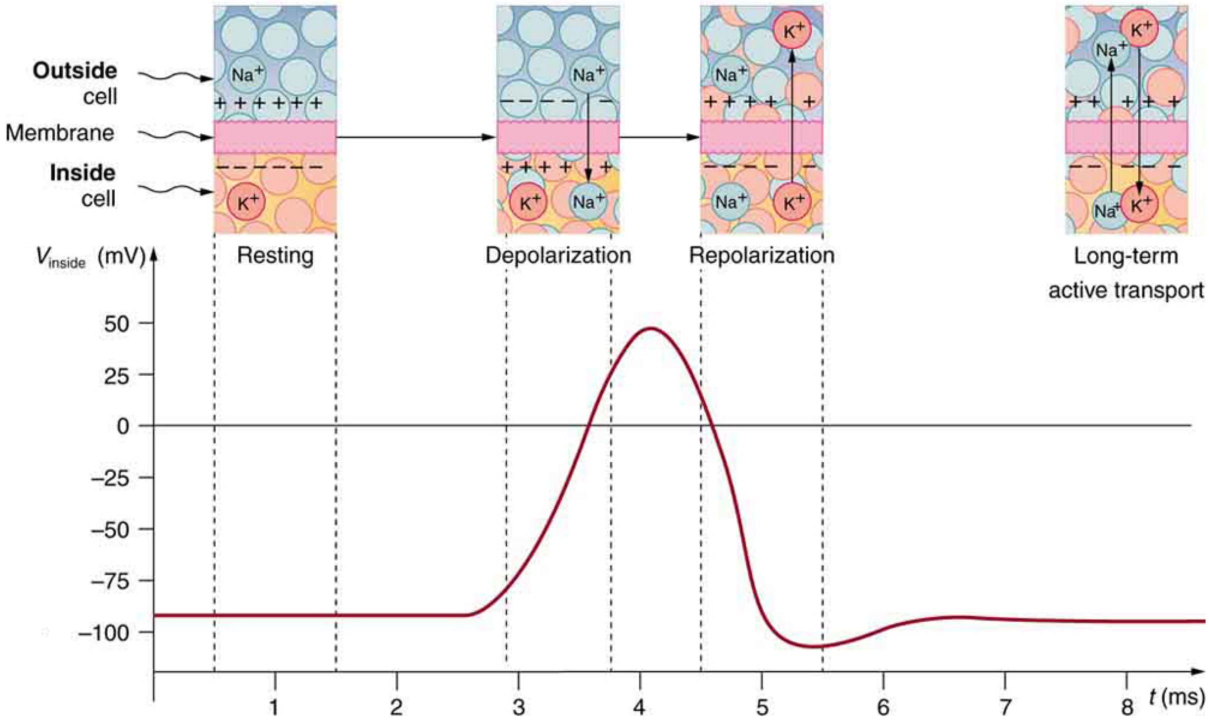


Figure 20.29 An action potential is the pulse of voltage inside a nerve cell graphed here. It is caused by movements of ions across the cell membrane as shown. Depolarization occurs when a stimulus makes the membrane permeable to Na^+ ions. Repolarization follows as the membrane again becomes impermeable to Na^+, and K^+ moves from high to low concentration. In the long term, active transport slowly maintains the concentration differences, but the cell may fire hundreds of times in rapid succession without seriously depleting them.

The separation of charge creates a potential difference of 70 to 90 mV across the cell membrane. While this is a small voltage, the resulting electric field ($E = V / d$) across the only 8-nm-thick membrane is immense (on the order of 11 MV/m!) and has fundamental effects on its structure and permeability. Now, if the exterior of a neuron is taken to be at 0 V, then the interior has a *resting potential* of about –90 mV. Such voltages are created across the membranes of almost all types of animal cells but are largest in nerve and muscle cells. In fact, fully 25% of the energy used by cells goes toward creating and maintaining these potentials.

Electric currents along the cell membrane are created by any stimulus that changes the membrane's permeability. The membrane thus temporarily becomes permeable to Na^+, which then rushes in, driven both by diffusion and the Coulomb force. This inrush of Na^+ first neutralizes the inside membrane, or *depolarizes* it, and then makes it slightly positive. The depolarization causes the membrane to again become impermeable to Na^+, and the movement of K^+ quickly returns the cell to its resting potential, or *repolarizes* it. This sequence of events results in a voltage pulse, called the *action potential*. (See **Figure 20.29**.) Only small fractions of the ions move, so that the cell can fire many hundreds of times without depleting the excess concentrations of Na^+ and K^+. Eventually, the cell must replenish these ions to maintain the concentration differences that create bioelectricity. This sodium-potassium pump is an example of *active transport*, wherein cell energy is used to move ions across membranes against diffusion gradients and the Coulomb force.

The action potential is a voltage pulse at one location on a cell membrane. How does it get transmitted along the cell membrane, and in particular down an axon, as a nerve impulse? The answer is that the changing voltage and electric fields affect the permeability of the adjacent cell membrane, so that the same process takes place there. The adjacent membrane depolarizes, affecting the membrane further down, and so on, as illustrated in **Figure 20.30**. Thus the action potential stimulated at one location triggers a *nerve impulse* that moves slowly (about 1 m/s) along the cell membrane.

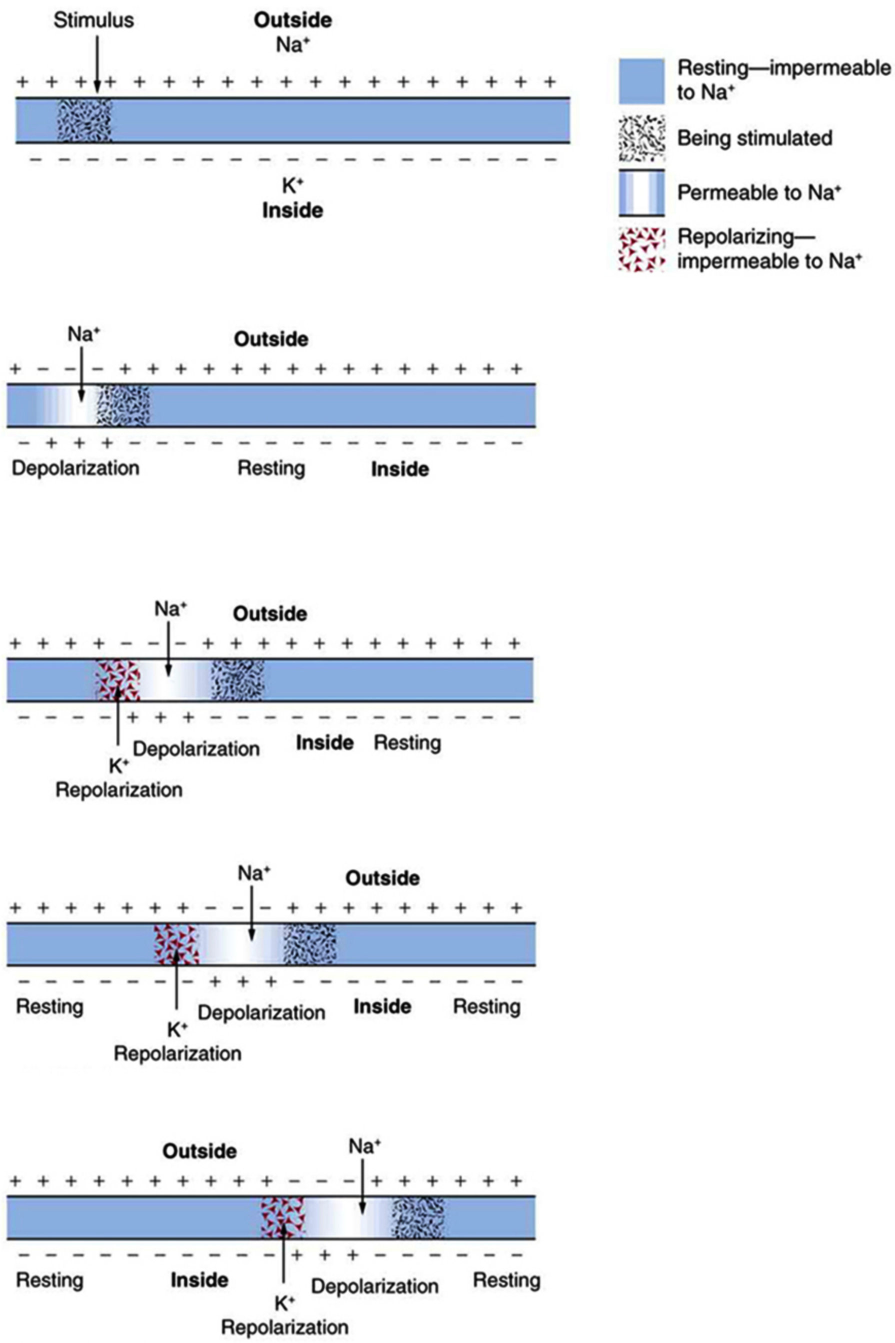


Figure 20.30 A nerve impulse is the propagation of an action potential along a cell membrane. A stimulus causes an action potential at one location, which changes the permeability of the adjacent membrane, causing an action potential there. This in turn affects the membrane further down, so that the action potential moves slowly (in electrical terms) along the cell membrane. Although the impulse is due to Na^+ and K^+ going across the membrane, it is equivalent to a wave of charge moving along the outside and inside of the membrane.

Some axons, like that in **Figure 20.27**, are sheathed with *myelin*, consisting of fat-containing cells. **Figure 20.31** shows an enlarged view of an axon having myelin sheaths characteristically separated by unmyelinated gaps (called nodes of Ranvier). This arrangement gives the axon a number of interesting properties. Since myelin is an insulator, it prevents signals from jumping between adjacent nerves (cross talk). Additionally, the myelinated regions transmit electrical signals at a very high speed, as an ordinary conductor or resistor would. There is no action potential in the myelinated regions, so that no cell energy is used in them. There is an IR signal loss in the myelin, but the signal is regenerated in the gaps, where the voltage pulse triggers the action potential at full voltage. So a myelinated axon transmits a nerve impulse faster, with less energy consumption, and is better protected from cross talk than an unmyelinated one. Not all axons are myelinated, so that cross talk and slow signal transmission are a characteristic of the normal operation of these axons, another variable in the nervous system.

The degeneration or destruction of the myelin sheaths that surround the nerve fibers impairs signal transmission and can lead to numerous neurological effects. One of the most prominent of these diseases comes from the body's own immune system attacking the myelin in the central nervous system—multiple sclerosis. MS symptoms include fatigue, vision problems, weakness of arms and legs, loss of balance, and tingling or numbness in one's extremities (neuropathy). It is more apt to strike younger adults, especially females. Causes might come from infection, environmental or geographic affects, or genetics. At the moment there is no known cure for MS.

Most animal cells can fire or create their own action potential. Muscle cells contract when they fire and are often induced to do so by a nerve impulse. In fact, nerve and muscle cells are physiologically similar, and there are even hybrid cells, such as in the heart, that have characteristics of both nerves and muscles. Some animals, like the infamous electric eel (see **Figure 20.32**), use muscles ganged so that their voltages add in order to create a shock great enough to stun prey.

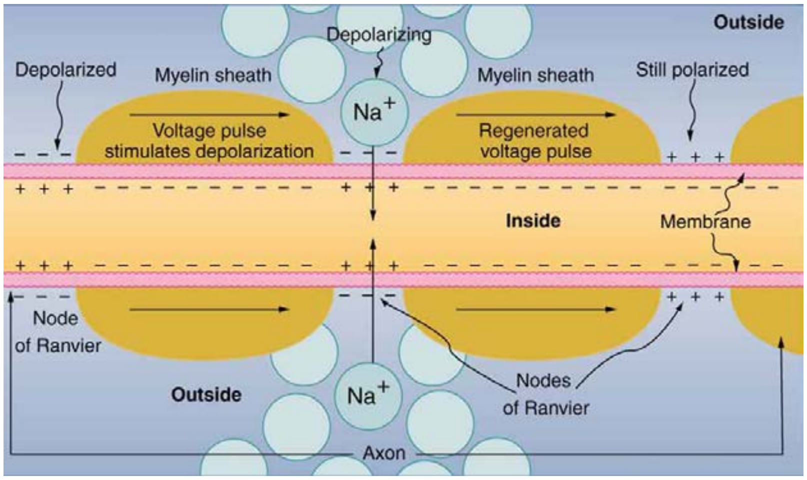


Figure 20.31 Propagation of a nerve impulse down a myelinated axon, from left to right. The signal travels very fast and without energy input in the myelinated regions, but it loses voltage. It is regenerated in the gaps. The signal moves faster than in unmyelinated axons and is insulated from signals in other nerves, limiting cross talk.

Figure 20.32 An electric eel flexes its muscles to create a voltage that stuns prey. (credit: chrisbb, Flickr)

Electrocardiograms

Just as nerve impulses are transmitted by depolarization and repolarization of adjacent membrane, the depolarization that causes muscle contraction can also stimulate adjacent muscle cells to depolarize (fire) and contract. Thus, a depolarization wave can be sent across the heart, coordinating its rhythmic contractions and enabling it to perform its vital function of propelling blood through the circulatory system. **Figure 20.33** is a simplified graphic of a depolarization wave spreading across the heart from the *sinoarterial (SA) node*, the heart's natural pacemaker.

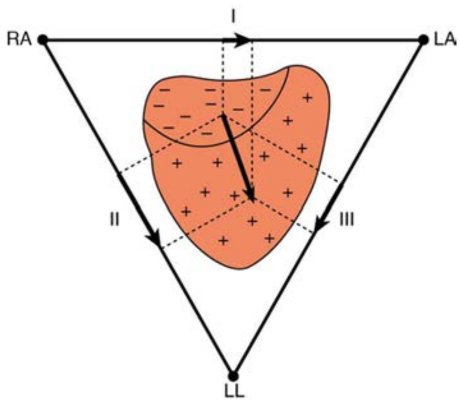


Figure 20.33 The outer surface of the heart changes from positive to negative during depolarization. This wave of depolarization is spreading from the top of the heart and is represented by a vector pointing in the direction of the wave. This vector is a voltage (potential difference) vector. Three electrodes, labeled RA, LA, and LL, are placed on the patient. Each pair (called leads I, II, and III) measures a component of the depolarization vector and is graphed in an ECG.

An **electrocardiogram (ECG)** is a record of the voltages created by the wave of depolarization and subsequent repolarization in the heart. Voltages between pairs of electrodes placed on the chest are vector components of the voltage wave on the heart. Standard ECGs have 12 or more electrodes, but only three are shown in **Figure 20.33** for clarity. Decades ago, three-electrode ECGs were performed by placing electrodes on the left and right arms and the left leg. The voltage between the right arm and the left leg is called the *lead II potential* and is the most often graphed. We shall examine the lead II potential as an indicator of heart-muscle function and see that it is coordinated with arterial blood pressure as well.

Heart function and its four-chamber action are explored in **Viscosity and Laminar Flow; Poiseuille's Law**. Basically, the right and left atria receive blood from the body and lungs, respectively, and pump the blood into the ventricles. The right and left ventricles, in turn, pump blood through the lungs and the rest of the body, respectively. Depolarization of the heart muscle causes it to contract. After contraction it is repolarized to ready it for the next beat. The ECG measures components of depolarization and repolarization of the heart muscle and can yield significant information on the functioning and malfunctioning of the heart.

Figure 20.34 shows an ECG of the lead II potential and a graph of the corresponding arterial blood pressure. The major features are labeled P, Q, R, S, and T. The *P wave* is generated by the depolarization and contraction of the atria as they pump blood into the ventricles. The *QRS complex* is created by the depolarization of the ventricles as they pump blood to the lungs and body. Since the shape of the heart and the path of the depolarization wave are not simple, the QRS complex has this typical shape and time span. The lead II QRS signal also masks the repolarization of the atria, which occur at the same time. Finally, the *T wave* is generated by the repolarization of the ventricles and is followed by the next P wave in the next heartbeat. Arterial blood pressure varies with each part of the heartbeat, with systolic (maximum) pressure occurring closely after the QRS complex, which signals contraction of the ventricles.

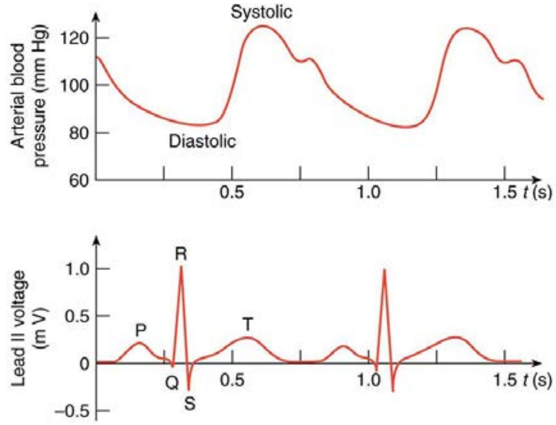


Figure 20.34 A lead II ECG with corresponding arterial blood pressure. The QRS complex is created by the depolarization and contraction of the ventricles and is followed shortly by the maximum or systolic blood pressure. See text for further description.

Taken together, the 12 leads of a state-of-the-art ECG can yield a wealth of information about the heart. For example, regions of damaged heart tissue, called infarcts, reflect electrical waves and are apparent in one or more lead potentials. Subtle changes due to slight or gradual damage to the heart are most readily detected by comparing a recent ECG to an older one. This is particularly the case since individual heart shape, size, and orientation can cause variations in ECGs from one individual to another. ECG technology has advanced to the point where a portable ECG monitor with a liquid crystal instant display and a printer can be carried to patients' homes or used in emergency vehicles. See **Figure 20.35**.

Figure 20.35 This NASA scientist and NEEMO 5 aquanaut's heart rate and other vital signs are being recorded by a portable device while living in an underwater habitat. (credit: NASA, Life Sciences Data Archive at Johnson Space Center, Houston, Texas)

PhET Explorations: Neuron

PhET Interactive Simulation

Figure 20.36 Neuron (http://cnx.org/content/m42352/1.3/neuron_en.jar)

Stimulate a neuron and monitor what happens. Pause, rewind, and move forward in time in order to observe the ions as they move across the neuron membrane.

Glossary

AC current: current that fluctuates sinusoidally with time, expressed as $I = I_0 \sin 2\pi ft$, where I is the current at time t, I_0 is the peak current, and f is the frequency in hertz

AC voltage: voltage that fluctuates sinusoidally with time, expressed as $V = V_0 \sin 2\pi ft$, where V is the voltage at time t, V_0 is the peak voltage, and f is the frequency in hertz

alternating current: (AC) the flow of electric charge that periodically reverses direction

ampere: (amp) the SI unit for current; 1 A = 1 C/s

bioelectricity: electrical effects in and created by biological systems

direct current: (DC) the flow of electric charge in only one direction

drift velocity: the average velocity at which free charges flow in response to an electric field

electric current: the rate at which charge flows, $I = \Delta Q/\Delta t$

electric power: the rate at which electrical energy is supplied by a source or dissipated by a device; it is the product of current times voltage

electrocardiogram (ECG): usually abbreviated ECG, a record of voltages created by depolarization and repolarization, especially in the heart

microshock sensitive: a condition in which a person's skin resistance is bypassed, possibly by a medical procedure, rendering the person vulnerable to electrical shock at currents about 1/1000 the normally required level

nerve conduction: the transport of electrical signals by nerve cells

Ohm's law: an empirical relation stating that the current I is proportional to the potential difference V, ∝ V; it is often written as $I = V/R$, where R is the resistance

ohmic: a type of a material for which Ohm's law is valid

ohm: the unit of resistance, given by 1Ω = 1 V/A

resistance: the electric property that impedes current; for ohmic materials, it is the ratio of voltage to current, $R = V/I$

resistivity: an intrinsic property of a material, independent of its shape or size, directly proportional to the resistance, denoted by ρ

rms current: the root mean square of the current, $I_{\text{rms}} = I_0 / \sqrt{2}$, where I_0 is the peak current, in an AC system

rms voltage: the root mean square of the voltage, $V_{\text{rms}} = V_0 / \sqrt{2}$, where V_0 is the peak voltage, in an AC system

semipermeable: property of a membrane that allows only certain types of ions to cross it

shock hazard: when electric current passes through a person

short circuit: also known as a "short," a low-resistance path between terminals of a voltage source

simple circuit: a circuit with a single voltage source and a single resistor

temperature coefficient of resistivity: an empirical quantity, denoted by α, which describes the change in resistance or resistivity of a material with temperature

thermal hazard: a hazard in which electric current causes undesired thermal effects

Section Summary

20.1 Current

- Electric current I is the rate at which charge flows, given by
$$I = \frac{\Delta Q}{\Delta t},$$
where ΔQ is the amount of charge passing through an area in time Δt .
- The direction of conventional current is taken as the direction in which positive charge moves.
- The SI unit for current is the ampere (A), where $1 \text{ A} = 1 \text{ C/s}.$
- Current is the flow of free charges, such as electrons and ions.
- Drift velocity v_{d} is the average speed at which these charges move.
- Current I is proportional to drift velocity v_{d} , as expressed in the relationship $I = nqAv_{\text{d}}$. Here, I is the current through a wire of cross-sectional area A . The wire's material has a free-charge density n , and each carrier has charge q and a drift velocity v_{d} .
- Electrical signals travel at speeds about 10^{12} times greater than the drift velocity of free electrons.

20.2 Ohm's Law: Resistance and Simple Circuits

- A simple circuit *is* one in which there is a single voltage source and a single resistance.
- One statement of Ohm's law gives the relationship between current I , voltage V , and resistance R in a simple circuit to be $I = \frac{V}{R}.$
- Resistance has units of ohms (Ω), related to volts and amperes by $1 \ \Omega = 1 \ \text{V/A}$.
- There is a voltage or IR drop across a resistor, caused by the current flowing through it, given by $V = IR$.

20.3 Resistance and Resistivity

- The resistance R of a cylinder of length L and cross-sectional area A is $R = \frac{\rho L}{A}$, where ρ is the resistivity of the material.
- Values of ρ in **Table 20.1** show that materials fall into three groups—*conductors, semiconductors, and insulators*.
- Temperature affects resistivity; for relatively small temperature changes ΔT, resistivity is $\rho = \rho_0(1 + \alpha \Delta T)$, where ρ_0 is the original resistivity and α is the temperature coefficient of resistivity.
- **Table 20.2** gives values for α, the temperature coefficient of resistivity.
- The resistance R of an object also varies with temperature: $R = R_0(1 + \alpha \Delta T)$, where R_0 is the original resistance, and R is the resistance after the temperature change.

20.4 Electric Power and Energy

- Electric power P is the rate (in watts) that energy is supplied by a source or dissipated by a device.
- Three expressions for electrical power are

$$P = IV,$$

$$P = \frac{V^2}{R},$$

and

$$P = I^2 R.$$

- The energy used by a device with a power P over a time t is $E = Pt$.

20.5 Alternating Current versus Direct Current

- Direct current (DC) is the flow of electric current in only one direction. It refers to systems where the source voltage is constant.
- The voltage source of an alternating current (AC) system puts out $V = V_0 \sin 2\pi ft$, where V is the voltage at time t, V_0 is the peak voltage, and f is the frequency in hertz.
- In a simple circuit, $I = V/R$ and AC current is $I = I_0 \sin 2\pi ft$, where I is the current at time t, and $I_0 = V_0/R$ is the peak current.
- The average AC power is $P_{\text{ave}} = \frac{1}{2} I_0 V_0$.
- Average (rms) current I_{rms} and average (rms) voltage V_{rms} are $I_{\text{rms}} = \frac{I_0}{\sqrt{2}}$ and $V_{\text{rms}} = \frac{V_0}{\sqrt{2}}$, where rms stands for root mean square.
- Thus, $P_{\text{ave}} = I_{\text{rms}} V_{\text{rms}}$.
- Ohm's law for AC is $I_{\text{rms}} = \frac{V_{\text{rms}}}{R}$.
- Expressions for the average power of an AC circuit are $P_{\text{ave}} = I_{\text{rms}} V_{\text{rms}}$, $P_{\text{ave}} = \frac{V_{\text{rms}}^2}{R}$, and $P_{\text{ave}} = I_{\text{rms}}^2 R$, analogous to the expressions for DC circuits.

20.6 Electric Hazards and the Human Body

- The two types of electric hazards are thermal (excessive power) and shock (current through a person).
- Shock severity is determined by current, path, duration, and AC frequency.
- **Table 20.3** lists shock hazards as a function of current.
- **Figure 20.25** graphs the threshold current for two hazards as a function of frequency.

20.7 Nerve Conduction–Electrocardiograms

- Electric potentials in neurons and other cells are created by ionic concentration differences across semipermeable membranes.
- Stimuli change the permeability and create action potentials that propagate along neurons.
- Myelin sheaths speed this process and reduce the needed energy input.
- This process in the heart can be measured with an electrocardiogram (ECG).

Conceptual Questions

20.1 Current

1. Can a wire carry a current and still be neutral—that is, have a total charge of zero? Explain.

2. Car batteries are rated in ampere-hours ($\text{A} \cdot \text{h}$). To what physical quantity do ampere-hours correspond (voltage, charge, . . .), and what relationship do ampere-hours have to energy content?

3. If two different wires having identical cross-sectional areas carry the same current, will the drift velocity be higher or lower in the better conductor? Explain in terms of the equation $v_\text{d} = \frac{I}{nqA}$, by considering how the density of charge carriers n relates to whether or not a material is a good conductor.

4. Why are two conducting paths from a voltage source to an electrical device needed to operate the device?

5. In cars, one battery terminal is connected to the metal body. How does this allow a single wire to supply current to electrical devices rather than two wires?

6. Why isn't a bird sitting on a high-voltage power line electrocuted? Contrast this with the situation in which a large bird hits two wires simultaneously with its wings.

20.2 Ohm's Law: Resistance and Simple Circuits

7. The IR drop across a resistor means that there is a change in potential or voltage across the resistor. Is there any change in current as it passes through a resistor? Explain.

8. How is the IR drop in a resistor similar to the pressure drop in a fluid flowing through a pipe?

20.3 Resistance and Resistivity

9. In which of the three semiconducting materials listed in **Table 20.1** do impurities supply free charges? (Hint: Examine the range of resistivity for each and determine whether the pure semiconductor has the higher or lower conductivity.)

10. Does the resistance of an object depend on the path current takes through it? Consider, for example, a rectangular bar—is its resistance the same along its length as across its width? (See **Figure 20.37**.)

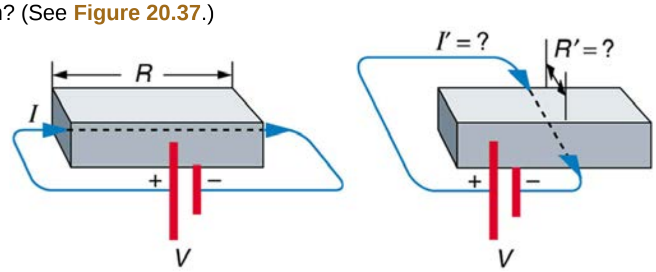


Figure 20.37 Does current taking two different paths through the same object encounter different resistance?

11. If aluminum and copper wires of the same length have the same resistance, which has the larger diameter? Why?

12. Explain why $R = R_0(1 + \alpha\Delta T)$ for the temperature variation of the resistance R of an object is not as accurate as $\rho = \rho_0(1 + \alpha\Delta T)$, which gives the temperature variation of resistivity ρ.

20.4 Electric Power and Energy

13. Why do incandescent lightbulbs grow dim late in their lives, particularly just before their filaments break?

14. The power dissipated in a resistor is given by $P = V^2 / R$, which means power decreases if resistance increases. Yet this power is also given by $P = I^2R$, which means power increases if resistance increases. Explain why there is no contradiction here.

20.5 Alternating Current versus Direct Current

15. Give an example of a use of AC power other than in the household. Similarly, give an example of a use of DC power other than that supplied by batteries.

16. Why do voltage, current, and power go through zero 120 times per second for 60-Hz AC electricity?

17. You are riding in a train, gazing into the distance through its window. As close objects streak by, you notice that the nearby fluorescent lights make *dashed* streaks. Explain.

20.6 Electric Hazards and the Human Body

18. Using an ohmmeter, a student measures the resistance between various points on his body. He finds that the resistance between two points on the same finger is about the same as the resistance between two points on opposite hands—both are several hundred thousand ohms. Furthermore, the resistance decreases when more skin is brought into contact with the probes of the ohmmeter. Finally, there is a dramatic drop in resistance (to a few thousand ohms) when the skin is wet. Explain these observations and their implications regarding skin and internal resistance of the human body.

19. What are the two major hazards of electricity?

20. Why isn't a short circuit a shock hazard?

21. What determines the severity of a shock? Can you say that a certain voltage is hazardous without further information?

22. An electrified needle is used to burn off warts, with the circuit being completed by having the patient sit on a large butt plate. Why is this plate large?

23. Some surgery is performed with high-voltage electricity passing from a metal scalpel through the tissue being cut. Considering the nature of electric fields at the surface of conductors, why would you expect most of the current to flow from the sharp edge of the scalpel? Do you think high- or low-frequency AC is used?

24. Some devices often used in bathrooms, such as hairdryers, often have safety messages saying "Do not use when the bathtub or basin is full of water." Why is this so?

25. We are often advised to not flick electric switches with wet hands, dry your hand first. We are also advised to never throw water on an electric fire. Why is this so?

26. Before working on a power transmission line, linemen will touch the line with the back of the hand as a final check that the voltage is zero. Why the back of the hand?

27. Why is the resistance of wet skin so much smaller than dry, and why do blood and other bodily fluids have low resistances?

28. Could a person on intravenous infusion (an IV) be microshock sensitive?

29. In view of the small currents that cause shock hazards and the larger currents that circuit breakers and fuses interrupt, how do they play a role in preventing shock hazards?

20.7 Nerve Conduction–Electrocardiograms

30. Note that in **Figure 20.28**, both the concentration gradient and the Coulomb force tend to move Na^+ ions into the cell. What prevents this?

31. Define depolarization, repolarization, and the action potential.

32. Explain the properties of myelinated nerves in terms of the insulating properties of myelin.

Problems & Exercises

20.1 Current

33. What is the current in milliamperes produced by the solar cells of a pocket calculator through which 4.00 C of charge passes in 4.00 h?

34. A total of 600 C of charge passes through a flashlight in 0.500 h. What is the average current?

35. What is the current when a typical static charge of $0.250\ \mu C$ moves from your finger to a metal doorknob in $1.00\ \mu s$?

36. Find the current when 2.00 nC jumps between your comb and hair over a $0.500 - \mu s$ time interval.

37. A large lightning bolt had a 20,000-A current and moved 30.0 C of charge. What was its duration?

38. The 200-A current through a spark plug moves 0.300 mC of charge. How long does the spark last?

39. (a) A defibrillator sends a 6.00-A current through the chest of a patient by applying a 10,000-V potential as in the figure below. What is the resistance of the path? (b) The defibrillator paddles make contact with the patient through a conducting gel that greatly reduces the path resistance. Discuss the difficulties that would ensue if a larger voltage were used to produce the same current through the patient, but with the path having perhaps 50 times the resistance. (Hint: The current must be about the same, so a higher voltage would imply greater power. Use this equation for power: $P = I^2R$.)

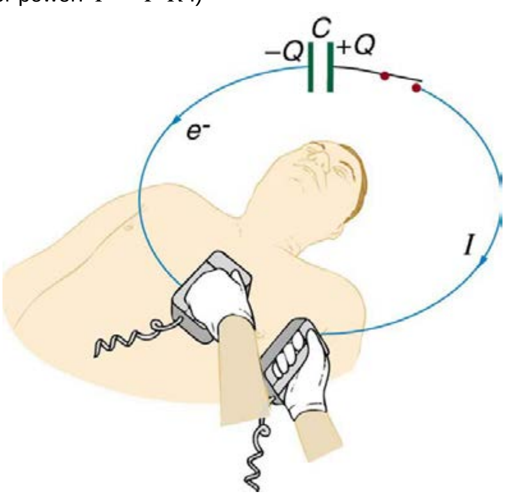


Figure 20.38 The capacitor in a defibrillation unit drives a current through the heart of a patient.

40. During open-heart surgery, a defibrillator can be used to bring a patient out of cardiac arrest. The resistance of the path is $500\ \Omega$ and a 10.0-mA current is needed. What voltage should be applied?

41. (a) A defibrillator passes 12.0 A of current through the torso of a person for 0.0100 s. How much charge moves? (b) How many electrons pass through the wires connected to the patient? (See figure two problems earlier.)

42. A clock battery wears out after moving 10,000 C of charge through the clock at a rate of 0.500 mA. (a) How long did the clock run? (b) How many electrons per second flowed?

43. The batteries of a submerged non-nuclear submarine supply 1000 A at full speed ahead. How long does it take to move Avogadro's number (6.02×10^{23}) of electrons at this rate?

44. Electron guns are used in X-ray tubes. The electrons are accelerated through a relatively large voltage and directed onto a metal target, producing X-rays. (a) How many electrons per second strike the target if the current is 0.500 mA? (b) What charge strikes the target in 0.750 s?

45. A large cyclotron directs a beam of He^{++} nuclei onto a target with a beam current of 0.250 mA. (a) How many He^{++} nuclei per second is this? (b) How long does it take for 1.00 C to strike the target? (c) How long before 1.00 mol of He^{++} nuclei strike the target?

46. Repeat the above example on **Example 20.3**, but for a wire made of silver and given there is one free electron per silver atom.

47. Using the results of the above example on **Example 20.3**, find the drift velocity in a copper wire of twice the diameter and carrying 20.0 A.

48. A 14-gauge copper wire has a diameter of 1.628 mm. What magnitude current flows when the drift velocity is 1.00 mm/s? (See above example on **Example 20.3** for useful information.)

49. SPEAR, a storage ring about 72.0 m in diameter at the Stanford Linear Accelerator (closed in 2009), has a 20.0-A circulating beam of electrons that are moving at nearly the speed of light. (See **Figure 20.39**.) How many electrons are in the beam?

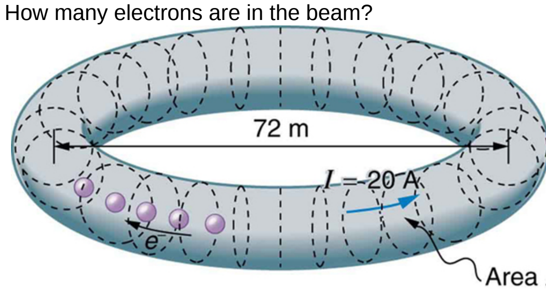


Figure 20.39 Electrons circulating in the storage ring called SPEAR constitute a 20.0-A current. Because they travel close to the speed of light, each electron completes many orbits in each second.

20.2 Ohm's Law: Resistance and Simple Circuits

50. What current flows through the bulb of a 3.00-V flashlight when its hot resistance is $3.60\ \Omega$?

51. Calculate the effective resistance of a pocket calculator that has a 1.35-V battery and through which 0.200 mA flows.

52. What is the effective resistance of a car's starter motor when 150 A flows through it as the car battery applies 11.0 V to the motor?

53. How many volts are supplied to operate an indicator light on a DVD player that has a resistance of $140\ \ \Omega$, given that 25.0 mA passes through it?

54. (a) Find the voltage drop in an extension cord having a $0.0600\text{-}\ \Omega$ resistance and through which 5.00 A is flowing. (b) A cheaper cord utilizes thinner wire and has a resistance of $0.300\ \ \Omega$. What is the voltage drop in it when 5.00 A flows? (c) Why is the voltage to whatever appliance is being used reduced by this amount? What is the effect on the appliance?

55. A power transmission line is hung from metal towers with glass insulators having a resistance of $1.00\times10^{9}\ \ \Omega$. What current flows through the insulator if the voltage is 200 kV? (Some high-voltage lines are DC.)

20.3 Resistance and Resistivity

56. What is the resistance of a 20.0-m-long piece of 12-gauge copper wire having a 2.053-mm diameter?

57. The diameter of 0-gauge copper wire is 8.252 mm. Find the resistance of a 1.00-km length of such wire used for power transmission.

58. If the 0.100-mm diameter tungsten filament in a light bulb is to have a resistance of $0.200\ \Omega$ at $20.0^\circ C$, how long should it be?

59. Find the ratio of the diameter of aluminum to copper wire, if they have the same resistance per unit length (as they might in household wiring).

60. What current flows through a 2.54-cm-diameter rod of pure silicon that is 20.0 cm long, when $1.00 \times 10^3\ \text{V}$ is applied to it? (Such a rod may be used to make nuclear-particle detectors, for example.)

61. (a) To what temperature must you raise a copper wire, originally at 20.0°C, to double its resistance, neglecting any changes in dimensions? (b) Does this happen in household wiring under ordinary circumstances?

62. A resistor made of Nichrome wire is used in an application where its resistance cannot change more than 1.00% from its value at 20.0°C. Over what temperature range can it be used?

63. Of what material is a resistor made if its resistance is 40.0% greater at 100°C than at 20.0°C?

64. An electronic device designed to operate at any temperature in the range from $-10.0^\circ\text{C to }55.0^\circ\text{C}$ contains pure carbon resistors. By what factor does their resistance increase over this range?

65. (a) Of what material is a wire made, if it is 25.0 m long with a 0.100 mm diameter and has a resistance of $77.7\ \Omega$ at 20.0°C? (b) What is its resistance at 150°C?

66. Assuming a constant temperature coefficient of resistivity, what is the maximum percent decrease in the resistance of a constantan wire starting at 20.0°C?

67. A wire is drawn through a die, stretching it to four times its original length. By what factor does its resistance increase?

68. A copper wire has a resistance of $0.500\ \Omega$ at 20.0°C, and an iron wire has a resistance of $0.525\ \Omega$ at the same temperature. At what temperature are their resistances equal?

69. (a) Digital medical thermometers determine temperature by measuring the resistance of a semiconductor device called a thermistor (which has $\alpha = -0.0600 / ^\circ\text{C}$) when it is at the same temperature as the patient. What is a patient's temperature if the thermistor's resistance at that temperature is 82.0% of its value at 37.0°C (normal body temperature)? (b) The negative value for α may not be maintained for very low temperatures. Discuss why and whether this is the case here. (Hint: Resistance can't become negative.)

70. Integrated Concepts

(a) Redo **Exercise 20.57** taking into account the thermal expansion of the tungsten filament. You may assume a thermal expansion coefficient of $12{\times}10^{-6} / ^\circ\text{C}$. (b) By what percentage does your answer differ from that in the example?

71. Unreasonable Results

(a) To what temperature must you raise a resistor made of constantan to double its resistance, assuming a constant temperature coefficient of resistivity? (b) To cut it in half? (c) What is unreasonable about these results? (d) Which assumptions are unreasonable, or which premises are inconsistent?

20.4 Electric Power and Energy

72. What is the power of a $1.00{\times}10^2\ \text{MV}$ lightning bolt having a current of $2.00 \times 10^4\ \text{A}$?

73. What power is supplied to the starter motor of a large truck that draws 250 A of current from a 24.0-V battery hookup?

74. A charge of 4.00 C of charge passes through a pocket calculator's solar cells in 4.00 h. What is the power output, given the calculator's voltage output is 3.00 V? (See **Figure 20.40**.)

Figure 20.40 The strip of solar cells just above the keys of this calculator convert light to electricity to supply its energy needs. (credit: Evan-Amos, Wikimedia Commons)

75. How many watts does a flashlight that has $6.00{\times}10^2\ \text{C}$ pass through it in 0.500 h use if its voltage is 3.00 V?

76. Find the power dissipated in each of these extension cords: (a) an extension cord having a $0.0600 \text{-}\ \Omega$ resistance and through which 5.00 A is flowing; (b) a cheaper cord utilizing thinner wire and with a resistance of $0.300\ \Omega$.

77. Verify that the units of a volt-ampere are watts, as implied by the equation $P = IV$.

78. Show that the units $1\ \text{V}^2/\ \Omega\ = 1\text{W}$, as implied by the equation $P = V^2 / R$.

79. Show that the units $1\ \text{A}^2 \cdot\ \Omega\ = 1\ \text{W}$, as implied by the equation $P = I^2 R$.

80. Verify the energy unit equivalence that $1\ \text{kW} \cdot \text{h} = 3.60{\times}10^6\ \text{J}$.

81. Electrons in an X-ray tube are accelerated through $1.00{\times}10^2\ \text{kV}$ and directed toward a target to produce X-rays. Calculate the power of the electron beam in this tube if it has a current of 15.0 mA.

82. An electric water heater consumes 5.00 kW for 2.00 h per day. What is the cost of running it for one year if electricity costs $12.0\ \text{cents/kW} \cdot \text{h}$? See **Figure 20.41**.

Figure 20.41 On-demand electric hot water heater. Heat is supplied to water only when needed. (credit: aviddavid, Flickr)

83. With a 1200-W toaster, how much electrical energy is needed to make a slice of toast (cooking time = 1 minute)? At $9.0\ \text{cents/kW} \cdot \text{h}$, how much does this cost?

84. What would be the maximum cost of a CFL such that the total cost (investment plus operating) would be the same for both CFL and incandescent 60-W bulbs? Assume the cost of the incandescent bulb is 25 cents and that electricity costs $10\ \text{cents/kWh}$. Calculate the cost for 1000 hours, as in the cost effectiveness of CFL example.

85. Some makes of older cars have 6.00-V electrical systems. (a) What is the hot resistance of a 30.0-W headlight in such a car? (b) What current flows through it?

86. Alkaline batteries have the advantage of putting out constant voltage until very nearly the end of their life. How long will an alkaline battery rated at $1.00 \text{ A} \cdot \text{h}$ and 1.58 V keep a 1.00-W flashlight bulb burning?

87. A cauterizer, used to stop bleeding in surgery, puts out 2.00 mA at 15.0 kV. (a) What is its power output? (b) What is the resistance of the path?

88. The average television is said to be on 6 hours per day. Estimate the yearly cost of electricity to operate 100 million TVs, assuming their power consumption averages 150 W and the cost of electricity averages $12.0 \text{ cents/kW} \cdot \text{h}$.

89. An old lightbulb draws only 50.0 W, rather than its original 60.0 W, due to evaporative thinning of its filament. By what factor is its diameter reduced, assuming uniform thinning along its length? Neglect any effects caused by temperature differences.

90. 00-gauge copper wire has a diameter of 9.266 mm. Calculate the power loss in a kilometer of such wire when it carries $1.00\times10^2 \text{ A}$.

91. Integrated Concepts

Cold vaporizers pass a current through water, evaporating it with only a small increase in temperature. One such home device is rated at 3.50 A and utilizes 120 V AC with 95.0% efficiency. (a) What is the vaporization rate in grams per minute? (b) How much water must you put into the vaporizer for 8.00 h of overnight operation? (See **Figure 20.42**.)

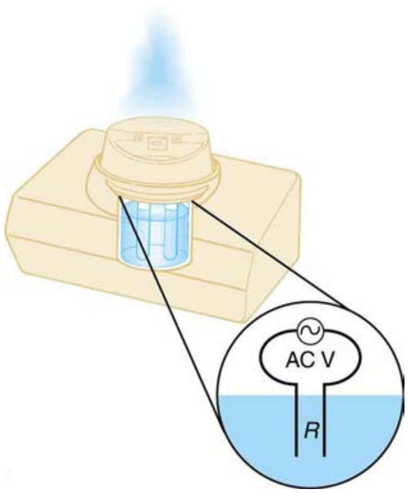


Figure 20.42 This cold vaporizer passes current directly through water, vaporizing it directly with relatively little temperature increase.

92. Integrated Concepts

(a) What energy is dissipated by a lightning bolt having a 20,000-A current, a voltage of $1.00\times10^2 \text{ MV}$, and a length of 1.00 ms? (b) What mass of tree sap could be raised from 18.0°C to its boiling point and then evaporated by this energy, assuming sap has the same thermal characteristics as water?

93. Integrated Concepts

What current must be produced by a 12.0-V battery-operated bottle warmer in order to heat 75.0 g of glass, 250 g of baby formula, and $3.00\times10^2 \text{ g}$ of aluminum from 20.0°C to 90.0°C in 5.00 min?

94. Integrated Concepts

How much time is needed for a surgical cauterizer to raise the temperature of 1.00 g of tissue from 37.0°C to 100°C and then boil away 0.500 g of water, if it puts out 2.00 mA at 15.0 kV? Ignore heat transfer to the surroundings.

95. Integrated Concepts

Hydroelectric generators (see **Figure 20.43**) at Hoover Dam produce a maximum current of $8.00\times10^3 \text{ A}$ at 250 kV. (a) What is the power output? (b) The water that powers the generators enters and leaves the system at low speed (thus its kinetic energy does not change) but loses 160 m in altitude. How many cubic meters per second are needed, assuming 85.0% efficiency?

Figure 20.43 Hydroelectric generators at the Hoover dam. (credit: Jon Sullivan)

96. Integrated Concepts

(a) Assuming 95.0% efficiency for the conversion of electrical power by the motor, what current must the 12.0-V batteries of a 750-kg electric car be able to supply: (a) To accelerate from rest to 25.0 m/s in 1.00 min? (b) To climb a $2.00\times10^2\text{-m}$-high hill in 2.00 min at a constant 25.0-m/s speed while exerting $5.00\times10^2 \text{ N}$ of force to overcome air resistance and friction? (c) To travel at a constant 25.0-m/s speed, exerting a $5.00\times10^2 \text{ N}$ force to overcome air resistance and friction? See **Figure 20.44**.

Figure 20.44 This REVAi, an electric car, gets recharged on a street in London. (credit: Frank Hebbert)

97. Integrated Concepts

A light-rail commuter train draws 630 A of 650-V DC electricity when accelerating. (a) What is its power consumption rate in kilowatts? (b) How long does it take to reach 20.0 m/s starting from rest if its loaded mass is $5.30\times10^4 \text{ kg}$, assuming 95.0% efficiency and constant power? (c) Find its average acceleration. (d) Discuss how the acceleration you found for the light-rail train compares to what might be typical for an automobile.

98. Integrated Concepts

(a) An aluminum power transmission line has a resistance of $0.0580 \ \Omega / \text{km}$. What is its mass per kilometer? (b) What is the mass per kilometer of a copper line having the same resistance? A lower resistance would shorten the heating time. Discuss the practical limits to speeding the heating by lowering the resistance.

99. Integrated Concepts

(a) An immersion heater utilizing 120 V can raise the temperature of a $1.00\times10^2\text{-g}$ aluminum cup containing 350 g of water from 20.0°C to 95.0°C in 2.00 min. Find its resistance, assuming it is constant during the process. (b) A lower resistance would shorten the heating time. Discuss the practical limits to speeding the heating by lowering the resistance.

100. Integrated Concepts

(a) What is the cost of heating a hot tub containing 1500 kg of water from 10.0°C to 40.0°C, assuming 75.0% efficiency to account for heat transfer to the surroundings? The cost of electricity is $12\ \text{cents/kW}\cdot\text{h}$. (b) What current was used by the 220-V AC electric heater, if this took 4.00 h?

101. Unreasonable Results

(a) What current is needed to transmit $1.00\times10^{2}\ \text{MW}$ of power at 480 V? (b) What power is dissipated by the transmission lines if they have a $1.00 - \Omega$ resistance? (c) What is unreasonable about this result? (d) Which assumptions are unreasonable, or which premises are inconsistent?

102. Unreasonable Results

(a) What current is needed to transmit $1.00\times10^{2}\ \text{MW}$ of power at 10.0 kV? (b) Find the resistance of 1.00 km of wire that would cause a 0.0100% power loss. (c) What is the diameter of a 1.00-km-long copper wire having this resistance? (d) What is unreasonable about these results? (e) Which assumptions are unreasonable, or which premises are inconsistent?

103. Construct Your Own Problem

Consider an electric immersion heater used to heat a cup of water to make tea. Construct a problem in which you calculate the needed resistance of the heater so that it increases the temperature of the water and cup in a reasonable amount of time. Also calculate the cost of the electrical energy used in your process. Among the things to be considered are the voltage used, the masses and heat capacities involved, heat losses, and the time over which the heating takes place. Your instructor may wish for you to consider a thermal safety switch (perhaps bimetallic) that will halt the process before damaging temperatures are reached in the immersion unit.

20.5 Alternating Current versus Direct Current

104. (a) What is the hot resistance of a 25-W light bulb that runs on 120-V AC? (b) If the bulb's operating temperature is 2700°C, what is its resistance at 2600°C?

105. Certain heavy industrial equipment uses AC power that has a peak voltage of 679 V. What is the rms voltage?

106. A certain circuit breaker trips when the rms current is 15.0 A. What is the corresponding peak current?

107. Military aircraft use 400-Hz AC power, because it is possible to design lighter-weight equipment at this higher frequency. What is the time for one complete cycle of this power?

108. A North American tourist takes his 25.0-W, 120-V AC razor to Europe, finds a special adapter, and plugs it into 240 V AC. Assuming constant resistance, what power does the razor consume as it is ruined?

109. In this problem, you will verify statements made at the end of the power losses for **Example 20.10**. (a) What current is needed to transmit 100 MW of power at a voltage of 25.0 kV? (b) Find the power loss in a $1.00 - \Omega$ transmission line. (c) What percent loss does this represent?

110. A small office-building air conditioner operates on 408-V AC and consumes 50.0 kW. (a) What is its effective resistance? (b) What is the cost of running the air conditioner during a hot summer month when it is on 8.00 h per day for 30 days and electricity costs $9.00\ \text{cents/kW}\cdot\text{h}$?

111. What is the peak power consumption of a 120-V AC microwave oven that draws 10.0 A?

112. What is the peak current through a 500-W room heater that operates on 120-V AC power?

113. Two different electrical devices have the same power consumption, but one is meant to be operated on 120-V AC and the other on 240-V AC. (a) What is the ratio of their resistances? (b) What is the ratio of their currents? (c) Assuming its resistance is unaffected, by what factor will the power increase if a 120-V AC device is connected to 240-V AC?

114. Nichrome wire is used in some radiative heaters. (a) Find the resistance needed if the average power output is to be 1.00 kW utilizing 120-V AC. (b) What length of Nichrome wire, having a cross-sectional area of 5.00mm^2, is needed if the operating temperature is 500° C? (c) What power will it draw when first switched on?

115. Find the time after $t = 0$ when the instantaneous voltage of 60-Hz AC first reaches the following values: (a) $V_0/2$ (b) V_0 (c) 0.

116. (a) At what two times in the first period following $t = 0$ does the instantaneous voltage in 60-Hz AC equal V_{rms}? (b) $-V_{\text{rms}}$?

20.6 Electric Hazards and the Human Body

117. (a) How much power is dissipated in a short circuit of 240-V AC through a resistance of $0.250\ \Omega$? (b) What current flows?

118. What voltage is involved in a 1.44-kW short circuit through a $0.100 - \Omega$ resistance?

119. Find the current through a person and identify the likely effect on her if she touches a 120-V AC source: (a) if she is standing on a rubber mat and offers a total resistance of $300\ \text{k}\Omega$; (b) if she is standing barefoot on wet grass and has a resistance of only $4000\ \text{k}\Omega$.

120. While taking a bath, a person touches the metal case of a radio. The path through the person to the drainpipe and ground has a resistance of $4000\ \Omega$. What is the smallest voltage on the case of the radio that could cause ventricular fibrillation?

121. Foolishly trying to fish a burning piece of bread from a toaster with a metal butter knife, a man comes into contact with 120-V AC. He does not even feel it since, luckily, he is wearing rubber-soled shoes. What is the minimum resistance of the path the current follows through the person?

122. (a) During surgery, a current as small as $20.0\ \mu\text{A}$ applied directly to the heart may cause ventricular fibrillation. If the resistance of the exposed heart is $300\ \Omega$, what is the smallest voltage that poses this danger? (b) Does your answer imply that special electrical safety precautions are needed?

123. (a) What is the resistance of a 220-V AC short circuit that generates a peak power of 96.8 kW? (b) What would the average power be if the voltage was 120 V AC?

124. A heart defibrillator passes 10.0 A through a patient's torso for 5.00 ms in an attempt to restore normal beating. (a) How much charge passed? (b) What voltage was applied if 500 J of energy was dissipated? (c) What was the path's resistance? (d) Find the temperature increase caused in the 8.00 kg of affected tissue.

125. Integrated Concepts

A short circuit in a 120-V appliance cord has a $0.500\text{-}\ \Omega$ resistance. Calculate the temperature rise of the 2.00 g of surrounding materials, assuming their specific heat capacity is $0.200\ \text{cal/g}\cdot^\circ\text{C}$ and that it takes 0.0500 s for a circuit breaker to interrupt the current. Is this likely to be damaging?

126. Construct Your Own Problem

Consider a person working in an environment where electric currents might pass through her body. Construct a problem in which you calculate the resistance of insulation needed to protect the person from harm. Among the things to be considered are the voltage to which the person might be exposed, likely body resistance (dry, wet, …), and acceptable currents (safe but sensed, safe and unfelt, …).

20.7 Nerve Conduction–Electrocardiograms

127. Integrated Concepts

Use the ECG in **Figure 20.34** to determine the heart rate in beats per minute assuming a constant time between beats.

128. Integrated Concepts

(a) Referring to **Figure 20.34**, find the time systolic pressure lags behind the middle of the QRS complex. (b) Discuss the reasons for the time lag.

21 CIRCUITS, BIOELECTRICITY, AND DC INSTRUMENTS

Figure 21.1 The complexity of the electric circuits in a computer is surpassed by those in the human brain. (credit: Airman 1st Class Mike Meares, United States Air Force)

Learning Objectives

21.1. Resistors in Series and Parallel
21.2. Electromotive Force: Terminal Voltage
21.3. Kirchhoff's Rules
21.4. DC Voltmeters and Ammeters
21.5. Null Measurements
21.6. DC Circuits Containing Resistors and Capacitors

Introduction to Circuits, Bioelectricity, and DC Instruments

Electric circuits are commonplace. Some are simple, such as those in flashlights. Others, such as those used in supercomputers, are extremely complex.

This collection of modules takes the topic of electric circuits a step beyond simple circuits. When the circuit is purely resistive, everything in this module applies to both DC and AC. Matters become more complex when capacitance is involved. We do consider what happens when capacitors are connected to DC voltage sources, but the interaction of capacitors and other nonresistive devices with AC is left for a later chapter. Finally, a number of important DC instruments, such as meters that measure voltage and current, are covered in this chapter.

21.1 Resistors in Series and Parallel

Most circuits have more than one component, called a **resistor** that limits the flow of charge in the circuit. A measure of this limit on charge flow is called **resistance**. The simplest combinations of resistors are the series and parallel connections illustrated in Figure 21.2. The total resistance of a combination of resistors depends on both their individual values and how they are connected.

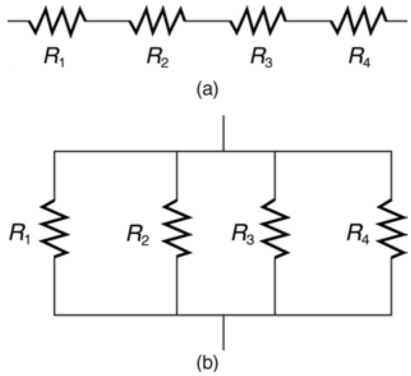


Figure 21.2 (a) A series connection of resistors. (b) A parallel connection of resistors.

Resistors in Series

When are resistors in **series**? Resistors are in series whenever the flow of charge, called the **current**, must flow through devices sequentially. For example, if current flows through a person holding a screwdriver and into the Earth, then R_1 in **Figure 21.2**(a) could be the resistance of the screwdriver's shaft, R_2 the resistance of its handle, R_3 the person's body resistance, and R_4 the resistance of her shoes.

Figure 21.3 shows resistors in series connected to a **voltage** source. It seems reasonable that the total resistance is the sum of the individual resistances, considering that the current has to pass through each resistor in sequence. (This fact would be an advantage to a person wishing to avoid an electrical shock, who could reduce the current by wearing high-resistance rubber-soled shoes. It could be a disadvantage if one of the resistances were a faulty high-resistance cord to an appliance that would reduce the operating current.)

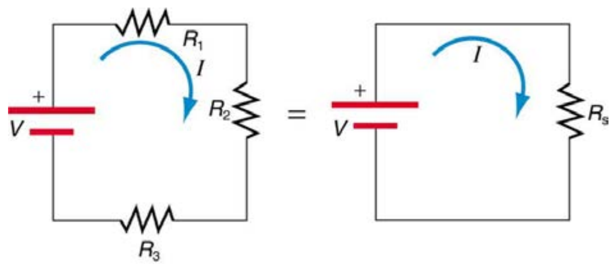


Figure 21.3 Three resistors connected in series to a battery (left) and the equivalent single or series resistance (right).

To verify that resistances in series do indeed add, let us consider the loss of electrical power, called a **voltage drop**, in each resistor in **Figure 21.3**. According to **Ohm's law**, the voltage drop, V, across a resistor when a current flows through it is calculated using the equation $V = IR$, where I equals the current in amps (A) and R is the resistance in ohms (Ω). Another way to think of this is that V is the voltage necessary to make a current I flow through a resistance R.

So the voltage drop across R_1 is $V_1 = IR_1$, that across R_2 is $V_2 = IR_2$, and that across R_3 is $V_3 = IR_3$. The sum of these voltages equals the voltage output of the source; that is,

$$V = V_1 + V_2 + V_3. \tag{21.1}$$

This equation is based on the conservation of energy and conservation of charge. Electrical potential energy can be described by the equation $PE = qV$, where q is the electric charge and V is the voltage. Thus the energy supplied by the source is qV, while that dissipated by the resistors is

$$qV_1 + qV_2 + qV_3. \tag{21.2}$$

Connections: Conservation Laws

The derivations of the expressions for series and parallel resistance are based on the laws of conservation of energy and conservation of charge, which state that total charge and total energy are constant in any process. These two laws are directly involved in all electrical phenomena and will be invoked repeatedly to explain both specific effects and the general behavior of electricity.

These energies must be equal, because there is no other source and no other destination for energy in the circuit. Thus, $qV = qV_1 + qV_2 + qV_3$. The charge q cancels, yielding $V = V_1 + V_2 + V_3$, as stated. (Note that the same amount of charge passes through the battery and each resistor in a given amount of time, since there is no capacitance to store charge, there is no place for charge to leak, and charge is conserved.)
Now substituting the values for the individual voltages gives

$$V = IR_1 + IR_2 + IR_3 = I(R_1 + R_2 + R_3). \tag{21.3}$$

Note that for the equivalent single series resistance R_s, we have

$$V = IR_s. \tag{21.4}$$

This implies that the total or equivalent series resistance R_s of three resistors is $R_s = R_1 + R_2 + R_3$.

This logic is valid in general for any number of resistors in series; thus, the total resistance R_s of a series connection is

$$R_s = R_1 + R_2 + R_3 + ..., \tag{21.5}$$

as proposed. Since all of the current must pass through each resistor, it experiences the resistance of each, and resistances in series simply add up.

Example 21.1 Calculating Resistance, Current, Voltage Drop, and Power Dissipation: Analysis of a Series Circuit

Suppose the voltage output of the battery in Figure 21.3 is 12.0 V, and the resistances are $R_1 = 1.00\ \Omega$, $R_2 = 6.00\ \Omega$, and $R_3 = 13.0\ \Omega$. (a) What is the total resistance? (b) Find the current. (c) Calculate the voltage drop in each resistor, and show these add to equal the voltage output of the source. (d) Calculate the power dissipated by each resistor. (e) Find the power output of the source, and show that it equals the total power dissipated by the resistors.

Strategy and Solution for (a)

The total resistance is simply the sum of the individual resistances, as given by this equation:

$$\begin{aligned} R_s &= R_1 + R_2 + R_3 \\ &= 1.00\ \Omega + 6.00\ \Omega + 13.0\ \Omega \\ &= 20.0\ \Omega. \end{aligned} \tag{21.6}$$

Strategy and Solution for (b)

The current is found using Ohm's law, $V = IR$. Entering the value of the applied voltage and the total resistance yields the current for the circuit:

$$I = \frac{V}{R_s} = \frac{12.0\text{ V}}{20.0\ \Omega} = 0.600\text{ A}. \tag{21.7}$$

Strategy and Solution for (c)

The voltage—or IR drop—in a resistor is given by Ohm's law. Entering the current and the value of the first resistance yields

$$V_1 = IR_1 = (0.600\text{ A})(1.0\ \Omega) = 0.600\text{ V}. \tag{21.8}$$

Similarly,

$$V_2 = IR_2 = (0.600\text{ A})(6.0\ \Omega) = 3.60\text{ V} \tag{21.9}$$

and

$$V_3 = IR_3 = (0.600\text{ A})(13.0\ \Omega) = 7.80\text{ V}. \tag{21.10}$$

Discussion for (c)

The three IR drops add to 12.0 V, as predicted:

$$V_1 + V_2 + V_3 = (0.600 + 3.60 + 7.80)\text{ V} = 12.0\text{ V}. \tag{21.11}$$

Strategy and Solution for (d)

The easiest way to calculate power in watts (W) dissipated by a resistor in a DC circuit is to use **Joule's law**, $P = IV$, where P is electric power. In this case, each resistor has the same full current flowing through it. By substituting Ohm's law $V = IR$ into Joule's law, we get the power dissipated by the first resistor as

$$P_1 = I^2R_1 = (0.600\text{ A})^2(1.00\ \Omega) = 0.360\text{ W}. \tag{21.12}$$

Similarly,

$$P_2 = I^2R_2 = (0.600\text{ A})^2(6.00\ \Omega) = 2.16\text{ W} \tag{21.13}$$

and

$$P_3 = I^2R_3 = (0.600\text{ A})^2(13.0\ \Omega) = 4.68\text{ W}. \tag{21.14}$$

Discussion for (d)

Power can also be calculated using either $P = IV$ or $P = \frac{V^2}{R}$, where V is the voltage drop across the resistor (not the full voltage of the source). The same values will be obtained.

Strategy and Solution for (e)

The easiest way to calculate power output of the source is to use $P = IV$, where V is the source voltage. This gives

$$P = (0.600\ \text{A})(12.0\ \text{V}) = 7.20\ \text{W}. \tag{21.15}$$

Discussion for (e)

Note, coincidentally, that the total power dissipated by the resistors is also 7.20 W, the same as the power put out by the source. That is,

$$P_1 + P_2 + P_3 = (0.360 + 2.16 + 4.68)\ \text{W} = 7.20\ \text{W}. \tag{21.16}$$

Power is energy per unit time (watts), and so conservation of energy requires the power output of the source to be equal to the total power dissipated by the resistors.

Major Features of Resistors in Series

1. Series resistances add: $R_\text{s} = R_1 + R_2 + R_3 + ...$
2. The same current flows through each resistor in series.
3. Individual resistors in series do not get the total source voltage, but divide it.

Resistors in Parallel

Figure 21.4 shows resistors in **parallel**, wired to a voltage source. Resistors are in parallel when each resistor is connected directly to the voltage source by connecting wires having negligible resistance. Each resistor thus has the full voltage of the source applied to it.

Each resistor draws the same current it would if it alone were connected to the voltage source (provided the voltage source is not overloaded). For example, an automobile's headlights, radio, and so on, are wired in parallel, so that they utilize the full voltage of the source and can operate completely independently. The same is true in your house, or any building. (See **Figure 21.4**(b).)

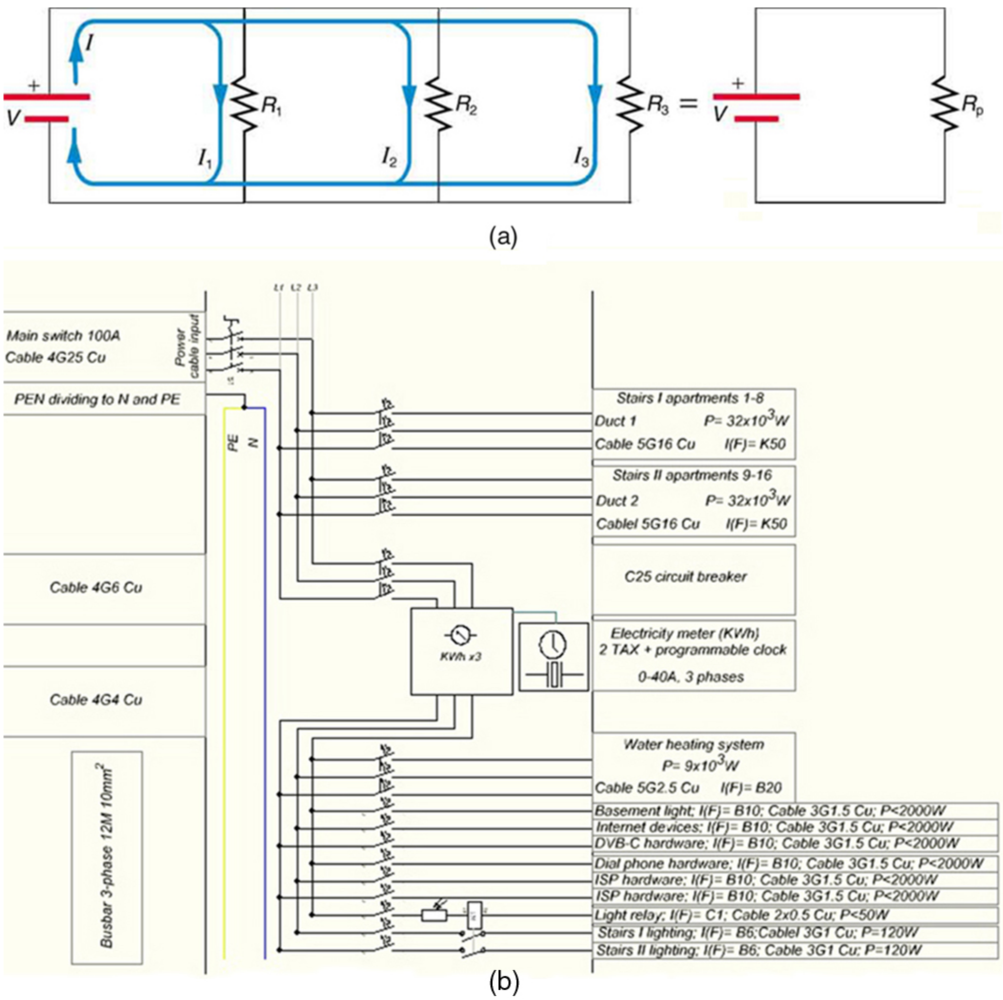


Figure 21.4 (a) Three resistors connected in parallel to a battery and the equivalent single or parallel resistance. (b) Electrical power setup in a house. (credit: Dmitry G, Wikimedia Commons)

To find an expression for the equivalent parallel resistance R_p , let us consider the currents that flow and how they are related to resistance. Since each resistor in the circuit has the full voltage, the currents flowing through the individual resistors are $I_1 = \frac{V}{R_1}$, $I_2 = \frac{V}{R_2}$, and $I_3 = \frac{V}{R_3}$.

Conservation of charge implies that the total current I produced by the source is the sum of these currents:

$$I = I_1 + I_2 + I_3. \tag{21.17}$$

Substituting the expressions for the individual currents gives

$$I = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right). \tag{21.18}$$

Note that Ohm's law for the equivalent single resistance gives

$$I = \frac{V}{R_\text{p}} = V\left(\frac{1}{R_\text{p}}\right). \tag{21.19}$$

The terms inside the parentheses in the last two equations must be equal. Generalizing to any number of resistors, the total resistance R_p of a parallel connection is related to the individual resistances by

$$\frac{1}{R_\text{p}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_{.3}} + ... \tag{21.20}$$

This relationship results in a total resistance R_p that is less than the smallest of the individual resistances. (This is seen in the next example.) When resistors are connected in parallel, more current flows from the source than would flow for any of them individually, and so the total resistance is lower.

Example 21.2 Calculating Resistance, Current, Power Dissipation, and Power Output: Analysis of a Parallel Circuit

Let the voltage output of the battery and resistances in the parallel connection in Figure 21.4 be the same as the previously considered series connection: $V = 12.0 \text{ V}$, $R_1 = 1.00 \ \Omega$, $R_2 = 6.00 \ \Omega$, and $R_3 = 13.0 \ \Omega$. (a) What is the total resistance? (b) Find the total current. (c) Calculate the currents in each resistor, and show these add to equal the total current output of the source. (d) Calculate the power dissipated by each resistor. (e) Find the power output of the source, and show that it equals the total power dissipated by the resistors.

Strategy and Solution for (a)

The total resistance for a parallel combination of resistors is found using the equation below. Entering known values gives

$$\frac{1}{R_\text{p}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{1.00 \ \Omega} + \frac{1}{6.00 \ \Omega} + \frac{1}{13.0 \ \Omega}. \tag{21.21}$$

Thus,

$$\frac{1}{R_\text{p}} = \frac{1.00}{\Omega} + \frac{0.1667}{\Omega} + \frac{0.07692}{\Omega} = \frac{1.2436}{\Omega}. \tag{21.22}$$

(Note that in these calculations, each intermediate answer is shown with an extra digit.)

We must invert this to find the total resistance R_p. This yields

$$R_\text{p} = \frac{1}{1.2436} \Omega \ = 0.8041 \ \Omega . \tag{21.23}$$

The total resistance with the correct number of significant digits is $R_\text{p} = 0.804 \ \Omega$.

Discussion for (a)

R_p is, as predicted, less than the smallest individual resistance.

Strategy and Solution for (b)

The total current can be found from Ohm's law, substituting R_p for the total resistance. This gives

$$I = \frac{V}{R_\text{p}} = \frac{12.0 \text{ V}}{0.8041 \ \Omega} = 14.92 \text{ A}. \tag{21.24}$$

Discussion for (b)

Current I for each device is much larger than for the same devices connected in series (see the previous example). A circuit with parallel connections has a smaller total resistance than the resistors connected in series.

Strategy and Solution for (c)

The individual currents are easily calculated from Ohm's law, since each resistor gets the full voltage. Thus,

$$I_1 = \frac{V}{R_1} = \frac{12.0 \text{ V}}{1.00 \ \Omega} = 12.0 \text{ A}. \tag{21.25}$$

Similarly,

$$I_2 = \frac{V}{R_2} = \frac{12.0\ \text{V}}{6.00\ \ \Omega} = 2.00\ \text{A} \tag{21.26}$$

and

$$I_3 = \frac{V}{R_3} = \frac{12.0\ \text{V}}{13.0\ \ \Omega} = 0.92\ \text{A}. \tag{21.27}$$

Discussion for (c)

The total current is the sum of the individual currents:

$$I_1 + I_2 + I_3 = 14.92\ \text{A}. \tag{21.28}$$

This is consistent with conservation of charge.

Strategy and Solution for (d)

The power dissipated by each resistor can be found using any of the equations relating power to current, voltage, and resistance, since all three are known. Let us use $P = \frac{V^2}{R}$, since each resistor gets full voltage. Thus,

$$P_1 = \frac{V^2}{R_1} = \frac{(12.0\ \text{V})^2}{1.00\ \ \Omega} = 144\ \text{W}. \tag{21.29}$$

Similarly,

$$P_2 = \frac{V^2}{R_2} = \frac{(12.0\ \text{V})^2}{6.00\ \ \Omega} = 24.0\ \text{W} \tag{21.30}$$

and

$$P_3 = \frac{V^2}{R_3} = \frac{(12.0\ \text{V})^2}{13.0\ \ \Omega} = 11.1\ \text{W}. \tag{21.31}$$

Discussion for (d)

The power dissipated by each resistor is considerably higher in parallel than when connected in series to the same voltage source.

Strategy and Solution for (e)

The total power can also be calculated in several ways. Choosing $P = IV$, and entering the total current, yields

$$P = IV = (14.92\ \text{A})(12.0\ \text{V}) = 179\ \text{W}. \tag{21.32}$$

Discussion for (e)

Total power dissipated by the resistors is also 179 W:

$$P_1 + P_2 + P_3 = 144\ \text{W} + 24.0\ \text{W} + 11.1\ \text{W} = 179\ \text{W}. \tag{21.33}$$

This is consistent with the law of conservation of energy.

Overall Discussion

Note that both the currents and powers in parallel connections are greater than for the same devices in series.

Major Features of Resistors in Parallel

1. Parallel resistance is found from $\frac{1}{R_\text{p}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ...$, and it is smaller than any individual resistance in the combination.
2. Each resistor in parallel has the same full voltage of the source applied to it. (Power distribution systems most often use parallel connections to supply the myriad devices served with the same voltage and to allow them to operate independently.)
3. Parallel resistors do not each get the total current; they divide it.

Combinations of Series and Parallel

More complex connections of resistors are sometimes just combinations of series and parallel. These are commonly encountered, especially when wire resistance is considered. In that case, wire resistance is in series with other resistances that are in parallel.

Combinations of series and parallel can be reduced to a single equivalent resistance using the technique illustrated in **Figure 21.5**. Various parts are identified as either series or parallel, reduced to their equivalents, and further reduced until a single resistance is left. The process is more time consuming than difficult.

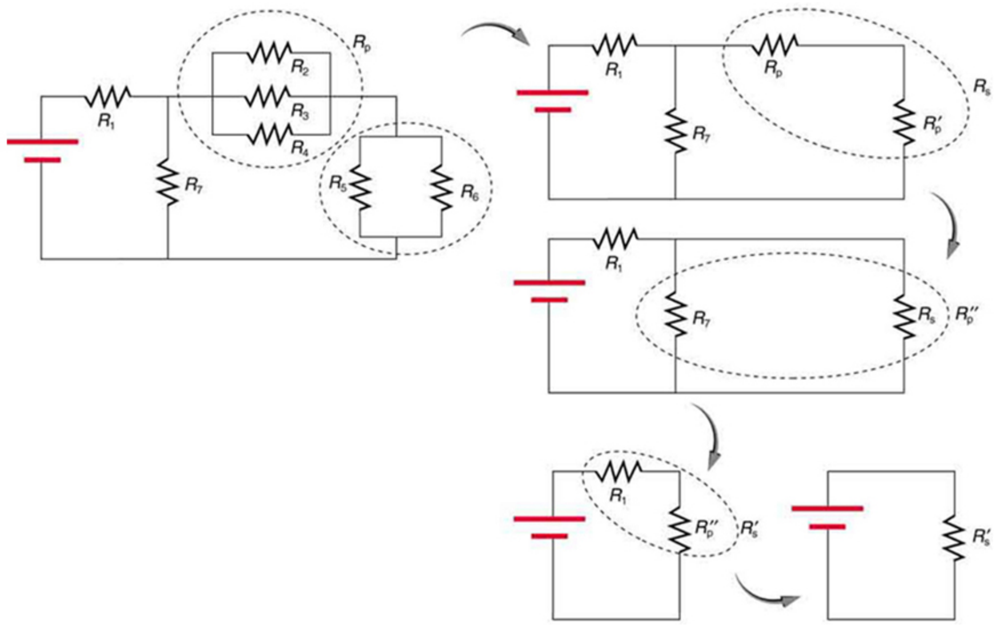


Figure 21.5 This combination of seven resistors has both series and parallel parts. Each is identified and reduced to an equivalent resistance, and these are further reduced until a single equivalent resistance is reached.

The simplest combination of series and parallel resistance, shown in **Figure 21.6**, is also the most instructive, since it is found in many applications. For example, R_1 could be the resistance of wires from a car battery to its electrical devices, which are in parallel. R_2 and R_3 could be the starter motor and a passenger compartment light. We have previously assumed that wire resistance is negligible, but, when it is not, it has important effects, as the next example indicates.

Example 21.3 Calculating Resistance, IR Drop, Current, and Power Dissipation: Combining Series and Parallel Circuits

Figure 21.6 shows the resistors from the previous two examples wired in a different way—a combination of series and parallel. We can consider R_1 to be the resistance of wires leading to R_2 and R_3. (a) Find the total resistance. (b) What is the IR drop in R_1? (c) Find the current I_2 through R_2. (d) What power is dissipated by R_2?

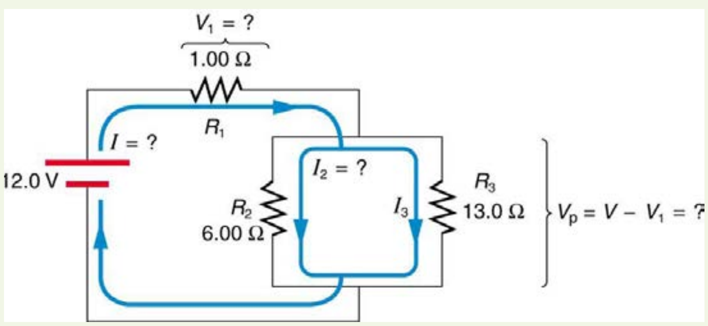


Figure 21.6 These three resistors are connected to a voltage source so that R_2 and R_3 are in parallel with one another and that combination is in series with R_1.

Strategy and Solution for (a)

To find the total resistance, we note that R_2 and R_3 are in parallel and their combination R_p is in series with R_1. Thus the total (equivalent) resistance of this combination is

$$R_\mathrm{tot} = R_1 + R_\mathrm{p}. \tag{21.34}$$

First, we find R_p using the equation for resistors in parallel and entering known values:

$$\frac{1}{R_\mathrm{p}} = \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{6.00\ \Omega} + \frac{1}{13.0\ \Omega} = \frac{0.2436}{\Omega}. \tag{21.35}$$

Inverting gives

$$R_\mathrm{p} = \frac{1}{0.2436}\,\Omega\ = 4.11\ \ \Omega\ . \tag{21.36}$$

So the total resistance is

$$R_{\text{tot}} = R_1 + R_{\text{p}} = 1.00\ \Omega + 4.11\ \Omega = 5.11\ \Omega. \tag{21.37}$$

Discussion for (a)

The total resistance of this combination is intermediate between the pure series and pure parallel values ($20.0\ \Omega$ and $0.804\ \Omega$, respectively) found for the same resistors in the two previous examples.

Strategy and Solution for (b)

To find the IR drop in R_1 , we note that the full current I flows through R_1 . Thus its IR drop is

$$V_1 = IR_1. \tag{21.38}$$

We must find I before we can calculate V_1 . The total current I is found using Ohm's law for the circuit. That is,

$$I = \frac{V}{R_{\text{tot}}} = \frac{12.0\text{ V}}{5.11\ \Omega} = 2.35\text{ A}. \tag{21.39}$$

Entering this into the expression above, we get

$$V_1 = IR_1 = (2.35\text{ A})(1.00\ \Omega) = 2.35\text{ V}. \tag{21.40}$$

Discussion for (b)

The voltage applied to R_2 and R_3 is less than the total voltage by an amount V_1 . When wire resistance is large, it can significantly affect the operation of the devices represented by R_2 and R_3 .

Strategy and Solution for (c)

To find the current through R_2 , we must first find the voltage applied to it. We call this voltage V_{p} , because it is applied to a parallel combination of resistors. The voltage applied to both R_2 and R_3 is reduced by the amount V_1 , and so it is

$$V_{\text{p}} = V - V_1 = 12.0\text{ V} - 2.35\text{ V} = 9.65\text{ V}. \tag{21.41}$$

Now the current I_2 through resistance R_2 is found using Ohm's law:

$$I_2 = \frac{V_{\text{p}}}{R_2} = \frac{9.65\text{ V}}{6.00\ \Omega} = 1.61\text{ A}. \tag{21.42}$$

Discussion for (c)

The current is less than the 2.00 A that flowed through R_2 when it was connected in parallel to the battery in the previous parallel circuit example.

Strategy and Solution for (d)

The power dissipated by R_2 is given by

$$P_2 = (I_2)^2 R_2 = (1.61\text{ A})^2(6.00\ \Omega) = 15.5\text{ W}. \tag{21.43}$$

Discussion for (d)

The power is less than the 24.0 W this resistor dissipated when connected in parallel to the 12.0-V source.

Practical Implications

One implication of this last example is that resistance in wires reduces the current and power delivered to a resistor. If wire resistance is relatively large, as in a worn (or a very long) extension cord, then this loss can be significant. If a large current is drawn, the IR drop in the wires can also be significant.

For example, when you are rummaging in the refrigerator and the motor comes on, the refrigerator light dims momentarily. Similarly, you can see the passenger compartment light dim when you start the engine of your car (although this may be due to resistance inside the battery itself).

What is happening in these high-current situations is illustrated in **Figure 21.7**. The device represented by R_3 has a very low resistance, and so when it is switched on, a large current flows. This increased current causes a larger IR drop in the wires represented by R_1 , reducing the voltage across the light bulb (which is R_2), which then dims noticeably.

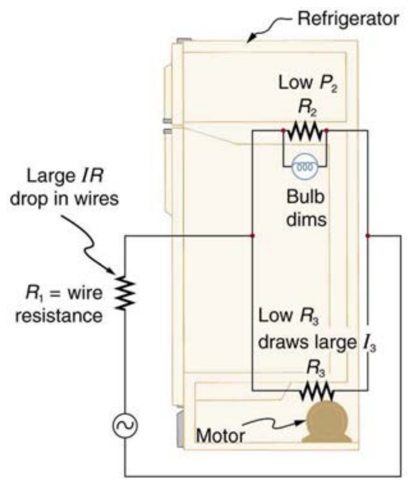


Figure 21.7 Why do lights dim when a large appliance is switched on? The answer is that the large current the appliance motor draws causes a significant IR drop in the wires and reduces the voltage across the light.

Check Your Understanding

Can any arbitrary combination of resistors be broken down into series and parallel combinations? See if you can draw a circuit diagram of resistors that cannot be broken down into combinations of series and parallel.

Solution

No, there are many ways to connect resistors that are not combinations of series and parallel, including loops and junctions. In such cases Kirchhoff's rules, to be introduced in **Kirchhoff's Rules**, will allow you to analyze the circuit.

Problem-Solving Strategies for Series and Parallel Resistors

1. Draw a clear circuit diagram, labeling all resistors and voltage sources. This step includes a list of the knowns for the problem, since they are labeled in your circuit diagram.
2. Identify exactly what needs to be determined in the problem (identify the unknowns). A written list is useful.
3. Determine whether resistors are in series, parallel, or a combination of both series and parallel. Examine the circuit diagram to make this assessment. Resistors are in series if the same current must pass sequentially through them.
4. Use the appropriate list of major features for series or parallel connections to solve for the unknowns. There is one list for series and another for parallel. If your problem has a combination of series and parallel, reduce it in steps by considering individual groups of series or parallel connections, as done in this module and the examples. Special note: When finding R, the reciprocal must be taken with care.
5. Check to see whether the answers are reasonable and consistent. Units and numerical results must be reasonable. Total series resistance should be greater, whereas total parallel resistance should be smaller, for example. Power should be greater for the same devices in parallel compared with series, and so on.

21.2 Electromotive Force: Terminal Voltage

When you forget to turn off your car lights, they slowly dim as the battery runs down. Why don't they simply blink off when the battery's energy is gone? Their gradual dimming implies that battery output voltage decreases as the battery is depleted.

Furthermore, if you connect an excessive number of 12-V lights in parallel to a car battery, they will be dim even when the battery is fresh and even if the wires to the lights have very low resistance. This implies that the battery's output voltage is reduced by the overload.

The reason for the decrease in output voltage for depleted or overloaded batteries is that all voltage sources have two fundamental parts—a source of electrical energy and an **internal resistance**. Let us examine both.

Electromotive Force

You can think of many different types of voltage sources. Batteries themselves come in many varieties. There are many types of mechanical/electrical generators, driven by many different energy sources, ranging from nuclear to wind. Solar cells create voltages directly from light, while thermoelectric devices create voltage from temperature differences.

A few voltage sources are shown in **Figure 21.8**. All such devices create a **potential difference** and can supply current if connected to a resistance. On the small scale, the potential difference creates an electric field that exerts force on charges, causing current. We thus use the name **electromotive force**, abbreviated emf.

Emf is not a force at all; it is a special type of potential difference. To be precise, the electromotive force (emf) is the potential difference of a source when no current is flowing. Units of emf are volts.

Figure 21.8 A variety of voltage sources (clockwise from top left): the Brazos Wind Farm in Fluvanna, Texas (credit: Leaflet, Wikimedia Commons); the Krasnoyarsk Dam in Russia (credit: Alex Polezhaev); a solar farm (credit: U.S. Department of Energy); and a group of nickel metal hydride batteries (credit: Tiaa Monto). The voltage output of each depends on its construction and load, and equals emf only if there is no load.

Electromotive force is directly related to the source of potential difference, such as the particular combination of chemicals in a battery. However, emf differs from the voltage output of the device when current flows. The voltage across the terminals of a battery, for example, is less than the emf when the battery supplies current, and it declines further as the battery is depleted or loaded down. However, if the device's output voltage can be measured without drawing current, then output voltage will equal emf (even for a very depleted battery).

Internal Resistance

As noted before, a 12-V truck battery is physically larger, contains more charge and energy, and can deliver a larger current than a 12-V motorcycle battery. Both are lead-acid batteries with identical emf, but, because of its size, the truck battery has a smaller internal resistance r . Internal resistance is the inherent resistance to the flow of current within the source itself.

Figure 21.9 is a schematic representation of the two fundamental parts of any voltage source. The emf (represented by a script E in the figure) and internal resistance r are in series. The smaller the internal resistance for a given emf, the more current and the more power the source can supply.

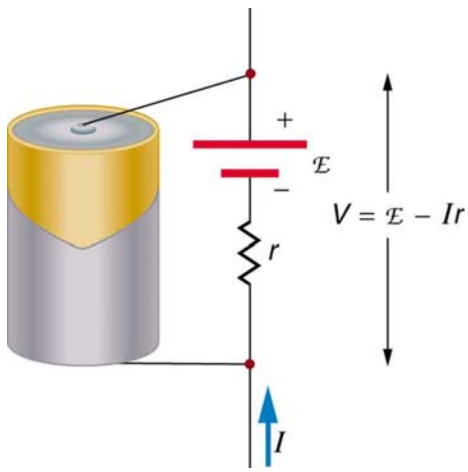


Figure 21.9 Any voltage source (in this case, a carbon-zinc dry cell) has an emf related to its source of potential difference, and an internal resistance r related to its construction. (Note that the script E stands for emf.). Also shown are the output terminals across which the terminal voltage V is measured. Since $V = \text{emf} - Ir$, terminal voltage equals emf only if there is no current flowing.

The internal resistance r can behave in complex ways. As noted, r increases as a battery is depleted. But internal resistance may also depend on the magnitude and direction of the current through a voltage source, its temperature, and even its history. The internal resistance of rechargeable nickel-cadmium cells, for example, depends on how many times and how deeply they have been depleted.

Things Great and Small: The Submicroscopic Origin of Battery Potential

Various types of batteries are available, with emfs determined by the combination of chemicals involved. We can view this as a molecular reaction (what much of chemistry is about) that separates charge.

The lead-acid battery used in cars and other vehicles is one of the most common types. A single cell (one of six) of this battery is seen in **Figure 21.10**. The cathode (positive) terminal of the cell is connected to a lead oxide plate, while the anode (negative) terminal is connected to a lead plate. Both plates are immersed in sulfuric acid, the electrolyte for the system.

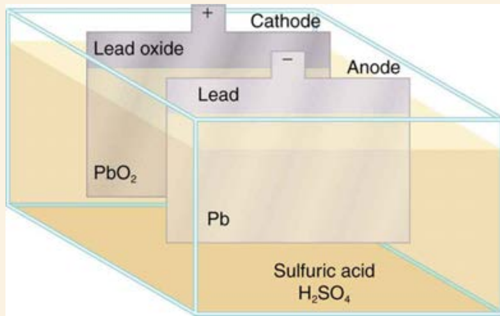


Figure 21.10 Artist's conception of a lead-acid cell. Chemical reactions in a lead-acid cell separate charge, sending negative charge to the anode, which is connected to the lead plates. The lead oxide plates are connected to the positive or cathode terminal of the cell. Sulfuric acid conducts the charge as well as participating in the chemical reaction.

The details of the chemical reaction are left to the reader to pursue in a chemistry text, but their results at the molecular level help explain the potential created by the battery. **Figure 21.11** shows the result of a single chemical reaction. Two electrons are placed on the anode, making it negative, provided that the cathode supplied two electrons. This leaves the cathode positively charged, because it has lost two electrons. In short, a separation of charge has been driven by a chemical reaction.

Note that the reaction will not take place unless there is a complete circuit to allow two electrons to be supplied to the cathode. Under many circumstances, these electrons come from the anode, flow through a resistance, and return to the cathode. Note also that since the chemical reactions involve substances with resistance, it is not possible to create the emf without an internal resistance.

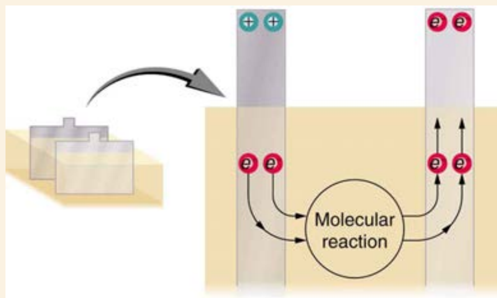


Figure 21.11 Artist's conception of two electrons being forced onto the anode of a cell and two electrons being removed from the cathode of the cell. The chemical reaction in a lead-acid battery places two electrons on the anode and removes two from the cathode. It requires a closed circuit to proceed, since the two electrons must be supplied to the cathode.

Why are the chemicals able to produce a unique potential difference? Quantum mechanical descriptions of molecules, which take into account the types of atoms and numbers of electrons in them, are able to predict the energy states they can have and the energies of reactions between them.

In the case of a lead-acid battery, an energy of 2 eV is given to each electron sent to the anode. Voltage is defined as the electrical potential energy divided by charge: $V = \frac{P_\mathrm{E}}{q}$. An electron volt is the energy given to a single electron by a voltage of 1 V. So the voltage here is 2 V, since 2 eV is given to each electron. It is the energy produced in each molecular reaction that produces the voltage. A different reaction produces a different energy and, hence, a different voltage.

Terminal Voltage

The voltage output of a device is measured across its terminals and, thus, is called its **terminal voltage** V . Terminal voltage is given by

$$V = \mathrm{emf} - Ir, \tag{21.44}$$

where r is the internal resistance and I is the current flowing at the time of the measurement.

I is positive if current flows away from the positive terminal, as shown in **Figure 21.9**. You can see that the larger the current, the smaller the terminal voltage. And it is likewise true that the larger the internal resistance, the smaller the terminal voltage.

Suppose a load resistance R_load is connected to a voltage source, as in **Figure 21.12**. Since the resistances are in series, the total resistance in the circuit is $R_\mathrm{load} + r$. Thus the current is given by Ohm's law to be

$$I = \frac{\mathrm{emf}}{R_\mathrm{load} + r}. \tag{21.45}$$

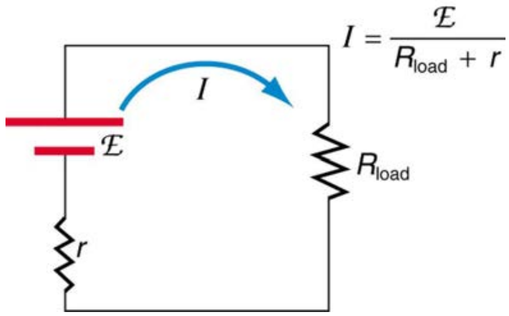


Figure 21.12 Schematic of a voltage source and its load R_{load} . Since the internal resistance r is in series with the load, it can significantly affect the terminal voltage and current delivered to the load. (Note that the script E stands for emf.)

We see from this expression that the smaller the internal resistance r, the greater the current the voltage source supplies to its load R_{load} . As batteries are depleted, r increases. If r becomes a significant fraction of the load resistance, then the current is significantly reduced, as the following example illustrates.

Example 21.4 Calculating Terminal Voltage, Power Dissipation, Current, and Resistance: Terminal Voltage and Load

A certain battery has a 12.0-V emf and an internal resistance of $0.100\ \Omega$. (a) Calculate its terminal voltage when connected to a $10.0\text{-}\Omega$ load. (b) What is the terminal voltage when connected to a $0.500\text{-}\Omega$ load? (c) What power does the $0.500\text{-}\Omega$ load dissipate? (d) If the internal resistance grows to $0.500\ \Omega$, find the current, terminal voltage, and power dissipated by a $0.500\text{-}\Omega$ load.

Strategy

The analysis above gave an expression for current when internal resistance is taken into account. Once the current is found, the terminal voltage can be calculated using the equation $V = \text{emf} - Ir$. Once current is found, the power dissipated by a resistor can also be found.

Solution for (a)

Entering the given values for the emf, load resistance, and internal resistance into the expression above yields

$$I = \frac{\text{emf}}{R_{\text{load}} + r} = \frac{12.0\text{ V}}{10.1\ \Omega} = 1.188\text{ A}. \tag{21.46}$$

Enter the known values into the equation $V = \text{emf} - Ir$ to get the terminal voltage:

$$\begin{aligned} V &= \text{emf} - Ir = 12.0\text{ V} - (1.188\text{ A})(0.100\ \Omega) \\ &= 11.9\text{ V}. \end{aligned} \tag{21.47}$$

Discussion for (a)

The terminal voltage here is only slightly lower than the emf, implying that $10.0\ \Omega$ is a light load for this particular battery.

Solution for (b)

Similarly, with $R_{\text{load}} = 0.500\ \Omega$, the current is

$$I = \frac{\text{emf}}{R_{\text{load}} + r} = \frac{12.0\text{ V}}{0.600\ \Omega} = 20.0\text{ A}. \tag{21.48}$$

The terminal voltage is now

$$\begin{aligned} V &= \text{emf} - Ir = 12.0\text{ V} - (20.0\text{ A})(0.100\ \Omega) \\ &= 10.0\text{ V}. \end{aligned} \tag{21.49}$$

Discussion for (b)

This terminal voltage exhibits a more significant reduction compared with emf, implying $0.500\ \Omega$ is a heavy load for this battery.

Solution for (c)

The power dissipated by the $0.500\text{ - }\Omega$ load can be found using the formula $P = I^2R$. Entering the known values gives

$$P_{\text{load}} = I^2R_{\text{load}} = (20.0\text{ A})^2(0.500\ \Omega) = 2.00\times10^2\text{ W}. \tag{21.50}$$

Discussion for (c)

Note that this power can also be obtained using the expressions $\frac{V^2}{R}$ or IV , where V is the terminal voltage (10.0 V in this case).

Solution for (d)

Here the internal resistance has increased, perhaps due to the depletion of the battery, to the point where it is as great as the load resistance. As before, we first find the current by entering the known values into the expression, yielding

$$I = \frac{\text{emf}}{R_{\text{load}} + r} = \frac{12.0\text{ V}}{1.00\ \ \Omega} = 12.0\text{ A.} \tag{21.51}$$

Now the terminal voltage is

$$\begin{aligned} V &= \text{emf} - Ir = 12.0\text{ V} - (12.0\text{ A})(0.500\ \Omega) \\ &= 6.00\text{ V,} \end{aligned} \tag{21.52}$$

and the power dissipated by the load is

$$P_{\text{load}} = I^2 R_{\text{load}} = (12.0\text{ A})^2 (0.500\ \ \Omega\ ) = 72.0\text{ W.} \tag{21.53}$$

Discussion for (d)

We see that the increased internal resistance has significantly decreased terminal voltage, current, and power delivered to a load.

Battery testers, such as those in **Figure 21.13**, use small load resistors to intentionally draw current to determine whether the terminal voltage drops below an acceptable level. They really test the internal resistance of the battery. If internal resistance is high, the battery is weak, as evidenced by its low terminal voltage.

Figure 21.13 These two battery testers measure terminal voltage under a load to determine the condition of a battery. The large device is being used by a U.S. Navy electronics technician to test large batteries aboard the aircraft carrier USS *Nimitz* and has a small resistance that can dissipate large amounts of power. (credit: U.S. Navy photo by Photographer's Mate Airman Jason A. Johnston) The small device is used on small batteries and has a digital display to indicate the acceptability of their terminal voltage. (credit: Keith Williamson)

Some batteries can be recharged by passing a current through them in the direction opposite to the current they supply to a resistance. This is done routinely in cars and batteries for small electrical appliances and electronic devices, and is represented pictorially in **Figure 21.14**. The voltage output of the battery charger must be greater than the emf of the battery to reverse current through it. This will cause the terminal voltage of the battery to be greater than the emf, since $V = \text{emf} - Ir$, and I is now negative.

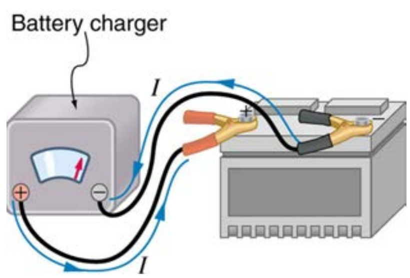


Figure 21.14 A car battery charger reverses the normal direction of current through a battery, reversing its chemical reaction and replenishing its chemical potential.

Multiple Voltage Sources

There are two voltage sources when a battery charger is used. Voltage sources connected in series are relatively simple. When voltage sources are in series, their internal resistances add and their emfs add algebraically. (See **Figure 21.15**.) Series connections of voltage sources are common—for example, in flashlights, toys, and other appliances. Usually, the cells are in series in order to produce a larger total emf.

But if the cells oppose one another, such as when one is put into an appliance backward, the total emf is less, since it is the algebraic sum of the individual emfs.

A battery is a multiple connection of voltaic cells, as shown in **Figure 21.16**. The disadvantage of series connections of cells is that their internal resistances add. One of the authors once owned a 1957 MGA that had two 6-V batteries in series, rather than a single 12-V battery. This arrangement produced a large internal resistance that caused him many problems in starting the engine.

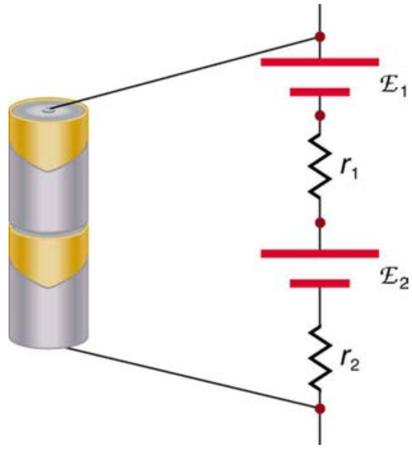


Figure 21.15 A series connection of two voltage sources. The emfs (each labeled with a script E) and internal resistances add, giving a total emf of $\text{emf}_1 + \text{emf}_2$ and a total internal resistance of $r_1 + r_2$.

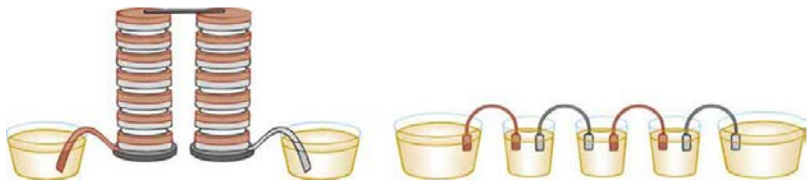


Figure 21.16 Batteries are multiple connections of individual cells, as shown in this modern rendition of an old print. Single cells, such as AA or C cells, are commonly called batteries, although this is technically incorrect.

If the *series* connection of two voltage sources is made into a complete circuit with the emfs in opposition, then a current of magnitude $I = \dfrac{(\text{emf}_1 - \text{emf}_2)}{r_1 + r_2}$ flows. See **Figure 21.17**, for example, which shows a circuit exactly analogous to the battery charger discussed above. If two voltage sources in series with emfs in the same sense are connected to a load R_{load}, as in **Figure 21.18**, then $I = \dfrac{(\text{emf}_1 + \text{emf}_2)}{r_1 + r_2 + R_{\text{load}}}$ flows.

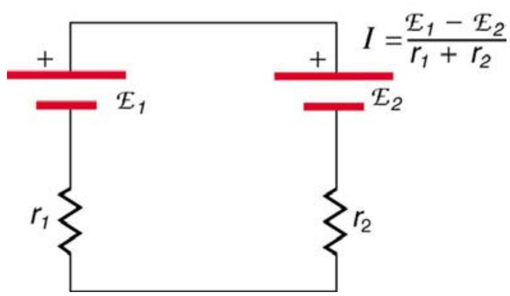


Figure 21.17 These two voltage sources are connected in series with their emfs in opposition. Current flows in the direction of the greater emf and is limited to $I = \dfrac{(\text{emf}_1 - \text{emf}_2)}{r_1 + r_2}$ by the sum of the internal resistances. (Note that each emf is represented by script E in the figure.) A battery charger connected to a battery is an example of such a connection. The charger must have a larger emf than the battery to reverse current through it.

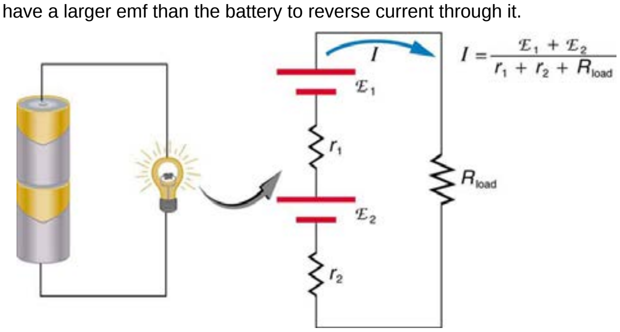


Figure 21.18 This schematic represents a flashlight with two cells (voltage sources) and a single bulb (load resistance) in series. The current that flows is $I = \dfrac{(\text{emf}_1 + \text{emf}_2)}{r_1 + r_2 + R_{\text{load}}}$. (Note that each emf is represented by script E in the figure.)

Take-Home Experiment: Flashlight Batteries

Find a flashlight that uses several batteries and find new and old batteries. Based on the discussions in this module, predict the brightness of the flashlight when different combinations of batteries are used. Do your predictions match what you observe? Now place new batteries in the flashlight and leave the flashlight switched on for several hours. Is the flashlight still quite bright? Do the same with the old batteries. Is the flashlight as bright when left on for the same length of time with old and new batteries? What does this say for the case when you are limited in the number of available new batteries?

Figure 21.19 shows two voltage sources with identical emfs in parallel and connected to a load resistance. In this simple case, the total emf is the same as the individual emfs. But the total internal resistance is reduced, since the internal resistances are in parallel. The parallel connection thus can produce a larger current.

Here, $I = \frac{\text{emf}}{(r_{\text{tot}} + R_{\text{load}})}$ flows through the load, and r_{tot} is less than those of the individual batteries. For example, some diesel-powered cars use two 12-V batteries in parallel; they produce a total emf of 12 V but can deliver the larger current needed to start a diesel engine.

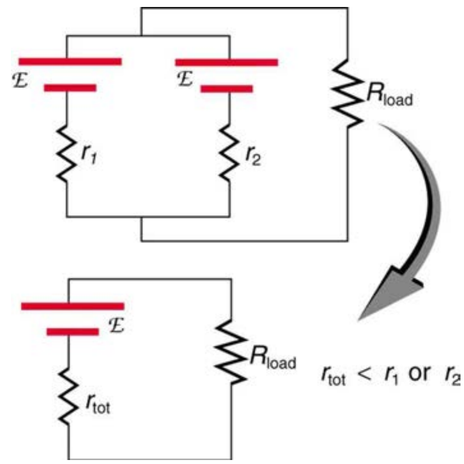


Figure 21.19 Two voltage sources with identical emfs (each labeled by script E) connected in parallel produce the same emf but have a smaller total internal resistance than the individual sources. Parallel combinations are often used to deliver more current. Here $I = \frac{\text{emf}}{(r_{\text{tot}} + R_{\text{load}})}$ flows through the load.

Animals as Electrical Detectors

A number of animals both produce and detect electrical signals. Fish, sharks, platypuses, and echidnas (spiny anteaters) all detect electric fields generated by nerve activity in prey. Electric eels produce their own emf through biological cells (electric organs) called electroplaques, which are arranged in both series and parallel as a set of batteries.

Electroplaques are flat, disk-like cells; those of the electric eel have a voltage of 0.15 V across each one. These cells are usually located toward the head or tail of the animal, although in the case of the electric eel, they are found along the entire body. The electroplaques in the South American eel are arranged in 140 rows, with each row stretching horizontally along the body and containing 5,000 electroplaques. This can yield an emf of approximately 600 V, and a current of 1 A—deadly.

The mechanism for detection of external electric fields is similar to that for producing nerve signals in the cell through depolarization and repolarization—the movement of ions across the cell membrane. Within the fish, weak electric fields in the water produce a current in a gel-filled canal that runs from the skin to sensing cells, producing a nerve signal. The Australian platypus, one of the very few mammals that lay eggs, can detect fields of 30 $\frac{\text{mV}}{\text{m}}$, while sharks have been found to be able to sense a field in their snouts as small as 100 $\frac{\text{mV}}{\text{m}}$ (**Figure 21.20**). Electric eels use their own electric fields produced by the electroplaques to stun their prey or enemies.

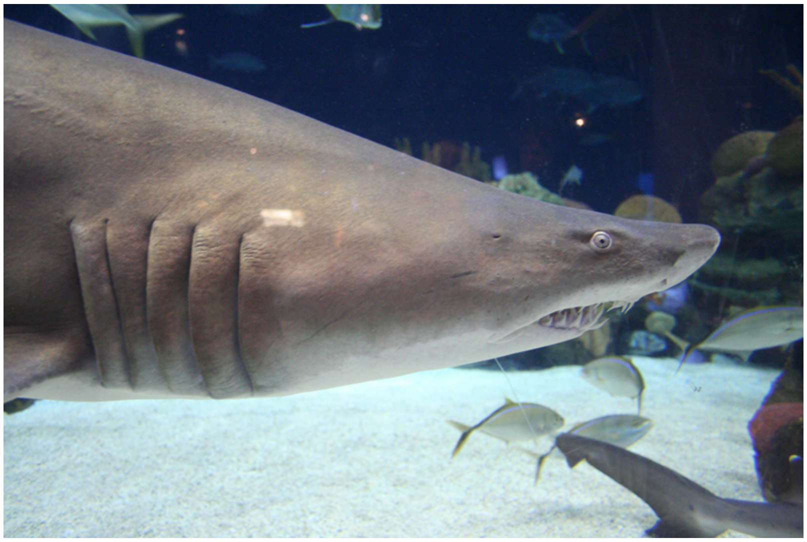


Figure 21.20 Sand tiger sharks (*Carcharias taurus*), like this one at the Minnesota Zoo, use electroreceptors in their snouts to locate prey. (credit: Jim Winstead, Flickr)

Solar Cell Arrays

Another example dealing with multiple voltage sources is that of combinations of solar cells—wired in both series and parallel combinations to yield a desired voltage and current. Photovoltaic generation (PV), the conversion of sunlight directly into electricity, is based upon the photoelectric effect, in which photons hitting the surface of a solar cell create an electric current in the cell.

Most solar cells are made from pure silicon—either as single-crystal silicon, or as a thin film of silicon deposited upon a glass or metal backing. Most single cells have a voltage output of about 0.5 V, while the current output is a function of the amount of sunlight upon the cell (the incident solar radiation—the insolation). Under bright noon sunlight, a current of about $100\ \text{mA/cm}^2$ of cell surface area is produced by typical single-crystal cells.

Individual solar cells are connected electrically in modules to meet electrical-energy needs. They can be wired together in series or in parallel—connected like the batteries discussed earlier. A solar-cell array or module usually consists of between 36 and 72 cells, with a power output of 50 W to 140 W.

The output of the solar cells is direct current. For most uses in a home, AC is required, so a device called an inverter must be used to convert the DC to AC. Any extra output can then be passed on to the outside electrical grid for sale to the utility.

Take-Home Experiment: Virtual Solar Cells

One can assemble a “virtual” solar cell array by using playing cards, or business or index cards, to represent a solar cell. Combinations of these cards in series and/or parallel can model the required array output. Assume each card has an output of 0.5 V and a current (under bright light) of 2 A. Using your cards, how would you arrange them to produce an output of 6 A at 3 V (18 W)?

Suppose you were told that you needed only 18 W (but no required voltage). Would you need more cards to make this arrangement?

21.3 Kirchhoff’s Rules

Many complex circuits, such as the one in **Figure 21.21**, cannot be analyzed with the series-parallel techniques developed in **Resistors in Series and Parallel** and **Electromotive Force: Terminal Voltage**. There are, however, two circuit analysis rules that can be used to analyze any circuit, simple or complex. These rules are special cases of the laws of conservation of charge and conservation of energy. The rules are known as **Kirchhoff’s rules**, after their inventor Gustav Kirchhoff (1824–1887).

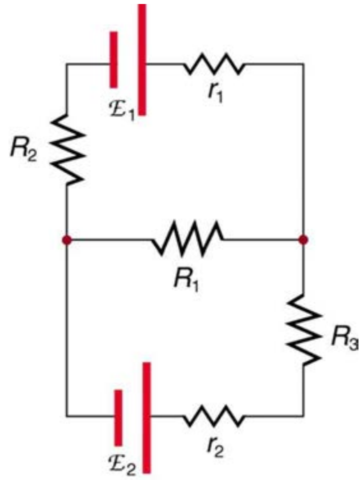


Figure 21.21 This circuit cannot be reduced to a combination of series and parallel connections. Kirchhoff's rules, special applications of the laws of conservation of charge and energy, can be used to analyze it. (Note: The script E in the figure represents electromotive force, emf.)

Kirchhoff's Rules

- Kirchhoff's first rule—the junction rule. The sum of all currents entering a junction must equal the sum of all currents leaving the junction.
- Kirchhoff's second rule—the loop rule. The algebraic sum of changes in potential around any closed circuit path (loop) must be zero.

Explanations of the two rules will now be given, followed by problem-solving hints for applying Kirchhoff's rules, and a worked example that uses them.

Kirchhoff's First Rule

Kirchhoff's first rule (the **junction rule**) is an application of the conservation of charge to a junction; it is illustrated in **Figure 21.22**. Current is the flow of charge, and charge is conserved; thus, whatever charge flows into the junction must flow out. Kirchhoff's first rule requires that $I_1 = I_2 + I_3$ (see figure). Equations like this can and will be used to analyze circuits and to solve circuit problems.

Making Connections: Conservation Laws

Kirchhoff's rules for circuit analysis are applications of **conservation laws** to circuits. The first rule is the application of conservation of charge, while the second rule is the application of conservation of energy. Conservation laws, even used in a specific application, such as circuit analysis, are so basic as to form the foundation of that application.

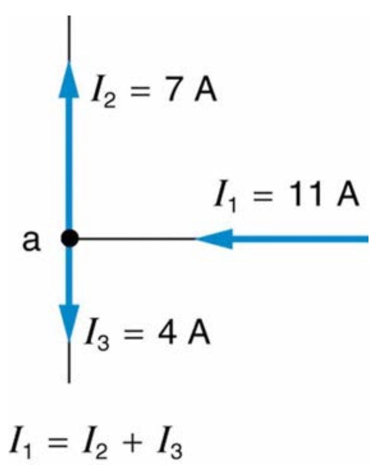


Figure 21.22 The junction rule. The diagram shows an example of Kirchhoff's first rule where the sum of the currents into a junction equals the sum of the currents out of a junction. In this case, the current going into the junction splits and comes out as two currents, so that $I_1 = I_2 + I_3$. Here I_1 must be 11 A, since I_2 is 7 A and I_3 is 4 A.

Kirchhoff's Second Rule

Kirchhoff's second rule (the **loop rule**) is an application of conservation of energy. The loop rule is stated in terms of potential, V , rather than potential energy, but the two are related since $\mathrm{PE_{elec}} = qV$. Recall that **emf** is the potential difference of a source when no current is flowing. In a closed loop, whatever energy is supplied by emf must be transferred into other forms by devices in the loop, since there are no other ways in which energy can be transferred into or out of the circuit. **Figure 21.23** illustrates the changes in potential in a simple series circuit loop.

Kirchhoff's second rule requires $\text{emf} - Ir - IR_1 - IR_2 = 0$. Rearranged, this is $\text{emf} = Ir + IR_1 + IR_2$, which means the emf equals the sum of the IR (voltage) drops in the loop.

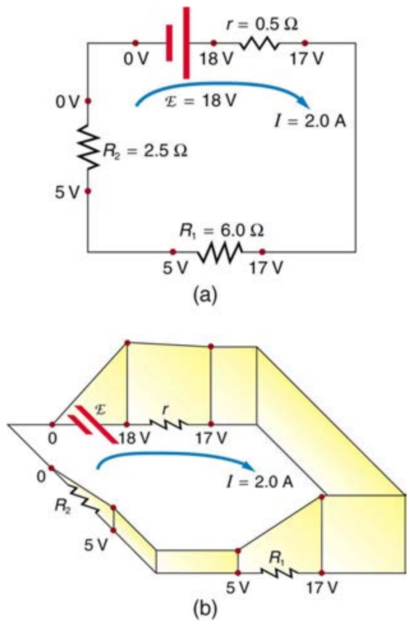


Figure 21.23 The loop rule. An example of Kirchhoff's second rule where the sum of the changes in potential around a closed loop must be zero. (a) In this standard schematic of a simple series circuit, the emf supplies 18 V, which is reduced to zero by the resistances, with 1 V across the internal resistance, and 12 V and 5 V across the two load resistances, for a total of 18 V. (b) This perspective view represents the potential as something like a roller coaster, where charge is raised in potential by the emf and lowered by the resistances. (Note that the script E stands for emf.)

Applying Kirchhoff's Rules

By applying Kirchhoff's rules, we generate equations that allow us to find the unknowns in circuits. The unknowns may be currents, emfs, or resistances. Each time a rule is applied, an equation is produced. If there are as many independent equations as unknowns, then the problem can be solved. There are two decisions you must make when applying Kirchhoff's rules. These decisions determine the signs of various quantities in the equations you obtain from applying the rules.

1. When applying Kirchhoff's first rule, the junction rule, you must label the current in each branch and decide in what direction it is going. For example, in **Figure 21.21**, **Figure 21.22**, and **Figure 21.23**, currents are labeled I_1, I_2, I_3, and I, and arrows indicate their directions. There is no risk here, for if you choose the wrong direction, the current will be of the correct magnitude but negative.
2. When applying Kirchhoff's second rule, the loop rule, you must identify a closed loop and decide in which direction to go around it, clockwise or counterclockwise. For example, in **Figure 21.23** the loop was traversed in the same direction as the current (clockwise). Again, there is no risk; going around the circuit in the opposite direction reverses the sign of every term in the equation, which is like multiplying both sides of the equation by -1.

Figure 21.24 and the following points will help you get the plus or minus signs right when applying the loop rule. Note that the resistors and emfs are traversed by going from a to b. In many circuits, it will be necessary to construct more than one loop. In traversing each loop, one needs to be consistent for the sign of the change in potential. (See **Example 21.5**.)

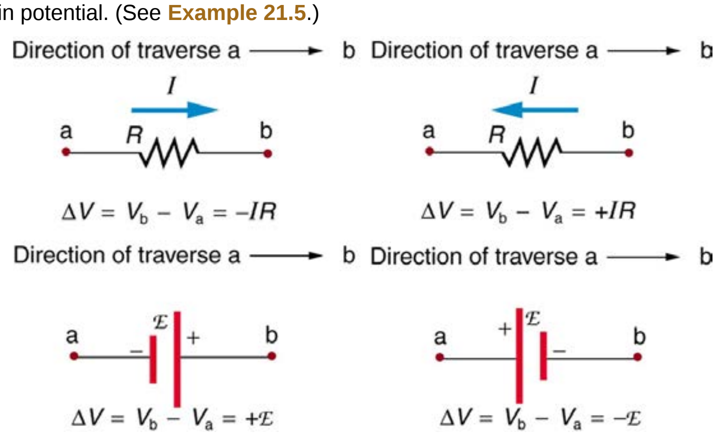


Figure 21.24 Each of these resistors and voltage sources is traversed from a to b. The potential changes are shown beneath each element and are explained in the text. (Note that the script E stands for emf.)

- When a resistor is traversed in the same direction as the current, the change in potential is $-IR$. (See **Figure 21.24**.)
- When a resistor is traversed in the direction opposite to the current, the change in potential is $+IR$. (See **Figure 21.24**.)
- When an emf is traversed from $-$ to + (the same direction it moves positive charge), the change in potential is +emf. (See **Figure 21.24**.)
- When an emf is traversed from + to $-$ (opposite to the direction it moves positive charge), the change in potential is $-$ emf. (See **Figure 21.24**.)

Example 21.5 Calculating Current: Using Kirchhoff's Rules

Find the currents flowing in the circuit in **Figure 21.25**.

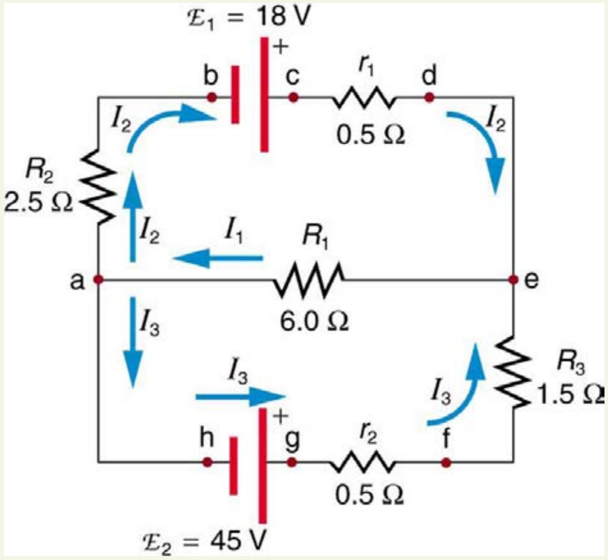


Figure 21.25 This circuit is similar to that in **Figure 21.21**, but the resistances and emfs are specified. (Each emf is denoted by script E.) The currents in each branch are labeled and assumed to move in the directions shown. This example uses Kirchhoff's rules to find the currents.

Strategy

This circuit is sufficiently complex that the currents cannot be found using Ohm's law and the series-parallel techniques—it is necessary to use Kirchhoff's rules. Currents have been labeled I_1 , I_2 , and I_3 in the figure and assumptions have been made about their directions. Locations on the diagram have been labeled with letters a through h. In the solution we will apply the junction and loop rules, seeking three independent equations to allow us to solve for the three unknown currents.

Solution

We begin by applying Kirchhoff's first or junction rule at point a. This gives

$$I_1 = I_2 + I_3, \tag{21.54}$$

since I_1 flows into the junction, while I_2 and I_3 flow out. Applying the junction rule at e produces exactly the same equation, so that no new information is obtained. This is a single equation with three unknowns—three independent equations are needed, and so the loop rule must be applied.

Now we consider the loop abcdea. Going from a to b, we traverse R_2 in the same (assumed) direction of the current I_2 , and so the change in potential is $-I_2R_2$. Then going from b to c, we go from $-$ to +, so that the change in potential is $+\text{emf}_1$. Traversing the internal resistance r_1 from c to d gives $-I_2r_1$. Completing the loop by going from d to a again traverses a resistor in the same direction as its current, giving a change in potential of $-I_1R_1$.

The loop rule states that the changes in potential sum to zero. Thus,

$$-I_2R_2 + \text{emf}_1 - I_2r_1 - I_1R_1 = -I_2(R_2 + r_1) + \text{emf}_1 - I_1R_1 = 0. \tag{21.55}$$

Substituting values from the circuit diagram for the resistances and emf, and canceling the ampere unit gives

$$-3I_2 + 18 - 6I_1 = 0. \tag{21.56}$$

Now applying the loop rule to aefgha (we could have chosen abcdefgha as well) similarly gives

$$+ I_1R_1 + I_3R_3 + I_3r_2 - \text{emf}_2 = +I_1R_1 + I_3\big(R_3 + r_2\big) - \text{emf}_2 = 0. \tag{21.57}$$

Note that the signs are reversed compared with the other loop, because elements are traversed in the opposite direction. With values entered, this becomes

$$+ 6I_1 + 2I_3 - 45 = 0. \tag{21.58}$$

These three equations are sufficient to solve for the three unknown currents. First, solve the second equation for I_2 :

$$I_2 = 6 - 2I_1. \tag{21.59}$$

Now solve the third equation for I_3 :

$$I_3 = 22.5 - 3I_1. \tag{21.60}$$

Substituting these two new equations into the first one allows us to find a value for I_1 :

$$I_1 = I_2 + I_3 = (6 - 2I_1) + (22.5 - 3I_1) = 28.5 - 5I_1. \tag{21.61}$$

Combining terms gives

$$6I_1 = 28.5, \text{ and} \tag{21.62}$$

$$I_1 = 4.75 \text{ A}. \tag{21.63}$$

Substituting this value for I_1 back into the fourth equation gives

$$I_2 = 6 - 2I_1 = 6 - 9.50 \tag{21.64}$$

$$I_2 = -3.50 \text{ A}. \tag{21.65}$$

The minus sign means I_2 flows in the direction opposite to that assumed in **Figure 21.25**.

Finally, substituting the value for I_1 into the fifth equation gives

$$I_3 = 22.5{-}3I_1 = 22.5 - 14.25 \tag{21.66}$$

$$I_3 = 8.25 \text{ A}. \tag{21.67}$$

Discussion

Just as a check, we note that indeed $I_1 = I_2 + I_3$. The results could also have been checked by entering all of the values into the equation for the abcdefgha loop.

Problem-Solving Strategies for Kirchhoff's Rules

1. Make certain there is a clear circuit diagram on which you can label all known and unknown resistances, emfs, and currents. If a current is unknown, you must assign it a direction. This is necessary for determining the signs of potential changes. If you assign the direction incorrectly, the current will be found to have a negative value—no harm done.
2. Apply the junction rule to any junction in the circuit. Each time the junction rule is applied, you should get an equation with a current that does not appear in a previous application—if not, then the equation is redundant.
3. Apply the loop rule to as many loops as needed to solve for the unknowns in the problem. (There must be as many independent equations as unknowns.) To apply the loop rule, you must choose a direction to go around the loop. Then carefully and consistently determine the signs of the potential changes for each element using the four bulleted points discussed above in conjunction with **Figure 21.24**.
4. Solve the simultaneous equations for the unknowns. This may involve many algebraic steps, requiring careful checking and rechecking.
5. Check to see whether the answers are reasonable and consistent. The numbers should be of the correct order of magnitude, neither exceedingly large nor vanishingly small. The signs should be reasonable—for example, no resistance should be negative. Check to see that the values obtained satisfy the various equations obtained from applying the rules. The currents should satisfy the junction rule, for example.

The material in this section is correct in theory. We should be able to verify it by making measurements of current and voltage. In fact, some of the devices used to make such measurements are straightforward applications of the principles covered so far and are explored in the next modules. As we shall see, a very basic, even profound, fact results—making a measurement alters the quantity being measured.

Check Your Understanding

Can Kirchhoff's rules be applied to simple series and parallel circuits or are they restricted for use in more complicated circuits that are not combinations of series and parallel?

Solution

Kirchhoff's rules can be applied to any circuit since they are applications to circuits of two conservation laws. Conservation laws are the most broadly applicable principles in physics. It is usually mathematically simpler to use the rules for series and parallel in simpler circuits so we emphasize Kirchhoff's rules for use in more complicated situations. But the rules for series and parallel can be derived from Kirchhoff's rules. Moreover, Kirchhoff's rules can be expanded to devices other than resistors and emfs, such as capacitors, and are one of the basic analysis devices in circuit analysis.

21.4 DC Voltmeters and Ammeters

Voltmeters measure voltage, whereas **ammeters** measure current. Some of the meters in automobile dashboards, digital cameras, cell phones, and tuner-amplifiers are voltmeters or ammeters. (See **Figure 21.26**.) The internal construction of the simplest of these meters and how they are connected to the system they monitor give further insight into applications of series and parallel connections.

Figure 21.26 The fuel and temperature gauges (far right and far left, respectively) in this 1996 Volkswagen are voltmeters that register the voltage output of "sender" units, which are hopefully proportional to the amount of gasoline in the tank and the engine temperature. (credit: Christian Giersing)

Voltmeters are connected in parallel with whatever device's voltage is to be measured. A parallel connection is used because objects in parallel experience the same potential difference. (See **Figure 21.27**, where the voltmeter is represented by the symbol V.)

Ammeters are connected in series with whatever device's current is to be measured. A series connection is used because objects in series have the same current passing through them. (See **Figure 21.28**, where the ammeter is represented by the symbol A.)

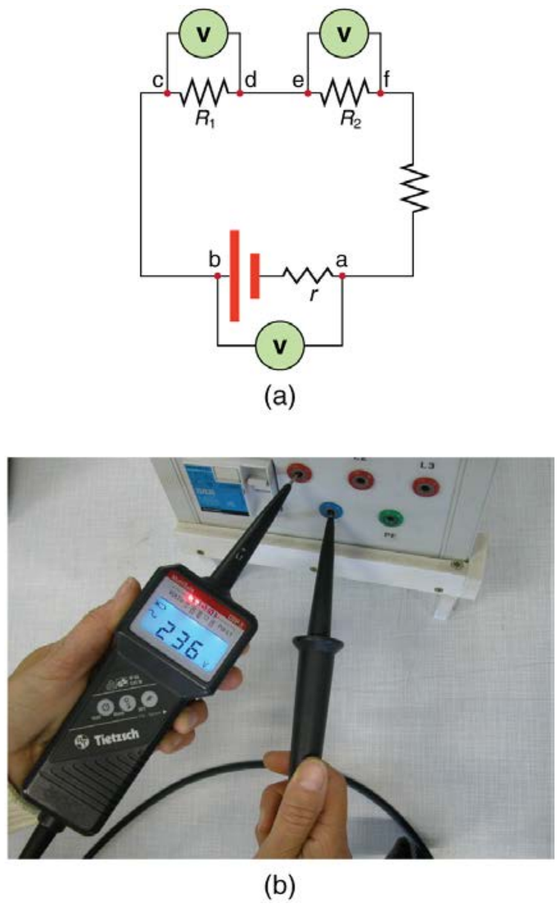


Figure 21.27 (a) To measure potential differences in this series circuit, the voltmeter (V) is placed in parallel with the voltage source or either of the resistors. Note that terminal voltage is measured between points a and b. It is not possible to connect the voltmeter directly across the emf without including its internal resistance, r . (b) A digital voltmeter in use. (credit: Messtechniker, Wikimedia Commons)

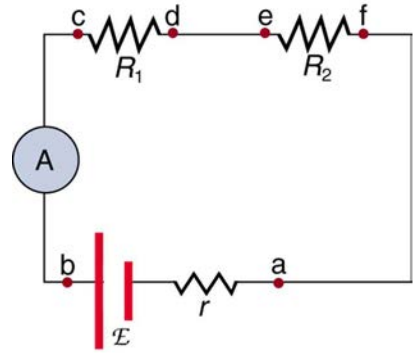


Figure 21.28 An ammeter (A) is placed in series to measure current. All of the current in this circuit flows through the meter. The ammeter would have the same reading if located between points d and e or between points f and a as it does in the position shown. (Note that the script capital E stands for emf, and r stands for the internal resistance of the source of potential difference.)

Analog Meters: Galvanometers

Analog meters have a needle that swivels to point at numbers on a scale, as opposed to **digital meters**, which have numerical readouts similar to a hand-held calculator. The heart of most analog meters is a device called a **galvanometer**, denoted by G. Current flow through a galvanometer, I_{G}, produces a proportional needle deflection. (This deflection is due to the force of a magnetic field upon a current-carrying wire.)

The two crucial characteristics of a given galvanometer are its resistance and current sensitivity. **Current sensitivity** is the current that gives a **full-scale deflection** of the galvanometer's needle, the maximum current that the instrument can measure. For example, a galvanometer with a current sensitivity of $50\ \mu\text{A}$ has a maximum deflection of its needle when $50\ \mu\text{A}$ flows through it, reads half-scale when $25\ \mu\text{A}$ flows through it, and so on.

If such a galvanometer has a $25\text{-}\ \Omega$ resistance, then a voltage of only $V = IR = (50\ \mu\text{A})(25\ \Omega) = 1.25\ \text{mV}$ produces a full-scale reading. By connecting resistors to this galvanometer in different ways, you can use it as either a voltmeter or ammeter that can measure a broad range of voltages or currents.

Galvanometer as Voltmeter

Figure 21.29 shows how a galvanometer can be used as a voltmeter by connecting it in series with a large resistance, R. The value of the resistance R is determined by the maximum voltage to be measured. Suppose you want 10 V to produce a full-scale deflection of a voltmeter containing a $25\text{-}\Omega$ galvanometer with a $50\text{-}\mu\text{A}$ sensitivity. Then 10 V applied to the meter must produce a current of $50\ \mu\text{A}$. The total resistance must be

$$R_{\text{tot}} = R + r = \frac{V}{I} = \frac{10\ \text{V}}{50\ \mu\text{A}} = 200\ \text{k}\Omega\text{, or} \tag{21.68}$$

$$R = R_{\text{tot}} - r = 200\ \text{k}\Omega - 25\ \ \Omega\ \approx 200\ \text{k}\ \Omega\ . \tag{21.69}$$

(R is so large that the galvanometer resistance, r, is nearly negligible.) Note that 5 V applied to this voltmeter produces a half-scale deflection by producing a $25\text{-}\mu\text{A}$ current through the meter, and so the voltmeter's reading is proportional to voltage as desired.

This voltmeter would not be useful for voltages less than about half a volt, because the meter deflection would be small and difficult to read accurately. For other voltage ranges, other resistances are placed in series with the galvanometer. Many meters have a choice of scales. That choice involves switching an appropriate resistance into series with the galvanometer.

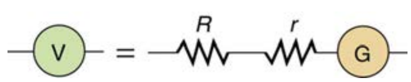


Figure 21.29 A large resistance R placed in series with a galvanometer G produces a voltmeter, the full-scale deflection of which depends on the choice of R. The larger the voltage to be measured, the larger R must be. (Note that r represents the internal resistance of the galvanometer.)

Galvanometer as Ammeter

The same galvanometer can also be made into an ammeter by placing it in parallel with a small resistance R, often called the **shunt resistance**, as shown in **Figure 21.30**. Since the shunt resistance is small, most of the current passes through it, allowing an ammeter to measure currents much greater than those producing a full-scale deflection of the galvanometer.

Suppose, for example, an ammeter is needed that gives a full-scale deflection for 1.0 A, and contains the same $25\text{-}\ \Omega$ galvanometer with its $50\text{-}\mu\text{A}$ sensitivity. Since R and r are in parallel, the voltage across them is the same.

These IR drops are $IR = I_{\text{G}}r$ so that $IR = \frac{I_{\text{G}}}{I} = \frac{R}{r}$. Solving for R, and noting that I_{G} is $50\ \mu\text{A}$ and I is 0.999950 A, we have

$$R = r\frac{I_{\text{G}}}{I} = (25\ \ \Omega\ )\frac{50\ \mu\text{A}}{0.999950\ \text{A}} = 1.25{\times}10^{-3}\ \ \Omega\ . \tag{21.70}$$

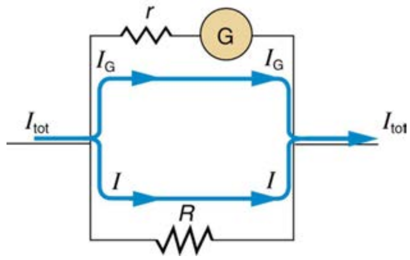


Figure 21.30 A small shunt resistance R placed in parallel with a galvanometer G produces an ammeter, the full-scale deflection of which depends on the choice of R . The larger the current to be measured, the smaller R must be. Most of the current (I) flowing through the meter is shunted through R to protect the galvanometer. (Note that r represents the internal resistance of the galvanometer.) Ammeters may also have multiple scales for greater flexibility in application. The various scales are achieved by switching various shunt resistances in parallel with the galvanometer—the greater the maximum current to be measured, the smaller the shunt resistance must be.

Taking Measurements Alters the Circuit

When you use a voltmeter or ammeter, you are connecting another resistor to an existing circuit and, thus, altering the circuit. Ideally, voltmeters and ammeters do not appreciably affect the circuit, but it is instructive to examine the circumstances under which they do or do not interfere.

First, consider the voltmeter, which is always placed in parallel with the device being measured. Very little current flows through the voltmeter if its resistance is a few orders of magnitude greater than the device, and so the circuit is not appreciably affected. (See **Figure 21.31**(a).) (A large resistance in parallel with a small one has a combined resistance essentially equal to the small one.) If, however, the voltmeter's resistance is comparable to that of the device being measured, then the two in parallel have a smaller resistance, appreciably affecting the circuit. (See **Figure 21.31**(b).) The voltage across the device is not the same as when the voltmeter is out of the circuit.

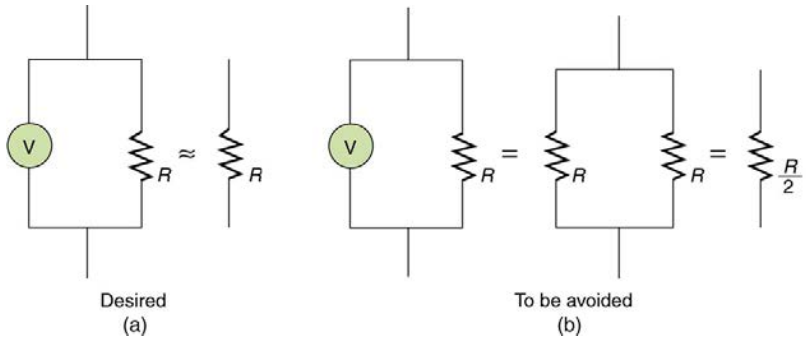


Figure 21.31 (a) A voltmeter having a resistance much larger than the device ($R_{\text{Voltmeter}} >> R$) with which it is in parallel produces a parallel resistance essentially the same as the device and does not appreciably affect the circuit being measured. (b) Here the voltmeter has the same resistance as the device ($R_{\text{Voltmeter}} \cong R$), so that the parallel resistance is half of what it is when the voltmeter is not connected. This is an example of a significant alteration of the circuit and is to be avoided.

An ammeter is placed in series in the branch of the circuit being measured, so that its resistance adds to that branch. Normally, the ammeter's resistance is very small compared with the resistances of the devices in the circuit, and so the extra resistance is negligible. (See **Figure 21.32**(a).) However, if very small load resistances are involved, or if the ammeter is not as low in resistance as it should be, then the total series resistance is significantly greater, and the current in the branch being measured is reduced. (See **Figure 21.32**(b).)

A practical problem can occur if the ammeter is connected incorrectly. If it was put in parallel with the resistor to measure the current in it, you could possibly damage the meter; the low resistance of the ammeter would allow most of the current in the circuit to go through the galvanometer, and this current would be larger since the effective resistance is smaller.

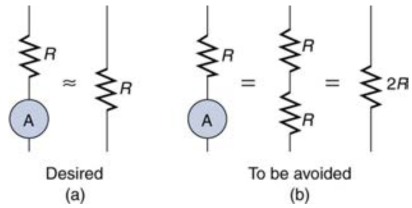


Figure 21.32 (a) An ammeter normally has such a small resistance that the total series resistance in the branch being measured is not appreciably increased. The circuit is essentially unaltered compared with when the ammeter is absent. (b) Here the ammeter's resistance is the same as that of the branch, so that the total resistance is doubled and the current is half what it is without the ammeter. This significant alteration of the circuit is to be avoided.

One solution to the problem of voltmeters and ammeters interfering with the circuits being measured is to use galvanometers with greater sensitivity. This allows construction of voltmeters with greater resistance and ammeters with smaller resistance than when less sensitive galvanometers are used.

There are practical limits to galvanometer sensitivity, but it is possible to get analog meters that make measurements accurate to a few percent. Note that the inaccuracy comes from altering the circuit, not from a fault in the meter.

Connections: Limits to Knowledge

Making a measurement alters the system being measured in a manner that produces uncertainty in the measurement. For macroscopic systems, such as the circuits discussed in this module, the alteration can usually be made negligibly small, but it cannot be eliminated entirely. For submicroscopic systems, such as atoms, nuclei, and smaller particles, measurement alters the system in a manner that cannot be made arbitrarily small. This actually limits knowledge of the system—even limiting what nature can know about itself. We shall see profound implications of this when the Heisenberg uncertainty principle is discussed in the modules on quantum mechanics.

There is another measurement technique based on drawing no current at all and, hence, not altering the circuit at all. These are called null measurements and are the topic of **Null Measurements**. Digital meters that employ solid-state electronics and null measurements can attain accuracies of one part in 10^6.

Check Your Understanding

Digital meters are able to detect smaller currents than analog meters employing galvanometers. How does this explain their ability to measure voltage and current more accurately than analog meters?

Solution

Since digital meters require less current than analog meters, they alter the circuit less than analog meters. Their resistance as a voltmeter can be far greater than an analog meter, and their resistance as an ammeter can be far less than an analog meter. Consult **Figure 21.27** and **Figure 21.28** and their discussion in the text.

PhET Explorations: Circuit Construction Kit (DC Only), Virtual Lab

Stimulate a neuron and monitor what happens. Pause, rewind, and move forward in time in order to observe the ions as they move across the neuron membrane.

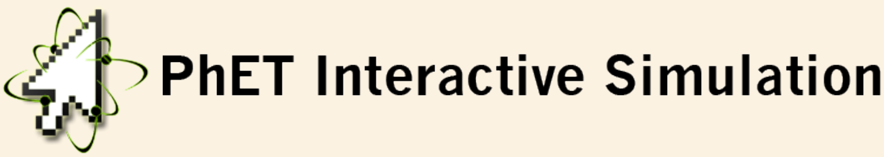


Figure 21.33 Circuit Construction Kit (DC Only), Virtual Lab (http://cnx.org/content/m42360/1.6/circuit-construction-kit-dc-virtual-lab_en.jar)

21.5 Null Measurements

Standard measurements of voltage and current alter the circuit being measured, introducing uncertainties in the measurements. Voltmeters draw some extra current, whereas ammeters reduce current flow. **Null measurements** balance voltages so that there is no current flowing through the measuring device and, therefore, no alteration of the circuit being measured.

Null measurements are generally more accurate but are also more complex than the use of standard voltmeters and ammeters, and they still have limits to their precision. In this module, we shall consider a few specific types of null measurements, because they are common and interesting, and they further illuminate principles of electric circuits.

The Potentiometer

Suppose you wish to measure the emf of a battery. Consider what happens if you connect the battery directly to a standard voltmeter as shown in **Figure 21.34**. (Once we note the problems with this measurement, we will examine a null measurement that improves accuracy.) As discussed before, the actual quantity measured is the terminal voltage V, which is related to the emf of the battery by $V = \text{emf} - Ir$, where I is the current that flows and r is the internal resistance of the battery.

The emf could be accurately calculated if r were very accurately known, but it is usually not. If the current I could be made zero, then $V = \text{emf}$, and so emf could be directly measured. However, standard voltmeters need a current to operate; thus, another technique is needed.

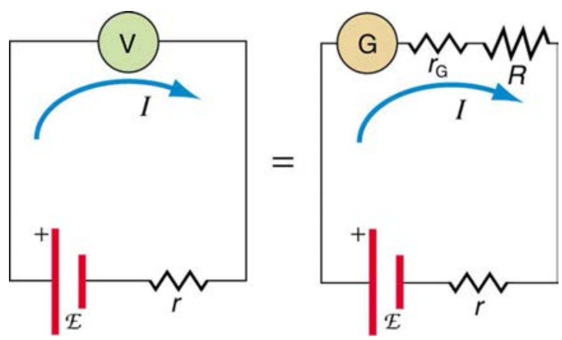


Figure 21.34 An analog voltmeter attached to a battery draws a small but nonzero current and measures a terminal voltage that differs from the emf of the battery. (Note that the script capital E symbolizes electromotive force, or emf.) Since the internal resistance of the battery is not known precisely, it is not possible to calculate the emf precisely.

A **potentiometer** is a null measurement device for measuring potentials (voltages). (See **Figure 21.35**.) A voltage source is connected to a resistor R, say, a long wire, and passes a constant current through it. There is a steady drop in potential (an IR drop) along the wire, so that a variable potential can be obtained by making contact at varying locations along the wire.

Figure 21.35(b) shows an unknown emf_x (represented by script E_x in the figure) connected in series with a galvanometer. Note that emf_x opposes the other voltage source. The location of the contact point (see the arrow on the drawing) is adjusted until the galvanometer reads zero.

When the galvanometer reads zero, $\text{emf}_\text{x} = IR_\text{x}$, where R_x is the resistance of the section of wire up to the contact point. Since no current flows through the galvanometer, none flows through the unknown emf, and so emf_x is directly sensed.

Now, a very precisely known standard emf_s is substituted for emf_x , and the contact point is adjusted until the galvanometer again reads zero, so that $\text{emf}_\text{s} = IR_\text{s}$. In both cases, no current passes through the galvanometer, and so the current I through the long wire is the same. Upon taking the ratio $\frac{\text{emf}_\text{x}}{\text{emf}_\text{s}}$, I cancels, giving

$$\frac{\text{emf}_\text{x}}{\text{emf}_\text{s}} = \frac{IR_\text{x}}{IR_\text{s}} = \frac{R_\text{x}}{R_\text{s}}. \tag{21.71}$$

Solving for emf_x gives

$$\text{emf}_\text{x} = \text{emf}_\text{s}\frac{R_\text{x}}{R_\text{s}}. \tag{21.72}$$

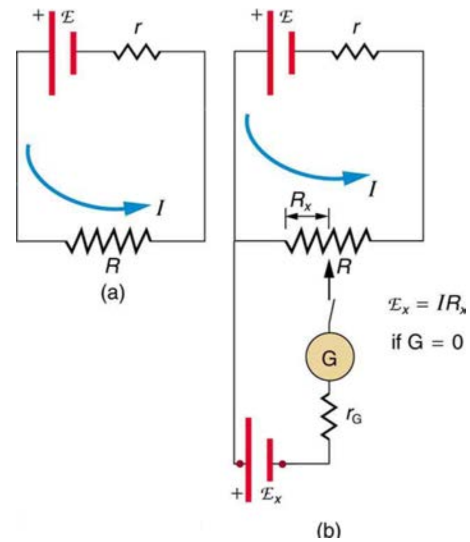


Figure 21.35 The potentiometer, a null measurement device. (a) A voltage source connected to a long wire resistor passes a constant current I through it. (b) An unknown emf (labeled script E_x in the figure) is connected as shown, and the point of contact along R is adjusted until the galvanometer reads zero. The segment of wire has a resistance R_x and script $E_\text{x} = IR_\text{x}$, where I is unaffected by the connection since no current flows through the galvanometer. The unknown emf is thus proportional to the resistance of the wire segment.

Because a long uniform wire is used for R , the ratio of resistances R_x / R_s is the same as the ratio of the lengths of wire that zero the galvanometer for each emf. The three quantities on the right-hand side of the equation are now known or measured, and emf_x can be calculated. The uncertainty in this calculation can be considerably smaller than when using a voltmeter directly, but it is not zero. There is always some uncertainty in the ratio of resistances R_x / R_s and in the standard emf_s . Furthermore, it is not possible to tell when the galvanometer reads exactly zero, which introduces error into both R_x and R_s , and may also affect the current I .

Resistance Measurements and the Wheatstone Bridge

There is a variety of so-called **ohmmeters** that purport to measure resistance. What the most common ohmmeters actually do is to apply a voltage to a resistance, measure the current, and calculate the resistance using Ohm's law. Their readout is this calculated resistance. Two configurations for ohmmeters using standard voltmeters and ammeters are shown in Figure 21.36. Such configurations are limited in accuracy, because the meters alter both the voltage applied to the resistor and the current that flows through it.

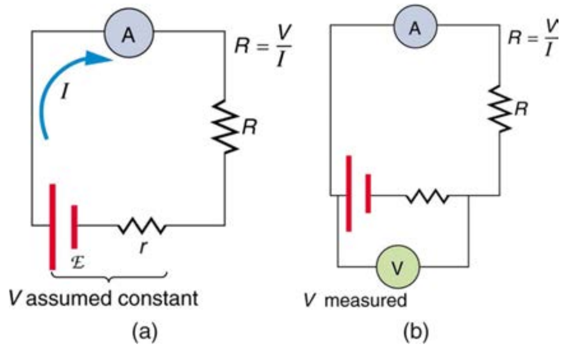


Figure 21.36 Two methods for measuring resistance with standard meters. (a) Assuming a known voltage for the source, an ammeter measures current, and resistance is calculated as $R = \frac{V}{I}$. (b) Since the terminal voltage V varies with current, it is better to measure it. V is most accurately known when I is small, but I itself is most accurately known when it is large.

The **Wheatstone bridge** is a null measurement device for calculating resistance by balancing potential drops in a circuit. (See **Figure 21.37**.) The device is called a bridge because the galvanometer forms a bridge between two branches. A variety of **bridge devices** are used to make null measurements in circuits.

Resistors R_1 and R_2 are precisely known, while the arrow through R_3 indicates that it is a variable resistance. The value of R_3 can be precisely read. With the unknown resistance R_x in the circuit, R_3 is adjusted until the galvanometer reads zero. The potential difference between points b and d is then zero, meaning that b and d are at the same potential. With no current running through the galvanometer, it has no effect on the rest of the circuit. So the branches abc and adc are in parallel, and each branch has the full voltage of the source. That is, the IR drops along abc and adc are the same. Since b and d are at the same potential, the IR drop along ad must equal the IR drop along ab. Thus,

$$I_1 R_1 = I_2 R_3. \tag{21.73}$$

Again, since b and d are at the same potential, the IR drop along dc must equal the IR drop along bc. Thus,

$$I_1 R_2 = I_2 R_x. \tag{21.74}$$

Taking the ratio of these last two expressions gives

$$\frac{I_1 R_1}{I_1 R_2} = \frac{I_2 R_3}{I_2 R_x}. \tag{21.75}$$

Canceling the currents and solving for R_x yields

$$R_x = R_3 \frac{R_2}{R_1}. \tag{21.76}$$

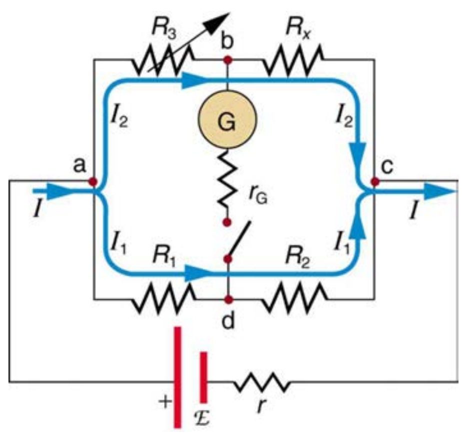


Figure 21.37 The Wheatstone bridge is used to calculate unknown resistances. The variable resistance R_3 is adjusted until the galvanometer reads zero with the switch closed. This simplifies the circuit, allowing R_x to be calculated based on the IR drops as discussed in the text.

This equation is used to calculate the unknown resistance when current through the galvanometer is zero. This method can be very accurate (often to four significant digits), but it is limited by two factors. First, it is not possible to get the current through the galvanometer to be exactly zero. Second, there are always uncertainties in R_1, R_2, and R_3, which contribute to the uncertainty in R_x.

Check Your Understanding

Identify other factors that might limit the accuracy of null measurements. Would the use of a digital device that is more sensitive than a galvanometer improve the accuracy of null measurements?

Solution

One factor would be resistance in the wires and connections in a null measurement. These are impossible to make zero, and they can change over time. Another factor would be temperature variations in resistance, which can be reduced but not completely eliminated by choice of material. Digital devices sensitive to smaller currents than analog devices do improve the accuracy of null measurements because they allow you to get the current closer to zero.

21.6 DC Circuits Containing Resistors and Capacitors

When you use a flash camera, it takes a few seconds to charge the capacitor that powers the flash. The light flash discharges the capacitor in a tiny fraction of a second. Why does charging take longer than discharging? This question and a number of other phenomena that involve charging and discharging capacitors are discussed in this module.

RC Circuits

An $\boldsymbol{RC}$ **circuit** is one containing a **resistor** R and a **capacitor** C. The capacitor is an electrical component that stores electric charge.

Figure 21.38 shows a simple RC circuit that employs a DC (direct current) voltage source. The capacitor is initially uncharged. As soon as the switch is closed, current flows to and from the initially uncharged capacitor. As charge increases on the capacitor plates, there is increasing opposition to the flow of charge by the repulsion of like charges on each plate.

In terms of voltage, this is because voltage across the capacitor is given by $V_\text{c} = Q/C$, where Q is the amount of charge stored on each plate and C is the **capacitance**. This voltage opposes the battery, growing from zero to the maximum emf when fully charged. The current thus decreases from its initial value of $I_0 = \frac{\text{emf}}{R}$ to zero as the voltage on the capacitor reaches the same value as the emf. When there is no current, there is no IR drop, and so the voltage on the capacitor must then equal the emf of the voltage source. This can also be explained with Kirchhoff's second rule (the loop rule), discussed in **Kirchhoff's Rules**, which says that the algebraic sum of changes in potential around any closed loop must be zero.

The initial current is $I_0 = \frac{\text{emf}}{R}$, because all of the IR drop is in the resistance. Therefore, the smaller the resistance, the faster a given capacitor will be charged. Note that the internal resistance of the voltage source is included in R, as are the resistances of the capacitor and the connecting wires. In the flash camera scenario above, when the batteries powering the camera begin to wear out, their internal resistance rises, reducing the current and lengthening the time it takes to get ready for the next flash.

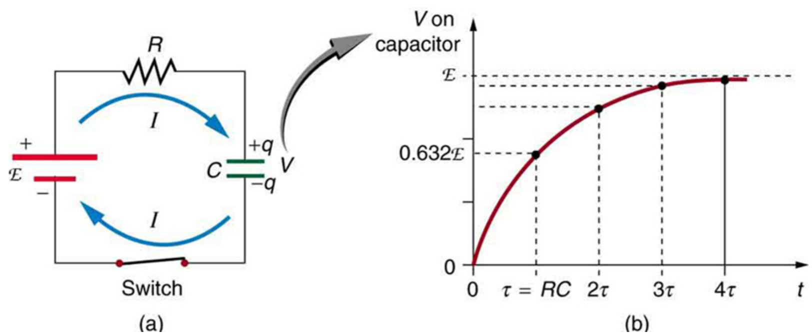


Figure 21.38 (a) An RC circuit with an initially uncharged capacitor. Current flows in the direction shown (opposite of electron flow) as soon as the switch is closed. Mutual repulsion of like charges in the capacitor progressively slows the flow as the capacitor is charged, stopping the current when the capacitor is fully charged and $Q = C \cdot \text{emf}$. (b) A graph of voltage across the capacitor versus time, with the switch closing at time $t = 0$. (Note that in the two parts of the figure, the capital script E stands for emf, q stands for the charge stored on the capacitor, and τ is the RC time constant.)

Voltage on the capacitor is initially zero and rises rapidly at first, since the initial current is a maximum. Figure 21.38(b) shows a graph of capacitor voltage versus time (t) starting when the switch is closed at $t = 0$. The voltage approaches emf asymptotically, since the closer it gets to emf the less current flows. The equation for voltage versus time when charging a capacitor C through a resistor R, derived using calculus, is

$$V = \text{emf}(1 - e^{-t/RC}) \text{ (charging)}, \tag{21.77}$$

where V is the voltage across the capacitor, emf is equal to the emf of the DC voltage source, and the exponential e = 2.718 … is the base of the natural logarithm. Note that the units of RC are seconds. We define

$$\tau = RC, \tag{21.78}$$

where τ (the Greek letter tau) is called the time constant for an RC circuit. As noted before, a small resistance R allows the capacitor to charge faster. This is reasonable, since a larger current flows through a smaller resistance. It is also reasonable that the smaller the capacitor C, the less time needed to charge it. Both factors are contained in $\tau = RC$.

More quantitatively, consider what happens when $t = \tau = RC$. Then the voltage on the capacitor is

$$V = \text{emf}\left(1 - e^{-1}\right) = \text{emf}(1 - 0.368) = 0.632 \cdot \text{emf}. \tag{21.79}$$

This means that in the time $\tau = RC$, the voltage rises to 0.632 of its final value. The voltage will rise 0.632 of the remainder in the next time τ . It is a characteristic of the exponential function that the final value is never reached, but 0.632 of the remainder to that value is achieved in every time, τ . In just a few multiples of the time constant τ , then, the final value is very nearly achieved, as the graph in **Figure 21.38**(b) illustrates.

Discharging a Capacitor

Discharging a capacitor through a resistor proceeds in a similar fashion, as **Figure 21.39** illustrates. Initially, the current is $I_0 = \frac{V_0}{R}$, driven by the initial voltage V_0 on the capacitor. As the voltage decreases, the current and hence the rate of discharge decreases, implying another exponential formula for V . Using calculus, the voltage V on a capacitor C being discharged through a resistor R is found to be

$$V = V\,e^{-t/RC}(\text{discharging}). \tag{21.80}$$

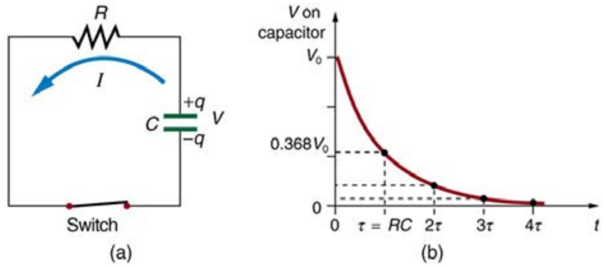


Figure 21.39 (a) Closing the switch discharges the capacitor C through the resistor R . Mutual repulsion of like charges on each plate drives the current. (b) A graph of voltage across the capacitor versus time, with $V = V_0$ at $t = 0$. The voltage decreases exponentially, falling a fixed fraction of the way to zero in each subsequent time constant τ .

The graph in **Figure 21.39**(b) is an example of this exponential decay. Again, the time constant is $\tau = RC$. A small resistance R allows the capacitor to discharge in a small time, since the current is larger. Similarly, a small capacitance requires less time to discharge, since less charge is stored. In the first time interval $\tau = RC$ after the switch is closed, the voltage falls to 0.368 of its initial value, since $V = V_0 \cdot e^{-1} = 0.368V_0$.

During each successive time τ , the voltage falls to 0.368 of its preceding value. In a few multiples of τ , the voltage becomes very close to zero, as indicated by the graph in **Figure 21.39**(b).

Now we can explain why the flash camera in our scenario takes so much longer to charge than discharge; the resistance while charging is significantly greater than while discharging. The internal resistance of the battery accounts for most of the resistance while charging. As the battery ages, the increasing internal resistance makes the charging process even slower. (You may have noticed this.)

The flash discharge is through a low-resistance ionized gas in the flash tube and proceeds very rapidly. Flash photographs, such as in **Figure 21.40**, can capture a brief instant of a rapid motion because the flash can be less than a microsecond in duration. Such flashes can be made extremely intense.

During World War II, nighttime reconnaissance photographs were made from the air with a single flash illuminating more than a square kilometer of enemy territory. The brevity of the flash eliminated blurring due to the surveillance aircraft's motion. Today, an important use of intense flash lamps is to pump energy into a laser. The short intense flash can rapidly energize a laser and allow it to reemit the energy in another form.

Figure 21.40 This stop-motion photograph of a rufous hummingbird (*Selasphorus rufus*) feeding on a flower was obtained with an extremely brief and intense flash of light powered by the discharge of a capacitor through a gas. (credit: Dean E. Biggins, U.S. Fish and Wildlife Service)

Example 21.6 Integrated Concept Problem: Calculating Capacitor Size—Strobe Lights

High-speed flash photography was pioneered by Doc Edgerton in the 1930s, while he was a professor of electrical engineering at MIT. You might have seen examples of his work in the amazing shots of hummingbirds in motion, a drop of milk splattering on a table, or a bullet penetrating an apple (see **Figure 21.40**). To stop the motion and capture these pictures, one needs a high-intensity, very short pulsed flash, as mentioned earlier in this module.

Suppose one wished to capture the picture of a bullet (moving at $5.0\times10^{2}\ \text{m/s}$) that was passing through an apple. The duration of the flash is related to the RC time constant, τ. What size capacitor would one need in the RC circuit to succeed, if the resistance of the flash tube was $10.0\ \Omega$? Assume the apple is a sphere with a diameter of $8.0\times10^{-2}\ \text{m}.$

Strategy

We begin by identifying the physical principles involved. This example deals with the strobe light, as discussed above. Figure 21.39 shows the circuit for this probe. The characteristic time τ of the strobe is given as $\tau = RC$.

Solution

We wish to find C, but we don't know τ. We want the flash to be on only while the bullet traverses the apple. So we need to use the kinematic equations that describe the relationship between distance x, velocity v, and time t:

$$x = vt \text{ or } t = \frac{x}{v}. \tag{21.81}$$

The bullet's velocity is given as $5.0\times10^{2}\ \text{m/s}$, and the distance x is $8.0\times10^{-2}\ \text{m}.$ The traverse time, then, is

$$t = \frac{x}{v} = \frac{8.0\times10^{-2}\ \text{m}}{5.0\times10^{2}\ \text{m/s}} = 1.6\times10^{-4}\ \text{s}. \tag{21.82}$$

We set this value for the crossing time t equal to τ. Therefore,

$$C = \frac{t}{R} = \frac{1.6\times10^{-4}\ \text{s}}{10.0\ \Omega} = 16\ \mu\text{F}. \tag{21.83}$$

(Note: Capacitance C is typically measured in farads, F, defined as Coulombs per volt. From the equation, we see that C can also be stated in units of seconds per ohm.)

Discussion

The flash interval of $160\ \mu\text{s}$ (the traverse time of the bullet) is relatively easy to obtain today. Strobe lights have opened up new worlds from science to entertainment. The information from the picture of the apple and bullet was used in the Warren Commission Report on the assassination of President John F. Kennedy in 1963 to confirm that only one bullet was fired.

RC Circuits for Timing

RC circuits are commonly used for timing purposes. A mundane example of this is found in the ubiquitous intermittent wiper systems of modern cars. The time between wipes is varied by adjusting the resistance in an RC circuit. Another example of an RC circuit is found in novelty jewelry, Halloween costumes, and various toys that have battery-powered flashing lights. (See Figure 21.41 for a timing circuit.)

A more crucial use of RC circuits for timing purposes is in the artificial pacemaker, used to control heart rate. The heart rate is normally controlled by electrical signals generated by the sino-atrial (SA) node, which is on the wall of the right atrium chamber. This causes the muscles to contract and pump blood. Sometimes the heart rhythm is abnormal and the heartbeat is too high or too low.

The artificial pacemaker is inserted near the heart to provide electrical signals to the heart when needed with the appropriate time constant. Pacemakers have sensors that detect body motion and breathing to increase the heart rate during exercise to meet the body's increased needs for blood and oxygen.

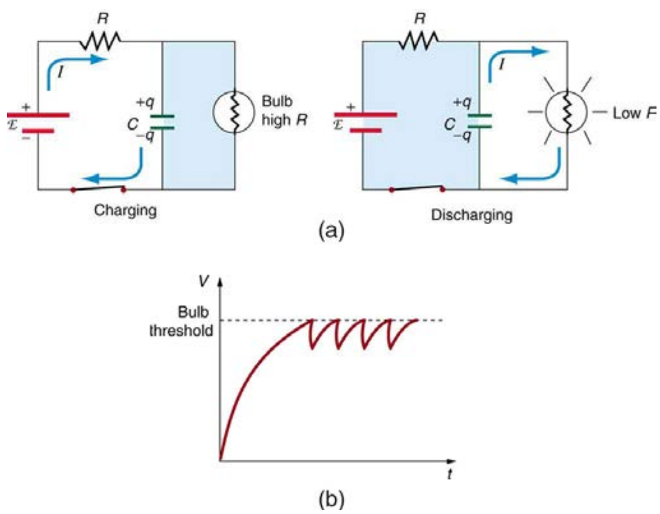


Figure 21.41 (a) The lamp in this RC circuit ordinarily has a very high resistance, so that the battery charges the capacitor as if the lamp were not there. When the voltage reaches a threshold value, a current flows through the lamp that dramatically reduces its resistance, and the capacitor discharges through the lamp as if the battery and charging resistor were not there. Once discharged, the process starts again, with the flash period determined by the RC constant τ. (b) A graph of voltage versus time for this circuit.

Example 21.7 Calculating Time: *RC* Circuit in a Heart Defibrillator

A heart defibrillator is used to resuscitate an accident victim by discharging a capacitor through the trunk of her body. A simplified version of the circuit is seen in **Figure 21.39**. (a) What is the time constant if an $8.00\text{-}\mu\text{F}$ capacitor is used and the path resistance through her body is $1.00\times10^3\ \Omega$? (b) If the initial voltage is 10.0 kV, how long does it take to decline to $5.00\times10^2\ \text{V}$?

Strategy

Since the resistance and capacitance are given, it is straightforward to multiply them to give the time constant asked for in part (a). To find the time for the voltage to decline to $5.00\times10^2\ \text{V}$, we repeatedly multiply the initial voltage by 0.368 until a voltage less than or equal to $5.00\times10^2\ \text{V}$ is obtained. Each multiplication corresponds to a time of τ seconds.

Solution for (a)

The time constant τ is given by the equation $\tau = RC$. Entering the given values for resistance and capacitance (and remembering that units for a farad can be expressed as $\text{s}/\ \Omega$) gives

$$\tau = RC = (1.00\times10^3\ \Omega)(8.00\ \mu F) = 8.00\ \text{ms}. \tag{21.84}$$

Solution for (b)

In the first 8.00 ms, the voltage (10.0 kV) declines to 0.368 of its initial value. That is:

$$V = 0.368V_0 = 3.680\times10^3\ \text{V at}\ t = 8.00\ \text{ms}. \tag{21.85}$$

(Notice that we carry an extra digit for each intermediate calculation.) After another 8.00 ms, we multiply by 0.368 again, and the voltage is

$$\begin{aligned} V' &= 0.368V \\ &= (0.368)\left(3.680\times10^3\ \text{V}\right) \\ &= 1.354\times10^3\ \text{V at}\ t = 16.0\ \text{ms}. \end{aligned} \tag{21.86}$$

Similarly, after another 8.00 ms, the voltage is

$$\begin{aligned} V'' &= 0.368V' = (0.368)(1.354\times10^3\ \text{V}) \\ &= 498\ \text{V at}\ t = 24.0\ \text{ms}. \end{aligned} \tag{21.87}$$

Discussion

So after only 24.0 ms, the voltage is down to 498 V, or 4.98% of its original value.Such brief times are useful in heart defibrillation, because the brief but intense current causes a brief but effective contraction of the heart. The actual circuit in a heart defibrillator is slightly more complex than the one in **Figure 21.39**, to compensate for magnetic and AC effects that will be covered in **Magnetism**.

Check Your Understanding

When is the potential difference across a capacitor an emf?

Solution

Only when the current being drawn from or put into the capacitor is zero. Capacitors, like batteries, have internal resistance, so their output voltage is not an emf unless current is zero. This is difficult to measure in practice so we refer to a capacitor’s voltage rather than its emf. But the source of potential difference in a capacitor is fundamental and it is an emf.

PhET Explorations: Circuit Construction Kit (DC only)

An electronics kit in your computer! Build circuits with resistors, light bulbs, batteries, and switches. Take measurements with the realistic ammeter and voltmeter. View the circuit as a schematic diagram, or switch to a life-like view.

PhET Interactive Simulation

Figure 21.42 Circuit Construction Kit (DC only) (http://cnx.org/content/m42363/1.5/circuit-construction-kit-dc_en.jar)

Glossary

ammeter: an instrument that measures current

analog meter: a measuring instrument that gives a readout in the form of a needle movement over a marked gauge

bridge device: a device that forms a bridge between two branches of a circuit; some bridge devices are used to make null measurements in circuits

capacitance: the maximum amount of electric potential energy that can be stored (or separated) for a given electric potential

capacitor: an electrical component used to store energy by separating electric charge on two opposing plates

conservation laws: require that energy and charge be conserved in a system

current sensitivity: the maximum current that a galvanometer can read

current: the flow of charge through an electric circuit past a given point of measurement

digital meter: a measuring instrument that gives a readout in a digital form

electromotive force (emf): the potential difference of a source of electricity when no current is flowing; measured in volts

full-scale deflection: the maximum deflection of a galvanometer needle, also known as current sensitivity; a galvanometer with a full-scale deflection of $50\ \mu\text{A}$ has a maximum deflection of its needle when $50\ \mu\text{A}$ flows through it

galvanometer: an analog measuring device, denoted by G, that measures current flow using a needle deflection caused by a magnetic field force acting upon a current-carrying wire

internal resistance: the amount of resistance within the voltage source

Joule's law: the relationship between potential electrical power, voltage, and resistance in an electrical circuit, given by: $P_e = IV$

junction rule: Kirchhoff's first rule, which applies the conservation of charge to a junction; current is the flow of charge; thus, whatever charge flows into the junction must flow out; the rule can be stated $I_1 = I_2 + I_3$

Kirchhoff's rules: a set of two rules, based on conservation of charge and energy, governing current and changes in potential in an electric circuit

loop rule: Kirchhoff's second rule, which states that in a closed loop, whatever energy is supplied by emf must be transferred into other forms by devices in the loop, since there are no other ways in which energy can be transferred into or out of the circuit. Thus, the emf equals the sum of the IR (voltage) drops in the loop and can be stated: $\text{emf} = Ir + IR_1 + IR_2$

null measurements: methods of measuring current and voltage more accurately by balancing the circuit so that no current flows through the measurement device

Ohm's law: the relationship between current, voltage, and resistance within an electrical circuit: $V = IR$

ohmmeter: an instrument that applies a voltage to a resistance, measures the current, calculates the resistance using Ohm's law, and provides a readout of this calculated resistance

parallel: the wiring of resistors or other components in an electrical circuit such that each component receives an equal voltage from the power source; often pictured in a ladder-shaped diagram, with each component on a rung of the ladder

potential difference: the difference in electric potential between two points in an electric circuit, measured in volts

potentiometer: a null measurement device for measuring potentials (voltages)

RC circuit: a circuit that contains both a resistor and a capacitor

resistance: causing a loss of electrical power in a circuit

resistor: a component that provides resistance to the current flowing through an electrical circuit

series: a sequence of resistors or other components wired into a circuit one after the other

shunt resistance: a small resistance R placed in parallel with a galvanometer G to produce an ammeter; the larger the current to be measured, the smaller R must be; most of the current flowing through the meter is shunted through R to protect the galvanometer

terminal voltage: the voltage measured across the terminals of a source of potential difference

voltage drop: the loss of electrical power as a current travels through a resistor, wire or other component

voltage: the electrical potential energy per unit charge; electric pressure created by a power source, such as a battery

voltmeter: an instrument that measures voltage

Wheatstone bridge: a null measurement device for calculating resistance by balancing potential drops in a circuit

Section Summary

21.1 Resistors in Series and Parallel

- The total resistance of an electrical circuit with resistors wired in a series is the sum of the individual resistances: $R_s = R_1 + R_2 + R_3 + \ldots$.
- Each resistor in a series circuit has the same amount of current flowing through it.

- The voltage drop, or power dissipation, across each individual resistor in a series is different, and their combined total adds up to the power source input.
- The total resistance of an electrical circuit with resistors wired in parallel is less than the lowest resistance of any of the components and can be determined using the formula:

$$\frac{1}{R_\text{p}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ...$$

- Each resistor in a parallel circuit has the same full voltage of the source applied to it.
- The current flowing through each resistor in a parallel circuit is different, depending on the resistance.
- If a more complex connection of resistors is a combination of series and parallel, it can be reduced to a single equivalent resistance by identifying its various parts as series or parallel, reducing each to its equivalent, and continuing until a single resistance is eventually reached.

21.2 Electromotive Force: Terminal Voltage

- All voltage sources have two fundamental parts—a source of electrical energy that has a characteristic electromotive force (emf), and an internal resistance r .
- The emf is the potential difference of a source when no current is flowing.
- The numerical value of the emf depends on the source of potential difference.
- The internal resistance r of a voltage source affects the output voltage when a current flows.
- The voltage output of a device is called its terminal voltage V and is given by $V = \text{emf} - Ir$, where I is the electric current and is positive when flowing away from the positive terminal of the voltage source.
- When multiple voltage sources are in series, their internal resistances add and their emfs add algebraically.
- Solar cells can be wired in series or parallel to provide increased voltage or current, respectively.

21.3 Kirchhoff's Rules

- Kirchhoff's rules can be used to analyze any circuit, simple or complex.
- Kirchhoff's first rule—the junction rule: The sum of all currents entering a junction must equal the sum of all currents leaving the junction.
- Kirchhoff's second rule—the loop rule: The algebraic sum of changes in potential around any closed circuit path (loop) must be zero.
- The two rules are based, respectively, on the laws of conservation of charge and energy.
- When calculating potential and current using Kirchhoff's rules, a set of conventions must be followed for determining the correct signs of various terms.
- The simpler series and parallel rules are special cases of Kirchhoff's rules.

21.4 DC Voltmeters and Ammeters

- Voltmeters measure voltage, and ammeters measure current.
- A voltmeter is placed in parallel with the voltage source to receive full voltage and must have a large resistance to limit its effect on the circuit.
- An ammeter is placed in series to get the full current flowing through a branch and must have a small resistance to limit its effect on the circuit.
- Both can be based on the combination of a resistor and a galvanometer, a device that gives an analog reading of current.
- Standard voltmeters and ammeters alter the circuit being measured and are thus limited in accuracy.

21.5 Null Measurements

- Null measurement techniques achieve greater accuracy by balancing a circuit so that no current flows through the measuring device.
- One such device, for determining voltage, is a potentiometer.
- Another null measurement device, for determining resistance, is the Wheatstone bridge.
- Other physical quantities can also be measured with null measurement techniques.

21.6 DC Circuits Containing Resistors and Capacitors

- An RC circuit is one that has both a resistor and a capacitor.
- The time constant τ for an RC circuit is $\tau = RC$.
- When an initially uncharged ($V_0 = 0$ at $t = 0$) capacitor in series with a resistor is charged by a DC voltage source, the voltage rises, asymptotically approaching the emf of the voltage source; as a function of time,

$$V = \text{emf}(1 - e^{-t/RC})(\text{charging}).$$

- Within the span of each time constant τ , the voltage rises by 0.632 of the remaining value, approaching the final voltage asymptotically.
- If a capacitor with an initial voltage V_0 is discharged through a resistor starting at $t = 0$, then its voltage decreases exponentially as given by

$$V = V_0 e^{-t/RC}(\text{discharging}).$$

- In each time constant τ , the voltage falls by 0.368 of its remaining initial value, approaching zero asymptotically.

Conceptual Questions

21.1 Resistors in Series and Parallel

1. A switch has a variable resistance that is nearly zero when closed and extremely large when open, and it is placed in series with the device it controls. Explain the effect the switch in **Figure 21.43** has on current when open and when closed.

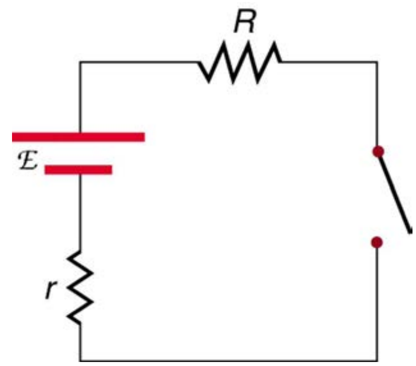


Figure 21.43 A switch is ordinarily in series with a resistance and voltage source. Ideally, the switch has nearly zero resistance when closed but has an extremely large resistance when open. (Note that in this diagram, the script E represents the voltage (or electromotive force) of the battery.)

2. What is the voltage across the open switch in **Figure 21.43**?

3. There is a voltage across an open switch, such as in **Figure 21.43**. Why, then, is the power dissipated by the open switch small?

4. Why is the power dissipated by a closed switch, such as in **Figure 21.43**, small?

5. A student in a physics lab mistakenly wired a light bulb, battery, and switch as shown in **Figure 21.44**. Explain why the bulb is on when the switch is open, and off when the switch is closed. (Do not try this—it is hard on the battery!)

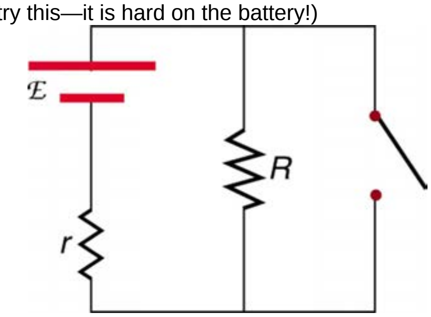


Figure 21.44 A wiring mistake put this switch in parallel with the device represented by R . (Note that in this diagram, the script E represents the voltage (or electromotive force) of the battery.)

6. Knowing that the severity of a shock depends on the magnitude of the current through your body, would you prefer to be in series or parallel with a resistance, such as the heating element of a toaster, if shocked by it? Explain.

7. Would your headlights dim when you start your car's engine if the wires in your automobile were superconductors? (Do not neglect the battery's internal resistance.) Explain.

8. Some strings of holiday lights are wired in series to save wiring costs. An old version utilized bulbs that break the electrical connection, like an open switch, when they burn out. If one such bulb burns out, what happens to the others? If such a string operates on 120 V and has 40 identical bulbs, what is the normal operating voltage of each? Newer versions use bulbs that short circuit, like a closed switch, when they burn out. If one such bulb burns out, what happens to the others? If such a string operates on 120 V and has 39 remaining identical bulbs, what is then the operating voltage of each?

9. If two household lightbulbs rated 60 W and 100 W are connected in series to household power, which will be brighter? Explain.

10. Suppose you are doing a physics lab that asks you to put a resistor into a circuit, but all the resistors supplied have a larger resistance than the requested value. How would you connect the available resistances to attempt to get the smaller value asked for?

11. Before World War II, some radios got power through a "resistance cord" that had a significant resistance. Such a resistance cord reduces the voltage to a desired level for the radio's tubes and the like, and it saves the expense of a transformer. Explain why resistance cords become warm and waste energy when the radio is on.

12. Some light bulbs have three power settings (not including zero), obtained from multiple filaments that are individually switched and wired in parallel. What is the minimum number of filaments needed for three power settings?

21.2 Electromotive Force: Terminal Voltage

13. Is every emf a potential difference? Is every potential difference an emf? Explain.

14. Explain which battery is doing the charging and which is being charged in **Figure 21.45**.

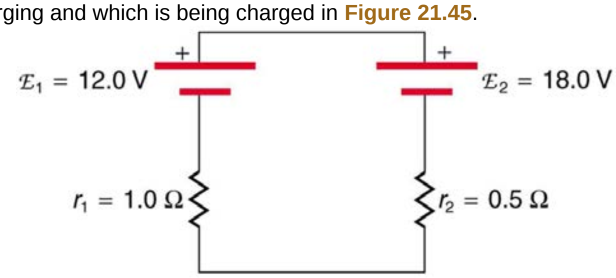


Figure 21.45

15. Given a battery, an assortment of resistors, and a variety of voltage and current measuring devices, describe how you would determine the internal resistance of the battery.

16. Two different 12-V automobile batteries on a store shelf are rated at 600 and 850 "cold cranking amps." Which has the smallest internal resistance?

17. What are the advantages and disadvantages of connecting batteries in series? In parallel?

18. Semitractor trucks use four large 12-V batteries. The starter system requires 24 V, while normal operation of the truck's other electrical components utilizes 12 V. How could the four batteries be connected to produce 24 V? To produce 12 V? Why is 24 V better than 12 V for starting the truck's engine (a very heavy load)?

21.3 Kirchhoff's Rules

19. Can all of the currents going into the junction in **Figure 21.46** be positive? Explain.

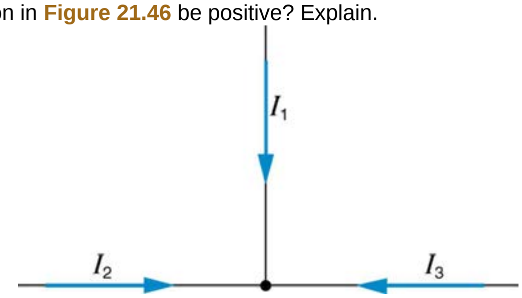


Figure 21.46

20. Apply the junction rule to junction b in **Figure 21.47**. Is any new information gained by applying the junction rule at e? (In the figure, each emf is represented by script E.)

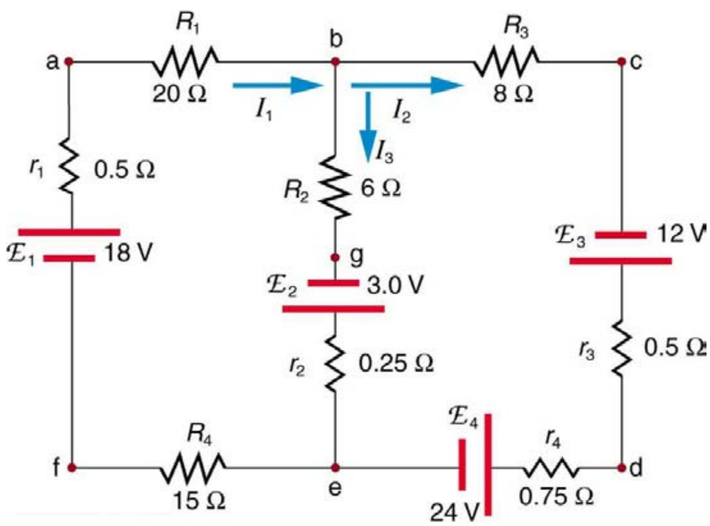


Figure 21.47

21. (a) What is the potential difference going from point a to point b in **Figure 21.47**? (b) What is the potential difference going from c to b? (c) From e to g? (d) From e to d?

22. Apply the loop rule to loop afedcba in **Figure 21.47**.

23. Apply the loop rule to loops abgefa and cbgedc in **Figure 21.47**.

21.4 DC Voltmeters and Ammeters

24. Why should you not connect an ammeter directly across a voltage source as shown in **Figure 21.48**? (Note that script E in the figure stands for emf.)

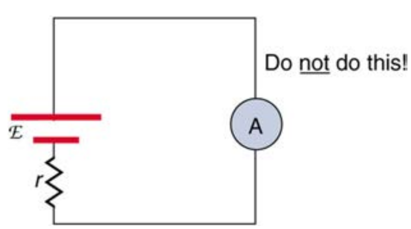


Figure 21.48

25. Suppose you are using a multimeter (one designed to measure a range of voltages, currents, and resistances) to measure current in a circuit and you inadvertently leave it in a voltmeter mode. What effect will the meter have on the circuit? What would happen if you were measuring voltage but accidentally put the meter in the ammeter mode?

26. Specify the points to which you could connect a voltmeter to measure the following potential differences in **Figure 21.49**: (a) the potential difference of the voltage source; (b) the potential difference across R_1 ; (c) across R_2 ; (d) across R_3 ; (e) across R_2 and R_3 . Note that there may be more than one answer to each part.

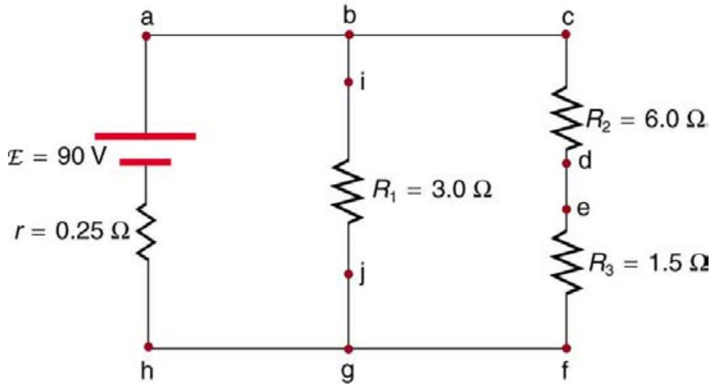


Figure 21.49

27. To measure currents in **Figure 21.49**, you would replace a wire between two points with an ammeter. Specify the points between which you would place an ammeter to measure the following: (a) the total current; (b) the current flowing through R_1 ; (c) through R_2 ; (d) through R_3 . Note that there may be more than one answer to each part.

21.5 Null Measurements

28. Why can a null measurement be more accurate than one using standard voltmeters and ammeters? What factors limit the accuracy of null measurements?

29. If a potentiometer is used to measure cell emfs on the order of a few volts, why is it most accurate for the standard emf_s to be the same order of magnitude and the resistances to be in the range of a few ohms?

21.6 DC Circuits Containing Resistors and Capacitors

30. Regarding the units involved in the relationship $\tau = RC$, verify that the units of resistance times capacitance are time, that is, $\Omega \cdot \text{F} = \text{s}$.

31. The RC time constant in heart defibrillation is crucial to limiting the time the current flows. If the capacitance in the defibrillation unit is fixed, how would you manipulate resistance in the circuit to adjust the RC constant τ ? Would an adjustment of the applied voltage also be needed to ensure that the current delivered has an appropriate value?

32. When making an ECG measurement, it is important to measure voltage variations over small time intervals. The time is limited by the RC constant of the circuit—it is not possible to measure time variations shorter than RC . How would you manipulate R and C in the circuit to allow the necessary measurements?

33. Draw two graphs of charge versus time on a capacitor. Draw one for charging an initially uncharged capacitor in series with a resistor, as in the circuit in **Figure 21.38**, starting from $\text{t} = 0$. Draw the other for discharging a capacitor through a resistor, as in the circuit in **Figure 21.39**, starting at $\text{t} = 0$, with an initial charge Q_0 . Show at least two intervals of τ .

34. When charging a capacitor, as discussed in conjunction with **Figure 21.38**, how long does it take for the voltage on the capacitor to reach emf? Is this a problem?

35. When discharging a capacitor, as discussed in conjunction with **Figure 21.39**, how long does it take for the voltage on the capacitor to reach zero? Is this a problem?

36. Referring to **Figure 21.38**, draw a graph of potential difference across the resistor versus time, showing at least two intervals of τ . Also draw a graph of current versus time for this situation.

37. A long, inexpensive extension cord is connected from inside the house to a refrigerator outside. The refrigerator doesn't run as it should. What might be the problem?

38. In **Figure 21.41**, does the graph indicate the time constant is shorter for discharging than for charging? Would you expect ionized gas to have low resistance? How would you adjust R to get a longer time between flashes? Would adjusting R affect the discharge time?

39. An electronic apparatus may have large capacitors at high voltage in the power supply section, presenting a shock hazard even when the apparatus is switched off. A "bleeder resistor" is therefore placed across such a capacitor, as shown schematically in **Figure 21.50**, to bleed the charge from it after the apparatus is off. Why must the bleeder resistance be much greater than the effective resistance of the rest of the circuit? How does this affect the time constant for discharging the capacitor?

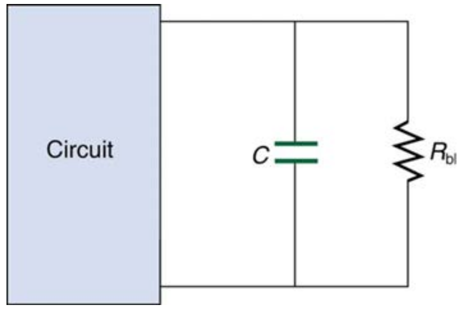


Figure 21.50 A bleeder resistor R_{bl} discharges the capacitor in this electronic device once it is switched off.

Problems & Exercises

21.1 Resistors in Series and Parallel

Note: Data taken from figures can be assumed to be accurate to three significant digits.

40. (a) What is the resistance of ten $275\text{-}\Omega$ resistors connected in series? (b) In parallel?

41. (a) What is the resistance of a $1.00\times10^2\text{-}\Omega$, a $2.50\text{-k}\Omega$, and a $4.00\text{-k }\Omega$ resistor connected in series? (b) In parallel?

42. What are the largest and smallest resistances you can obtain by connecting a $36.0\text{-}\Omega$, a $50.0\text{-}\Omega$, and a $700\text{-}\Omega$ resistor together?

43. An 1800-W toaster, a 1400-W electric frying pan, and a 75-W lamp are plugged into the same outlet in a 15-A, 120-V circuit. (The three devices are in parallel when plugged into the same socket.). (a) What current is drawn by each device? (b) Will this combination blow the 15-A fuse?

44. Your car's 30.0-W headlight and 2.40-kW starter are ordinarily connected in parallel in a 12.0-V system. What power would one headlight and the starter consume if connected in series to a 12.0-V battery? (Neglect any other resistance in the circuit and any change in resistance in the two devices.)

45. (a) Given a 48.0-V battery and $24.0\text{-}\Omega$ and $96.0\text{-}\Omega$ resistors, find the current and power for each when connected in series. (b) Repeat when the resistances are in parallel.

46. Referring to the example combining series and parallel circuits and **Figure 21.6**, calculate I_3 in the following two different ways: (a) from the known values of I and I_2; (b) using Ohm's law for R_3. In both parts explicitly show how you follow the steps in the **Problem-Solving Strategies for Series and Parallel Resistors**.

47. Referring to **Figure 21.6**: (a) Calculate P_3 and note how it compares with P_3 found in the first two example problems in this module. (b) Find the total power supplied by the source and compare it with the sum of the powers dissipated by the resistors.

48. Refer to **Figure 21.7** and the discussion of lights dimming when a heavy appliance comes on. (a) Given the voltage source is 120 V, the wire resistance is $0.400\ \Omega$, and the bulb is nominally 75.0 W, what power will the bulb dissipate if a total of 15.0 A passes through the wires when the motor comes on? Assume negligible change in bulb resistance. (b) What power is consumed by the motor?

49. A 240-kV power transmission line carrying $5.00\times10^2\ \text{A}$ is hung from grounded metal towers by ceramic insulators, each having a $1.00\times10^9\text{-}\Omega$ resistance. **Figure 21.51**. (a) What is the resistance to ground of 100 of these insulators? (b) Calculate the power dissipated by 100 of them. (c) What fraction of the power carried by the line is this? Explicitly show how you follow the steps in the **Problem-Solving Strategies for Series and Parallel Resistors**.

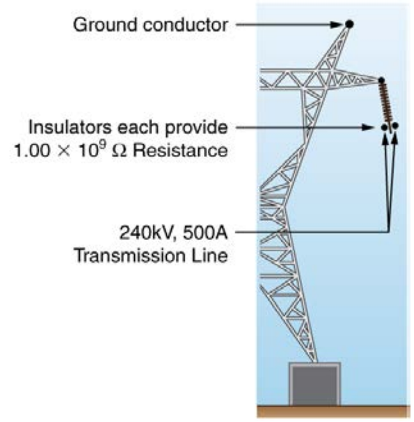


Figure 21.51 High-voltage (240-kV) transmission line carrying $5.00\times10^2\ \text{A}$ is hung from a grounded metal transmission tower. The row of ceramic insulators provide $1.00\times10^9\ \Omega$ of resistance each.

50. Show that if two resistors R_1 and R_2 are combined and one is much greater than the other ($R_1 >> R_2$): (a) Their series resistance is very nearly equal to the greater resistance R_1. (b) Their parallel resistance is very nearly equal to smaller resistance R_2.

51. Unreasonable Results

Two resistors, one having a resistance of $145\ \Omega$, are connected in parallel to produce a total resistance of $150\ \Omega$. (a) What is the value of the second resistance? (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

52. Unreasonable Results

Two resistors, one having a resistance of $900\ \text{k}\Omega$, are connected in series to produce a total resistance of $0.500\ \text{M}\Omega$. (a) What is the value of the second resistance? (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

21.2 Electromotive Force: Terminal Voltage

53. Standard automobile batteries have six lead-acid cells in series, creating a total emf of 12.0 V. What is the emf of an individual lead-acid cell?

54. Carbon-zinc dry cells (sometimes referred to as non-alkaline cells) have an emf of 1.54 V, and they are produced as single cells or in various combinations to form other voltages. (a) How many 1.54-V cells are needed to make the common 9-V battery used in many small electronic devices? (b) What is the actual emf of the approximately 9-V battery? (c) Discuss how internal resistance in the series connection of cells will affect the terminal voltage of this approximately 9-V battery.

55. What is the output voltage of a 3.0000-V lithium cell in a digital wristwatch that draws 0.300 mA, if the cell's internal resistance is $2.00\ \Omega$?

56. (a) What is the terminal voltage of a large 1.54-V carbon-zinc dry cell used in a physics lab to supply 2.00 A to a circuit, if the cell's internal resistance is $0.100\ \Omega$? (b) How much electrical power does the cell produce? (c) What power goes to its load?

57. What is the internal resistance of an automobile battery that has an emf of 12.0 V and a terminal voltage of 15.0 V while a current of 8.00 A is charging it?

58. (a) Find the terminal voltage of a 12.0-V motorcycle battery having a $0.600\text{-}\Omega$ internal resistance, if it is being charged by a current of 10.0 A. (b) What is the output voltage of the battery charger?

59. A car battery with a 12-V emf and an internal resistance of $0.050\ \Omega$ is being charged with a current of 60 A. Note that in this process the battery is being charged. (a) What is the potential difference across its terminals? (b) At what rate is thermal energy being dissipated in the battery? (c) At what rate is electric energy being converted to chemical energy? (d) What are the answers to (a) and (b) when the battery is used to supply 60 A to the starter motor?

60. The hot resistance of a flashlight bulb is $2.30\ \Omega$, and it is run by a 1.58-V alkaline cell having a $0.100\text{-}\Omega$ internal resistance. (a) What current flows? (b) Calculate the power supplied to the bulb using $I^2 R_{\text{bulb}}$. (c) Is this power the same as calculated using $\dfrac{V^2}{R_{\text{bulb}}}$?

61. The label on a portable radio recommends the use of rechargeable nickel-cadmium cells (nicads), although they have a 1.25-V emf while alkaline cells have a 1.58-V emf. The radio has a $3.20\text{-}\Omega$ resistance. (a) Draw a circuit diagram of the radio and its batteries. Now, calculate the power delivered to the radio. (b) When using Nicad cells each having an internal resistance of $0.0400\ \Omega$. (c) When using alkaline cells each having an internal resistance of $0.200\ \Omega$. (d) Does this difference seem significant, considering that the radio's effective resistance is lowered when its volume is turned up?

62. An automobile starter motor has an equivalent resistance of $0.0500\ \Omega$ and is supplied by a 12.0-V battery with a $0.0100\text{-}\Omega$ internal resistance. (a) What is the current to the motor? (b) What voltage is applied to it? (c) What power is supplied to the motor? (d) Repeat these calculations for when the battery connections are corroded and add $0.0900\ \Omega$ to the circuit. (Significant problems are caused by even small amounts of unwanted resistance in low-voltage, high-current applications.)

63. A child's electronic toy is supplied by three 1.58-V alkaline cells having internal resistances of $0.0200\ \Omega$ in series with a 1.53-V carbon-zinc dry cell having a $0.100\text{-}\Omega$ internal resistance. The load resistance is $10.0\ \Omega$. (a) Draw a circuit diagram of the toy and its batteries. (b) What current flows? (c) How much power is supplied to the load? (d) What is the internal resistance of the dry cell if it goes bad, resulting in only 0.500 W being supplied to the load?

64. (a) What is the internal resistance of a voltage source if its terminal voltage drops by 2.00 V when the current supplied increases by 5.00 A? (b) Can the emf of the voltage source be found with the information supplied?

65. A person with body resistance between his hands of $10.0\ \text{k}\ \Omega$ accidentally grasps the terminals of a 20.0-kV power supply. (Do NOT do this!) (a) Draw a circuit diagram to represent the situation. (b) If the internal resistance of the power supply is $2000\ \Omega$, what is the current through his body? (c) What is the power dissipated in his body? (d) If the power supply is to be made safe by increasing its internal resistance, what should the internal resistance be for the maximum current in this situation to be 1.00 mA or less? (e) Will this modification compromise the effectiveness of the power supply for driving low-resistance devices? Explain your reasoning.

66. Electric fish generate current with biological cells called electroplaques, which are physiological emf devices. The electroplaques in the South American eel are arranged in 140 rows, each row stretching horizontally along the body and each containing 5000 electroplaques. Each electroplaque has an emf of 0.15 V and internal resistance of $0.25\ \Omega$. If the water surrounding the fish has resistance of $800\ \Omega$, how much current can the eel produce in water from near its head to near its tail?

67. Integrated Concepts

A 12.0-V emf automobile battery has a terminal voltage of 16.0 V when being charged by a current of 10.0 A. (a) What is the battery's internal resistance? (b) What power is dissipated inside the battery? (c) At what rate (in $^{\circ}\text{C/min}$) will its temperature increase if its mass is 20.0 kg and it has a specific heat of $0.300\ \text{kcal/kg}\cdot{}^{\circ}\text{C}$, assuming no heat escapes?

68. Unreasonable Results

A 1.58-V alkaline cell with a $0.200\text{-}\Omega$ internal resistance is supplying 8.50 A to a load. (a) What is its terminal voltage? (b) What is the value of the load resistance? (c) What is unreasonable about these results? (d) Which assumptions are unreasonable or inconsistent?

69. Unreasonable Results

(a) What is the internal resistance of a 1.54-V dry cell that supplies 1.00 W of power to a $15.0\text{-}\Omega$ bulb? (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

21.3 Kirchhoff's Rules

70. Apply the loop rule to loop abcdefgha in **Figure 21.25**.

71. Apply the loop rule to loop aedcba in **Figure 21.25**.

72. Verify the second equation in **Example 21.5** by substituting the values found for the currents I_1 and I_2 .

73. Verify the third equation in **Example 21.5** by substituting the values found for the currents I_1 and I_3 .

74. Apply the junction rule at point a in **Figure 21.52**.

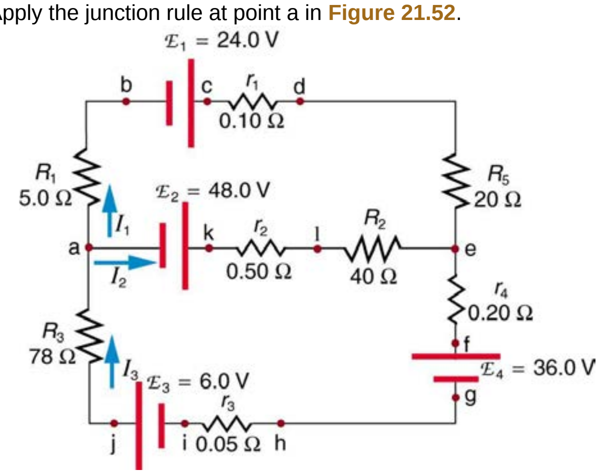


Figure 21.52

75. Apply the loop rule to loop abcdefghija in **Figure 21.52**.

76. Apply the loop rule to loop akledcba in **Figure 21.52**.

77. Find the currents flowing in the circuit in **Figure 21.52**. Explicitly show how you follow the steps in the **Problem-Solving Strategies for Series and Parallel Resistors**.

78. Solve **Example 21.5**, but use loop abcdefgha instead of loop akledcba. Explicitly show how you follow the steps in the **Problem-Solving Strategies for Series and Parallel Resistors**.

79. Find the currents flowing in the circuit in **Figure 21.47**.

80. Unreasonable Results

Consider the circuit in **Figure 21.53**, and suppose that the emfs are unknown and the currents are given to be $I_1 = 5.00\ \text{A}$, $I_2 = 3.0\ \text{A}$, and $I_3 = -2.00\ \text{A}$. (a) Could you find the emfs? (b) What is wrong with the assumptions?

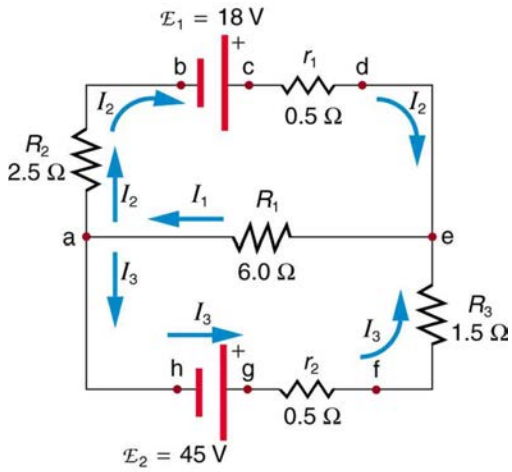


Figure 21.53

21.4 DC Voltmeters and Ammeters

81. What is the sensitivity of the galvanometer (that is, what current gives a full-scale deflection) inside a voltmeter that has a $1.00\text{-M}\ \Omega$ resistance on its 30.0-V scale?

82. What is the sensitivity of the galvanometer (that is, what current gives a full-scale deflection) inside a voltmeter that has a $25.0\text{-k}\ \Omega$ resistance on its 100-V scale?

83. Find the resistance that must be placed in series with a $25.0\text{-}\Omega$ galvanometer having a $50.0\text{-}\mu\text{A}$ sensitivity (the same as the one discussed in the text) to allow it to be used as a voltmeter with a 0.100-V full-scale reading.

84. Find the resistance that must be placed in series with a $25.0\text{-}\Omega$ galvanometer having a $50.0\text{-}\mu\text{A}$ sensitivity (the same as the one discussed in the text) to allow it to be used as a voltmeter with a 3000-V full-scale reading. Include a circuit diagram with your solution.

85. Find the resistance that must be placed in parallel with a $25.0\text{-}\Omega$ galvanometer having a $50.0\text{-}\mu\text{A}$ sensitivity (the same as the one discussed in the text) to allow it to be used as an ammeter with a 10.0-A full-scale reading. Include a circuit diagram with your solution.

86. Find the resistance that must be placed in parallel with a $25.0\text{-}\Omega$ galvanometer having a $50.0\text{-}\mu\text{A}$ sensitivity (the same as the one discussed in the text) to allow it to be used as an ammeter with a 300-mA full-scale reading.

87. Find the resistance that must be placed in series with a $10.0\text{-}\Omega$ galvanometer having a $100\text{-}\mu\text{A}$ sensitivity to allow it to be used as a voltmeter with: (a) a 300-V full-scale reading, and (b) a 0.300-V full-scale reading.

88. Find the resistance that must be placed in parallel with a $10.0\text{-}\Omega$ galvanometer having a $100\text{-}\mu\text{A}$ sensitivity to allow it to be used as an ammeter with: (a) a 20.0-A full-scale reading, and (b) a 100-mA full-scale reading.

89. Suppose you measure the terminal voltage of a 1.585-V alkaline cell having an internal resistance of $0.100\ \Omega$ by placing a $1.00\text{-k}\ \Omega$ voltmeter across its terminals. (See Figure 21.54.) (a) What current flows? (b) Find the terminal voltage. (c) To see how close the measured terminal voltage is to the emf, calculate their ratio.

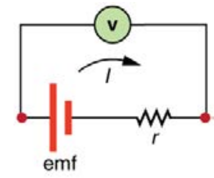


Figure 21.54

90. Suppose you measure the terminal voltage of a 3.200-V lithium cell having an internal resistance of $5.00\ \Omega$ by placing a $1.00\text{-k}\ \Omega$ voltmeter across its terminals. (a) What current flows? (b) Find the terminal voltage. (c) To see how close the measured terminal voltage is to the emf, calculate their ratio.

91. A certain ammeter has a resistance of $5.00\times10^{-5}\ \Omega$ on its 3.00-A scale and contains a $10.0\text{-}\Omega$ galvanometer. What is the sensitivity of the galvanometer?

92. A $1.00\text{-M}\Omega$ voltmeter is placed in parallel with a $75.0\text{-k}\ \Omega$ resistor in a circuit. (a) Draw a circuit diagram of the connection. (b) What is the resistance of the combination? (c) If the voltage across the combination is kept the same as it was across the $75.0\text{-k}\ \Omega$ resistor alone, what is the percent increase in current? (d) If the current through the combination is kept the same as it was through the $75.0\text{-k}\ \Omega$ resistor alone, what is the percentage decrease in voltage? (e) Are the changes found in parts (c) and (d) significant? Discuss.

93. A $0.0200\text{-}\Omega$ ammeter is placed in series with a $10.00\text{-}\Omega$ resistor in a circuit. (a) Draw a circuit diagram of the connection. (b) Calculate the resistance of the combination. (c) If the voltage is kept the same across the combination as it was through the $10.00\text{-}\Omega$ resistor alone, what is the percent decrease in current? (d) If the current is kept the same through the combination as it was through the $10.00\text{-}\Omega$ resistor alone, what is the percent increase in voltage? (e) Are the changes found in parts (c) and (d) significant? Discuss.

94. Unreasonable Results

Suppose you have a $40.0\text{-}\Omega$ galvanometer with a $25.0\text{-}\mu\text{A}$ sensitivity. (a) What resistance would you put in series with it to allow it to be used as a voltmeter that has a full-scale deflection for 0.500 mV? (b) What is unreasonable about this result? (c) Which assumptions are responsible?

95. Unreasonable Results

(a) What resistance would you put in parallel with a $40.0\text{-}\Omega$ galvanometer having a $25.0\text{-}\mu\text{A}$ sensitivity to allow it to be used as an ammeter that has a full-scale deflection for $10.0\text{-}\mu\text{A}$? (b) What is unreasonable about this result? (c) Which assumptions are responsible?

21.5 Null Measurements

96. What is the emf_x of a cell being measured in a potentiometer, if the standard cell's emf is 12.0 V and the potentiometer balances for $R_\text{x} = 5.000\ \Omega$ and $R_\text{s} = 2.500\ \Omega$?

97. Calculate the emf_x of a dry cell for which a potentiometer is balanced when $R_\text{x} = 1.200\ \Omega$, while an alkaline standard cell with an emf of 1.600 V requires $R_\text{s} = 1.247\ \Omega$ to balance the potentiometer.

98. When an unknown resistance R_x is placed in a Wheatstone bridge, it is possible to balance the bridge by adjusting R_3 to be $2500\ \Omega$. What is R_x if $\dfrac{R_2}{R_1} = 0.625$?

99. To what value must you adjust R_3 to balance a Wheatstone bridge, if the unknown resistance R_x is $100\ \Omega$, R_1 is $50.0\ \Omega$, and R_2 is $175\ \Omega$?

100. (a) What is the unknown emf_x in a potentiometer that balances when R_x is $10.0\ \Omega$, and balances when R_s is $15.0\ \Omega$ for a standard 3.000-V emf? (b) The same emf_x is placed in the same potentiometer, which now balances when R_s is $15.0\ \Omega$ for a standard emf of 3.100 V. At what resistance R_x will the potentiometer balance?

101. Suppose you want to measure resistances in the range from $10.0\ \Omega$ to $10.0\ \mathrm{k}\Omega$ using a Wheatstone bridge that has $\frac{R_2}{R_1} = 2.000$. Over what range should R_3 be adjustable?

21.6 DC Circuits Containing Resistors and Capacitors

102. The timing device in an automobile's intermittent wiper system is based on an RC time constant and utilizes a $0.500\text{-}\mu\mathrm{F}$ capacitor and a variable resistor. Over what range must R be made to vary to achieve time constants from 2.00 to 15.0 s?

103. A heart pacemaker fires 72 times a minute, each time a 25.0-nF capacitor is charged (by a battery in series with a resistor) to 0.632 of its full voltage. What is the value of the resistance?

104. The duration of a photographic flash is related to an RC time constant, which is $0.100\ \mu\mathrm{s}$ for a certain camera. (a) If the resistance of the flash lamp is $0.0400\ \Omega$ during discharge, what is the size of the capacitor supplying its energy? (b) What is the time constant for charging the capacitor, if the charging resistance is $800\ \mathrm{k}\Omega$?

105. A 2.00- and a $7.50\text{-}\mu\mathrm{F}$ capacitor can be connected in series or parallel, as can a 25.0- and a $100\text{-k}\Omega$ resistor. Calculate the four RC time constants possible from connecting the resulting capacitance and resistance in series.

106. After two time constants, what percentage of the final voltage, emf, is on an initially uncharged capacitor C, charged through a resistance R?

107. A $500\text{-}\Omega$ resistor, an uncharged $1.50\text{-}\mu\mathrm{F}$ capacitor, and a 6.16-V emf are connected in series. (a) What is the initial current? (b) What is the RC time constant? (c) What is the current after one time constant? (d) What is the voltage on the capacitor after one time constant?

108. A heart defibrillator being used on a patient has an RC time constant of 10.0 ms due to the resistance of the patient and the capacitance of the defibrillator. (a) If the defibrillator has an $8.00\text{-}\mu\mathrm{F}$ capacitance, what is the resistance of the path through the patient? (You may neglect the capacitance of the patient and the resistance of the defibrillator.) (b) If the initial voltage is 12.0 kV, how long does it take to decline to $6.00\times10^2\ \mathrm{V}$?

109. An ECG monitor must have an RC time constant less than $1.00\times10^2\ \mu\mathrm{s}$ to be able to measure variations in voltage over small time intervals. (a) If the resistance of the circuit (due mostly to that of the patient's chest) is $1.00\ \mathrm{k}\Omega$, what is the maximum capacitance of the circuit? (b) Would it be difficult in practice to limit the capacitance to less than the value found in (a)?

110. Figure 21.55 shows how a bleeder resistor is used to discharge a capacitor after an electronic device is shut off, allowing a person to work on the electronics with less risk of shock. (a) What is the time constant? (b) How long will it take to reduce the voltage on the capacitor to 0.250% (5% of 5%) of its full value once discharge begins? (c) If the capacitor is charged to a voltage V_0 through a $100\text{-}\Omega$ resistance, calculate the time it takes to rise to $0.865V_0$ (This is about two time constants.)

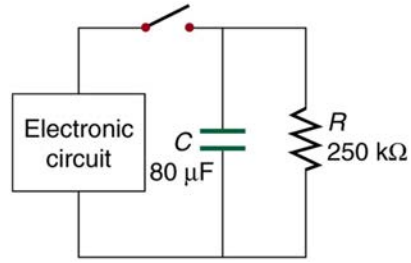


Figure 21.55

111. Using the exact exponential treatment, find how much time is required to discharge a $250\text{-}\mu\mathrm{F}$ capacitor through a $500\text{-}\Omega$ resistor down to 1.00% of its original voltage.

112. Using the exact exponential treatment, find how much time is required to charge an initially uncharged 100-pF capacitor through a $75.0\text{-M}\ \Omega$ resistor to 90.0% of its final voltage.

113. Integrated Concepts

If you wish to take a picture of a bullet traveling at 500 m/s, then a very brief flash of light produced by an RC discharge through a flash tube can limit blurring. Assuming 1.00 mm of motion during one RC constant is acceptable, and given that the flash is driven by a $600\text{-}\mu\mathrm{F}$ capacitor, what is the resistance in the flash tube?

114. Integrated Concepts

A flashing lamp in a Christmas earring is based on an RC discharge of a capacitor through its resistance. The effective duration of the flash is 0.250 s, during which it produces an average 0.500 W from an average 3.00 V. (a) What energy does it dissipate? (b) How much charge moves through the lamp? (c) Find the capacitance. (d) What is the resistance of the lamp?

115. Integrated Concepts

A $160\text{-}\mu\mathrm{F}$ capacitor charged to 450 V is discharged through a $31.2\text{-k}\ \Omega$ resistor. (a) Find the time constant. (b) Calculate the temperature increase of the resistor, given that its mass is 2.50 g and its specific heat is $1.67\frac{\mathrm{kJ}}{\mathrm{kg}\cdot{}^\circ\mathrm{C}}$, noting that most of the thermal energy is retained in the short time of the discharge. (c) Calculate the new resistance, assuming it is pure carbon. (d) Does this change in resistance seem significant?

116. Unreasonable Results

(a) Calculate the capacitance needed to get an RC time constant of $1.00\times10^3\ \mathrm{s}$ with a $0.100\text{-}\Omega$ resistor. (b) What is unreasonable about this result? (c) Which assumptions are responsible?

117. Construct Your Own Problem

Consider a camera's flash unit. Construct a problem in which you calculate the size of the capacitor that stores energy for the flash lamp. Among the things to be considered are the voltage applied to the capacitor, the energy needed in the flash and the associated charge needed on the capacitor, the resistance of the flash lamp during discharge, and the desired RC time constant.

118. Construct Your Own Problem

Consider a rechargeable lithium cell that is to be used to power a camcorder. Construct a problem in which you calculate the internal resistance of the cell during normal operation. Also, calculate the minimum voltage output of a battery charger to be used to recharge your lithium cell. Among the things to be considered are the emf and useful terminal voltage of a lithium cell and the current it should be able to supply to a camcorder.

22 MAGNETISM

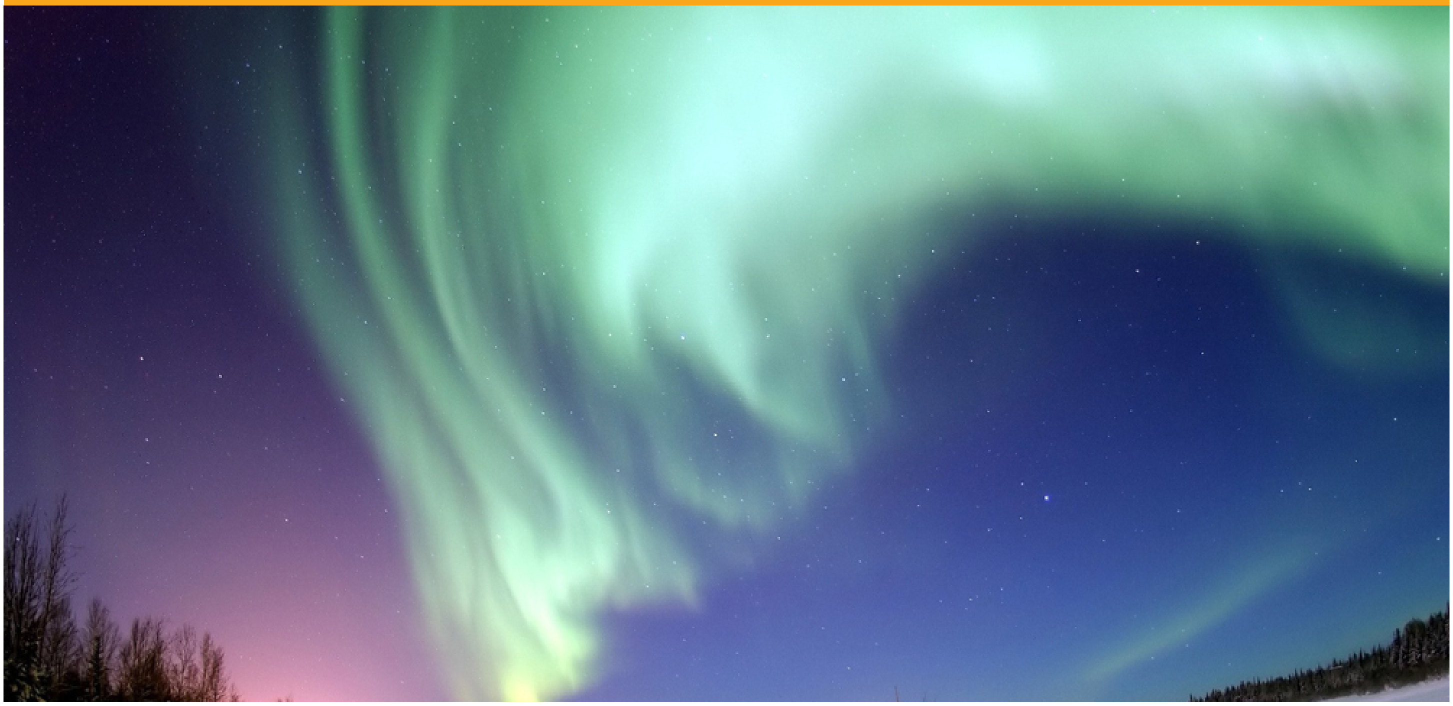


Figure 22.1 The magnificent spectacle of the Aurora Borealis, or northern lights, glows in the northern sky above Bear Lake near Eielson Air Force Base, Alaska. Shaped by the Earth's magnetic field, this light is produced by radiation spewed from solar storms. (credit: Senior Airman Joshua Strang, via Flickr)

Learning Objectives

Introduction to Magnetism

One evening, an Alaskan sticks a note to his refrigerator with a small magnet. Through the kitchen window, the Aurora Borealis glows in the night sky. This grand spectacle is shaped by the same force that holds the note to the refrigerator.

People have been aware of magnets and magnetism for thousands of years. The earliest records date to well before the time of Christ, particularly in a region of Asia Minor called Magnesia (the name of this region is the source of words like *magnetic*). Magnetic rocks found in Magnesia, which is now part of western Turkey, stimulated interest during ancient times. A practical application for magnets was found later, when they were employed as navigational compasses. The use of magnets in compasses resulted not only in improved long-distance sailing, but also in the names of "north" and "south" being given to the two types of magnetic poles.

Today magnetism plays many important roles in our lives. Physicists' understanding of magnetism has enabled the development of technologies that affect our everyday lives. The iPod in your purse or backpack, for example, wouldn't have been possible without the applications of magnetism and electricity on a small scale.

The discovery that weak changes in a magnetic field in a thin film of iron and chromium could bring about much larger changes in electrical resistance was one of the first large successes of nanotechnology. The 2007 Nobel Prize in Physics went to Albert Fert from France and Peter Grunberg from Germany for this discovery of *giant magnetoresistance* and its applications to computer memory.

All electric motors, with uses as diverse as powering refrigerators, starting cars, and moving elevators, contain magnets. Generators, whether producing hydroelectric power or running bicycle lights, use magnetic fields. Recycling facilities employ magnets to separate iron from other refuse. Hundreds of millions of dollars are spent annually on magnetic containment of fusion as a future energy source. Magnetic resonance imaging (MRI) has become an important diagnostic tool in the field of medicine, and the use of magnetism to explore brain activity is a subject of contemporary research and development. The list of applications also includes computer hard drives, tape recording, detection of inhaled asbestos, and levitation of high-speed trains. Magnetism is used to explain atomic energy levels, cosmic rays, and charged particles trapped in the Van Allen belts. Once again, we will find all these disparate phenomena are linked by a small number of underlying physical principles.

Figure 22.2 Engineering of technology like iPods would not be possible without a deep understanding magnetism. (credit: Jesse! S?, Flickr)

22.1 Magnets

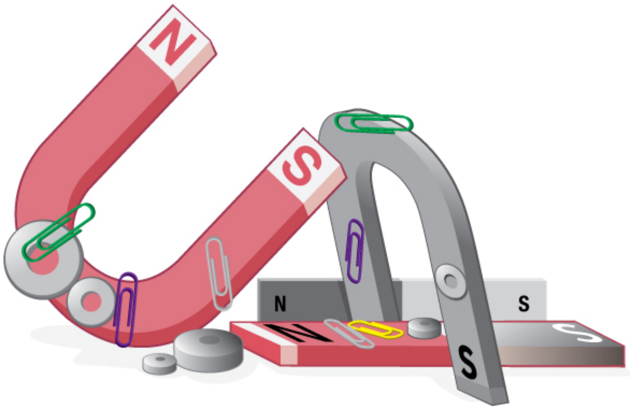


Figure 22.3 Magnets come in various shapes, sizes, and strengths. All have both a north pole and a south pole. There is never an isolated pole (a monopole).

All magnets attract iron, such as that in a refrigerator door. However, magnets may attract or repel other magnets. Experimentation shows that all magnets have two poles. If freely suspended, one pole will point toward the north. The two poles are thus named the **north magnetic pole** and the **south magnetic pole** (or more properly, north-seeking and south-seeking poles, for the attractions in those directions).

Universal Characteristics of Magnets and Magnetic Poles

It is a universal characteristic of all magnets that *like poles repel and unlike poles attract*. (Note the similarity with electrostatics: unlike charges attract and like charges repel.)

Further experimentation shows that it is *impossible to separate north and south poles* in the manner that + and − charges can be separated.

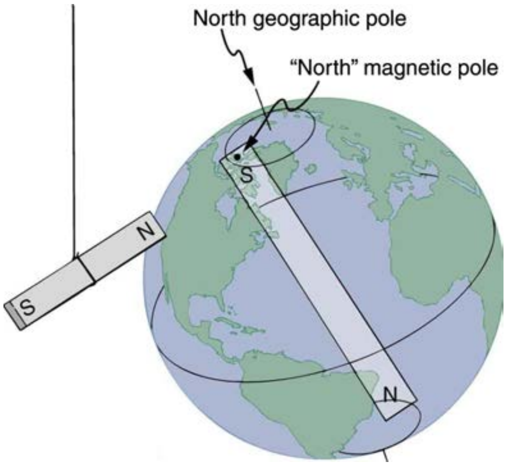


Figure 22.4 One end of a bar magnet is suspended from a thread that points toward north. The magnet's two poles are labeled N and S for north-seeking and south-seeking poles, respectively.

Misconception Alert: Earth's Geographic North Pole Hides an S

The Earth acts like a very large bar magnet with its south-seeking pole near the geographic North Pole. That is why the north pole of your compass is attracted toward the geographic north pole of the Earth—because the magnetic pole that is near the geographic North Pole is actually a south magnetic pole! Confusion arises because the geographic term "North Pole" has come to be used (incorrectly) for the magnetic pole that is near the North Pole. Thus, "North magnetic pole" is actually a misnomer—it should be called the South magnetic pole.

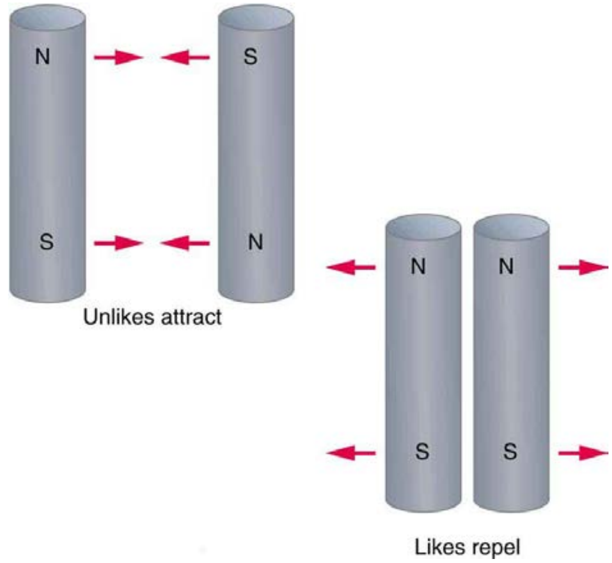


Figure 22.5 Unlike poles attract, whereas like poles repel.

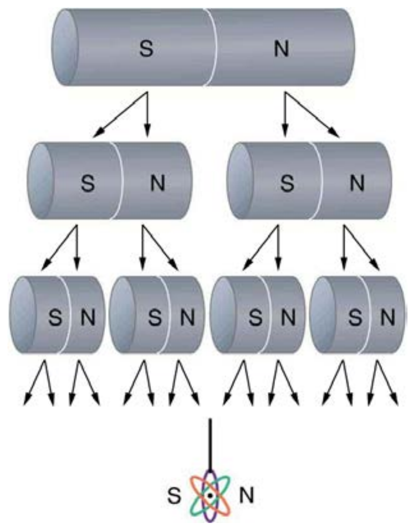


Figure 22.6 North and south poles always occur in pairs. Attempts to separate them result in more pairs of poles. If we continue to split the magnet, we will eventually get down to an iron atom with a north pole and a south pole—these, too, cannot be separated.

The fact that magnetic poles always occur in pairs of north and south is true from the very large scale—for example, sunspots always occur in pairs that are north and south magnetic poles—all the way down to the very small scale. Magnetic atoms have both a north pole and a south pole, as do many types of subatomic particles, such as electrons, protons, and neutrons.

Making Connections: Take-Home Experiment—Refrigerator Magnets

We know that like magnetic poles repel and unlike poles attract. See if you can show this for two refrigerator magnets. Will the magnets stick if you turn them over? Why do they stick to the door anyway? What can you say about the magnetic properties of the door next to the magnet? Do refrigerator magnets stick to metal or plastic spoons? Do they stick to all types of metal?

22.2 Ferromagnets and Electromagnets

Ferromagnets

Only certain materials, such as iron, cobalt, nickel, and gadolinium, exhibit strong magnetic effects. Such materials are called **ferromagnetic**, after the Latin word for iron, *ferrum*. A group of materials made from the alloys of the rare earth elements are also used as strong and permanent magnets; a popular one is neodymium. Other materials exhibit weak magnetic effects, which are detectable only with sensitive instruments. Not only do ferromagnetic materials respond strongly to magnets (the way iron is attracted to magnets), they can also be **magnetized** themselves—that is, they can be induced to be magnetic or made into permanent magnets.

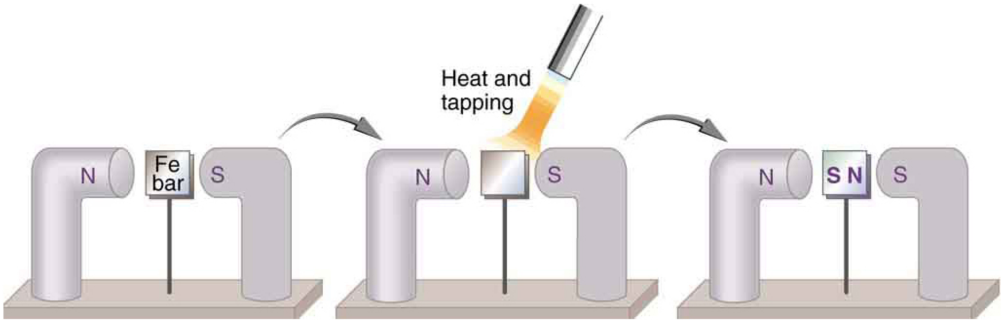


Figure 22.7 An unmagnetized piece of iron is placed between two magnets, heated, and then cooled, or simply tapped when cold. The iron becomes a permanent magnet with the poles aligned as shown: its south pole is adjacent to the north pole of the original magnet, and its north pole is adjacent to the south pole of the original magnet. Note that there are attractive forces between the magnets.

When a magnet is brought near a previously unmagnetized ferromagnetic material, it causes local magnetization of the material with unlike poles closest, as in **Figure 22.7**. (This results in the attraction of the previously unmagnetized material to the magnet.) What happens on a microscopic scale is illustrated in **Figure 22.8**. The regions within the material called **domains** act like small bar magnets. Within domains, the poles of individual atoms are aligned. Each atom acts like a tiny bar magnet. Domains are small and randomly oriented in an unmagnetized ferromagnetic object. In response to an external magnetic field, the domains may grow to millimeter size, aligning themselves as shown in **Figure 22.8**(b). This induced magnetization can be made permanent if the material is heated and then cooled, or simply tapped in the presence of other magnets.

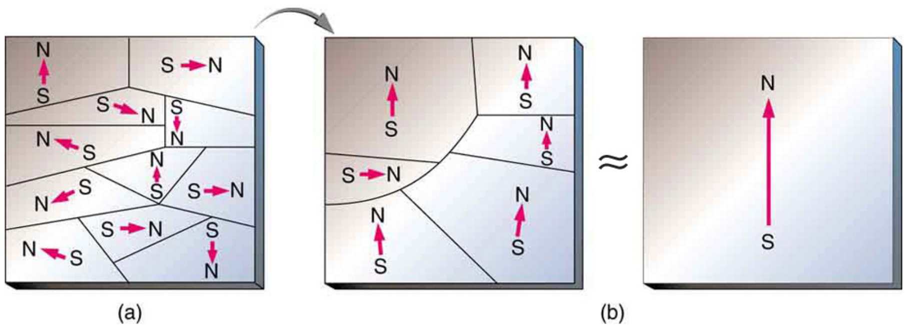


Figure 22.8 (a) An unmagnetized piece of iron (or other ferromagnetic material) has randomly oriented domains. (b) When magnetized by an external field, the domains show greater alignment, and some grow at the expense of others. Individual atoms are aligned within domains; each atom acts like a tiny bar magnet.

Conversely, a permanent magnet can be demagnetized by hard blows or by heating it in the absence of another magnet. Increased thermal motion at higher temperature can disrupt and randomize the orientation and the size of the domains. There is a well-defined temperature for ferromagnetic materials, which is called the **Curie temperature**, above which they cannot be magnetized. The Curie temperature for iron is 1043 K (770°C), which is well above room temperature. There are several elements and alloys that have Curie temperatures much lower than room temperature and are ferromagnetic only below those temperatures.

Electromagnets

Early in the 19th century, it was discovered that electrical currents cause magnetic effects. The first significant observation was by the Danish scientist Hans Christian Oersted (1777–1851), who found that a compass needle was deflected by a current-carrying wire. This was the first significant evidence that the movement of charges had any connection with magnets. **Electromagnetism** is the use of electric current to make magnets. These temporarily induced magnets are called **electromagnets**. Electromagnets are employed for everything from a wrecking yard crane that lifts scrapped cars to controlling the beam of a 90-km-circumference particle accelerator to the magnets in medical imaging machines (See **Figure 22.9**).

Figure 22.9 Instrument for magnetic resonance imaging (MRI). The device uses a superconducting cylindrical coil for the main magnetic field. The patient goes into this “tunnel” on the gurney. (credit: Bill McChesney, Flickr)

Figure 22.10 shows that the response of iron filings to a current-carrying coil and to a permanent bar magnet. The patterns are similar. In fact, electromagnets and ferromagnets have the same basic characteristics—for example, they have north and south poles that cannot be separated and for which like poles repel and unlike poles attract.

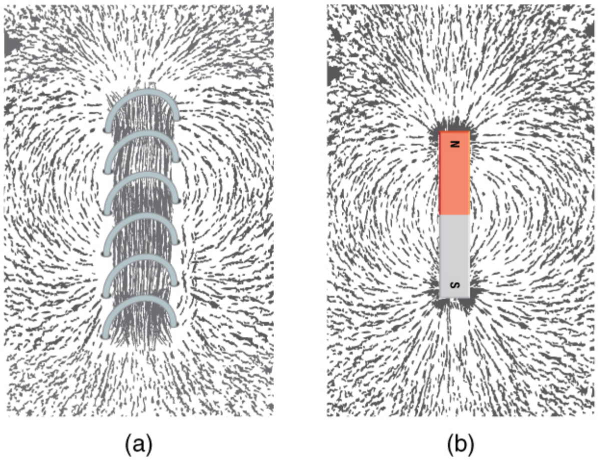


Figure 22.10 Iron filings near (a) a current-carrying coil and (b) a magnet act like tiny compass needles, showing the shape of their fields. Their response to a current-carrying coil and a permanent magnet is seen to be very similar, especially near the ends of the coil and the magnet.

Combining a ferromagnet with an electromagnet can produce particularly strong magnetic effects. (See **Figure 22.11**.) Whenever strong magnetic effects are needed, such as lifting scrap metal, or in particle accelerators, electromagnets are enhanced by ferromagnetic materials. Limits to how strong the magnets can be made are imposed by coil resistance (it will overheat and melt at sufficiently high current), and so superconducting magnets may be employed. These are still limited, because superconducting properties are destroyed by too great a magnetic field.

Figure 22.11 An electromagnet with a ferromagnetic core can produce very strong magnetic effects. Alignment of domains in the core produces a magnet, the poles of which are aligned with the electromagnet.

Figure 22.12 shows a few uses of combinations of electromagnets and ferromagnets. Ferromagnetic materials can act as memory devices, because the orientation of the magnetic fields of small domains can be reversed or erased. Magnetic information storage on videotapes and computer hard drives are among the most common applications. This property is vital in our digital world.

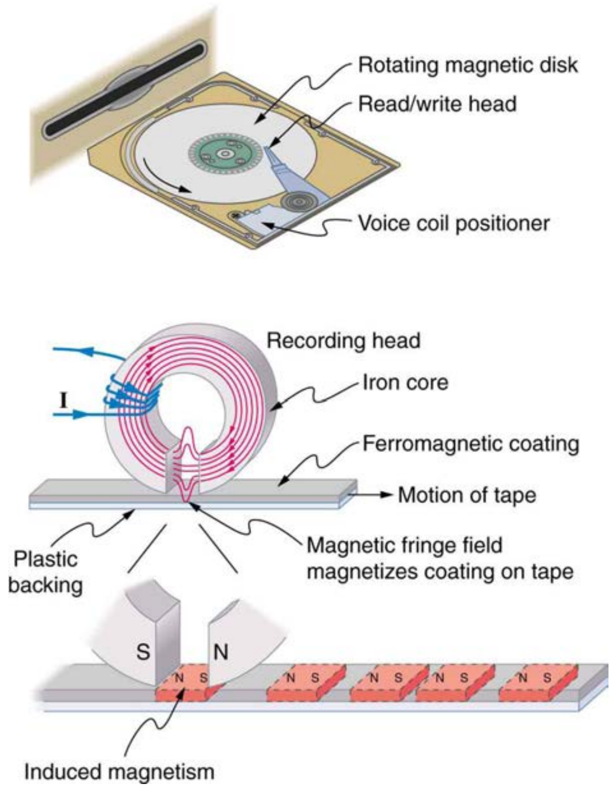


Figure 22.12 An electromagnet induces regions of permanent magnetism on a floppy disk coated with a ferromagnetic material. The information stored here is digital (a region is either magnetic or not); in other applications, it can be analog (with a varying strength), such as on audiotapes.

Current: The Source of All Magnetism

An electromagnet creates magnetism with an electric current. In later sections we explore this more quantitatively, finding the strength and direction of magnetic fields created by various currents. But what about ferromagnets? **Figure 22.13** shows models of how electric currents create magnetism at the submicroscopic level. (Note that we cannot directly observe the paths of individual electrons about atoms, and so a model or visual image, consistent with all direct observations, is made. We can directly observe the electron's orbital angular momentum, its spin momentum, and subsequent magnetic moments, all of which are explained with electric-current-creating subatomic magnetism.) Currents, including those associated with other submicroscopic particles like protons, allow us to explain ferromagnetism and all other magnetic effects. Ferromagnetism, for example, results from an internal cooperative alignment of electron spins, possible in some materials but not in others.

Crucial to the statement that electric current is the source of all magnetism is the fact that it is impossible to separate north and south magnetic poles. (This is far different from the case of positive and negative charges, which are easily separated.) A current loop always produces a magnetic dipole—that is, a magnetic field that acts like a north pole and south pole pair. Since isolated north and south magnetic poles, called **magnetic monopoles**, are not observed, currents are used to explain all magnetic effects. If magnetic monopoles did exist, then we would have to modify this underlying connection that all magnetism is due to electrical current. There is no known reason that magnetic monopoles should not exist—they are simply never observed—and so searches at the subnuclear level continue. If they do *not* exist, we would like to find out why not. If they *do* exist, we would like to see evidence of them.

Electric Currents and Magnetism

Electric current is the source of all magnetism.

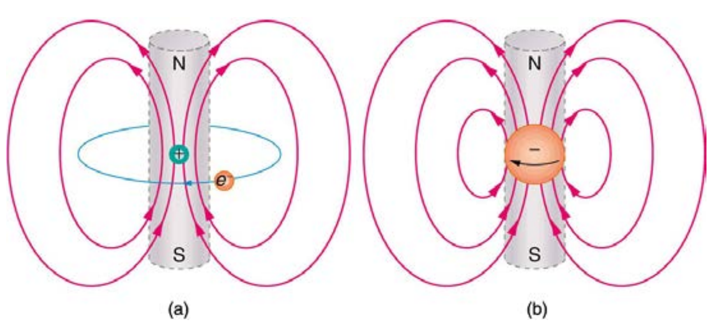


Figure 22.13 (a) In the planetary model of the atom, an electron orbits a nucleus, forming a closed-current loop and producing a magnetic field with a north pole and a south pole. (b) Electrons have spin and can be crudely pictured as rotating charge, forming a current that produces a magnetic field with a north pole and a south pole. Neither the planetary model nor the image of a spinning electron is completely consistent with modern physics. However, they do provide a useful way of understanding phenomena.

PhET Explorations: Magnets and Electromagnets

Explore the interactions between a compass and bar magnet. Discover how you can use a battery and wire to make a magnet! Can you make it a stronger magnet? Can you make the magnetic field reverse?

PhET Interactive Simulation

Figure 22.14 Magnets and Electromagnets (http://cnx.org/content/m42368/1.4/magnets-and-electromagnets_en.jar)

22.3 Magnetic Fields and Magnetic Field Lines

Einstein is said to have been fascinated by a compass as a child, perhaps musing on how the needle felt a force without direct physical contact. His ability to think deeply and clearly about action at a distance, particularly for gravitational, electric, and magnetic forces, later enabled him to create his revolutionary theory of relativity. Since magnetic forces act at a distance, we define a **magnetic field** to represent magnetic forces. The pictorial representation of **magnetic field lines** is very useful in visualizing the strength and direction of the magnetic field. As shown in **Figure 22.15**, the **direction of magnetic field lines** is defined to be the direction in which the north end of a compass needle points. The magnetic field is traditionally called the ***B*-field**.

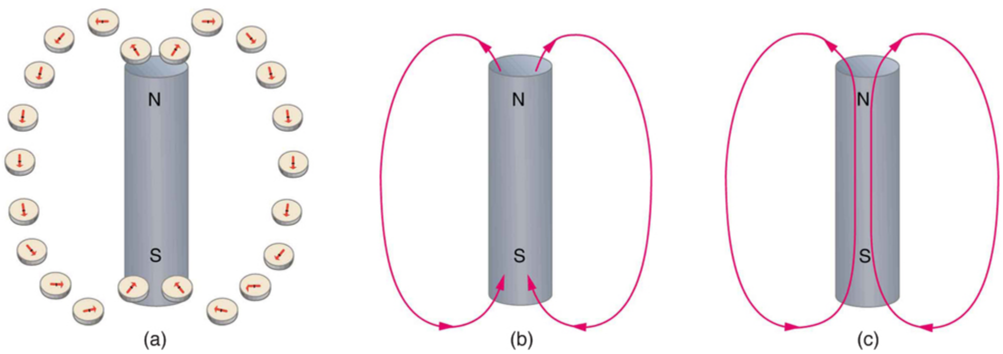


Figure 22.15 Magnetic field lines are defined to have the direction that a small compass points when placed at a location. (a) If small compasses are used to map the magnetic field around a bar magnet, they will point in the directions shown: away from the north pole of the magnet, toward the south pole of the magnet. (Recall that the Earth's north magnetic pole is really a south pole in terms of definitions of poles on a bar magnet.) (b) Connecting the arrows gives continuous magnetic field lines. The strength of the field is proportional to the closeness (or density) of the lines. (c) If the interior of the magnet could be probed, the field lines would be found to form continuous closed loops.

Small compasses used to test a magnetic field will not disturb it. (This is analogous to the way we tested electric fields with a small test charge. In both cases, the fields represent only the object creating them and not the probe testing them.) **Figure 22.16** shows how the magnetic field appears for a current loop and a long straight wire, as could be explored with small compasses. A small compass placed in these fields will align itself parallel to the field line at its location, with its north pole pointing in the direction of *B*. Note the symbols used for field into and out of the paper.

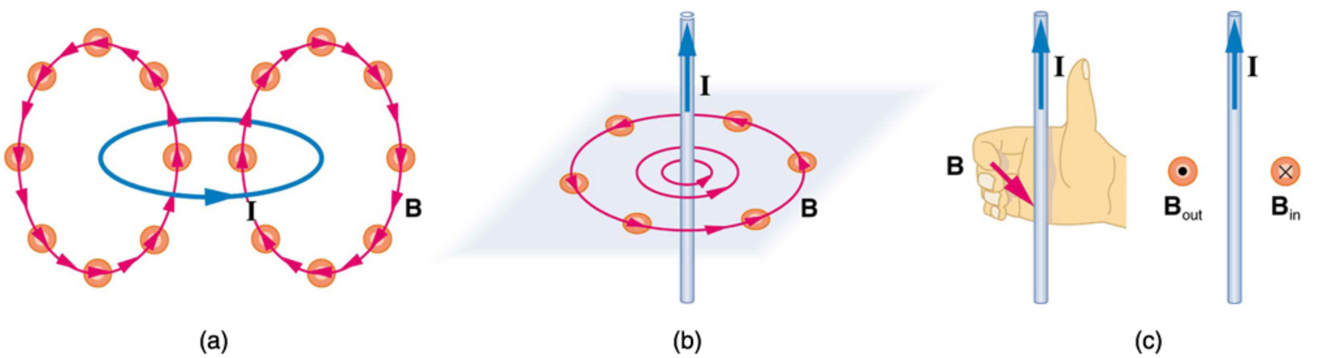


Figure 22.16 Small compasses could be used to map the fields shown here. (a) The magnetic field of a circular current loop is similar to that of a bar magnet. (b) A long and straight wire creates a field with magnetic field lines forming circular loops. (c) When the wire is in the plane of the paper, the field is perpendicular to the paper. Note that the symbols used for the field pointing inward (like the tail of an arrow) and the field pointing outward (like the tip of an arrow).

Making Connections: Concept of a Field

A field is a way of mapping forces surrounding any object that can act on another object at a distance without apparent physical connection. The field represents the object generating it. Gravitational fields map gravitational forces, electric fields map electrical forces, and magnetic fields map magnetic forces.

Extensive exploration of magnetic fields has revealed a number of hard-and-fast rules. We use magnetic field lines to represent the field (the lines are a pictorial tool, not a physical entity in and of themselves). The properties of magnetic field lines can be summarized by these rules:

1. The direction of the magnetic field is tangent to the field line at any point in space. A small compass will point in the direction of the field line.

2. The strength of the field is proportional to the closeness of the lines. It is exactly proportional to the number of lines per unit area perpendicular to the lines (called the areal density).
3. Magnetic field lines can never cross, meaning that the field is unique at any point in space.
4. Magnetic field lines are continuous, forming closed loops without beginning or end. They go from the north pole to the south pole.

The last property is related to the fact that the north and south poles cannot be separated. It is a distinct difference from electric field lines, which begin and end on the positive and negative charges. If magnetic monopoles existed, then magnetic field lines would begin and end on them.

22.4 Magnetic Field Strength: Force on a Moving Charge in a Magnetic Field

What is the mechanism by which one magnet exerts a force on another? The answer is related to the fact that all magnetism is caused by current, the flow of charge. *Magnetic fields exert forces on moving charges*, and so they exert forces on other magnets, all of which have moving charges.

Right Hand Rule 1

The magnetic force on a moving charge is one of the most fundamental known. Magnetic force is as important as the electrostatic or Coulomb force. Yet the magnetic force is more complex, in both the number of factors that affects it and in its direction, than the relatively simple Coulomb force. The magnitude of the **magnetic force** F on a charge q moving at a speed v in a magnetic field of strength B is given by

$$F = qvB\sin\theta, \tag{22.1}$$

where θ is the angle between the directions of $\mathbf{v}$ and $\mathbf{B}$. This force is often called the **Lorentz force**. In fact, this is how we define the magnetic field strength B—in terms of the force on a charged particle moving in a magnetic field. The SI unit for magnetic field strength B is called the **tesla** (T) after the eccentric but brilliant inventor Nikola Tesla (1856–1943). To determine how the tesla relates to other SI units, we solve $F = qvB\sin\theta$ for B.

$$B = \frac{F}{qv\sin\theta} \tag{22.2}$$

Because $\sin\theta$ is unitless, the tesla is

$$1\text{ T} = \frac{1\text{ N}}{\text{C}\cdot\text{m/s}} = \frac{1\text{ N}}{\text{A}\cdot\text{m}} \tag{22.3}$$

(note that C/s = A).

Another smaller unit, called the **gauss** (G), where $1\text{ G} = 10^{-4}\text{ T}$, is sometimes used. The strongest permanent magnets have fields near 2 T; superconducting electromagnets may attain 10 T or more. The Earth's magnetic field on its surface is only about $5\times10^{-5}\text{ T}$, or 0.5 G.

The *direction* of the magnetic force $\mathbf{F}$ is perpendicular to the plane formed by $\mathbf{v}$ and $\mathbf{B}$, as determined by the **right hand rule 1** (or RHR-1), which is illustrated in **Figure 22.17**. RHR-1 states that, to determine the direction of the magnetic force on a positive moving charge, you point the thumb of the right hand in the direction of $\mathbf{v}$, the fingers in the direction of $\mathbf{B}$, and a perpendicular to the palm points in the direction of $\mathbf{F}$. One way to remember this is that there is one velocity, and so the thumb represents it. There are many field lines, and so the fingers represent them. The force is in the direction you would push with your palm. The force on a negative charge is in exactly the opposite direction to that on a positive charge.

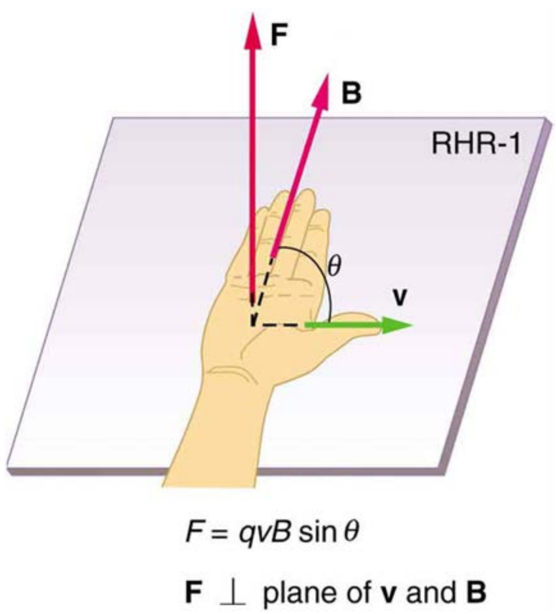


Figure 22.17 Magnetic fields exert forces on moving charges. This force is one of the most basic known. The direction of the magnetic force on a moving charge is perpendicular to the plane formed by $\mathbf{v}$ and $\mathbf{B}$ and follows right hand rule–1 (RHR-1) as shown. The magnitude of the force is proportional to q, v, B, and the sine of the angle between $\mathbf{v}$ and $\mathbf{B}$.

Making Connections: Charges and Magnets

There is no magnetic force on static charges. However, there is a magnetic force on moving charges. When charges are stationary, their electric fields do not affect magnets. But, when charges move, they produce magnetic fields that exert forces on other magnets. When there is relative motion, a connection between electric and magnetic fields emerges—each affects the other.

Example 22.1 Calculating Magnetic Force: Earth’s Magnetic Field on a Charged Glass Rod

With the exception of compasses, you seldom see or personally experience forces due to the Earth’s small magnetic field. To illustrate this, suppose that in a physics lab you rub a glass rod with silk, placing a 20-nC positive charge on it. Calculate the force on the rod due to the Earth’s magnetic field, if you throw it with a horizontal velocity of 10 m/s due west in a place where the Earth’s field is due north parallel to the ground. (The direction of the force is determined with right hand rule 1 as shown in **Figure 22.18**.)

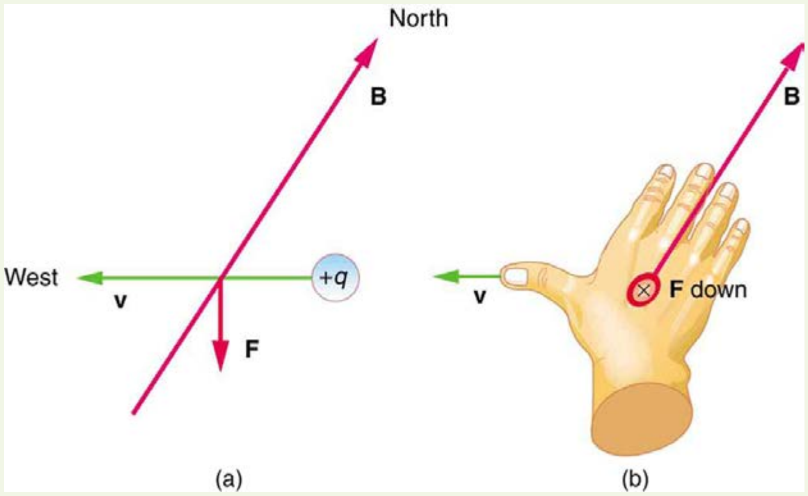


Figure 22.18 A positively charged object moving due west in a region where the Earth’s magnetic field is due north experiences a force that is straight down as shown. A negative charge moving in the same direction would feel a force straight up.

Strategy

We are given the charge, its velocity, and the magnetic field strength and direction. We can thus use the equation $F = qvB \sin \theta$ to find the force.

Solution

The magnetic force is

$$F = qvb \sin \theta. \tag{22.4}$$

We see that $\sin \theta = 1$, since the angle between the velocity and the direction of the field is 90° . Entering the other given quantities yields

$$\begin{aligned} F &= \left(20\times10^{-9}\ \text{C}\right)(10\ \text{m/s})\left(5\times10^{-5}\ \text{T}\right) \\ &= 1\times10^{-11}\ (\text{C}\cdot\text{m/s})\left(\frac{\text{N}}{\text{C}\cdot\text{m/s}}\right) = 1\times10^{-11}\ \text{N}. \end{aligned} \tag{22.5}$$

Discussion

This force is completely negligible on any macroscopic object, consistent with experience. (It is calculated to only one digit, since the Earth’s field varies with location and is given to only one digit.) The Earth’s magnetic field, however, does produce very important effects, particularly on submicroscopic particles. Some of these are explored in **Force on a Moving Charge in a Magnetic Field: Examples and Applications**.

22.5 Force on a Moving Charge in a Magnetic Field: Examples and Applications

Magnetic force can cause a charged particle to move in a circular or spiral path. Cosmic rays are energetic charged particles in outer space, some of which approach the Earth. They can be forced into spiral paths by the Earth’s magnetic field. Protons in giant accelerators are kept in a circular path by magnetic force. The bubble chamber photograph in **Figure 22.19** shows charged particles moving in such curved paths. The curved paths of charged particles in magnetic fields are the basis of a number of phenomena and can even be used analytically, such as in a mass spectrometer.

Figure 22.19 Trails of bubbles are produced by high-energy charged particles moving through the superheated liquid hydrogen in this artist's rendition of a bubble chamber. There is a strong magnetic field perpendicular to the page that causes the curved paths of the particles. The radius of the path can be used to find the mass, charge, and energy of the particle.

So does the magnetic force cause circular motion? Magnetic force is always perpendicular to velocity, so that it does no work on the charged particle. The particle's kinetic energy and speed thus remain constant. The direction of motion is affected, but not the speed. This is typical of uniform circular motion. The simplest case occurs when a charged particle moves perpendicular to a uniform B -field, such as shown in **Figure 22.20**. (If this takes place in a vacuum, the magnetic field is the dominant factor determining the motion.) Here, the magnetic force supplies the centripetal force $F_c = mv^2 / r$. Noting that $\sin \theta = 1$, we see that $F = qvB$.

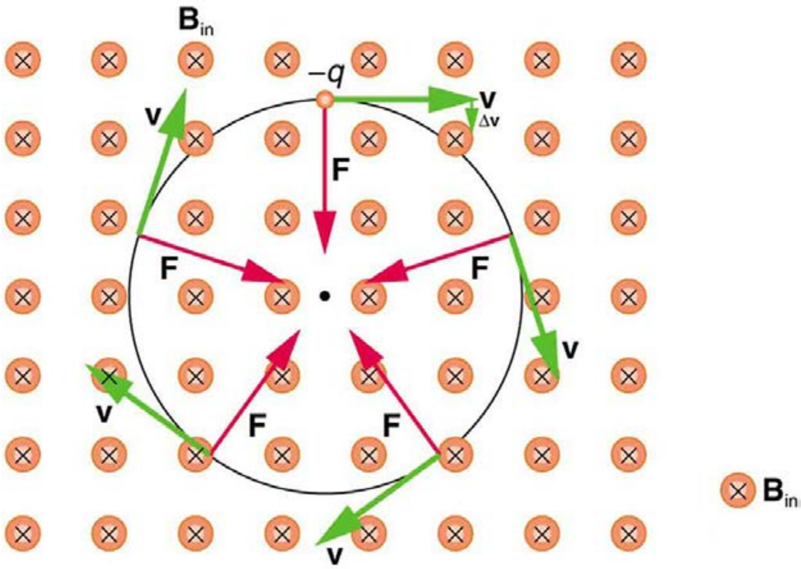


Figure 22.20 A negatively charged particle moves in the plane of the page in a region where the magnetic field is perpendicular into the page (represented by the small circles with x's—like the tails of arrows). The magnetic force is perpendicular to the velocity, and so velocity changes in direction but not magnitude. Uniform circular motion results.

Because the magnetic force F supplies the centripetal force F_c , we have

$$qvB = \frac{mv^2}{r}. \tag{22.6}$$

Solving for r yields

$$r = \frac{mv}{qB}. \tag{22.7}$$

Here, r is the radius of curvature of the path of a charged particle with mass m and charge q , moving at a speed v perpendicular to a magnetic field of strength B . If the velocity is not perpendicular to the magnetic field, then v is the component of the velocity perpendicular to the field. The component of the velocity parallel to the field is unaffected, since the magnetic force is zero for motion parallel to the field. This produces a spiral motion rather than a circular one.

Example 22.2 Calculating the Curvature of the Path of an Electron Moving in a Magnetic Field: A Magnet on a TV Screen

A magnet brought near an old-fashioned TV screen such as in Figure 22.21 (TV sets with cathode ray tubes instead of LCD screens) severely distorts its picture by altering the path of the electrons that make its phosphors glow. ***(Don't try this at home, as it will permanently magnetize and ruin the TV.)*** To illustrate this, calculate the radius of curvature of the path of an electron having a velocity of 6.00×10^7 m/s (corresponding to the accelerating voltage of about 10.0 kV used in some TVs) perpendicular to a magnetic field of strength $B = 0.500$ T (obtainable with permanent magnets).

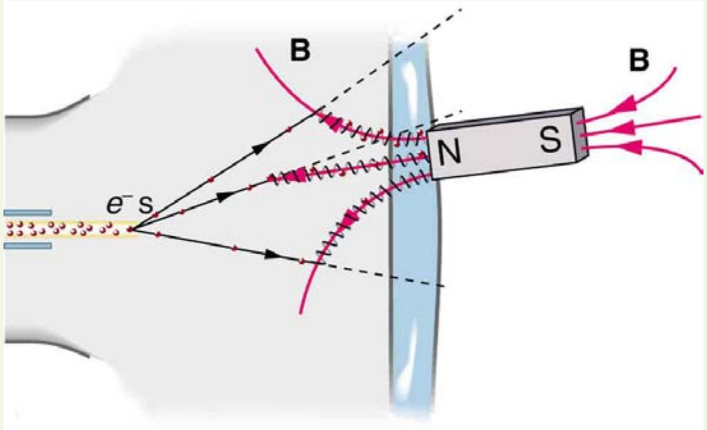


Figure 22.21 Side view showing what happens when a magnet comes in contact with a computer monitor or TV screen. Electrons moving toward the screen spiral about magnetic field lines, maintaining the component of their velocity parallel to the field lines. This distorts the image on the screen.

Strategy

We can find the radius of curvature r directly from the equation $r = \frac{mv}{qB}$, since all other quantities in it are given or known.

Solution

Using known values for the mass and charge of an electron, along with the given values of v and B gives us

$$\begin{aligned} r = \frac{mv}{qB} &= \frac{\left(9.11\times10^{-31}\text{ kg}\right)\left(6.00\times10^{7}\text{ m/s}\right)}{\left(1.60\times10^{-19}\text{ C}\right)(0.500\text{ T})} \\ &= 6.83\times10^{-4}\text{ m} \end{aligned} \tag{22.8}$$

or

$$r = 0.683\text{ mm}. \tag{22.9}$$

Discussion

The small radius indicates a large effect. The electrons in the TV picture tube are made to move in very tight circles, greatly altering their paths and distorting the image.

Figure 22.22 shows how electrons not moving perpendicular to magnetic field lines follow the field lines. The component of velocity parallel to the lines is unaffected, and so the charges spiral along the field lines. If field strength increases in the direction of motion, the field will exert a force to slow the charges, forming a kind of magnetic mirror, as shown below.

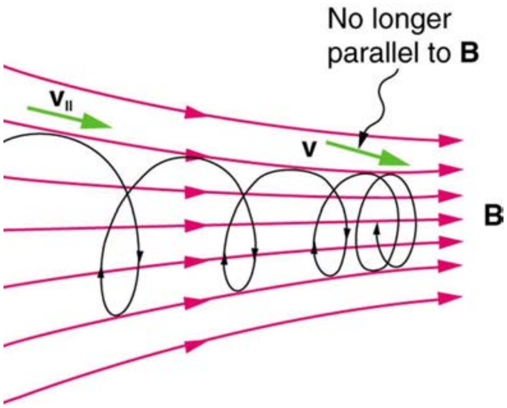


Figure 22.22 When a charged particle moves along a magnetic field line into a region where the field becomes stronger, the particle experiences a force that reduces the component of velocity parallel to the field. This force slows the motion along the field line and here reverses it, forming a "magnetic mirror."

The properties of charged particles in magnetic fields are related to such different things as the Aurora Australis or Aurora Borealis and particle accelerators. *Charged particles approaching magnetic field lines may get trapped in spiral orbits about the lines rather than crossing them*, as seen above. Some cosmic rays, for example, follow the Earth's magnetic field lines, entering the atmosphere near the magnetic poles and causing the southern or northern lights through their ionization of molecules in the atmosphere. This glow of energized atoms and molecules is seen in **Figure 22.1**. Those particles that approach middle latitudes must cross magnetic field lines, and many are prevented from penetrating the atmosphere. Cosmic rays are a component of background radiation; consequently, they give a higher radiation dose at the poles than at the equator.

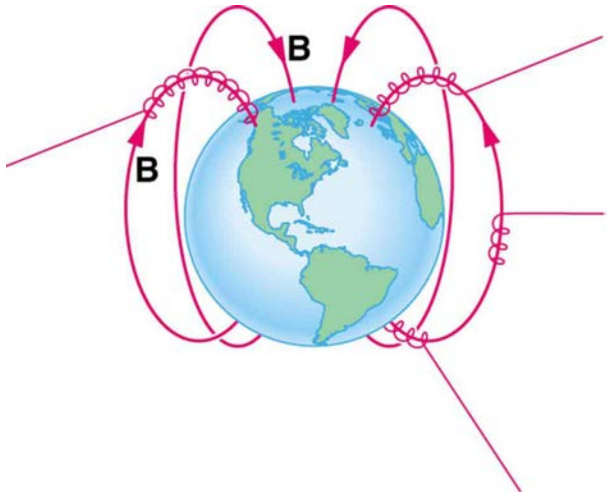


Figure 22.23 Energetic electrons and protons, components of cosmic rays, from the Sun and deep outer space often follow the Earth's magnetic field lines rather than cross them. (Recall that the Earth's north magnetic pole is really a south pole in terms of a bar magnet.)

Some incoming charged particles become trapped in the Earth's magnetic field, forming two belts above the atmosphere known as the Van Allen radiation belts after the discoverer James A. Van Allen, an American astrophysicist. (See **Figure 22.24**.) Particles trapped in these belts form radiation fields (similar to nuclear radiation) so intense that manned space flights avoid them and satellites with sensitive electronics are kept out of them. In the few minutes it took lunar missions to cross the Van Allen radiation belts, astronauts received radiation doses more than twice the allowed annual exposure for radiation workers. Other planets have similar belts, especially those having strong magnetic fields like Jupiter.

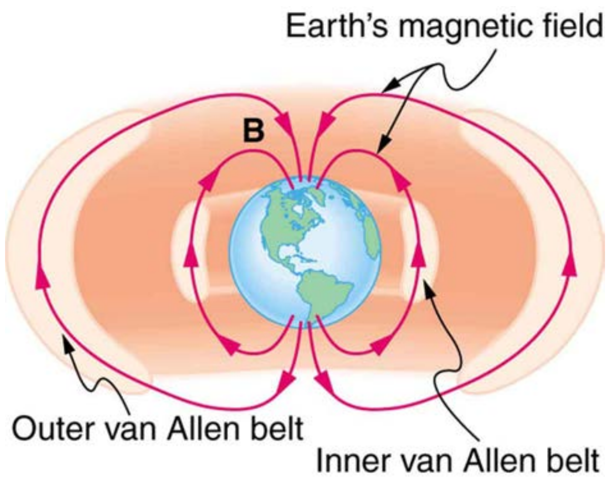


Figure 22.24 The Van Allen radiation belts are two regions in which energetic charged particles are trapped in the Earth's magnetic field. One belt lies about 300 km above the Earth's surface, the other about 16,000 km. Charged particles in these belts migrate along magnetic field lines and are partially reflected away from the poles by the stronger fields there. The charged particles that enter the atmosphere are replenished by the Sun and sources in deep outer space.

Back on Earth, we have devices that employ magnetic fields to contain charged particles. Among them are the giant particle accelerators that have been used to explore the substructure of matter. (See **Figure 22.25**.) Magnetic fields not only control the direction of the charged particles, they also are used to focus particles into beams and overcome the repulsion of like charges in these beams.

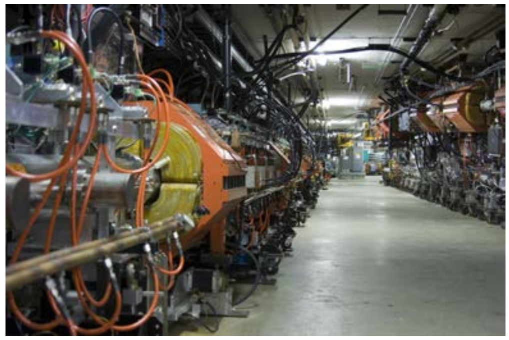


Figure 22.25 The Fermilab facility in Illinois has a large particle accelerator (the most powerful in the world until 2008) that employs magnetic fields (magnets seen here in orange) to contain and direct its beam. This and other accelerators have been in use for several decades and have allowed us to discover some of the laws underlying all matter. (credit: ammcrim, Flickr)

Thermonuclear fusion (like that occurring in the Sun) is a hope for a future clean energy source. One of the most promising devices is the *tokamak*, which uses magnetic fields to contain (or trap) and direct the reactive charged particles. (See **Figure 22.26**.) Less exotic, but more immediately practical, amplifiers in microwave ovens use a magnetic field to contain oscillating electrons. These oscillating electrons generate the microwaves sent into the oven.

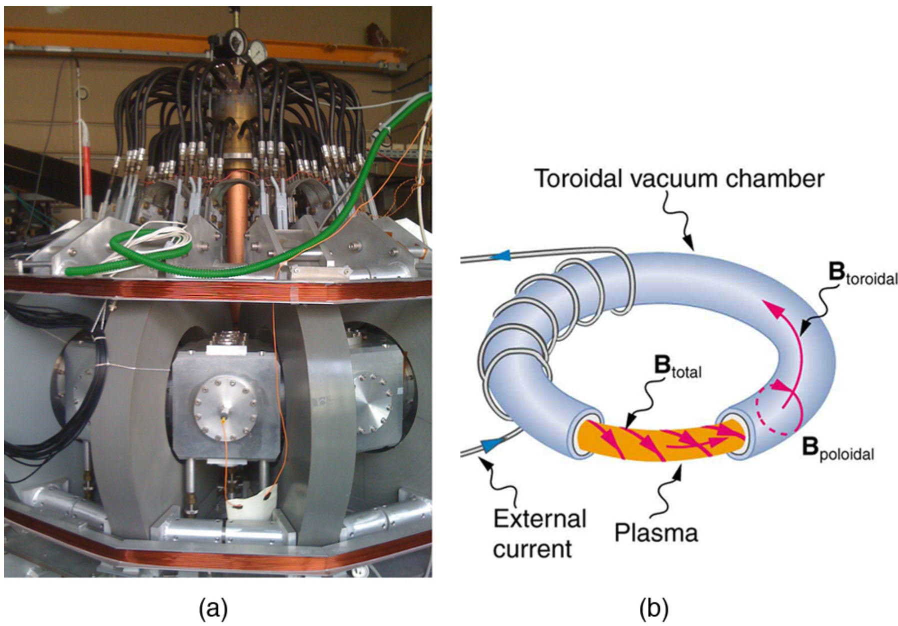


Figure 22.26 Tokamaks such as the one shown in the figure are being studied with the goal of economical production of energy by nuclear fusion. Magnetic fields in the doughnut-shaped device contain and direct the reactive charged particles. (credit: David Mellis, Flickr)

Mass spectrometers have a variety of designs, and many use magnetic fields to measure mass. The curvature of a charged particle’s path in the field is related to its mass and is measured to obtain mass information. (See **More Applications of Magnetism**.) Historically, such techniques were employed in the first direct observations of electron charge and mass. Today, mass spectrometers (sometimes coupled with gas chromatographs) are used to determine the make-up and sequencing of large biological molecules.

22.6 The Hall Effect

We have seen effects of a magnetic field on free-moving charges. The magnetic field also affects charges moving in a conductor. One result is the Hall effect, which has important implications and applications.

Figure 22.27 shows what happens to charges moving through a conductor in a magnetic field. The field is perpendicular to the electron drift velocity and to the width of the conductor. Note that conventional current is to the right in both parts of the figure. In part (a), electrons carry the current and move to the left. In part (b), positive charges carry the current and move to the right. Moving electrons feel a magnetic force toward one side of the conductor, leaving a net positive charge on the other side. This separation of charge *creates a voltage* ε, known as the **Hall emf**, *across* the conductor. The creation of a voltage *across* a current-carrying conductor by a magnetic field is known as the **Hall effect**, after Edwin Hall, the American physicist who discovered it in 1879.

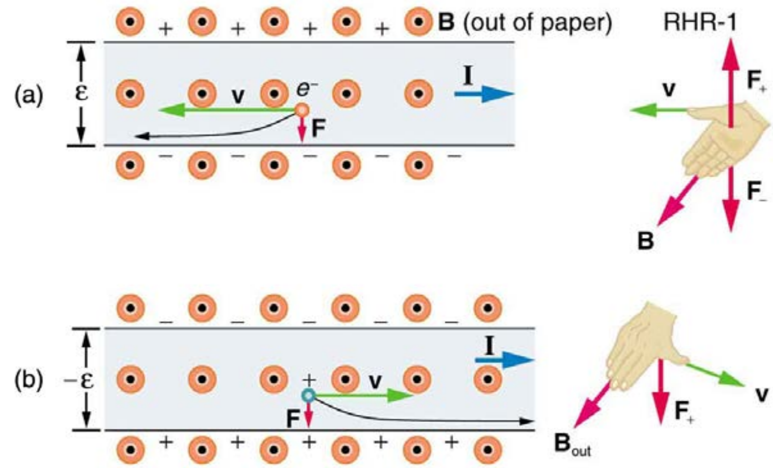


Figure 22.27 The Hall effect. (a) Electrons move to the left in this flat conductor (conventional current to the right). The magnetic field is directly out of the page, represented by circled dots; it exerts a force on the moving charges, causing a voltage ε, the Hall emf, across the conductor. (b) Positive charges moving to the right (conventional current also to the right) are moved to the side, producing a Hall emf of the opposite sign, $-\varepsilon$. Thus, if the direction of the field and current are known, the sign of the charge carriers can be determined from the Hall effect.

One very important use of the Hall effect is to determine whether positive or negative charges carries the current. Note that in **Figure 22.27**(b), where positive charges carry the current, the Hall emf has the sign opposite to when negative charges carry the current. Historically, the Hall effect was used to show that electrons carry current in metals and it also shows that positive charges carry current in some semiconductors. The Hall effect is used today as a research tool to probe the movement of charges, their drift velocities and densities, and so on, in materials. In 1980, it was discovered that the Hall effect is quantized, an example of quantum behavior in a macroscopic object.

The Hall effect has other uses that range from the determination of blood flow rate to precision measurement of magnetic field strength. To examine these quantitatively, we need an expression for the Hall emf, ε, across a conductor. Consider the balance of forces on a moving charge in a situation where B, v, and l are mutually perpendicular, such as shown in **Figure 22.28**. Although the magnetic force moves negative charges to one side, they cannot build up without limit. The electric field caused by their separation opposes the magnetic force, $F = qvB$, and the electric force, $F_\mathrm{e} = qE$, eventually grows to equal it. That is,

$$qE = qvB \tag{22.10}$$

or

$$E = vB. \tag{22.11}$$

Note that the electric field E is uniform across the conductor because the magnetic field B is uniform, as is the conductor. For a uniform electric field, the relationship between electric field and voltage is $E = \varepsilon / l$, where l is the width of the conductor and ε is the Hall emf. Entering this into the last expression gives

$$\frac{\varepsilon}{l} = vB. \tag{22.12}$$

Solving this for the Hall emf yields

$$\varepsilon = Blv \text{ (}B, v, \text{ and } l, \text{ mutually perpendicular),} \tag{22.13}$$

where ε is the Hall effect voltage across a conductor of width l through which charges move at a speed v.

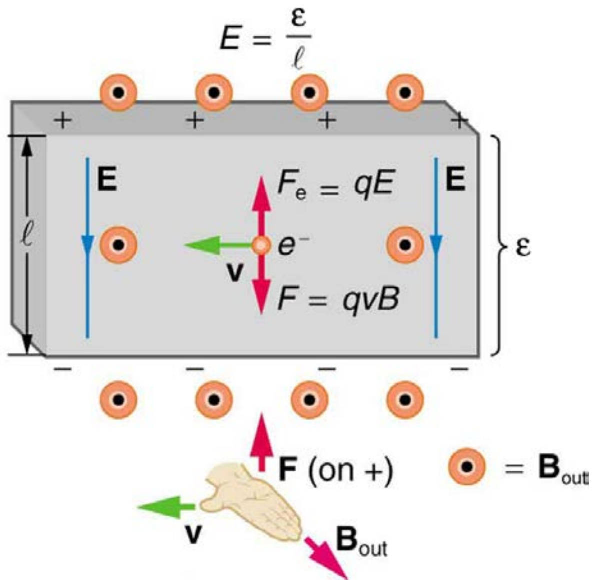


Figure 22.28 The Hall emf ε produces an electric force that balances the magnetic force on the moving charges. The magnetic force produces charge separation, which builds up until it is balanced by the electric force, an equilibrium that is quickly reached.

One of the most common uses of the Hall effect is in the measurement of magnetic field strength B. Such devices, called *Hall probes*, can be made very small, allowing fine position mapping. Hall probes can also be made very accurate, usually accomplished by careful calibration. Another application of the Hall effect is to measure fluid flow in any fluid that has free charges (most do). (See Figure 22.29.) A magnetic field applied perpendicular to the flow direction produces a Hall emf ε as shown. Note that the sign of ε depends not on the sign of the charges, but only on the directions of B and v. The magnitude of the Hall emf is $\varepsilon = Blv$, where l is the pipe diameter, so that the average velocity v can be determined from ε providing the other factors are known.

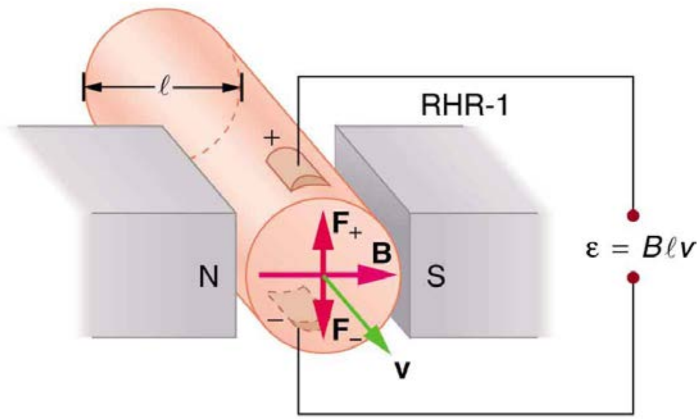


Figure 22.29 The Hall effect can be used to measure fluid flow in any fluid having free charges, such as blood. The Hall emf ε is measured across the tube perpendicular to the applied magnetic field and is proportional to the average velocity v.

Example 22.3 Calculating the Hall emf: Hall Effect for Blood Flow

A Hall effect flow probe is placed on an artery, applying a 0.100-T magnetic field across it, in a setup similar to that in Figure 22.29. What is the Hall emf, given the vessel's inside diameter is 4.00 mm and the average blood velocity is 20.0 cm/s?

Strategy

Because B, v, and l are mutually perpendicular, the equation $\varepsilon = Blv$ can be used to find ε.

Solution

Entering the given values for B, v, and l gives

$$\begin{aligned} \varepsilon &= Blv = (0.100\text{ T})\left(4.00\times10^{-3}\text{ m}\right)(0.200\text{ m/s}) \\ &= 80.0\ \mu\text{V} \end{aligned} \tag{22.14}$$

Discussion

This is the average voltage output. Instantaneous voltage varies with pulsating blood flow. The voltage is small in this type of measurement. ε is particularly difficult to measure, because there are voltages associated with heart action (ECG voltages) that are on the order of millivolts. In practice, this difficulty is overcome by applying an AC magnetic field, so that the Hall emf is AC with the same frequency. An amplifier can be very selective in picking out only the appropriate frequency, eliminating signals and noise at other frequencies.

22.7 Magnetic Force on a Current-Carrying Conductor

Because charges ordinarily cannot escape a conductor, the magnetic force on charges moving in a conductor is transmitted to the conductor itself.

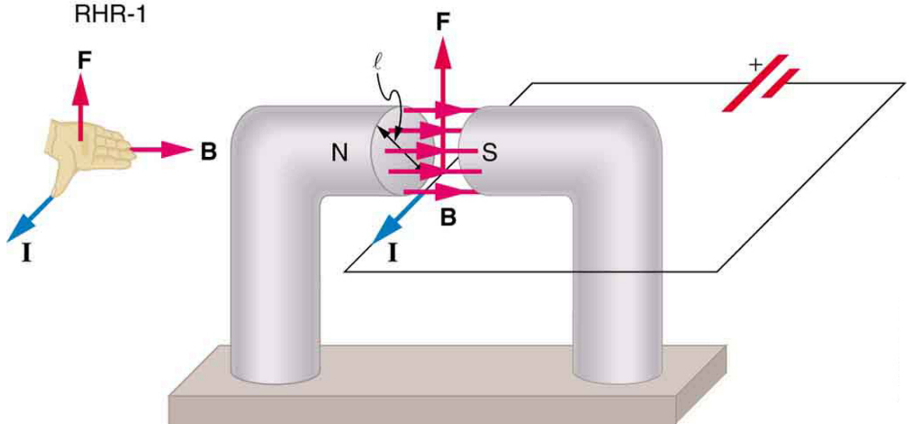


Figure 22.30 The magnetic field exerts a force on a current-carrying wire in a direction given by the right hand rule 1 (the same direction as that on the individual moving charges). This force can easily be large enough to move the wire, since typical currents consist of very large numbers of moving charges.

We can derive an expression for the magnetic force on a current by taking a sum of the magnetic forces on individual charges. (The forces add because they are in the same direction.) The force on an individual charge moving at the drift velocity v_{d} is given by $F = qv_{\text{d}}B \sin \theta$. Taking B to be uniform over a length of wire l and zero elsewhere, the total magnetic force on the wire is then $F = (qv_{\text{d}}B \sin \theta)(N)$, where N is the number of charge carriers in the section of wire of length l . Now, $N = nV$, where n is the number of charge carriers per unit volume and V is the volume of wire in the field. Noting that $V = Al$, where A is the cross-sectional area of the wire, then the force on the wire is $F = (qv_{\text{d}}B \sin \theta)(nAl)$. Gathering terms,

$$F = (nqAv_{\text{d}})lB \sin \theta. \tag{22.15}$$

Because $nqAv_{\text{d}} = I$ (see **Current**),

$$F = IlB \sin \theta \tag{22.16}$$

is the equation for *magnetic force on a length* l *of wire carrying a current* I *in a uniform magnetic field* B , as shown in **Figure 22.31**. If we divide both sides of this expression by l , we find that the magnetic force per unit length of wire in a uniform field is $\frac{F}{l} = IB \sin \theta$. The direction of this force is given by RHR-1, with the thumb in the direction of the current I . Then, with the fingers in the direction of B , a perpendicular to the palm points in the direction of F , as in **Figure 22.31**.

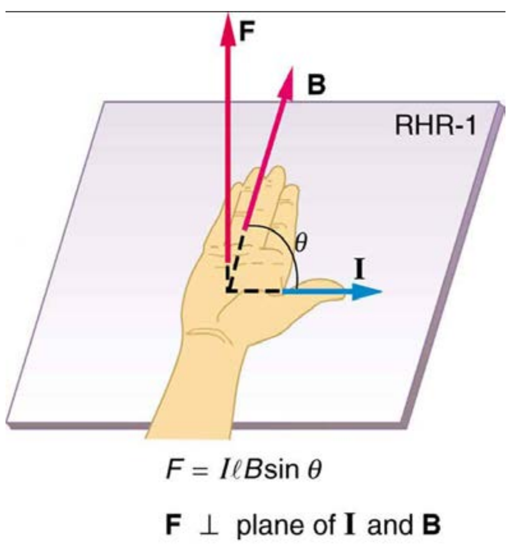


Figure 22.31 The force on a current-carrying wire in a magnetic field is $F = IlB \sin \theta$. Its direction is given by RHR-1.

Example 22.4 Calculating Magnetic Force on a Current-Carrying Wire: A Strong Magnetic Field

Calculate the force on the wire shown in **Figure 22.30**, given $B = 1.50\text{ T}$, $l = 5.00\text{ cm}$, and $I = 20.0\text{ A}$.

Strategy

The force can be found with the given information by using $F = IlB\sin\theta$ and noting that the angle θ between I and B is 90°, so that $\sin\theta = 1$.

Solution

Entering the given values into $F = IlB\sin\theta$ yields

$$F = IlB\sin\theta = (20.0\text{ A})(0.0500\text{ m})(1.50\text{ T})(1). \tag{22.17}$$

The units for tesla are $1\text{ T} = \dfrac{\text{N}}{\text{A}\cdot\text{m}}$; thus,

$$F = 1.50\text{ N}. \tag{22.18}$$

Discussion

This large magnetic field creates a significant force on a small length of wire.

Magnetic force on current-carrying conductors is used to convert electric energy to work. (Motors are a prime example—they employ loops of wire and are considered in the next section.) Magnetohydrodynamics (MHD) is the technical name given to a clever application where magnetic force pumps fluids without moving mechanical parts. (See **Figure 22.32**.)

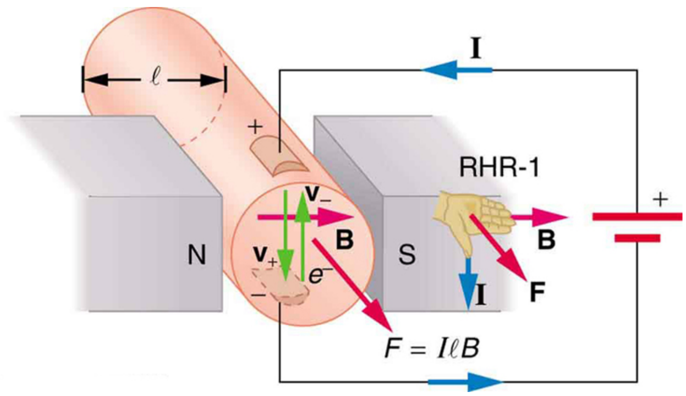


Figure 22.32 Magnetohydrodynamics. The magnetic force on the current passed through this fluid can be used as a nonmechanical pump.

A strong magnetic field is applied across a tube and a current is passed through the fluid at right angles to the field, resulting in a force on the fluid parallel to the tube axis as shown. The absence of moving parts makes this attractive for moving a hot, chemically active substance, such as the liquid sodium employed in some nuclear reactors. Experimental artificial hearts are testing with this technique for pumping blood, perhaps circumventing the adverse effects of mechanical pumps. (Cell membranes, however, are affected by the large fields needed in MHD, delaying its practical application in humans.) MHD propulsion for nuclear submarines has been proposed, because it could be considerably quieter than conventional propeller drives. The deterrent value of nuclear submarines is based on their ability to hide and survive a first or second nuclear strike. As we slowly disassemble our nuclear weapons arsenals, the submarine branch will be the last to be decommissioned because of this ability (See **Figure 22.33**.) Existing MHD drives are heavy and inefficient—much development work is needed.

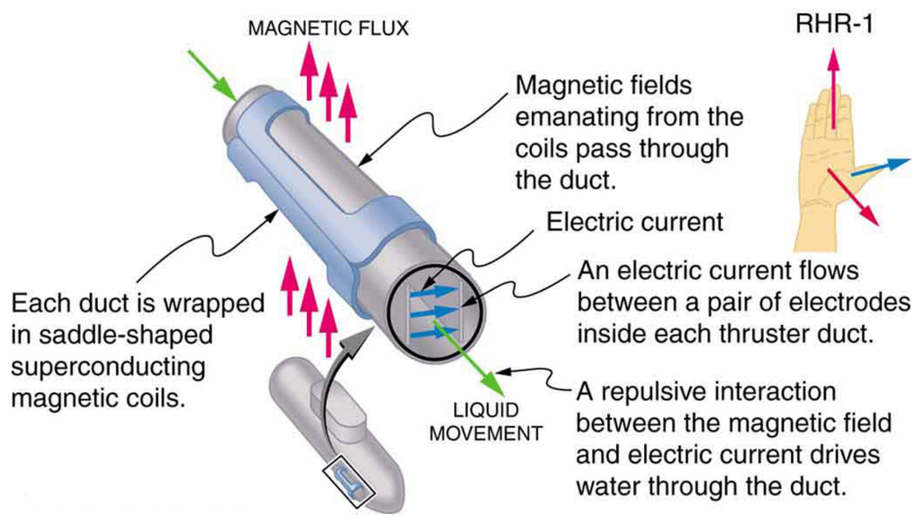


Figure 22.33 An MHD propulsion system in a nuclear submarine could produce significantly less turbulence than propellers and allow it to run more silently. The development of a silent drive submarine was dramatized in the book and the film *The Hunt for Red October*.

22.8 Torque on a Current Loop: Motors and Meters

Motors are the most common application of magnetic force on current-carrying wires. Motors have loops of wire in a magnetic field. When current is passed through the loops, the magnetic field exerts torque on the loops, which rotates a shaft. Electrical energy is converted to mechanical work in the process. (See **Figure 22.34**.)

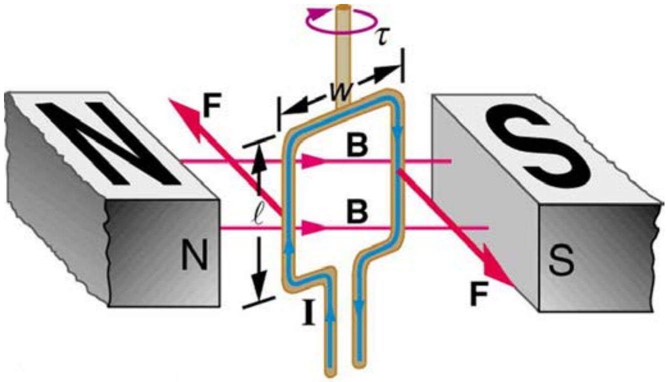


Figure 22.34 Torque on a current loop. A current-carrying loop of wire attached to a vertically rotating shaft feels magnetic forces that produce a clockwise torque as viewed from above.

Let us examine the force on each segment of the loop in **Figure 22.34** to find the torques produced about the axis of the vertical shaft. (This will lead to a useful equation for the torque on the loop.) We take the magnetic field to be uniform over the rectangular loop, which has width w and height l. First, we note that the forces on the top and bottom segments are vertical and, therefore, parallel to the shaft, producing no torque. Those vertical forces are equal in magnitude and opposite in direction, so that they also produce no net force on the loop. **Figure 22.35** shows views of the loop from above. Torque is defined as $\tau = rF \sin\theta$, where F is the force, r is the distance from the pivot that the force is applied, and θ is the angle between r and F. As seen in **Figure 22.35**(a), right hand rule 1 gives the forces on the sides to be equal in magnitude and opposite in direction, so that the net force is again zero. However, each force produces a clockwise torque. Since $r = w/2$, the torque on each vertical segment is $(w/2)F \sin\theta$, and the two add to give a total torque.

$$\tau = \frac{w}{2}F \sin\theta + \frac{w}{2}F \sin\theta = wF \sin\theta \tag{22.19}$$

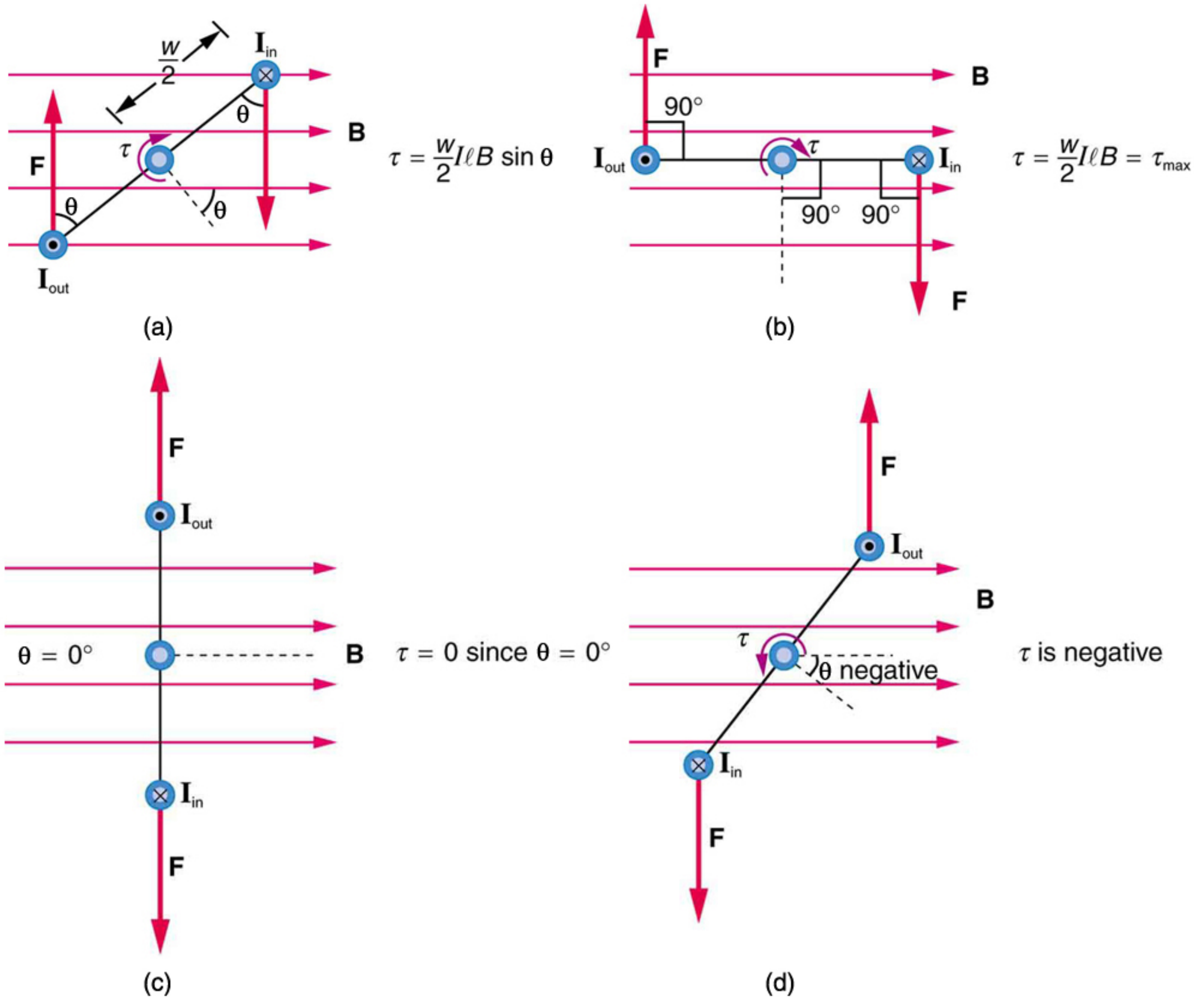


Figure 22.35 Top views of a current-carrying loop in a magnetic field. (a) The equation for torque is derived using this view. Note that the perpendicular to the loop makes an angle θ with the field that is the same as the angle between $w/2$ and $\mathbf{F}$. (b) The maximum torque occurs when θ is a right angle and $\sin\theta = 1$. (c) Zero (minimum) torque occurs when θ is zero and $\sin\theta = 0$. (d) The torque reverses once the loop rotates past $\theta = 0$.

Now, each vertical segment has a length l that is perpendicular to B, so that the force on each is $F = IlB$. Entering F into the expression for torque yields

$$\tau = wIlB\sin\theta. \tag{22.20}$$

If we have a multiple loop of N turns, we get N times the torque of one loop. Finally, note that the area of the loop is $A = wl$; the expression for the torque becomes

$$\tau = NIAB\sin\theta. \tag{22.21}$$

This is the torque on a current-carrying loop in a uniform magnetic field. This equation can be shown to be valid for a loop of any shape. The loop carries a current I, has N turns, each of area A, and the perpendicular to the loop makes an angle θ with the field B. The net force on the loop is zero.

Example 22.5 Calculating Torque on a Current-Carrying Loop in a Strong Magnetic Field

Find the maximum torque on a 100-turn square loop of a wire of 10.0 cm on a side that carries 15.0 A of current in a 2.00-T field.

Strategy

Torque on the loop can be found using $\tau = NIAB\sin\theta$. Maximum torque occurs when $\theta = 90^\circ$ and $\sin\theta = 1$.

Solution

For $\sin\theta = 1$, the maximum torque is

$$\tau_{\text{max}} = NIAB. \tag{22.22}$$

Entering known values yields

$$\begin{aligned}\tau_{\text{max}} &= (100)(15.0\text{ A})\left(0.100\text{ m}^2\right)(2.00\text{ T}) \\ &= 30.0\text{ N}\cdot\text{m}.\end{aligned} \tag{22.23}$$

Discussion

This torque is large enough to be useful in a motor.

The torque found in the preceding example is the maximum. As the coil rotates, the torque decreases to zero at $\theta = 0$. The torque then *reverses* its direction once the coil rotates past $\theta = 0$. (See **Figure 22.35**(d).) This means that, unless we do something, the coil will oscillate back and forth about equilibrium at $\theta = 0$. To get the coil to continue rotating in the same direction, we can reverse the current as it passes through $\theta = 0$ with automatic switches called *brushes*. (See **Figure 22.36**.)

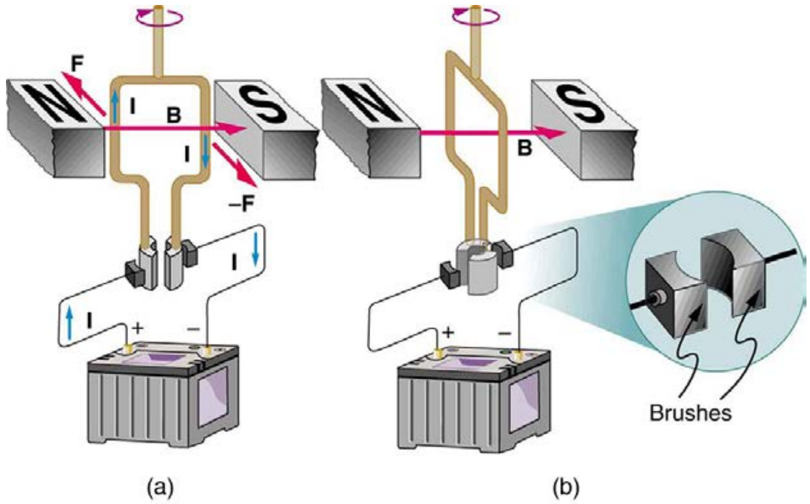


Figure 22.36 (a) As the angular momentum of the coil carries it through $\theta = 0$, the brushes reverse the current to keep the torque clockwise. (b) The coil will rotate continuously in the clockwise direction, with the current reversing each half revolution to maintain the clockwise torque.

Meters, such as those in analog fuel gauges on a car, are another common application of magnetic torque on a current-carrying loop. **Figure 22.37** shows that a meter is very similar in construction to a motor. The meter in the figure has its magnets shaped to limit the effect of θ by making B perpendicular to the loop over a large angular range. Thus the torque is proportional to I and not θ . A linear spring exerts a counter-torque that balances the current-produced torque. This makes the needle deflection proportional to I . If an exact proportionality cannot be achieved, the gauge reading can be calibrated. To produce a galvanometer for use in analog voltmeters and ammeters that have a low resistance and respond to small currents, we use a large loop area A , high magnetic field B , and low-resistance coils.

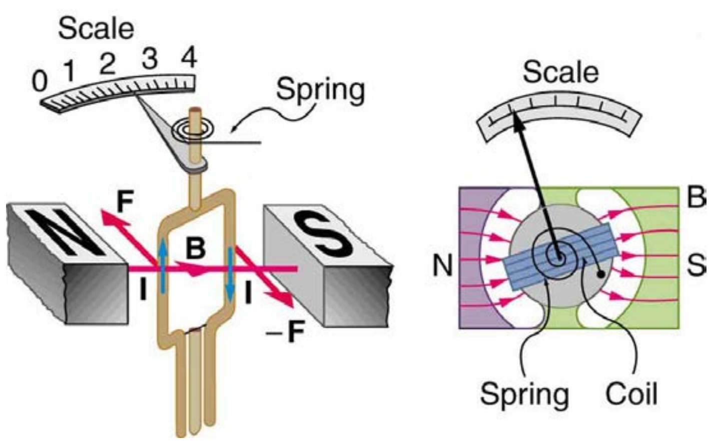


Figure 22.37 Meters are very similar to motors but only rotate through a part of a revolution. The magnetic poles of this meter are shaped to keep the component of B perpendicular to the loop constant, so that the torque does not depend on θ and the deflection against the return spring is proportional only to the current I .

22.9 Magnetic Fields Produced by Currents: Ampere's Law

How much current is needed to produce a significant magnetic field, perhaps as strong as the Earth's field? Surveyors will tell you that overhead electric power lines create magnetic fields that interfere with their compass readings. Indeed, when Oersted discovered in 1820 that a current in a wire affected a compass needle, he was not dealing with extremely large currents. How does the shape of wires carrying current affect the shape of the magnetic field created? We noted earlier that a current loop created a magnetic field similar to that of a bar magnet, but what about a straight wire or a toroid (doughnut)? How is the direction of a current-created field related to the direction of the current? Answers to these questions are explored in this section, together with a brief discussion of the law governing the fields created by currents.

Magnetic Field Created by a Long Straight Current-Carrying Wire: Right Hand Rule 2

Magnetic fields have both direction and magnitude. As noted before, one way to explore the direction of a magnetic field is with compasses, as shown for a long straight current-carrying wire in **Figure 22.38**. Hall probes can determine the magnitude of the field. The field around a long straight wire is found to be in circular loops. The **right hand rule 2** (RHR-2) emerges from this exploration and is valid for any current segment—*point the thumb in the direction of the current, and the fingers curl in the direction of the magnetic field loops* created by it.

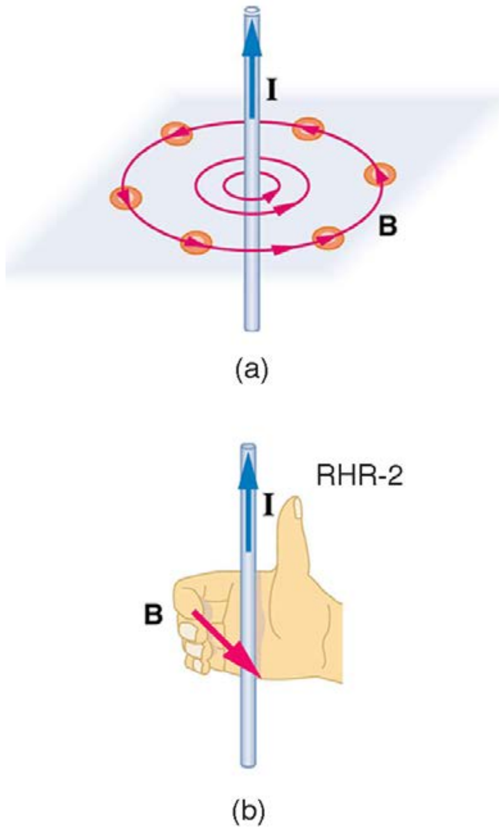


Figure 22.38 (a) Compasses placed near a long straight current-carrying wire indicate that field lines form circular loops centered on the wire. (b) Right hand rule 2 states that, if the right hand thumb points in the direction of the current, the fingers curl in the direction of the field. This rule is consistent with the field mapped for the long straight wire and is valid for any current segment.

The **magnetic field strength (magnitude) produced by a long straight current-carrying wire** is found by experiment to be

$$B = \frac{\mu_0 I}{2\pi r} \text{ (long straight wire)}, \tag{22.24}$$

where I is the current, r is the shortest distance to the wire, and the constant $\mu_0 = 4\pi \times 10^{-7}\ \text{T} \cdot \text{m/A}$ is the **permeability of free space**. (μ_0 is one of the basic constants in nature. We will see later that μ_0 is related to the speed of light.) Since the wire is very long, the magnitude of the field depends only on distance from the wire r, not on position along the wire.

Example 22.6 Calculating Current that Produces a Magnetic Field

Find the current in a long straight wire that would produce a magnetic field twice the strength of the Earth's at a distance of 5.0 cm from the wire.

Strategy

The Earth's field is about $5.0\times10^{-5}\ \text{T}$, and so here B due to the wire is taken to be $1.0\times10^{-4}\ \text{T}$. The equation $B = \frac{\mu_0 I}{2\pi r}$ can be used to find I, since all other quantities are known.

Solution

Solving for I and entering known values gives

$$\begin{aligned} I &= \frac{2\pi r B}{\mu_0} = \frac{2\pi\left(5.0\times10^{-2}\ \text{m}\right)\left(1.0\times10^{-4}\ \text{T}\right)}{4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}} \\ &= 25\ \text{A}. \end{aligned} \tag{22.25}$$

Discussion

So a moderately large current produces a significant magnetic field at a distance of 5.0 cm from a long straight wire. Note that the answer is stated to only two digits, since the Earth's field is specified to only two digits in this example.

Ampere's Law and Others

The magnetic field of a long straight wire has more implications than you might at first suspect. *Each segment of current produces a magnetic field like that of a long straight wire, and the total field of any shape current is the vector sum of the fields due to each segment.* The formal statement of the direction and magnitude of the field due to each segment is called the **Biot-Savart law**. Integral calculus is needed to sum the field for an arbitrary shape current. This results in a more complete law, called **Ampere's law**, which relates magnetic field and current in a general way. Ampere's law in turn is a part of **Maxwell's equations**, which give a complete theory of all electromagnetic phenomena. Considerations of how Maxwell's equations appear to different observers led to the modern theory of relativity, and the realization that electric and magnetic fields are different manifestations of the same thing. Most of this is beyond the scope of this text in both mathematical level, requiring calculus, and in the amount of space that can be devoted to it. But for the interested student, and particularly for those who continue in physics, engineering, or similar pursuits, delving into these matters further will reveal descriptions of nature that are elegant as well as profound. In this text, we shall keep the general features in mind, such as RHR-2 and the rules for magnetic field lines listed in **Magnetic Fields and Magnetic Field Lines**, while concentrating on the fields created in certain important situations.

Making Connections: Relativity

Hearing all we do about Einstein, we sometimes get the impression that he invented relativity out of nothing. On the contrary, one of Einstein's motivations was to solve difficulties in knowing how different observers see magnetic and electric fields.

Magnetic Field Produced by a Current-Carrying Circular Loop

The magnetic field near a current-carrying loop of wire is shown in **Figure 22.39**. Both the direction and the magnitude of the magnetic field produced by a current-carrying loop are complex. RHR-2 can be used to give the direction of the field near the loop, but mapping with compasses and the rules about field lines given in **Magnetic Fields and Magnetic Field Lines** are needed for more detail. There is a simple formula for the **magnetic field strength at the center of a circular loop**. It is

$$B = \frac{\mu_0 I}{2R} \text{ (at center of loop)}, \tag{22.26}$$

where R is the radius of the loop. This equation is very similar to that for a straight wire, but it is valid *only* at the center of a circular loop of wire. The similarity of the equations does indicate that similar field strength can be obtained at the center of a loop. One way to get a larger field is to have N loops; then, the field is $B = N\mu_0 I / (2R)$. Note that the larger the loop, the smaller the field at its center, because the current is farther away.

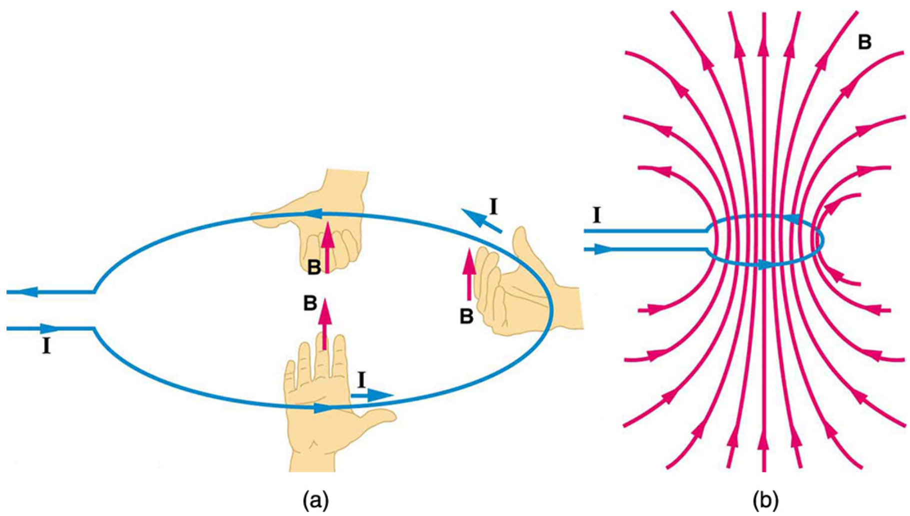


Figure 22.39 (a) RHR-2 gives the direction of the magnetic field inside and outside a current-carrying loop. (b) More detailed mapping with compasses or with a Hall probe completes the picture. The field is similar to that of a bar magnet.

Magnetic Field Produced by a Current-Carrying Solenoid

A **solenoid** is a long coil of wire (with many turns or loops, as opposed to a flat loop). Because of its shape, the field inside a solenoid can be very uniform, and also very strong. The field just outside the coils is nearly zero. **Figure 22.40** shows how the field looks and how its direction is given by RHR-2.

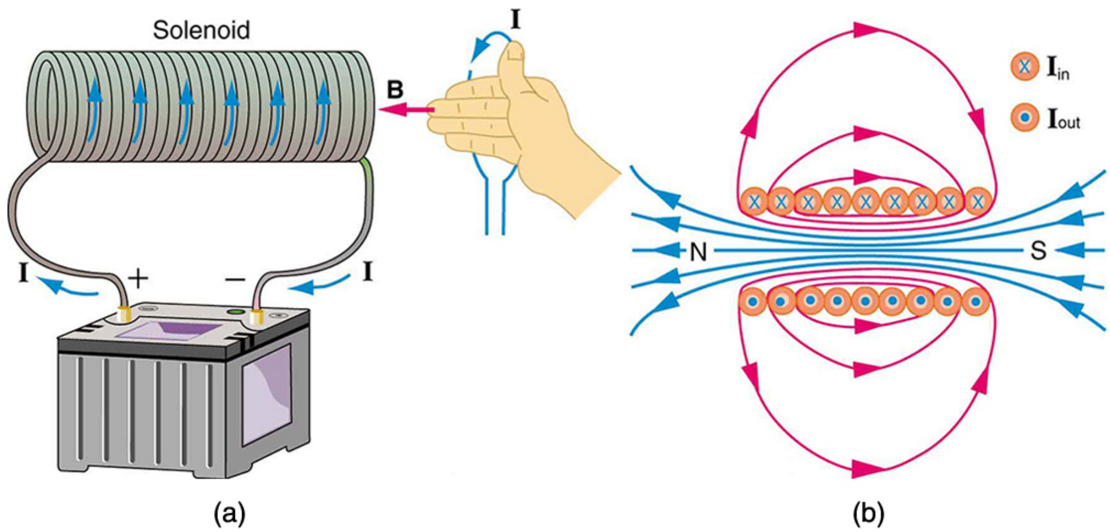


Figure 22.40 (a) Because of its shape, the field inside a solenoid of length l is remarkably uniform in magnitude and direction, as indicated by the straight and uniformly spaced field lines. The field outside the coils is nearly zero. (b) This cutaway shows the magnetic field generated by the current in the solenoid.

The magnetic field inside of a current-carrying solenoid is very uniform in direction and magnitude. Only near the ends does it begin to weaken and change direction. The field outside has similar complexities to flat loops and bar magnets, but the **magnetic field strength inside a solenoid** is simply

$$B = \mu_0 nI \text{ (inside a solenoid)}, \tag{22.27}$$

where n is the number of loops per unit length of the solenoid $(n = N / l$, with N being the number of loops and l the length). Note that B is the field strength anywhere in the uniform region of the interior and not just at the center. Large uniform fields spread over a large volume are possible with solenoids, as **Example 22.7** implies.

Example 22.7 Calculating Field Strength inside a Solenoid

What is the field inside a 2.00-m-long solenoid that has 2000 loops and carries a 1600-A current?

Strategy

To find the field strength inside a solenoid, we use $B = \mu_0 nI$. First, we note the number of loops per unit length is

$$n^{-1} = \frac{N}{l} = \frac{2000}{2.00\text{ m}} = 1000\text{ m}^{-1} = 10\text{ cm}^{-1}. \tag{22.28}$$

Solution

Substituting known values gives

$$\begin{aligned} B &= \mu_0 nI = \left(4\pi\times10^{-7}\text{ T}\cdot\text{m/A}\right)\left(1000\text{ m}^{-1}\right)(1600\text{ A}) \\ &= 2.01\text{ T}. \end{aligned} \tag{22.29}$$

Discussion

This is a large field strength that could be established over a large-diameter solenoid, such as in medical uses of magnetic resonance imaging (MRI). The very large current is an indication that the fields of this strength are not easily achieved, however. Such a large current through 1000 loops squeezed into a meter’s length would produce significant heating. Higher currents can be achieved by using superconducting wires, although this is expensive. There is an upper limit to the current, since the superconducting state is disrupted by very large magnetic fields.

There are interesting variations of the flat coil and solenoid. For example, the toroidal coil used to confine the reactive particles in tokamaks is much like a solenoid bent into a circle. The field inside a toroid is very strong but circular. Charged particles travel in circles, following the field lines, and collide with one another, perhaps inducing fusion. But the charged particles do not cross field lines and escape the toroid. A whole range of coil shapes are used to produce all sorts of magnetic field shapes. Adding ferromagnetic materials produces greater field strengths and can have a significant effect on the shape of the field. Ferromagnetic materials tend to trap magnetic fields (the field lines bend into the ferromagnetic material, leaving weaker fields outside it) and are used as shields for devices that are adversely affected by magnetic fields, including the Earth’s magnetic field.

PhET Explorations: Generator

Generate electricity with a bar magnet! Discover the physics behind the phenomena by exploring magnets and how you can use them to make a bulb light.

PhET Interactive Simulation

Figure 22.41 Generator (http://cnx.org/content/m42382/1.2/generator_en.jar)

22.10 Magnetic Force between Two Parallel Conductors

You might expect that there are significant forces between current-carrying wires, since ordinary currents produce significant magnetic fields and these fields exert significant forces on ordinary currents. But you might not expect that the force between wires is used to *define* the ampere. It might also surprise you to learn that this force has something to do with why large circuit breakers burn up when they attempt to interrupt large currents.

The force between two long straight and parallel conductors separated by a distance r can be found by applying what we have developed in preceding sections. Figure 22.42 shows the wires, their currents, the fields they create, and the subsequent forces they exert on one another. Let us consider the field produced by wire 1 and the force it exerts on wire 2 (call the force F_2). The field due to I_1 at a distance r is given to be

$$B_1 = \frac{\mu_0 I_1}{2\pi r}. \tag{22.30}$$

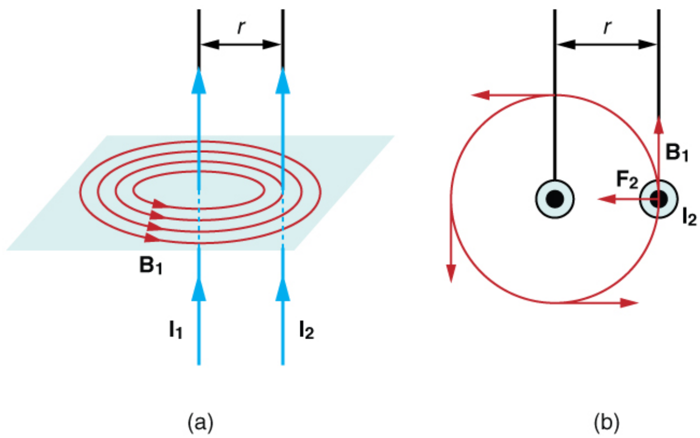


Figure 22.42 (a) The magnetic field produced by a long straight conductor is perpendicular to a parallel conductor, as indicated by RHR-2. (b) A view from above of the two wires shown in (a), with one magnetic field line shown for each wire. RHR-1 shows that the force between the parallel conductors is attractive when the currents are in the same direction. A similar analysis shows that the force is repulsive between currents in opposite directions.

This field is uniform along wire 2 and perpendicular to it, and so the force F_2 it exerts on wire 2 is given by $F = IlB \sin\theta$ with $\sin\theta = 1$:

$$F_2 = I_2 l B_1. \tag{22.31}$$

By Newton's third law, the forces on the wires are equal in magnitude, and so we just write F for the magnitude of F_2. (Note that $F_1 = -F_2$.) Since the wires are very long, it is convenient to think in terms of F / l, the force per unit length. Substituting the expression for B_1 into the last equation and rearranging terms gives

$$\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi r}. \tag{22.32}$$

F / l is the force per unit length between two parallel currents I_1 and I_2 separated by a distance r. The force is attractive if the currents are in the same direction and repulsive if they are in opposite directions.

This force is responsible for the *pinch effect* in electric arcs and plasmas. The force exists whether the currents are in wires or not. In an electric arc, where currents are moving parallel to one another, there is an attraction that squeezes currents into a smaller tube. In large circuit breakers, like those used in neighborhood power distribution systems, the pinch effect can concentrate an arc between plates of a switch trying to break a large current, burn holes, and even ignite the equipment. Another example of the pinch effect is found in the solar plasma, where jets of ionized material, such as solar flares, are shaped by magnetic forces.

The *operational definition of the ampere* is based on the force between current-carrying wires. Note that for parallel wires separated by 1 meter with each carrying 1 ampere, the force per meter is

$$\frac{F}{l} = \frac{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)(1\ \text{A})^2}{(2\pi)(1\ \text{m})} = 2\times10^{-7}\ \text{N/m}. \tag{22.33}$$

Since μ_0 is exactly $4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}$ by definition, and because $1\ \text{T} = 1\ \text{N/(A}\cdot\text{m)}$, the force per meter is exactly $2\times10^{-7}\ \text{N/m}$. This is the basis of the operational definition of the ampere.

The Ampere

The official definition of the ampere is:

One ampere of current through each of two parallel conductors of infinite length, separated by one meter in empty space free of other magnetic fields, causes a force of exactly $2\times10^{-7}\text{ N/m}$ on each conductor.

Infinite-length straight wires are impractical and so, in practice, a current balance is constructed with coils of wire separated by a few centimeters. Force is measured to determine current. This also provides us with a method for measuring the coulomb. We measure the charge that flows for a current of one ampere in one second. That is, $1\text{ C} = 1\text{ A}\cdot\text{s}$. For both the ampere and the coulomb, the method of measuring force between conductors is the most accurate in practice.

22.11 More Applications of Magnetism

Mass Spectrometry

The curved paths followed by charged particles in magnetic fields can be put to use. A charged particle moving perpendicular to a magnetic field travels in a circular path having a radius r.

$$r = \frac{mv}{qB} \tag{22.34}$$

It was noted that this relationship could be used to measure the mass of charged particles such as ions. A mass spectrometer is a device that measures such masses. Most mass spectrometers use magnetic fields for this purpose, although some of them have extremely sophisticated designs. Since there are five variables in the relationship, there are many possibilities. However, if v, q, and B can be fixed, then the radius of the path r is simply proportional to the mass m of the charged particle. Let us examine one such mass spectrometer that has a relatively simple design. (See **Figure 22.43**.) The process begins with an ion source, a device like an electron gun. The ion source gives ions their charge, accelerates them to some velocity v, and directs a beam of them into the next stage of the spectrometer. This next region is a *velocity selector* that only allows particles with a particular value of v to get through.

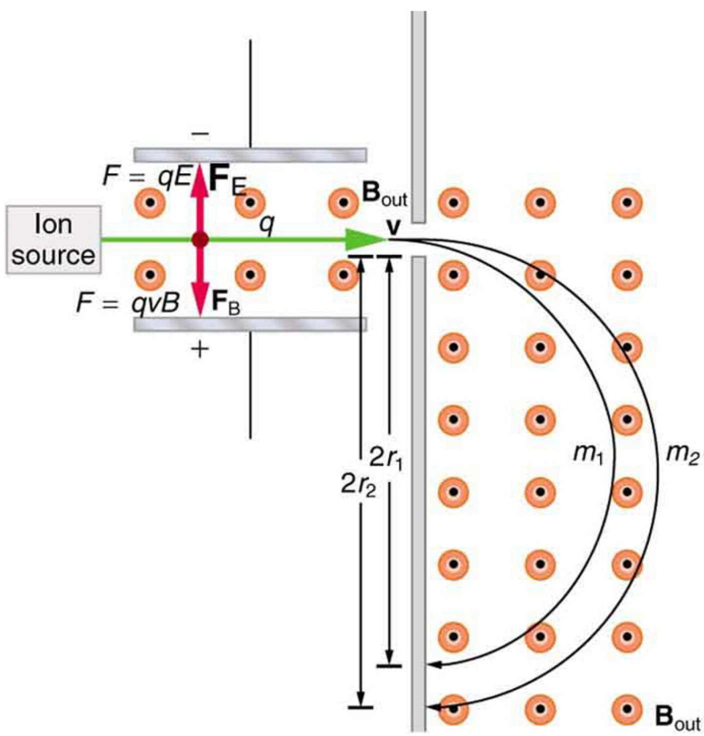


Figure 22.43 This mass spectrometer uses a velocity selector to fix v so that the radius of the path is proportional to mass.

The velocity selector has both an electric field and a magnetic field, perpendicular to one another, producing forces in opposite directions on the ions. Only those ions for which the forces balance travel in a straight line into the next region. If the forces balance, then the electric force $F = qE$ equals the magnetic force $F = qvB$, so that $qE = qvB$. Noting that q cancels, we see that

$$v = \frac{E}{B} \tag{22.35}$$

is the velocity particles must have to make it through the velocity selector, and further, that v can be selected by varying E and B. In the final region, there is only a uniform magnetic field, and so the charged particles move in circular arcs with radii proportional to particle mass. The paths also depend on charge q, but since q is in multiples of electron charges, it is easy to determine and to discriminate between ions in different charge states.

Mass spectrometry today is used extensively in chemistry and biology laboratories to identify chemical and biological substances according to their mass-to-charge ratios. In medicine, mass spectrometers are used to measure the concentration of isotopes used as tracers. Usually, biological molecules such as proteins are very large, so they are broken down into smaller fragments before analyzing. Recently, large virus particles have been analyzed as a whole on mass spectrometers. Sometimes a gas chromatograph or high-performance liquid chromatograph provides an initial separation of the large molecules, which are then input into the mass spectrometer.

Cathode Ray Tubes—CRTs—and the Like

What do non-flat-screen TVs, old computer monitors, x-ray machines, and the 2-mile-long Stanford Linear Accelerator have in common? All of them accelerate electrons, making them different versions of the electron gun. Many of these devices use magnetic fields to steer the accelerated electrons. **Figure 22.44** shows the construction of the type of cathode ray tube (CRT) found in some TVs, oscilloscopes, and old computer monitors. Two pairs of coils are used to steer the electrons, one vertically and the other horizontally, to their desired destination.

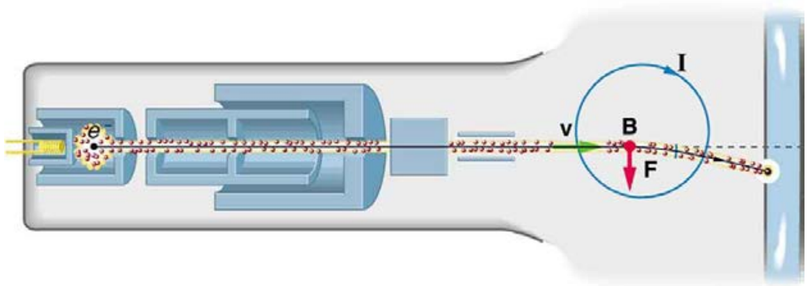


Figure 22.44 The cathode ray tube (CRT) is so named because rays of electrons originate at the cathode in the electron gun. Magnetic coils are used to steer the beam in many CRTs. In this case, the beam is moved down. Another pair of horizontal coils would steer the beam horizontally.

Magnetic Resonance Imaging

Magnetic resonance imaging (MRI) is one of the most useful and rapidly growing medical imaging tools. It non-invasively produces two-dimensional and three-dimensional images of the body that provide important medical information with none of the hazards of x-rays. MRI is based on an effect called **nuclear magnetic resonance (NMR)** in which an externally applied magnetic field interacts with the nuclei of certain atoms, particularly those of hydrogen (protons). These nuclei possess their own small magnetic fields, similar to those of electrons and the current loops discussed earlier in this chapter.

When placed in an external magnetic field, such nuclei experience a torque that pushes or aligns the nuclei into one of two new energy states—depending on the orientation of its spin (analogous to the N pole and S pole in a bar magnet). Transitions from the lower to higher energy state can be achieved by using an external radio frequency signal to “flip” the orientation of the small magnets. (This is actually a quantum mechanical process. The direction of the nuclear magnetic field is quantized as is energy in the radio waves. We will return to these topics in later chapters.) The specific frequency of the radio waves that are absorbed and reemitted depends sensitively on the type of nucleus, the chemical environment, and the external magnetic field strength. Therefore, this is a *resonance* phenomenon in which *nuclei* in a *magnetic* field act like resonators (analogous to those discussed in the treatment of sound in **Oscillatory Motion and Waves**) that absorb and reemit only certain frequencies. Hence, the phenomenon is named *nuclear magnetic resonance (NMR).*

NMR has been used for more than 50 years as an analytical tool. It was formulated in 1946 by F. Bloch and E. Purcell, with the 1952 Nobel Prize in Physics going to them for their work. Over the past two decades, NMR has been developed to produce detailed images in a process now called magnetic resonance imaging (MRI), a name coined to avoid the use of the word “nuclear” and the concomitant implication that nuclear radiation is involved. (It is not.) The 2003 Nobel Prize in Medicine went to P. Lauterbur and P. Mansfield for their work with MRI applications.

The largest part of the MRI unit is a superconducting magnet that creates a magnetic field, typically between 1 and 2 T in strength, over a relatively large volume. MRI images can be both highly detailed and informative about structures and organ functions. It is helpful that normal and non-normal tissues respond differently for slight changes in the magnetic field. In most medical images, the protons that are hydrogen nuclei are imaged. (About 2/3 of the atoms in the body are hydrogen.) Their location and density give a variety of medically useful information, such as organ function, the condition of tissue (as in the brain), and the shape of structures, such as vertebral disks and knee-joint surfaces. MRI can also be used to follow the movement of certain ions across membranes, yielding information on active transport, osmosis, dialysis, and other phenomena. With excellent spatial resolution, MRI can provide information about tumors, strokes, shoulder injuries, infections, etc.

An image requires position information as well as the density of a nuclear type (usually protons). By varying the magnetic field slightly over the volume to be imaged, the resonant frequency of the protons is made to vary with position. Broadcast radio frequencies are swept over an appropriate range and nuclei absorb and reemit them only if the nuclei are in a magnetic field with the correct strength. The imaging receiver gathers information through the body almost point by point, building up a tissue map. The reception of reemitted radio waves as a function of frequency thus gives position information. These “slices” or cross sections through the body are only several mm thick. The intensity of the reemitted radio waves is proportional to the concentration of the nuclear type being flipped, as well as information on the chemical environment in that area of the body. Various techniques are available for enhancing contrast in images and for obtaining more information. Scans called T1, T2, or proton density scans rely on different relaxation mechanisms of nuclei. Relaxation refers to the time it takes for the protons to return to equilibrium after the external field is turned off. This time depends upon tissue type and status (such as inflammation).

While MRI images are superior to x rays for certain types of tissue and have none of the hazards of x rays, they do not completely supplant x-ray images. MRI is less effective than x rays for detecting breaks in bone, for example, and in imaging breast tissue, so the two diagnostic tools complement each other. MRI images are also expensive compared to simple x-ray images and tend to be used most often where they supply information not readily obtained from x rays. Another disadvantage of MRI is that the patient is totally enclosed with detectors close to the body for about 30 minutes or more, leading to claustrophobia. It is also difficult for the obese patient to be in the magnet tunnel. New “open-MRI” machines are now available in which the magnet does not completely surround the patient.

Over the last decade, the development of much faster scans, called “functional MRI” (fMRI), has allowed us to map the functioning of various regions in the brain responsible for thought and motor control. This technique measures the change in blood flow for activities (thought, experiences, action) in the brain. The nerve cells increase their consumption of oxygen when active. Blood hemoglobin releases oxygen to active nerve cells and has somewhat different magnetic properties when oxygenated than when deoxygenated. With MRI, we can measure this and detect a blood oxygen-dependent signal. Most of the brain scans today use fMRI.

Other Medical Uses of Magnetic Fields

Currents in nerve cells and the heart create magnetic fields like any other currents. These can be measured but with some difficulty since their strengths are about 10^{-6} to 10^{-8} *less* than the Earth's magnetic field. Recording of the heart's magnetic field as it beats is called a **magnetocardiogram (MCG)**, while measurements of the brain's magnetic field is called a **magnetoencephalogram (MEG)**. Both give information that differs from that obtained by measuring the electric fields of these organs (ECGs and EEGs), but they are not yet of sufficient importance to make these difficult measurements common.

In both of these techniques, the sensors do not touch the body. MCG can be used in fetal studies, and is probably more sensitive than echocardiography. MCG also looks at the heart's electrical activity whose voltage output is too small to be recorded by surface electrodes as in EKG. It has the potential of being a rapid scan for early diagnosis of cardiac ischemia (obstruction of blood flow to the heart) or problems with the fetus.

MEG can be used to identify abnormal electrical discharges in the brain that produce weak magnetic signals. Therefore, it looks at brain activity, not just brain structure. It has been used for studies of Alzheimer's disease and epilepsy. Advances in instrumentation to measure very small magnetic fields have allowed these two techniques to be used more in recent years. What is used is a sensor called a SQUID, for superconducting quantum interference device. This operates at liquid helium temperatures and can measure magnetic fields thousands of times smaller than the Earth's.

Finally, there is a burgeoning market for magnetic cures in which magnets are applied in a variety of ways to the body, from magnetic bracelets to magnetic mattresses. The best that can be said for such practices is that they are apparently harmless, unless the magnets get close to the patient's computer or magnetic storage disks. Claims are made for a broad spectrum of benefits from cleansing the blood to giving the patient more energy, but clinical studies have not verified these claims, nor is there an identifiable mechanism by which such benefits might occur.

PhET Explorations: Magnet and Compass

Ever wonder how a compass worked to point you to the Arctic? Explore the interactions between a compass and bar magnet, and then add the Earth and find the surprising answer! Vary the magnet's strength, and see how things change both inside and outside. Use the field meter to measure how the magnetic field changes.

PhET Interactive Simulation

Figure 22.45 Magnet and Compass (http://cnx.org/content/m42388/1.4/magnet-and-compass_en.jar)

Glossary

***B*-field:** another term for magnetic field

Ampere's law: the physical law that states that the magnetic field around an electric current is proportional to the current; each segment of current produces a magnetic field like that of a long straight wire, and the total field of any shape current is the vector sum of the fields due to each segment

Biot-Savart law: a physical law that describes the magnetic field generated by an electric current in terms of a specific equation

Curie temperature: the temperature above which a ferromagnetic material cannot be magnetized

direction of magnetic field lines: the direction that the north end of a compass needle points

domains: regions within a material that behave like small bar magnets

electromagnet: an object that is temporarily magnetic when an electrical current is passed through it

electromagnetism: the use of electrical currents to induce magnetism

ferromagnetic: materials, such as iron, cobalt, nickel, and gadolinium, that exhibit strong magnetic effects

gauss: G, the unit of the magnetic field strength; $1\text{ G} = 10^{-4}\text{ T}$

Hall effect: the creation of voltage across a current-carrying conductor by a magnetic field

Hall emf: the electromotive force created by a current-carrying conductor by a magnetic field, $\varepsilon = Blv$

Lorentz force: the force on a charge moving in a magnetic field

Maxwell's equations: a set of four equations that describe electromagnetic phenomena

magnetic field lines: the pictorial representation of the strength and the direction of a magnetic field

magnetic field strength (magnitude) produced by a long straight current-carrying wire: defined as $B = \frac{\mu_0 I}{2\pi r}$, where I is the current, r is the shortest distance to the wire, and μ_0 is the permeability of free space

magnetic field strength at the center of a circular loop: defined as $B = \frac{\mu_0 I}{2R}$ where R is the radius of the loop

magnetic field strength inside a solenoid: defined as $B = \mu_0 nI$ where n is the number of loops per unit length of the solenoid $(n = N / l$, with N being the number of loops and l the length)

magnetic field: the representation of magnetic forces

magnetic force: the force on a charge produced by its motion through a magnetic field; the Lorentz force

magnetic monopoles: an isolated magnetic pole; a south pole without a north pole, or vice versa (no magnetic monopole has ever been observed)

magnetic resonance imaging (MRI): a medical imaging technique that uses magnetic fields create detailed images of internal tissues and organs

magnetized: to be turned into a magnet; to be induced to be magnetic

magnetocardiogram (MCG): a recording of the heart's magnetic field as it beats

magnetoencephalogram (MEG): a measurement of the brain's magnetic field

meter: common application of magnetic torque on a current-carrying loop that is very similar in construction to a motor; by design, the torque is proportional to I and not θ, so the needle deflection is proportional to the current

motor: loop of wire in a magnetic field; when current is passed through the loops, the magnetic field exerts torque on the loops, which rotates a shaft; electrical energy is converted to mechanical work in the process

north magnetic pole: the end or the side of a magnet that is attracted toward Earth's geographic north pole

nuclear magnetic resonance (NMR): a phenomenon in which an externally applied magnetic field interacts with the nuclei of certain atoms

permeability of free space: the measure of the ability of a material, in this case free space, to support a magnetic field; the constant $\mu_0 = 4\pi \times 10^{-7}\ \text{T} \cdot \text{m/A}$

right hand rule 1 (RHR-1): the rule to determine the direction of the magnetic force on a positive moving charge: when the thumb of the right hand points in the direction of the charge's velocity $\mathbf{v}$ and the fingers point in the direction of the magnetic field $\mathbf{B}$, then the force on the charge is perpendicular and away from the palm; the force on a negative charge is perpendicular and into the palm

right hand rule 2 (RHR-2): a rule to determine the direction of the magnetic field induced by a current-carrying wire: Point the thumb of the right hand in the direction of current, and the fingers curl in the direction of the magnetic field loops

solenoid: a thin wire wound into a coil that produces a magnetic field when an electric current is passed through it

south magnetic pole: the end or the side of a magnet that is attracted toward Earth's geographic south pole

tesla: T, the SI unit of the magnetic field strength; $1\ \text{T} = \frac{1\ \text{N}}{\text{A} \cdot \text{m}}$

Section Summary

22.1 Magnets

- Magnetism is a subject that includes the properties of magnets, the effect of the magnetic force on moving charges and currents, and the creation of magnetic fields by currents.
- There are two types of magnetic poles, called the north magnetic pole and south magnetic pole.
- North magnetic poles are those that are attracted toward the Earth's geographic north pole.
- Like poles repel and unlike poles attract.
- Magnetic poles always occur in pairs of north and south—it is not possible to isolate north and south poles.

22.2 Ferromagnets and Electromagnets

- Magnetic poles always occur in pairs of north and south—it is not possible to isolate north and south poles.
- All magnetism is created by electric current.
- Ferromagnetic materials, such as iron, are those that exhibit strong magnetic effects.
- The atoms in ferromagnetic materials act like small magnets (due to currents within the atoms) and can be aligned, usually in millimeter-sized regions called domains.
- Domains can grow and align on a larger scale, producing permanent magnets. Such a material is magnetized, or induced to be magnetic.
- Above a material's Curie temperature, thermal agitation destroys the alignment of atoms, and ferromagnetism disappears.
- Electromagnets employ electric currents to make magnetic fields, often aided by induced fields in ferromagnetic materials.

22.3 Magnetic Fields and Magnetic Field Lines

- Magnetic fields can be pictorially represented by magnetic field lines, the properties of which are as follows:

1. The field is tangent to the magnetic field line.
2. Field strength is proportional to the line density.
3. Field lines cannot cross.
4. Field lines are continuous loops.

22.4 Magnetic Field Strength: Force on a Moving Charge in a Magnetic Field

- Magnetic fields exert a force on a moving charge *q*, the magnitude of which is

$$F = qvB\sin\theta,$$

where θ is the angle between the directions of v and B.

- The SI unit for magnetic field strength B is the tesla (T), which is related to other units by

$$1\text{ T} = \frac{1\text{ N}}{\text{C}\cdot\text{m/s}} = \frac{1\text{ N}}{\text{A}\cdot\text{m}}.$$

- The *direction* of the force on a moving charge is given by right hand rule 1 (RHR-1): Point the thumb of the right hand in the direction of v, the fingers in the direction of B, and a perpendicular to the palm points in the direction of F.
- The force is perpendicular to the plane formed by $\mathbf{v}$ and $\mathbf{B}$. Since the force is zero if $\mathbf{v}$ is parallel to $\mathbf{B}$, charged particles often follow magnetic field lines rather than cross them.

22.5 Force on a Moving Charge in a Magnetic Field: Examples and Applications

- Magnetic force can supply centripetal force and cause a charged particle to move in a circular path of radius

$$r = \frac{mv}{qB},$$

where v is the component of the velocity perpendicular to B for a charged particle with mass m and charge q.

22.6 The Hall Effect

- The Hall effect is the creation of voltage ε, known as the Hall emf, across a current-carrying conductor by a magnetic field.
- The Hall emf is given by

$$\varepsilon = Blv\ (B,\ v,\text{ and } l,\text{ mutually perpendicular})$$

for a conductor of width l through which charges move at a speed v.

22.7 Magnetic Force on a Current-Carrying Conductor

- The magnetic force on current-carrying conductors is given by

$$F = IlB\sin\theta,$$

where I is the current, l is the length of a straight conductor in a uniform magnetic field B, and θ is the angle between I and B. The force follows RHR-1 with the thumb in the direction of I.

22.8 Torque on a Current Loop: Motors and Meters

- The torque τ on a current-carrying loop of any shape in a uniform magnetic field. is

$$\tau = NIAB\sin\theta,$$

where N is the number of turns, I is the current, A is the area of the loop, B is the magnetic field strength, and θ is the angle between the perpendicular to the loop and the magnetic field.

22.9 Magnetic Fields Produced by Currents: Ampere's Law

- The strength of the magnetic field created by current in a long straight wire is given by

$$B = \frac{\mu_0 I}{2\pi r}(\text{long straight wire}),$$

where I is the current, r is the shortest distance to the wire, and the constant $\mu_0 = 4\pi\times10^{-7}\text{ T}\cdot\text{m/A}$ is the permeability of free space.

- The direction of the magnetic field created by a long straight wire is given by right hand rule 2 (RHR-2): *Point the thumb of the right hand in the direction of current, and the fingers curl in the direction of the magnetic field loops* created by it.
- The magnetic field created by current following any path is the sum (or integral) of the fields due to segments along the path (magnitude and direction as for a straight wire), resulting in a general relationship between current and field known as Ampere's law.
- The magnetic field strength at the center of a circular loop is given by

$$B = \frac{\mu_0 I}{2R}(\text{at center of loop}),$$

where R is the radius of the loop. This equation becomes $B = \mu_0 nI/(2R)$ for a flat coil of N loops. RHR-2 gives the direction of the field about the loop. A long coil is called a solenoid.

- The magnetic field strength inside a solenoid is

$$B = \mu_0 nI\ \ (\text{inside a solenoid}),$$

where n is the number of loops per unit length of the solenoid. The field inside is very uniform in magnitude and direction.

22.10 Magnetic Force between Two Parallel Conductors

- The force between two parallel currents I_1 and I_2, separated by a distance r, has a magnitude per unit length given by

$$\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi r}.$$

- The force is attractive if the currents are in the same direction, repulsive if they are in opposite directions.

22.11 More Applications of Magnetism

- Crossed (perpendicular) electric and magnetic fields act as a velocity filter, giving equal and opposite forces on any charge with velocity perpendicular to the fields and of magnitude

$$v = \frac{E}{B}.$$

Conceptual Questions

22.1 Magnets

1. Volcanic and other such activity at the mid-Atlantic ridge extrudes material to fill the gap between separating tectonic plates associated with continental drift. The magnetization of rocks is found to reverse in a coordinated manner with distance from the ridge. What does this imply about the Earth's magnetic field and how could the knowledge of the spreading rate be used to give its historical record?

22.3 Magnetic Fields and Magnetic Field Lines

2. Explain why the magnetic field would not be unique (that is, not have a single value) at a point in space where magnetic field lines might cross. (Consider the direction of the field at such a point.)

3. List the ways in which magnetic field lines and electric field lines are similar. For example, the field direction is tangent to the line at any point in space. Also list the ways in which they differ. For example, electric force is parallel to electric field lines, whereas magnetic force on moving charges is perpendicular to magnetic field lines.

4. Noting that the magnetic field lines of a bar magnet resemble the electric field lines of a pair of equal and opposite charges, do you expect the magnetic field to rapidly decrease in strength with distance from the magnet? Is this consistent with your experience with magnets?

5. Is the Earth's magnetic field parallel to the ground at all locations? If not, where is it parallel to the surface? Is its strength the same at all locations? If not, where is it greatest?

22.4 Magnetic Field Strength: Force on a Moving Charge in a Magnetic Field

6. If a charged particle moves in a straight line through some region of space, can you say that the magnetic field in that region is necessarily zero?

22.5 Force on a Moving Charge in a Magnetic Field: Examples and Applications

7. How can the motion of a charged particle be used to distinguish between a magnetic and an electric field?

8. High-velocity charged particles can damage biological cells and are a component of radiation exposure in a variety of locations ranging from research facilities to natural background. Describe how you could use a magnetic field to shield yourself.

9. If a cosmic ray proton approaches the Earth from outer space along a line toward the center of the Earth that lies in the plane of the equator, in what direction will it be deflected by the Earth's magnetic field? What about an electron? A neutron?

10. What are the signs of the charges on the particles in Figure 22.46?

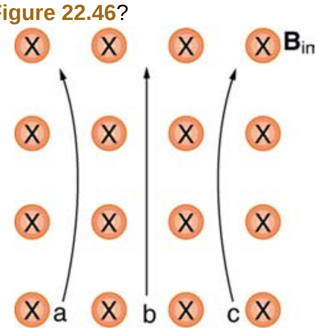


Figure 22.46

11. Which of the particles in Figure 22.47 has the greatest velocity, assuming they have identical charges and masses?

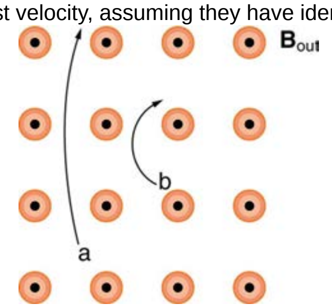


Figure 22.47

12. Which of the particles in Figure 22.47 has the greatest mass, assuming all have identical charges and velocities?

13. While operating, a high-precision TV monitor is placed on its side during maintenance. The image on the monitor changes color and blurs slightly. Discuss the possible relation of these effects to the Earth's magnetic field.

22.6 The Hall Effect

14. Discuss how the Hall effect could be used to obtain information on free charge density in a conductor. (Hint: Consider how drift velocity and current are related.)

22.7 Magnetic Force on a Current-Carrying Conductor

15. Draw a sketch of the situation in **Figure 22.30** showing the direction of electrons carrying the current, and use RHR-1 to verify the direction of the force on the wire.

16. Verify that the direction of the force in an MHD drive, such as that in **Figure 22.32**, does not depend on the sign of the charges carrying the current across the fluid.

17. Why would a magnetohydrodynamic drive work better in ocean water than in fresh water? Also, why would superconducting magnets be desirable?

18. Which is more likely to interfere with compass readings, AC current in your refrigerator or DC current when you start your car? Explain.

22.8 Torque on a Current Loop: Motors and Meters

19. Draw a diagram and use RHR-1 to show that the forces on the top and bottom segments of the motor's current loop in **Figure 22.34** are vertical and produce no torque about the axis of rotation.

22.9 Magnetic Fields Produced by Currents: Ampere's Law

20. Make a drawing and use RHR-2 to find the direction of the magnetic field of a current loop in a motor (such as in **Figure 22.34**). Then show that the direction of the torque on the loop is the same as produced by like poles repelling and unlike poles attracting.

22.10 Magnetic Force between Two Parallel Conductors

21. Is the force attractive or repulsive between the hot and neutral lines hung from power poles? Why?

22. If you have three parallel wires in the same plane, as in **Figure 22.48**, with currents in the outer two running in opposite directions, is it possible for the middle wire to be repelled by both? Attracted by both? Explain.

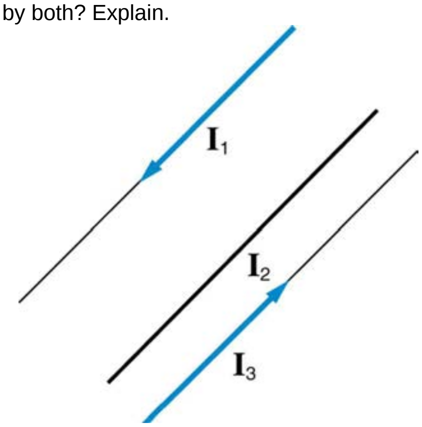


Figure 22.48 Three parallel coplanar wires with currents in the outer two in opposite directions.

23. Suppose two long straight wires run perpendicular to one another without touching. Does one exert a net force on the other? If so, what is its direction? Does one exert a net torque on the other? If so, what is its direction? Justify your responses by using the right hand rules.

24. Use the right hand rules to show that the force between the two loops in **Figure 22.49** is attractive if the currents are in the same direction and repulsive if they are in opposite directions. Is this consistent with like poles of the loops repelling and unlike poles of the loops attracting? Draw sketches to justify your answers.

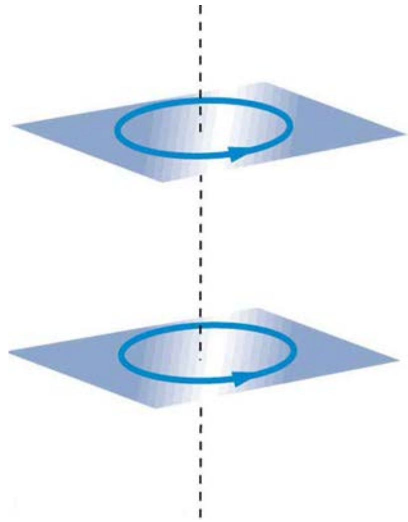


Figure 22.49 Two loops of wire carrying currents can exert forces and torques on one another.

25. If one of the loops in Figure 22.49 is tilted slightly relative to the other and their currents are in the same direction, what are the directions of the torques they exert on each other? Does this imply that the poles of the bar magnet-like fields they create will line up with each other if the loops are allowed to rotate?

26. Electric field lines can be shielded by the Faraday cage effect. Can we have magnetic shielding? Can we have gravitational shielding?

22.11 More Applications of Magnetism

27. Measurements of the weak and fluctuating magnetic fields associated with brain activity are called magnetoencephalograms (MEGs). Do the brain's magnetic fields imply coordinated or uncoordinated nerve impulses? Explain.

28. Discuss the possibility that a Hall voltage would be generated on the moving heart of a patient during MRI imaging. Also discuss the same effect on the wires of a pacemaker. (The fact that patients with pacemakers are not given MRIs is significant.)

29. A patient in an MRI unit turns his head quickly to one side and experiences momentary dizziness and a strange taste in his mouth. Discuss the possible causes.

30. You are told that in a certain region there is either a uniform electric or magnetic field. What measurement or observation could you make to determine the type? (Ignore the Earth's magnetic field.)

31. An example of magnetohydrodynamics (MHD) comes from the flow of a river (salty water). This fluid interacts with the Earth's magnetic field to produce a potential difference between the two river banks. How would you go about calculating the potential difference?

32. Draw gravitational field lines between 2 masses, electric field lines between a positive and a negative charge, electric field lines between 2 positive charges and magnetic field lines around a magnet. Qualitatively describe the differences between the fields and the entities responsible for the field lines.

Problems & Exercises

22.4 Magnetic Field Strength: Force on a Moving Charge in a Magnetic Field

33. What is the direction of the magnetic force on a positive charge that moves as shown in each of the six cases shown in Figure 22.50?

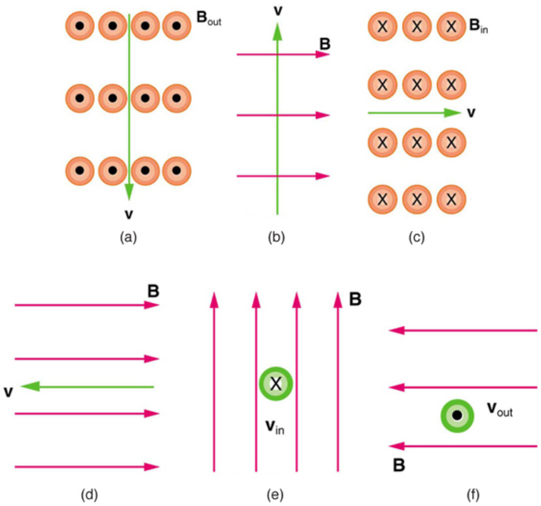


Figure 22.50

34. Repeat Exercise 22.33 for a negative charge.

35. What is the direction of the velocity of a negative charge that experiences the magnetic force shown in each of the three cases in Figure 22.51, assuming it moves perpendicular to $\mathbf{B}$?

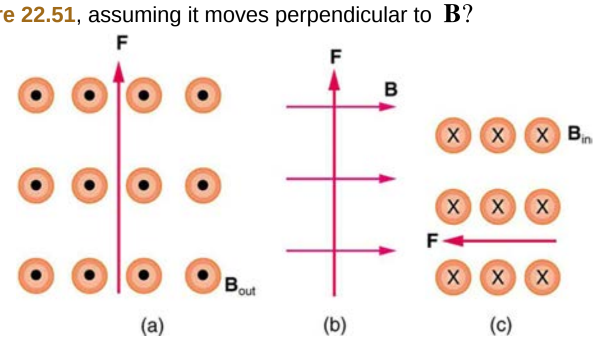


Figure 22.51

36. Repeat Exercise 22.35 for a positive charge.

37. What is the direction of the magnetic field that produces the magnetic force on a positive charge as shown in each of the three cases in the figure below, assuming $\mathbf{B}$ is perpendicular to $\mathbf{v}$?

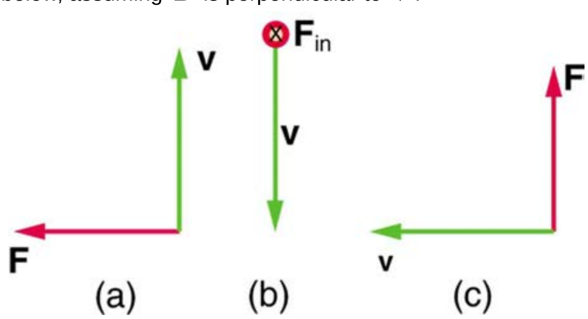


Figure 22.52

38. Repeat Exercise 22.37 for a negative charge.

39. What is the maximum force on an aluminum rod with a $0.100\text{-}\mu\text{C}$ charge that you pass between the poles of a 1.50-T permanent magnet at a speed of 5.00 m/s? In what direction is the force?

40. (a) Aircraft sometimes acquire small static charges. Suppose a supersonic jet has a $0.500\text{-}\mu\text{C}$ charge and flies due west at a speed of 660 m/s over the Earth's south magnetic pole, where the $8.00\times10^{-5}\text{-T}$ magnetic field points straight up. What are the direction and the magnitude of the magnetic force on the plane? (b) Discuss whether the value obtained in part (a) implies this is a significant or negligible effect.

41. (a) A cosmic ray proton moving toward the Earth at $5.00\times10^{7}\text{ m/s}$ experiences a magnetic force of $1.70\times10^{-16}\text{ N}$. What is the strength of the magnetic field if there is a 45° angle between it and the proton's velocity? (b) Is the value obtained in part (a) consistent with the known strength of the Earth's magnetic field on its surface? Discuss.

42. An electron moving at $4.00\times10^{3}\text{ m/s}$ in a 1.25-T magnetic field experiences a magnetic force of $1.40\times10^{-16}\text{ N}$. What angle does the velocity of the electron make with the magnetic field? There are two answers.

43. (a) A physicist performing a sensitive measurement wants to limit the magnetic force on a moving charge in her equipment to less than $1.00\times10^{-12}\text{ N}$. What is the greatest the charge can be if it moves at a maximum speed of 30.0 m/s in the Earth's field? (b) Discuss whether it would be difficult to limit the charge to less than the value found in (a) by comparing it with typical static electricity and noting that static is often absent.

22.5 Force on a Moving Charge in a Magnetic Field: Examples and Applications

If you need additional support for these problems, see More Applications of Magnetism.

44. A cosmic ray electron moves at $7.50\times10^{6}\text{ m/s}$ perpendicular to the Earth's magnetic field at an altitude where field strength is $1.00\times10^{-5}\text{ T}$. What is the radius of the circular path the electron follows?

45. A proton moves at $7.50\times10^{7}\text{ m/s}$ perpendicular to a magnetic field. The field causes the proton to travel in a circular path of radius 0.800 m. What is the field strength?

46. (a) Viewers of *Star Trek* hear of an antimatter drive on the Starship *Enterprise*. One possibility for such a futuristic energy source is to store antimatter charged particles in a vacuum chamber, circulating in a magnetic field, and then extract them as needed. Antimatter annihilates with normal matter, producing pure energy. What strength magnetic field is needed to hold antiprotons, moving at $5.00\times10^{7}\text{ m/s}$ in a circular path 2.00 m in radius? Antiprotons have the same mass as protons but the opposite (negative) charge. (b) Is this field strength obtainable with today's technology or is it a futuristic possibility?

47. (a) An oxygen-16 ion with a mass of $2.66\times10^{-26}\text{ kg}$ travels at $5.00\times10^{6}\text{ m/s}$ perpendicular to a 1.20-T magnetic field, which makes it move in a circular arc with a 0.231-m radius. What positive charge is on the ion? (b) What is the ratio of this charge to the charge of an electron? (c) Discuss why the ratio found in (b) should be an integer.

48. What radius circular path does an electron travel if it moves at the same speed and in the same magnetic field as the proton in Exercise 22.45?

49. A velocity selector in a mass spectrometer uses a 0.100-T magnetic field. (a) What electric field strength is needed to select a speed of $4.00\times10^{6}\text{ m/s}$? (b) What is the voltage between the plates if they are separated by 1.00 cm?

50. An electron in a TV CRT moves with a speed of $6.00\times10^{7}\text{ m/s}$, in a direction perpendicular to the Earth's field, which has a strength of $5.00\times10^{-5}\text{ T}$. (a) What strength electric field must be applied perpendicular to the Earth's field to make the electron moves in a straight line? (b) If this is done between plates separated by 1.00 cm, what is the voltage applied? (Note that TVs are usually surrounded by a ferromagnetic material to shield against external magnetic fields and avoid the need for such a correction.)

51. (a) At what speed will a proton move in a circular path of the same radius as the electron in **Exercise 22.44**? (b) What would the radius of the path be if the proton had the same speed as the electron? (c) What would the radius be if the proton had the same kinetic energy as the electron? (d) The same momentum?

52. A mass spectrometer is being used to separate common oxygen-16 from the much rarer oxygen-18, taken from a sample of old glacial ice. (The relative abundance of these oxygen isotopes is related to climatic temperature at the time the ice was deposited.) The ratio of the masses of these two ions is 16 to 18, the mass of oxygen-16 is $2.66\times10^{-26}\text{ kg},$ and they are singly charged and travel at $5.00\times10^{6}\text{ m/s}$ in a 1.20-T magnetic field. What is the separation between their paths when they hit a target after traversing a semicircle?

53. (a) Triply charged uranium-235 and uranium-238 ions are being separated in a mass spectrometer. (The much rarer uranium-235 is used as reactor fuel.) The masses of the ions are $3.90\times10^{-25}\text{ kg}$ and $3.95\times10^{-25}\text{ kg}$, respectively, and they travel at $3.00\times10^{5}\text{ m/s}$ in a 0.250-T field. What is the separation between their paths when they hit a target after traversing a semicircle? (b) Discuss whether this distance between their paths seems to be big enough to be practical in the separation of uranium-235 from uranium-238.

22.6 The Hall Effect

54. A large water main is 2.50 m in diameter and the average water velocity is 6.00 m/s. Find the Hall voltage produced if the pipe runs perpendicular to the Earth's $5.00\times10^{-5}\text{-T}$ field.

55. What Hall voltage is produced by a 0.200-T field applied across a 2.60-cm-diameter aorta when blood velocity is 60.0 cm/s?

56. (a) What is the speed of a supersonic aircraft with a 17.0-m wingspan, if it experiences a 1.60-V Hall voltage between its wing tips when in level flight over the north magnetic pole, where the Earth's field strength is $8.00\times10^{-5}\text{ T}$? (b) Explain why very little current flows as a result of this Hall voltage.

57. A nonmechanical water meter could utilize the Hall effect by applying a magnetic field across a metal pipe and measuring the Hall voltage produced. What is the average fluid velocity in a 3.00-cm-diameter pipe, if a 0.500-T field across it creates a 60.0-mV Hall voltage?

58. Calculate the Hall voltage induced on a patient's heart while being scanned by an MRI unit. Approximate the conducting path on the heart wall by a wire 7.50 cm long that moves at 10.0 cm/s perpendicular to a 1.50-T magnetic field.

59. A Hall probe calibrated to read $1.00\ \mu\text{V}$ when placed in a 2.00-T field is placed in a 0.150-T field. What is its output voltage?

60. Using information in **Example 20.6**, what would the Hall voltage be if a 2.00-T field is applied across a 10-gauge copper wire (2.588 mm in diameter) carrying a 20.0-A current?

61. Show that the Hall voltage across wires made of the same material, carrying identical currents, and subjected to the same magnetic field is inversely proportional to their diameters. (Hint: Consider how drift velocity depends on wire diameter.)

62. A patient with a pacemaker is mistakenly being scanned for an MRI image. A 10.0-cm-long section of pacemaker wire moves at a speed of 10.0 cm/s perpendicular to the MRI unit's magnetic field and a 20.0-mV Hall voltage is induced. What is the magnetic field strength?

22.7 Magnetic Force on a Current-Carrying Conductor

63. What is the direction of the magnetic force on the current in each of the six cases in **Figure 22.53**?

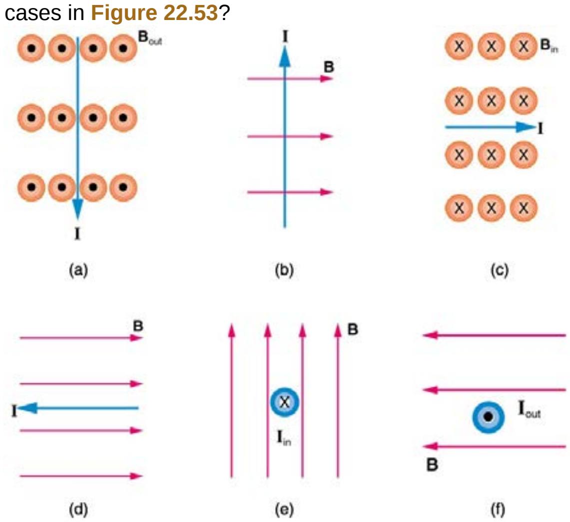


Figure 22.53

64. What is the direction of a current that experiences the magnetic force shown in each of the three cases in **Figure 22.54**, assuming the current runs perpendicular to B?

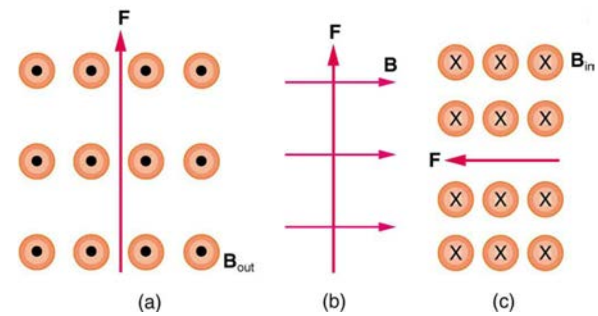


Figure 22.54

65. What is the direction of the magnetic field that produces the magnetic force shown on the currents in each of the three cases in **Figure 22.55**, assuming $\mathbf{B}$ is perpendicular to $\mathbf{I}$?

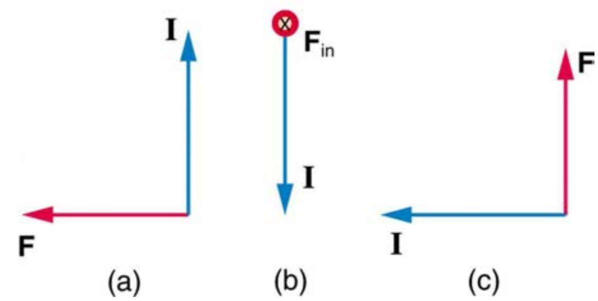


Figure 22.55

66. (a) What is the force per meter on a lightning bolt at the equator that carries 20,000 A perpendicular to the Earth's $3.00\times10^{-5}\text{-T}$ field? (b) What is the direction of the force if the current is straight up and the Earth's field direction is due north, parallel to the ground?

67. (a) A DC power line for a light-rail system carries 1000 A at an angle of 30.0° to the Earth's $5.00\times10^{-5}\text{ -T}$ field. What is the force on a 100-m section of this line? (b) Discuss practical concerns this presents, if any.

68. What force is exerted on the water in an MHD drive utilizing a 25.0-cm-diameter tube, if 100-A current is passed across the tube that is perpendicular to a 2.00-T magnetic field? (The relatively small size of this force indicates the need for very large currents and magnetic fields to make practical MHD drives.)

69. A wire carrying a 30.0-A current passes between the poles of a strong magnet that is perpendicular to its field and experiences a 2.16-N force on the 4.00 cm of wire in the field. What is the average field strength?

70. (a) A 0.750-m-long section of cable carrying current to a car starter motor makes an angle of 60° with the Earth's $5.50\times10^{-5}\ \text{T}$ field. What is the current when the wire experiences a force of $7.00\times10^{-3}\ \text{N}$? (b) If you run the wire between the poles of a strong horseshoe magnet, subjecting 5.00 cm of it to a 1.75-T field, what force is exerted on this segment of wire?

71. (a) What is the angle between a wire carrying an 8.00-A current and the 1.20-T field it is in if 50.0 cm of the wire experiences a magnetic force of 2.40 N? (b) What is the force on the wire if it is rotated to make an angle of 90° with the field?

72. The force on the rectangular loop of wire in the magnetic field in Figure 22.56 can be used to measure field strength. The field is uniform, and the plane of the loop is perpendicular to the field. (a) What is the direction of the magnetic force on the loop? Justify the claim that the forces on the sides of the loop are equal and opposite, independent of how much of the loop is in the field and do not affect the net force on the loop. (b) If a current of 5.00 A is used, what is the force per tesla on the 20.0-cm-wide loop?

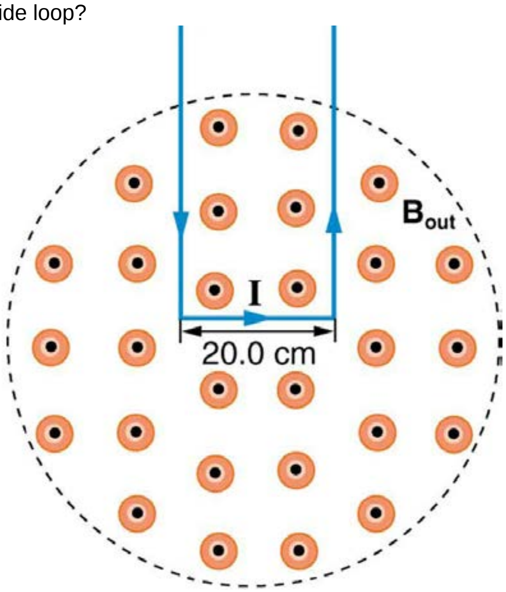


Figure 22.56 A rectangular loop of wire carrying a current is perpendicular to a magnetic field. The field is uniform in the region shown and is zero outside that region.

22.8 Torque on a Current Loop: Motors and Meters

73. (a) By how many percent is the torque of a motor decreased if its permanent magnets lose 5.0% of their strength? (b) How many percent would the current need to be increased to return the torque to original values?

74. (a) What is the maximum torque on a 150-turn square loop of wire 18.0 cm on a side that carries a 50.0-A current in a 1.60-T field? (b) What is the torque when θ is 10.9°?

75. Find the current through a loop needed to create a maximum torque of $9.00\ \text{N}\cdot\text{m}.$ The loop has 50 square turns that are 15.0 cm on a side and is in a uniform 0.800-T magnetic field.

76. Calculate the magnetic field strength needed on a 200-turn square loop 20.0 cm on a side to create a maximum torque of $300\ \text{N}\cdot\text{m}$ if the loop is carrying 25.0 A.

77. Since the equation for torque on a current-carrying loop is $\tau = NIAB\sin\theta$, the units of $\text{N}\cdot\text{m}$ must equal units of $\text{A}\cdot\text{m}^2\ \text{T}$. Verify this.

78. (a) At what angle θ is the torque on a current loop 90.0% of maximum? (b) 50.0% of maximum? (c) 10.0% of maximum?

79. A proton has a magnetic field due to its spin on its axis. The field is similar to that created by a circular current loop $0.650\times10^{-15}\ \text{m}$ in radius with a current of $1.05\times10^{4}\ \text{A}$ (no kidding). Find the maximum torque on a proton in a 2.50-T field. (This is a significant torque on a small particle.)

80. (a) A 200-turn circular loop of radius 50.0 cm is vertical, with its axis on an east-west line. A current of 100 A circulates clockwise in the loop when viewed from the east. The Earth's field here is due north, parallel to the ground, with a strength of $3.00\times10^{-5}\ \text{T}$. What are the direction and magnitude of the torque on the loop? (b) Does this device have any practical applications as a motor?

81. Repeat Exercise 22.73, but with the loop lying flat on the ground with its current circulating counterclockwise (when viewed from above) in a location where the Earth's field is north, but at an angle 45.0° below the horizontal and with a strength of $6.00\times10^{-5}\ \text{T}$.

22.10 Magnetic Force between Two Parallel Conductors

82. (a) The hot and neutral wires supplying DC power to a light-rail commuter train carry 800 A and are separated by 75.0 cm. What is the magnitude and direction of the force between 50.0 m of these wires? (b) Discuss the practical consequences of this force, if any.

83. The force per meter between the two wires of a jumper cable being used to start a stalled car is 0.225 N/m. (a) What is the current in the wires, given they are separated by 2.00 cm? (b) Is the force attractive or repulsive?

84. A 2.50-m segment of wire supplying current to the motor of a submerged submarine carries 1000 A and feels a 4.00-N repulsive force from a parallel wire 5.00 cm away. What is the direction and magnitude of the current in the other wire?

85. The wire carrying 400 A to the motor of a commuter train feels an attractive force of $4.00\times10^{-3}\ \text{N/m}$ due to a parallel wire carrying 5.00 A to a headlight. (a) How far apart are the wires? (b) Are the currents in the same direction?

86. An AC appliance cord has its hot and neutral wires separated by 3.00 mm and carries a 5.00-A current. (a) What is the average force per meter between the wires in the cord? (b) What is the maximum force per meter between the wires? (c) Are the forces attractive or repulsive? (d) Do appliance cords need any special design features to compensate for these forces?

87. Figure 22.57 shows a long straight wire near a rectangular current loop. What is the direction and magnitude of the total force on the loop?

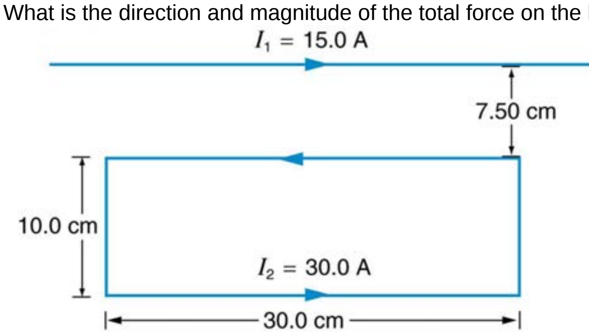


Figure 22.57

88. Find the direction and magnitude of the force that each wire experiences in Figure 22.58(a) by, using vector addition.

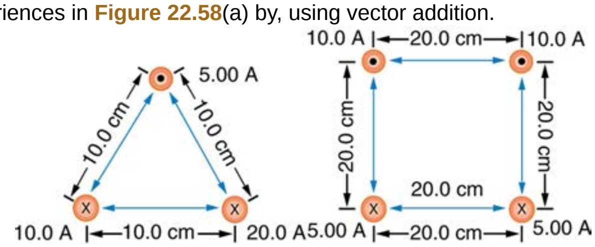


Figure 22.58

89. Find the direction and magnitude of the force that each wire experiences in **Figure 22.58**(b), using vector addition.

22.11 More Applications of Magnetism

90. Indicate whether the magnetic field created in each of the three situations shown in **Figure 22.59** is into or out of the page on the left and right of the current.

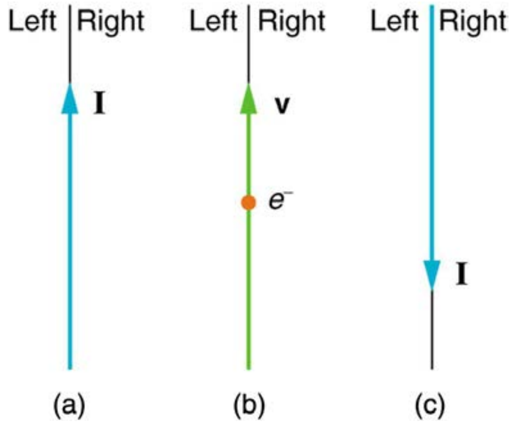


Figure 22.59

91. What are the directions of the fields in the center of the loop and coils shown in **Figure 22.60**?

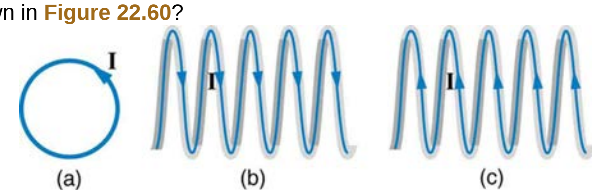


Figure 22.60

92. What are the directions of the currents in the loop and coils shown in **Figure 22.61**?

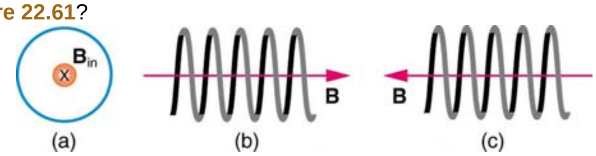


Figure 22.61

93. To see why an MRI utilizes iron to increase the magnetic field created by a coil, calculate the current needed in a 400-loop-per-meter circular coil 0.660 m in radius to create a 1.20-T field (typical of an MRI instrument) at its center with no iron present. The magnetic field of a proton is approximately like that of a circular current loop $0.650\times10^{-15}\text{ m}$ in radius carrying $1.05\times10^{4}\text{ A}$. What is the field at the center of such a loop?

94. Inside a motor, 30.0 A passes through a 250-turn circular loop that is 10.0 cm in radius. What is the magnetic field strength created at its center?

95. Nonnuclear submarines use batteries for power when submerged. (a) Find the magnetic field 50.0 cm from a straight wire carrying 1200 A from the batteries to the drive mechanism of a submarine. (b) What is the field if the wires to and from the drive mechanism are side by side? (c) Discuss the effects this could have for a compass on the submarine that is not shielded.

96. How strong is the magnetic field inside a solenoid with 10,000 turns per meter that carries 20.0 A?

97. What current is needed in the solenoid described in **Exercise 22.90** to produce a magnetic field 10^4 times the Earth's magnetic field of $5.00\times10^{-5}\text{ T}$?

98. How far from the starter cable of a car, carrying 150 A, must you be to experience a field less than the Earth's $(5.00\times10^{-5}\text{ T})$? Assume a long straight wire carries the current. (In practice, the body of your car shields the dashboard compass.)

99. Measurements affect the system being measured, such as the current loop in **Figure 22.56**. (a) Estimate the field the loop creates by calculating the field at the center of a circular loop 20.0 cm in diameter carrying 5.00 A. (b) What is the smallest field strength this loop can be used to measure, if its field must alter the measured field by less than 0.0100%?

100. **Figure 22.62** shows a long straight wire just touching a loop carrying a current I_1. Both lie in the same plane. (a) What direction must the current I_2 in the straight wire have to create a field at the center of the loop in the direction opposite to that created by the loop? (b) What is the ratio of I_1/I_2 that gives zero field strength at the center of the loop? (c) What is the direction of the field directly above the loop under this circumstance?

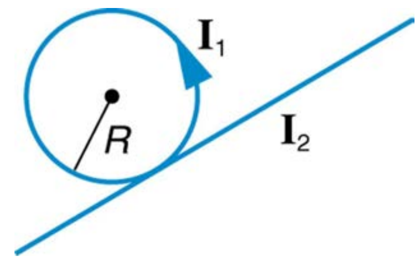


Figure 22.62

101. Find the magnitude and direction of the magnetic field at the point equidistant from the wires in **Figure 22.58**(a), using the rules of vector addition to sum the contributions from each wire.

102. Find the magnitude and direction of the magnetic field at the point equidistant from the wires in **Figure 22.58**(b), using the rules of vector addition to sum the contributions from each wire.

103. What current is needed in the top wire in **Figure 22.58**(a) to produce a field of zero at the point equidistant from the wires, if the currents in the bottom two wires are both 10.0 A into the page?

104. Calculate the size of the magnetic field 20 m below a high voltage power line. The line carries 450 MW at a voltage of 300,000 V.

105. Integrated Concepts

(a) A pendulum is set up so that its bob (a thin copper disk) swings between the poles of a permanent magnet as shown in **Figure 22.63**. What is the magnitude and direction of the magnetic force on the bob at the lowest point in its path, if it has a positive $0.250\ \mu\text{C}$ charge and is released from a height of 30.0 cm above its lowest point? The magnetic field strength is 1.50 T. (b) What is the acceleration of the bob at the bottom of its swing if its mass is 30.0 grams and it is hung from a flexible string? Be certain to include a free-body diagram as part of your analysis.

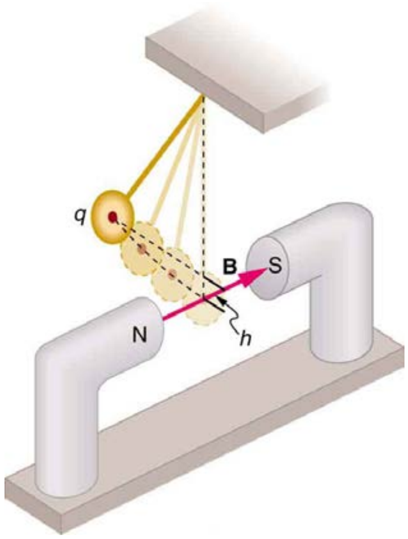


Figure 22.63

106. Integrated Concepts

(a) What voltage will accelerate electrons to a speed of $6.00\times10^{-7}\ \text{m/s}$? (b) Find the radius of curvature of the path of a *proton* accelerated through this potential in a 0.500-T field and compare this with the radius of curvature of an electron accelerated through the same potential.

107. Integrated Concepts

Find the radius of curvature of the path of a 25.0-MeV proton moving perpendicularly to the 1.20-T field of a cyclotron.

108. Integrated Concepts

To construct a nonmechanical water meter, a 0.500-T magnetic field is placed across the supply water pipe to a home and the Hall voltage is recorded. (a) Find the flow rate in liters per second through a 3.00-cm-diameter pipe if the Hall voltage is 60.0 mV. (b) What would the Hall voltage be for the same flow rate through a 10.0-cm-diameter pipe with the same field applied?

109. Integrated Concepts

(a) Using the values given for an MHD drive in **Exercise 22.91**, and assuming the force is uniformly applied to the fluid, calculate the pressure created in N/m^2. (b) Is this a significant fraction of an atmosphere?

110. Integrated Concepts

(a) Calculate the maximum torque on a 50-turn, 1.50 cm radius circular current loop carrying $50\ \mu\text{A}$ in a 0.500-T field. (b) If this coil is to be used in a galvanometer that reads $50\ \mu\text{A}$ full scale, what force constant spring must be used, if it is attached 1.00 cm from the axis of rotation and is stretched by the 60° arc moved?

111. Integrated Concepts

A current balance used to define the ampere is designed so that the current through it is constant, as is the distance between wires. Even so, if the wires change length with temperature, the force between them will change. What percent change in force per degree will occur if the wires are copper?

112. Integrated Concepts

(a) Show that the period of the circular orbit of a charged particle moving perpendicularly to a uniform magnetic field is $T = 2\pi m/(qB)$. (b) What is the frequency f ? (c) What is the angular velocity ω ? Note that these results are independent of the velocity and radius of the orbit and, hence, of the energy of the particle. (**Figure 22.64**.)

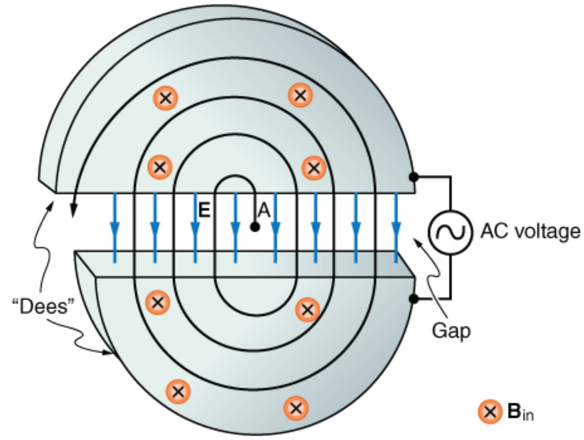


Figure 22.64 Cyclotrons accelerate charged particles orbiting in a magnetic field by placing an AC voltage on the metal Dees, between which the particles move, so that energy is added twice each orbit. The frequency is constant, since it is independent of the particle energy—the radius of the orbit simply increases with energy until the particles approach the edge and are extracted for various experiments and applications.

113. Integrated Concepts

A cyclotron accelerates charged particles as shown in **Figure 22.64**. Using the results of the previous problem, calculate the frequency of the accelerating voltage needed for a proton in a 1.20-T field.

114. Integrated Concepts

(a) A 0.140-kg baseball, pitched at 40.0 m/s horizontally and perpendicular to the Earth's horizontal $5.00\times10^{-5}\ \text{T}$ field, has a 100-nC charge on it. What distance is it deflected from its path by the magnetic force, after traveling 30.0 m horizontally? (b) Would you suggest this as a secret technique for a pitcher to throw curve balls?

115. Integrated Concepts

(a) What is the direction of the force on a wire carrying a current due east in a location where the Earth's field is due north? Both are parallel to the ground. (b) Calculate the force per meter if the wire carries 20.0 A and the field strength is $3.00\times10^{-5}\ \text{T}$. (c) What diameter copper wire would have its weight supported by this force? (d) Calculate the resistance per meter and the voltage per meter needed.

116. Integrated Concepts

One long straight wire is to be held directly above another by repulsion between their currents. The lower wire carries 100 A and the wire 7.50 cm above it is 10-gauge (2.588 mm diameter) copper wire. (a) What current must flow in the upper wire, neglecting the Earth's field? (b) What is the smallest current if the Earth's $3.00\times10^{-5}\ \text{T}$ field is parallel to the ground and is not neglected? (c) Is the supported wire in a stable or unstable equilibrium if displaced vertically? If displaced horizontally?

117. Unreasonable Results

(a) Find the charge on a baseball, thrown at 35.0 m/s perpendicular to the Earth's $5.00\times10^{-5}\ \text{T}$ field, that experiences a 1.00-N magnetic force. (b) What is unreasonable about this result? (c) Which assumption or premise is responsible?

118. Unreasonable Results

A charged particle having mass $6.64\times10^{-27}\ \text{kg}$ (that of a helium atom) moving at $8.70\times10^{5}\ \text{m/s}$ perpendicular to a 1.50-T magnetic field travels in a circular path of radius 16.0 mm. (a) What is the charge of the particle? (b) What is unreasonable about this result? (c) Which assumptions are responsible?

119. Unreasonable Results

An inventor wants to generate 120-V power by moving a 1.00-m-long wire perpendicular to the Earth's $5.00\times10^{-5}\ \text{T}$ field. (a) Find the speed with which the wire must move. (b) What is unreasonable about this result? (c) Which assumption is responsible?

120. Unreasonable Results

Frustrated by the small Hall voltage obtained in blood flow measurements, a medical physicist decides to increase the applied magnetic field strength to get a 0.500-V output for blood moving at 30.0 cm/s in a 1.50-cm-diameter vessel. (a) What magnetic field strength is needed? (b) What is unreasonable about this result? (c) Which premise is responsible?

121. Unreasonable Results

A surveyor 100 m from a long straight 200-kV DC power line suspects that its magnetic field may equal that of the Earth and affect compass readings. (a) Calculate the current in the wire needed to create a $5.00\times10^{-5}\ \text{T}$ field at this distance. (b) What is unreasonable about this result? (c) Which assumption or premise is responsible?

122. Construct Your Own Problem

Consider a mass separator that applies a magnetic field perpendicular to the velocity of ions and separates the ions based on the radius of curvature of their paths in the field. Construct a problem in which you calculate the magnetic field strength needed to separate two ions that differ in mass, but not charge, and have the same initial velocity. Among the things to consider are the types of ions, the velocities they can be given before entering the magnetic field, and a reasonable value for the radius of curvature of the paths they follow. In addition, calculate the separation distance between the ions at the point where they are detected.

123. Construct Your Own Problem

Consider using the torque on a current-carrying coil in a magnetic field to detect relatively small magnetic fields (less than the field of the Earth, for example). Construct a problem in which you calculate the maximum torque on a current-carrying loop in a magnetic field. Among the things to be considered are the size of the coil, the number of loops it has, the current you pass through the coil, and the size of the field you wish to detect. Discuss whether the torque produced is large enough to be effectively measured. Your instructor may also wish for you to consider the effects, if any, of the field produced by the coil on the surroundings that could affect detection of the small field.

23 ELECTROMAGNETIC INDUCTION, AC CIRCUITS, AND ELECTRICAL TECHNOLOGIES

Figure 23.1 This wind turbine in the Thames Estuary in the UK is an example of induction at work. Wind pushes the blades of the turbine, spinning a shaft attached to magnets. The magnets spin around a conductive coil, inducing an electric current in the coil, and eventually feeding the electrical grid. (credit: phault, Flickr)

Learning Objectives

Introduction to Electromagnetic Induction, AC Circuits and Electrical Technologies

Nature's displays of symmetry are beautiful and alluring. A butterfly's wings exhibit an appealing symmetry in a complex system. (See **Figure 23.2**.) The laws of physics display symmetries at the most basic level—these symmetries are a source of wonder and imply deeper meaning. Since we place a high value on symmetry, we look for it when we explore nature. The remarkable thing is that we find it.

Figure 23.2 Physics, like this butterfly, has inherent symmetries. (credit: Thomas Bresson)

The hint of symmetry between electricity and magnetism found in the preceding chapter will be elaborated upon in this chapter. Specifically, we know that a current creates a magnetic field. If nature is symmetric here, then perhaps a magnetic field can create a current. The Hall effect is a voltage caused by a magnetic force. That voltage could drive a current. Historically, it was very shortly after Oersted discovered currents cause magnetic fields that other scientists asked the following question: Can magnetic fields cause currents? The answer was soon found by experiment to be yes. In 1831, some 12 years after Oersted's discovery, the English scientist Michael Faraday (1791–1862) and the American scientist Joseph Henry (1797–1878) independently demonstrated that magnetic fields can produce currents. The basic process of generating emfs (electromotive force) and, hence, currents with magnetic fields is known as **induction**; this process is also called magnetic induction to distinguish it from charging by induction, which utilizes the Coulomb force.

Today, currents induced by magnetic fields are essential to our technological society. The ubiquitous generator—found in automobiles, on bicycles, in nuclear power plants, and so on—uses magnetism to generate current. Other devices that use magnetism to induce currents include pickup coils in electric guitars, transformers of every size, certain microphones, airport security gates, and damping mechanisms on sensitive chemical balances. Not so familiar perhaps, but important nevertheless, is that the behavior of AC circuits depends strongly on the effect of magnetic fields on currents.

23.1 Induced Emf and Magnetic Flux

The apparatus used by Faraday to demonstrate that magnetic fields can create currents is illustrated in **Figure 23.3**. When the switch is closed, a magnetic field is produced in the coil on the top part of the iron ring and transmitted to the coil on the bottom part of the ring. The galvanometer is used to detect any current induced in the coil on the bottom. It was found that each time the switch is closed, the galvanometer detects a current in one direction in the coil on the bottom. (You can also observe this in a physics lab.) Each time the switch is opened, the galvanometer detects a current in the opposite direction. Interestingly, if the switch remains closed or open for any length of time, there is no current through the galvanometer. *Closing and opening the switch* induces the current. It is the *change* in magnetic field that creates the current. More basic than the current that flows is the emfthat causes it. The current is a result of an *emf induced by a changing magnetic field*, whether or not there is a path for current to flow.

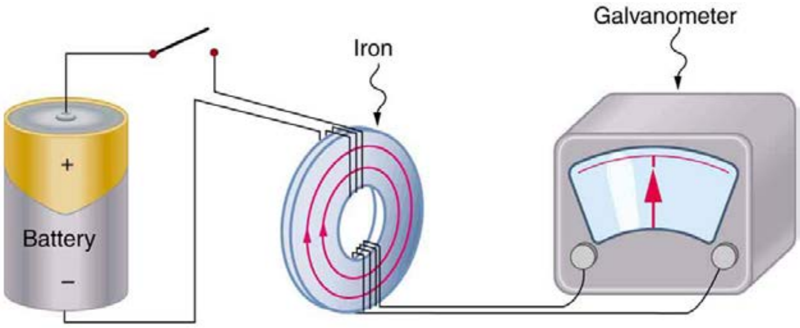


Figure 23.3 Faraday's apparatus for demonstrating that a magnetic field can produce a current. A change in the field produced by the top coil induces an emf and, hence, a current in the bottom coil. When the switch is opened and closed, the galvanometer registers currents in opposite directions. No current flows through the galvanometer when the switch remains closed or open.

An experiment easily performed and often done in physics labs is illustrated in **Figure 23.4**. An emf is induced in the coil when a bar magnet is pushed in and out of it. Emfs of opposite signs are produced by motion in opposite directions, and the emfs are also reversed by reversing poles. The same results are produced if the coil is moved rather than the magnet—it is the relative motion that is important. The faster the motion, the greater the emf, and there is no emf when the magnet is stationary relative to the coil.

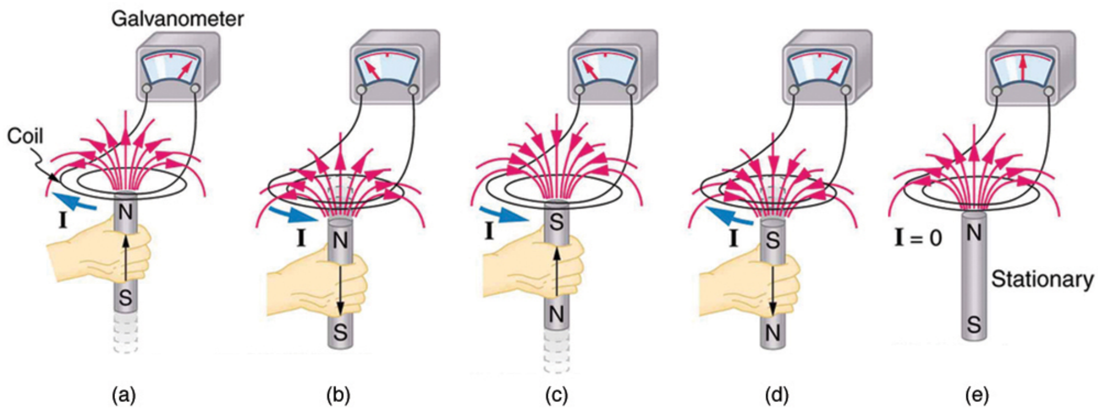


Figure 23.4 Movement of a magnet relative to a coil produces emfs as shown. The same emfs are produced if the coil is moved relative to the magnet. The greater the speed, the greater the magnitude of the emf, and the emf is zero when there is no motion.

The method of inducing an emf used in most electric generators is shown in **Figure 23.5**. A coil is rotated in a magnetic field, producing an alternating current emf, which depends on rotation rate and other factors that will be explored in later sections. Note that the generator is remarkably similar in construction to a motor (another symmetry).

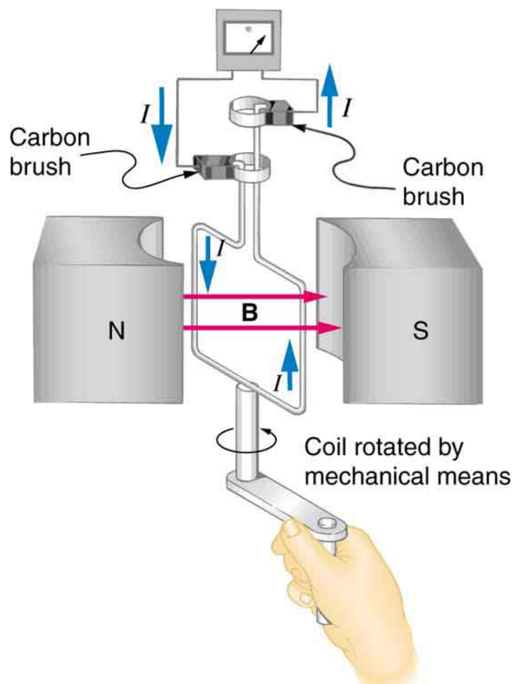


Figure 23.5 Rotation of a coil in a magnetic field produces an emf. This is the basic construction of a generator, where work done to turn the coil is converted to electric energy. Note the generator is very similar in construction to a motor.

So we see that changing the magnitude or direction of a magnetic field produces an emf. Experiments revealed that there is a crucial quantity called the **magnetic flux**, Φ , given by

$$\Phi = BA\cos\theta, \tag{23.1}$$

where B is the magnetic field strength over an area A , at an angle θ with the perpendicular to the area as shown in **Figure 23.6**. **Any change in magnetic flux $\boldsymbol{\Phi}$ induces an emf.** This process is defined to be **electromagnetic induction**. Units of magnetic flux Φ are $\mathrm{T \cdot m^2}$. As seen in **Figure 23.6**, $B\cos\theta = B_{\perp}$, which is the component of B perpendicular to the area A . Thus magnetic flux is $\Phi = B_{\perp}A$, the product of the area and the component of the magnetic field perpendicular to it.

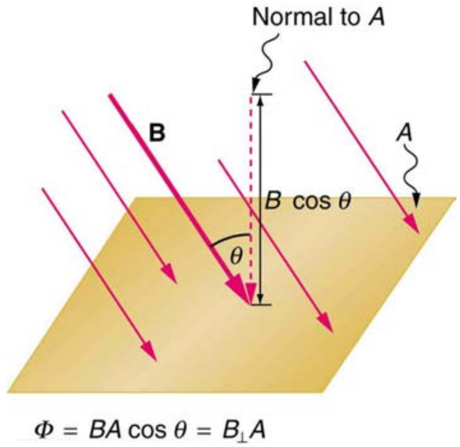


Figure 23.6 Magnetic flux Φ is related to the magnetic field and the area over which it exists. The flux $\Phi = BA \cos \theta$ is related to induction; any change in Φ induces an emf.

All induction, including the examples given so far, arises from some change in magnetic flux Φ. For example, Faraday changed B and hence Φ when opening and closing the switch in his apparatus (shown in **Figure 23.3**). This is also true for the bar magnet and coil shown in **Figure 23.4**. When rotating the coil of a generator, the angle θ and, hence, Φ is changed. Just how great an emf and what direction it takes depend on the change in Φ and how rapidly the change is made, as examined in the next section.

23.2 Faraday's Law of Induction: Lenz's Law

Faraday's and Lenz's Law

Faraday's experiments showed that the emf induced by a change in magnetic flux depends on only a few factors. First, emf is directly proportional to the change in flux $\Delta\Phi$. Second, emf is greatest when the change in time Δt is smallest—that is, emf is inversely proportional to Δt. Finally, if a coil has N turns, an emf will be produced that is N times greater than for a single coil, so that emf is directly proportional to N. The equation for the emf induced by a change in magnetic flux is

$$\text{emf} = -N\frac{\Delta\Phi}{\Delta t}. \tag{23.2}$$

This relationship is known as **Faraday's law of induction**. The units for emf are volts, as is usual.

The minus sign in Faraday's law of induction is very important. The minus means that *the emf creates a current I and magnetic field B that oppose the change in flux $\Delta\Phi$ —this is known as Lenz's law*. The direction (given by the minus sign) of the emfis so important that it is called **Lenz's law** after the Russian Heinrich Lenz (1804–1865), who, like Faraday and Henry,independently investigated aspects of induction. Faraday was aware of the direction, but Lenz stated it so clearly that he is credited for its discovery. (See **Figure 23.7**.)

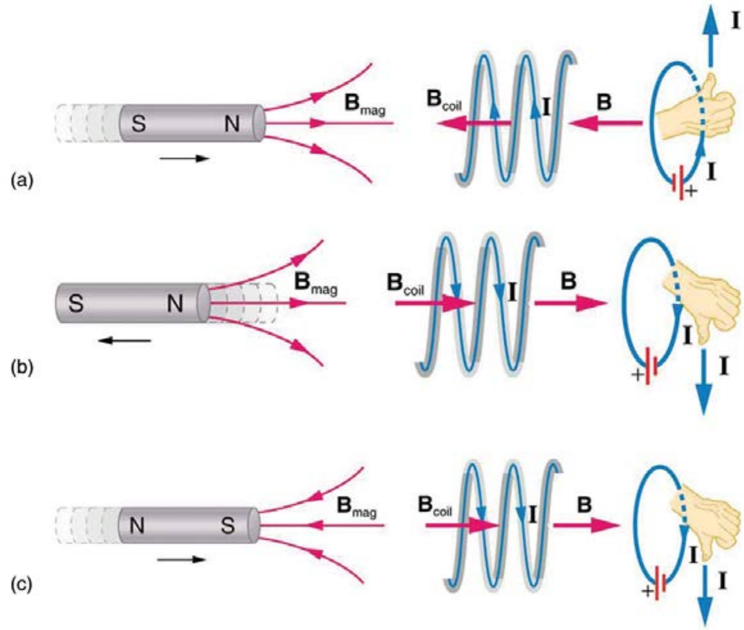


Figure 23.7 (a) When this bar magnet is thrust into the coil, the strength of the magnetic field increases in the coil. The current induced in the coil creates another field, in the opposite direction of the bar magnet's to oppose the increase. This is one aspect of *Lenz's law—induction opposes any change in flux*. (b) and (c) are two other situations. Verify for yourself that the direction of the induced B_{coil} shown indeed opposes the change in flux and that the current direction shown is consistent with RHR-2.

Problem-Solving Strategy for Lenz's Law

To use Lenz's law to determine the directions of the induced magnetic fields, currents, and emfs:

1. Make a sketch of the situation for use in visualizing and recording directions.
2. Determine the direction of the magnetic field B.
3. Determine whether the flux is increasing or decreasing.
4. Now determine the direction of the induced magnetic field B. It opposes the *change* in flux by adding or subtracting from the original field.
5. Use RHR-2 to determine the direction of the induced current I that is responsible for the induced magnetic field B.
6. The direction (or polarity) of the induced emf will now drive a current in this direction and can be represented as current emerging from the positive terminal of the emf and returning to its negative terminal.

For practice, apply these steps to the situations shown in **Figure 23.7** and to others that are part of the following text material.

Applications of Electromagnetic Induction

There are many applications of Faraday's Law of induction, as we will explore in this chapter and others. At this juncture, let us mention several that have to do with data storage and magnetic fields. A very important application has to do with audio and video *recording tapes*. A plastic tape, coated with iron oxide, moves past a recording head. This recording head is basically a round iron ring about which is wrapped a coil of wire—an electromagnet (**Figure 23.8**). A signal in the form of a varying input current from a microphone or camera goes to the recording head. These signals (which are a function of the signal amplitude and frequency) produce varying magnetic fields at the recording head. As the tape moves past the recording head, the magnetic field orientations of the iron oxide molecules on the tape are changed thus recording the signal. In the playback mode, the magnetized tape is run past another head, similar in structure to the recording head. The different magnetic field orientations of the iron oxide molecules on the tape induces an emf in the coil of wire in the playback head. This signal then is sent to a loudspeaker or video player.

Figure 23.8 Recording and playback heads used with audio and video magnetic tapes. (credit: Steve Jurvetson)

Similar principles apply to computer hard drives, except at a much faster rate. Here recordings are on a coated, spinning disk. Read heads historically were made to work on the principle of induction. However, the input information is carried in digital rather than analog form – a series of 0's or 1's are written upon the spinning hard drive. Today, most hard drive readout devices do not work on the principle of induction, but use a technique known as *giant magnetoresistance*. (The discovery that weak changes in a magnetic field in a thin film of iron and chromium could bring about much larger changes in electrical resistance was one of the first large successes of nanotechnology.) Another application of induction is found on the magnetic stripe on the back of your personal credit card as used at the grocery store or the ATM machine. This works on the same principle as the audio or video tape mentioned in the last paragraph in which a head reads personal information from your card.

Another application of electromagnetic induction is when electrical signals need to be transmitted across a barrier. Consider the *cochlear implant* shown below. Sound is picked up by a microphone on the outside of the skull and is used to set up a varying magnetic field. A current is induced in a receiver secured in the bone beneath the skin and transmitted to electrodes in the inner ear. Electromagnetic induction can be used in other instances where electric signals need to be conveyed across various media.

Figure 23.9 Electromagnetic induction used in transmitting electric currents across mediums. The device on the baby's head induces an electrical current in a receiver secured in the bone beneath the skin. (credit: Bjorn Knetsch)

Another contemporary area of research in which electromagnetic induction is being successfully implemented (and with substantial potential) is transcranial magnetic simulation. A host of disorders, including depression and hallucinations can be traced to irregular localized electrical activity in the brain. In *transcranial magnetic stimulation*, a rapidly varying and very localized magnetic field is placed close to certain sites identified in the brain. Weak electric currents are induced in the identified sites and can result in recovery of electrical functioning in the brain tissue.

Sleep apnea ("the cessation of breath") affects both adults and infants (especially premature babies and it may be a cause of sudden infant deaths [SID]). In such individuals, breath can stop repeatedly during their sleep. A cessation of more than 20 seconds can be very dangerous. Stroke, heart failure, and tiredness are just some of the possible consequences for a person having sleep apnea. The concern in infants is the stopping of breath for these longer times. One type of monitor to alert parents when a child is not breathing uses electromagnetic induction. A wire wrapped around the infant's chest has an alternating current running through it. The expansion and contraction of the infant's chest as the infant breathes changes the area through the coil. A pickup coil located nearby has an alternating current induced in it due to the changing magnetic field of the initial wire. If the child stops breathing, there will be a change in the induced current, and so a parent can be alerted.

Making Connections: Conservation of Energy

Lenz's law is a manifestation of the conservation of energy. The induced emf produces a current that opposes the change in flux, because a change in flux means a change in energy. Energy can enter or leave, but not instantaneously. Lenz's law is a consequence. As the change begins, the law says induction opposes and, thus, slows the change. In fact, if the induced emf were in the same direction as the change in flux, there would be a positive feedback that would give us free energy from no apparent source—conservation of energy would be violated.

Example 23.1 Calculating Emf: How Great Is the Induced Emf?

Calculate the magnitude of the induced emf when the magnet in **Figure 23.7**(a) is thrust into the coil, given the following information: the single loop coil has a radius of 6.00 cm and the average value of $B \cos \theta$ (this is given, since the bar magnet's field is complex) increases from 0.0500 T to 0.250 T in 0.100 s.

Strategy

To find the *magnitude* of emf, we use Faraday's law of induction as stated by $\text{emf} = -N\dfrac{\Delta \Phi}{\Delta t}$, but without the minus sign that indicates direction:

$$\text{emf} = N\frac{\Delta \Phi}{\Delta t}. \tag{23.3}$$

Solution

We are given that $N = 1$ and $\Delta t = 0.100\text{ s}$, but we must determine the change in flux $\Delta \Phi$ before we can find emf. Since the area of the loop is fixed, we see that

$$\Delta \Phi = \Delta(BA \cos \theta) = A\Delta(B \cos \theta). \tag{23.4}$$

Now $\Delta(B \cos \theta) = 0.200\text{ T}$, since it was given that $B \cos \theta$ changes from 0.0500 to 0.250 T. The area of the loop is $A = \pi r^2 = (3.14...)(0.060\text{ m})^2 = 1.13\times10^{-2}\text{ m}^2$. Thus,

$$\Delta \Phi = (1.13\times10^{-2}\text{ m}^2)(0.200\text{ T}). \tag{23.5}$$

Entering the determined values into the expression for emf gives

$$\text{Emf} = N\frac{\Delta \Phi}{\Delta t} = \frac{(1.13\times10^{-2}\text{ m}^2)(0.200\text{ T})}{0.100\text{ s}} = 22.6\text{ mV}. \tag{23.6}$$

Discussion

While this is an easily measured voltage, it is certainly not large enough for most practical applications. More loops in the coil, a stronger magnet, and faster movement make induction the practical source of voltages that it is.

PhET Explorations: Faraday's Electromagnetic Lab

Play with a bar magnet and coils to learn about Faraday's law. Move a bar magnet near one or two coils to make a light bulb glow. View the magnetic field lines. A meter shows the direction and magnitude of the current. View the magnetic field lines or use a meter to show the direction and magnitude of the current. You can also play with electromagnets, generators and transformers!

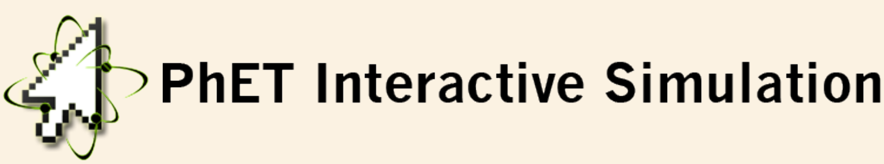


Figure 23.10 Faraday's Electromagnetic Lab (http://cnx.org/content/m42392/1.4/faraday_en.jar)

23.3 Motional Emf

As we have seen, any change in magnetic flux induces an emf opposing that change—a process known as induction. Motion is one of the major causes of induction. For example, a magnet moved toward a coil induces an emf, and a coil moved toward a magnet produces a similar emf. In this section, we concentrate on motion in a magnetic field that is stationary relative to the Earth, producing what is loosely called *motional emf*.

One situation where motional emf occurs is known as the Hall effect and has already been examined. Charges moving in a magnetic field experience the magnetic force $F = qvB \sin \theta$, which moves opposite charges in opposite directions and produces an $\text{emf} = B\ell v$. We saw that the Hall effect has applications, including measurements of B and v. We will now see that the Hall effect is one aspect of the broader phenomenon of induction, and we will find that motional emf can be used as a power source.

Consider the situation shown in **Figure 23.11**. A rod is moved at a speed v along a pair of conducting rails separated by a distance ℓ in a uniform magnetic field B. The rails are stationary relative to B and are connected to a stationary resistor R. The resistor could be anything from a light bulb to a voltmeter. Consider the area enclosed by the moving rod, rails, and resistor. B is perpendicular to this area, and the area is increasing as the rod moves. Thus the magnetic flux enclosed by the rails, rod, and resistor is increasing. When flux changes, an emf is induced according to Faraday's law of induction.

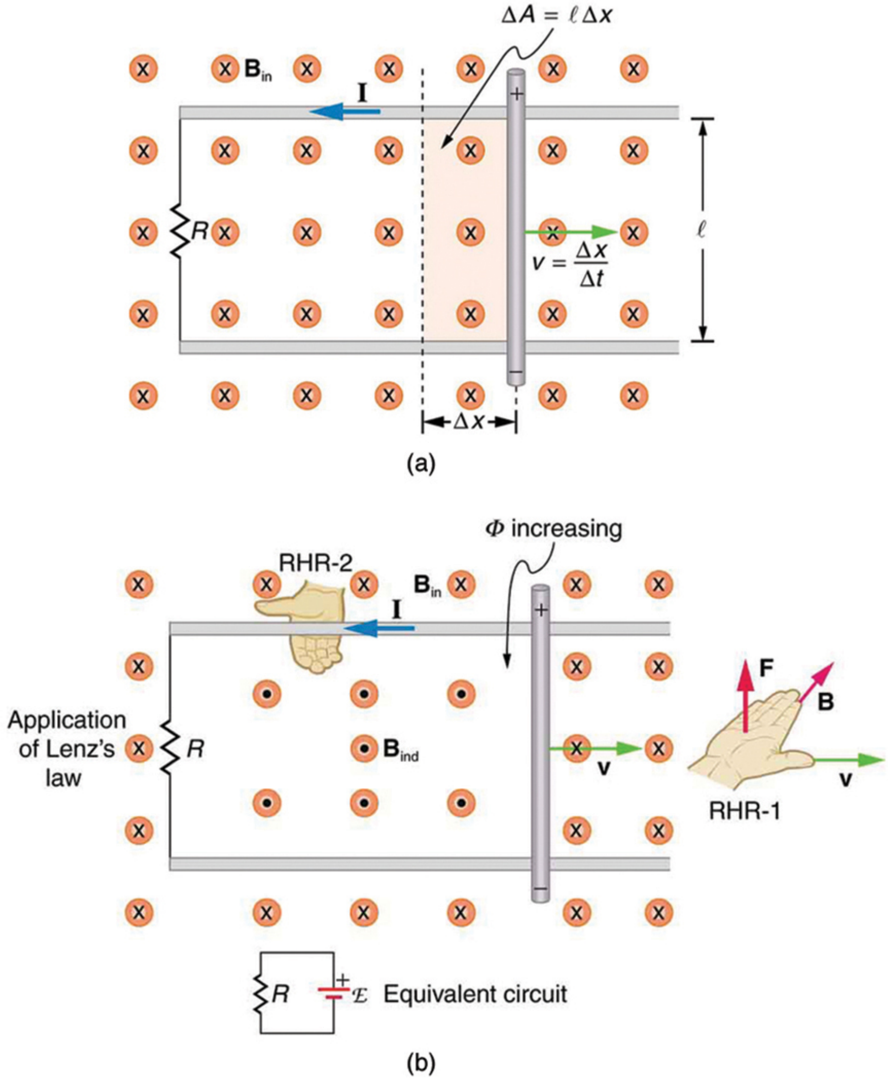


Figure 23.11 (a) A motional $\text{emf} = B\ell v$ is induced between the rails when this rod moves to the right in the uniform magnetic field. The magnetic field B is into the page, perpendicular to the moving rod and rails and, hence, to the area enclosed by them. (b) Lenz's law gives the directions of the induced field and current, and the polarity of the induced emf. Since the flux is increasing, the induced field is in the opposite direction, or out of the page. RHR-2 gives the current direction shown, and the polarity of the rod will drive such a current. RHR-1 also indicates the same polarity for the rod. (Note that the script E symbol used in the equivalent circuit at the bottom of part (b) represents emf.)

To find the magnitude of emf induced along the moving rod, we use Faraday's law of induction without the sign:

$$\text{emf} = N\frac{\Delta\Phi}{\Delta t}. \tag{23.7}$$

Here and below, "emf" implies the magnitude of the emf. In this equation, $N = 1$ and the flux $\Phi = BA\cos\theta$. We have $\theta = 0^\circ$ and $\cos\theta = 1$, since B is perpendicular to A. Now $\Delta\Phi = \Delta(BA) = B\Delta A$, since B is uniform. Note that the area swept out by the rod is $\Delta A = \ell\,\Delta\,x$. Entering these quantities into the expression for emf yields

$$\text{emf} = \frac{B\Delta A}{\Delta t} = B\frac{\ell\Delta x}{\Delta t}. \tag{23.8}$$

Finally, note that $\Delta x / \Delta t = v$, the velocity of the rod. Entering this into the last expression shows that

$$\text{emf} = B\ell v \qquad (B,\ \ell,\ \text{and } v \text{ perpendicular}) \tag{23.9}$$

is the motional emf. This is the same expression given for the Hall effect previously.

Making Connections: Unification of Forces

There are many connections between the electric force and the magnetic force. The fact that a moving electric field produces a magnetic field and, conversely, a moving magnetic field produces an electric field is part of why electric and magnetic forces are now considered to be different manifestations of the same force. This classic unification of electric and magnetic forces into what is called the electromagnetic force is the inspiration for contemporary efforts to unify other basic forces.

To find the direction of the induced field, the direction of the current, and the polarity of the induced emf, we apply Lenz's law as explained in **Faraday's Law of Induction: Lenz's Law**. (See **Figure 23.11**(b).) Flux is increasing, since the area enclosed is increasing. Thus the induced field must oppose the existing one and be out of the page. And so the RHR-2 requires that *I* be counterclockwise, which in turn means the top of the rod is positive as shown.

Motional emf also occurs if the magnetic field moves and the rod (or other object) is stationary relative to the Earth (or some observer). We have seen an example of this in the situation where a moving magnet induces an emf in a stationary coil. It is the relative motion that is important. What is emerging in these observations is a connection between magnetic and electric fields. A moving magnetic field produces an electric field through its induced emf. We already have seen that a moving electric field produces a magnetic field—moving charge implies moving electric field and moving charge produces a magnetic field.

Motional emfs in the Earth's weak magnetic field are not ordinarily very large, or we would notice voltage along metal rods, such as a screwdriver, during ordinary motions. For example, a simple calculation of the motional emf of a 1 m rod moving at 3.0 m/s perpendicular to the Earth's field gives $\text{emf} = B\ell v = (5.0\times10^{-5}\ \text{T})(1.0\ \text{m})(3.0\ \text{m/s}) = 150\ \mu\text{V}$. This small value is consistent with experience. There is a spectacular exception, however. In 1992 and 1996, attempts were made with the space shuttle to create large motional emfs. The Tethered Satellite was to be let out on a 20 km length of wire as shown in **Figure 23.12**, to create a 5 kV emf by moving at orbital speed through the Earth's field. This emf could be used to convert some of the shuttle's kinetic and potential energy into electrical energy if a complete circuit could be made. To complete the circuit, the stationary ionosphere was to supply a return path for the current to flow. (The ionosphere is the rarefied and partially ionized atmosphere at orbital altitudes. It conducts because of the ionization. The ionosphere serves the same function as the stationary rails and connecting resistor in **Figure 23.11**, without which there would not be a complete circuit.) Drag on the current in the cable due to the magnetic force $F = I\ell B \sin\theta$ does the work that reduces the shuttle's kinetic and potential energy and allows it to be converted to electrical energy. The tests were both unsuccessful. In the first, the cable hung up and could only be extended a couple of hundred meters; in the second, the cable broke when almost fully extended. **Example 23.2** indicates feasibility in principle.

Example 23.2 Calculating the Large Motional Emf of an Object in Orbit

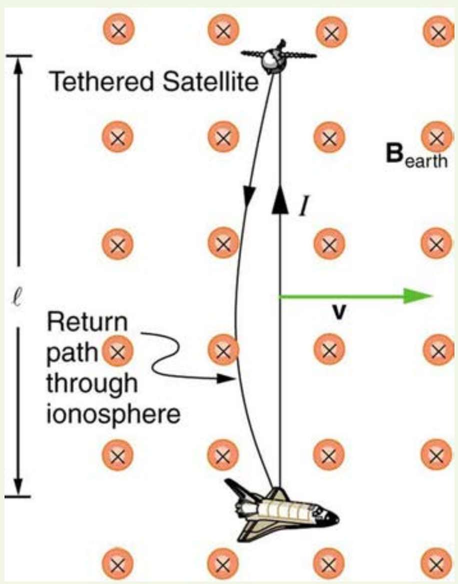


Figure 23.12 Motional emf as electrical power conversion for the space shuttle is the motivation for the Tethered Satellite experiment. A 5 kV emf was predicted to be induced in the 20 km long tether while moving at orbital speed in the Earth's magnetic field. The circuit is completed by a return path through the stationary ionosphere.

Calculate the motional emf induced along a 20.0 km long conductor moving at an orbital speed of 7.80 km/s perpendicular to the Earth's $5.00\times10^{-5}\ \text{T}$ magnetic field.

Strategy

This is a straightforward application of the expression for motional emf— $\text{emf} = B\ell v$.

Solution

Entering the given values into $\text{emf} = B\ell v$ gives

$$\begin{aligned}\text{emf} &= B\ell v \\ &= (5.00\times10^{-5}\ \text{T})(2.0\times10^{4}\ \text{m})(7.80\times10^{3}\ \text{m/s}) \\ &= 7.80\times10^{3}\ \text{V}.\end{aligned} \tag{23.10}$$

Discussion

The value obtained is greater than the 5 kV measured voltage for the shuttle experiment, since the actual orbital motion of the tether is not perpendicular to the Earth's field. The 7.80 kV value is the maximum emf obtained when $\theta = 90^\circ$ and $\sin\theta = 1$.

23.4 Eddy Currents and Magnetic Damping

Eddy Currents and Magnetic Damping

As discussed in **Motional Emf**, motional emf is induced when a conductor moves in a magnetic field or when a magnetic field moves relative to a conductor. If motional emf can cause a current loop in the conductor, we refer to that current as an **eddy current**. Eddy currents can produce significant drag, called **magnetic damping**, on the motion involved. Consider the apparatus shown in **Figure 23.13**, which swings a pendulum bob between the poles of a strong magnet. (This is another favorite physics lab activity.) If the bob is metal, there is significant drag on the bob as it enters and leaves the field, quickly damping the motion. If, however, the bob is a slotted metal plate, as shown in **Figure 23.13**(b), there is a much smaller effect due to the magnet. There is no discernible effect on a bob made of an insulator. Why is there drag in both directions, and are there any uses for magnetic drag?

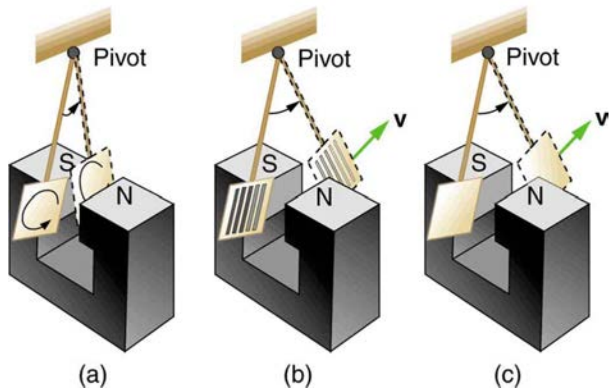


Figure 23.13 A common physics demonstration device for exploring eddy currents and magnetic damping. (a) The motion of a metal pendulum bob swinging between the poles of a magnet is quickly damped by the action of eddy currents. (b) There is little effect on the motion of a slotted metal bob, implying that eddy currents are made less effective. (c) There is also no magnetic damping on a nonconducting bob, since the eddy currents are extremely small.

Figure 23.14 shows what happens to the metal plate as it enters and leaves the magnetic field. In both cases, it experiences a force opposing its motion. As it enters from the left, flux increases, and so an eddy current is set up (Faraday's law) in the counterclockwise direction (Lenz's law), as shown. Only the right-hand side of the current loop is in the field, so that there is an unopposed force on it to the left (RHR-1). When the metal plate is completely inside the field, there is no eddy current if the field is uniform, since the flux remains constant in this region. But when the plate leaves the field on the right, flux decreases, causing an eddy current in the clockwise direction that, again, experiences a force to the left, further slowing the motion. A similar analysis of what happens when the plate swings from the right toward the left shows that its motion is also damped when entering and leaving the field.

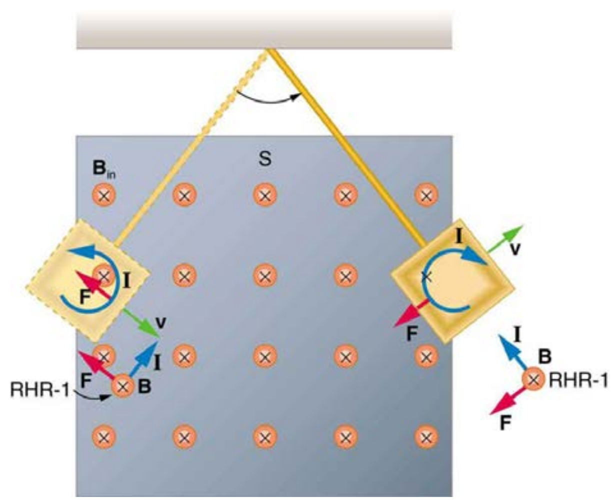


Figure 23.14 A more detailed look at the conducting plate passing between the poles of a magnet. As it enters and leaves the field, the change in flux produces an eddy current. Magnetic force on the current loop opposes the motion. There is no current and no magnetic drag when the plate is completely inside the uniform field.

When a slotted metal plate enters the field, as shown in **Figure 23.15**, an emf is induced by the change in flux, but it is less effective because the slots limit the size of the current loops. Moreover, adjacent loops have currents in opposite directions, and their effects cancel. When an insulating material is used, the eddy current is extremely small, and so magnetic damping on insulators is negligible. If eddy currents are to be avoided in conductors, then they can be slotted or constructed of thin layers of conducting material separated by insulating sheets.

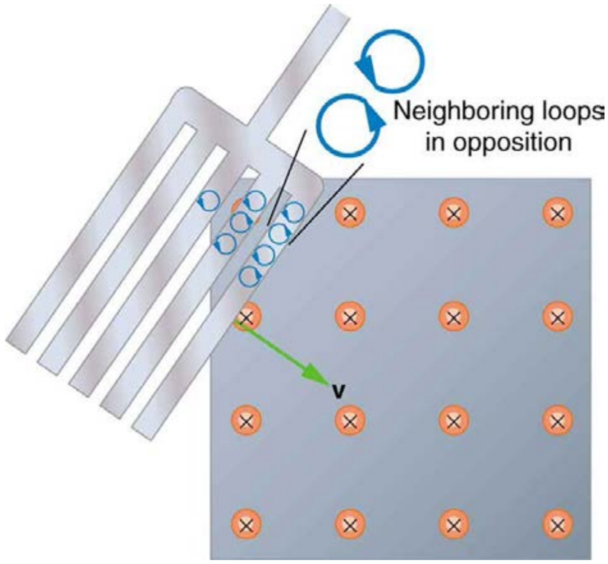


Figure 23.15 Eddy currents induced in a slotted metal plate entering a magnetic field form small loops, and the forces on them tend to cancel, thereby making magnetic drag almost zero.

Applications of Magnetic Damping

One use of magnetic damping is found in sensitive laboratory balances. To have maximum sensitivity and accuracy, the balance must be as friction-free as possible. But if it is friction-free, then it will oscillate for a very long time. Magnetic damping is a simple and ideal solution. With magnetic damping, drag is proportional to speed and becomes zero at zero velocity. Thus the oscillations are quickly damped, after which the damping force disappears, allowing the balance to be very sensitive. (See **Figure 23.16**.) In most balances, magnetic damping is accomplished with a conducting disc that rotates in a fixed field.

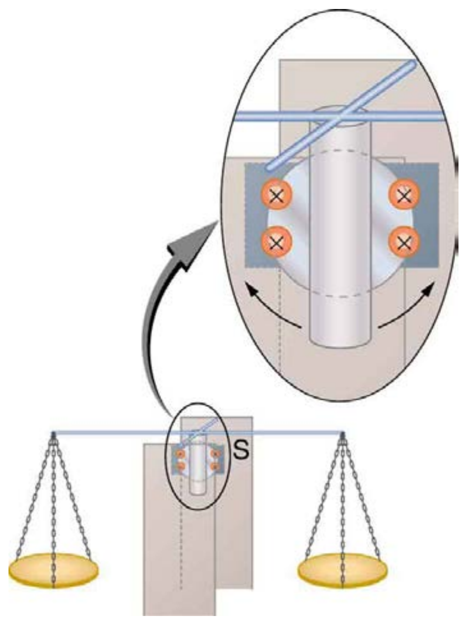


Figure 23.16 Magnetic damping of this sensitive balance slows its oscillations. Since Faraday's law of induction gives the greatest effect for the most rapid change, damping is greatest for large oscillations and goes to zero as the motion stops.

Since eddy currents and magnetic damping occur only in conductors, recycling centers can use magnets to separate metals from other materials. Trash is dumped in batches down a ramp, beneath which lies a powerful magnet. Conductors in the trash are slowed by magnetic damping while nonmetals in the trash move on, separating from the metals. (See **Figure 23.17**.) This works for all metals, not just ferromagnetic ones. A magnet can separate out the ferromagnetic materials alone by acting on stationary trash.

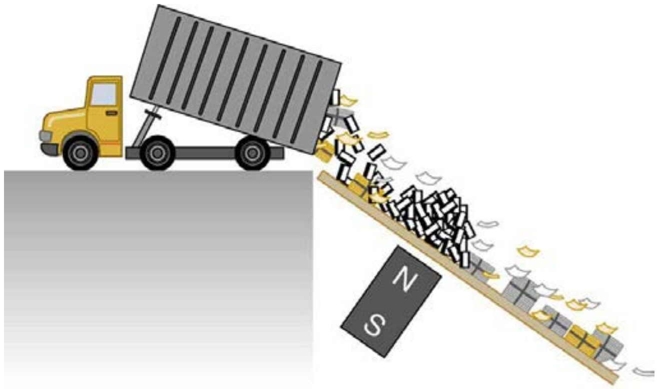


Figure 23.17 Metals can be separated from other trash by magnetic drag. Eddy currents and magnetic drag are created in the metals sent down this ramp by the powerful magnet beneath it. Nonmetals move on.

Other major applications of eddy currents are in metal detectors and braking systems in trains and roller coasters. Portable metal detectors (**Figure 23.18**) consist of a primary coil carrying an alternating current and a secondary coil in which a current is induced. An eddy current will be induced in a piece of metal close to the detector which will cause a change in the induced current within the secondary coil, leading to some sort of signal like a shrill noise. Braking using eddy currents is safer because factors such as rain do not affect the braking and the braking is smoother. However, eddy currents cannot bring the motion to a complete stop, since the force produced decreases with speed. Thus, speed can be reduced from say 20 m/s to 5 m/s, but another form of braking is needed to completely stop the vehicle. Generally, powerful rare earth magnets such as neodymium magnets are used in roller coasters. **Figure 23.19** shows rows of magnets in such an application. The vehicle has metal fins (normally containing copper) which pass through the magnetic field slowing the vehicle down in much the same way as with the pendulum bob shown in **Figure 23.13**.

Figure 23.18 A soldier in Iraq uses a metal detector to search for explosives and weapons. (credit: U.S. Army)

Figure 23.19 The rows of rare earth magnets (protruding horizontally) are used for magnetic braking in roller coasters. (credit: Stefan Scheer, Wikimedia Commons)

Induction cooktops have electromagnets under their surface. The magnetic field is varied rapidly producing eddy currents in the base of the pot, causing the pot and its contents to increase in temperature. Induction cooktops have high efficiencies and good response times but the base of the pot needs to be ferromagnetic, iron or steel for induction to work.

23.5 Electric Generators

Electric generators induce an emf by rotating a coil in a magnetic field, as briefly discussed in **Induced Emf and Magnetic Flux**. We will now explore generators in more detail. Consider the following example.

Example 23.3 Calculating the Emf Induced in a Generator Coil

The generator coil shown in **Figure 23.20** is rotated through one-fourth of a revolution (from $\theta = 0^\circ$ to $\theta = 90^\circ$) in 15.0 ms. The 200-turn circular coil has a 5.00 cm radius and is in a uniform 1.25 T magnetic field. What is the average emf induced?

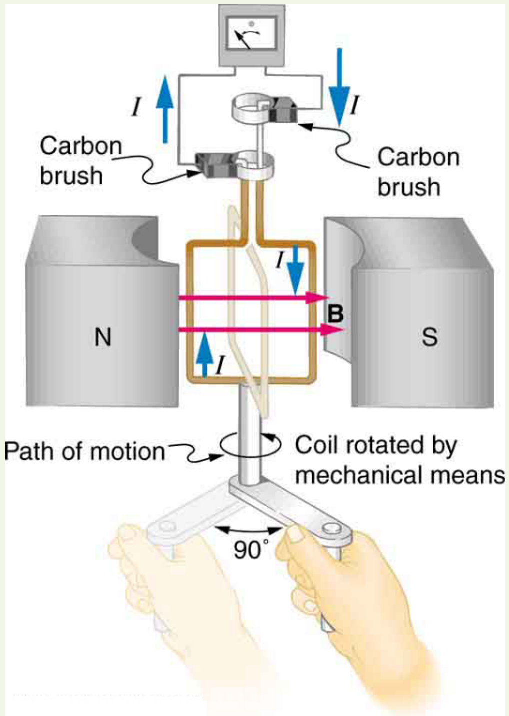


Figure 23.20 When this generator coil is rotated through one-fourth of a revolution, the magnetic flux Φ changes from its maximum to zero, inducing an emf.

Strategy

We use Faraday's law of induction to find the average emf induced over a time Δt :

$$\text{emf} = -N\frac{\Delta\Phi}{\Delta t}. \tag{23.11}$$

We know that $N = 200$ and $\Delta t = 15.0\ \text{ms}$, and so we must determine the change in flux $\Delta\Phi$ to find emf.

Solution

Since the area of the loop and the magnetic field strength are constant, we see that

$$\Delta\Phi = \Delta(BA\cos\theta) = AB\Delta(\cos\theta). \tag{23.12}$$

Now, $\Delta(\cos\theta) = -1.0$, since it was given that θ goes from 0° to 90° . Thus $\Delta\Phi = -AB$, and

$$\text{emf} = N\frac{AB}{\Delta t}. \tag{23.13}$$

The area of the loop is $A = \pi r^2 = (3.14...)(0.0500\ \text{m})^2 = 7.85\times10^{-3}\ \text{m}^2$. Entering this value gives

$$\text{emf} = 200\frac{(7.85\times10^{-3}\ \text{m}^2)(1.25\ \text{T})}{15.0\times10^{-3}\ \text{s}} = 131\ \text{V}. \tag{23.14}$$

Discussion

This is a practical average value, similar to the 120 V used in household power.

The emf calculated in **Example 23.3** is the average over one-fourth of a revolution. What is the emf at any given instant? It varies with the angle between the magnetic field and a perpendicular to the coil. We can get an expression for emf as a function of time by considering the motional emf on a rotating rectangular coil of width w and height ℓ in a uniform magnetic field, as illustrated in **Figure 23.21**.

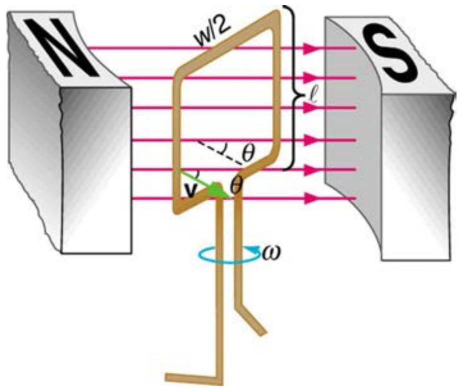


Figure 23.21 A generator with a single rectangular coil rotated at constant angular velocity in a uniform magnetic field produces an emf that varies sinusoidally in time. Note the generator is similar to a motor, except the shaft is rotated to produce a current rather than the other way around.

Charges in the wires of the loop experience the magnetic force, because they are moving in a magnetic field. Charges in the vertical wires experience forces parallel to the wire, causing currents. But those in the top and bottom segments feel a force perpendicular to the wire, which does not cause a current. We can thus find the induced emf by considering only the side wires. Motional emf is given to be $\text{emf} = B\ell v$, where the velocity *v* is perpendicular to the magnetic field B. Here the velocity is at an angle θ with B, so that its component perpendicular to B is $v \sin\theta$ (see **Figure 23.21**). Thus in this case the emf induced on each side is $\text{emf} = B\ell v \sin\theta$, and they are in the same direction. The total emf around the loop is then

$$\text{emf} = 2B\ell v \sin\theta. \tag{23.15}$$

This expression is valid, but it does not give emf as a function of time. To find the time dependence of emf, we assume the coil rotates at a constant angular velocity ω. The angle θ is related to angular velocity by $\theta = \omega t$, so that

$$\text{emf} = 2B\ell v \sin\omega t. \tag{23.16}$$

Now, linear velocity v is related to angular velocity ω by $v = r\omega$. Here $r = w/2$, so that $v = (w/2)\omega$, and

$$\text{emf} = 2B\ell\frac{w}{2}\omega \sin\omega t = (\ell w)B\omega \sin\omega t. \tag{23.17}$$

Noting that the area of the loop is $A = \ell w$, and allowing for N loops, we find that

$$\text{emf} = NAB\omega \sin\omega t \tag{23.18}$$

is the **emf induced in a generator coil** of N turns and area A rotating at a constant angular velocity ω in a uniform magnetic field B. This can also be expressed as

$$\text{emf} = \text{emf}_0 \sin\omega t, \tag{23.19}$$

where

$$\text{emf}_0 = NAB\omega \tag{23.20}$$

is the maximum **(peak) emf**. Note that the frequency of the oscillation is $f = \omega/2\pi$, and the period is $T = 1/f = 2\pi/\omega$. **Figure 23.22** shows a graph of emf as a function of time, and it now seems reasonable that AC voltage is sinusoidal.

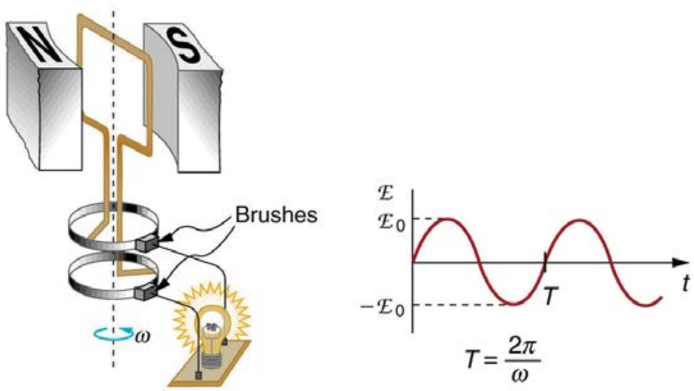


Figure 23.22 The emf of a generator is sent to a light bulb with the system of rings and brushes shown. The graph gives the emf of the generator as a function of time. emf_0 is the peak emf. The period is $T = 1/f = 2\pi/\omega$, where f is the frequency. Note that the script E stands for emf.

The fact that the peak emf, $\text{emf}_0 = NAB\omega$, makes good sense. The greater the number of coils, the larger their area, and the stronger the field, the greater the output voltage. It is interesting that the faster the generator is spun (greater ω), the greater the emf. This is noticeable on bicycle generators—at least the cheaper varieties. One of the authors as a juvenile found it amusing to ride his bicycle fast enough to burn out his lights, until he had to ride home lightless one dark night.

Figure 23.23 shows a scheme by which a generator can be made to produce pulsed DC. More elaborate arrangements of multiple coils and split rings can produce smoother DC, although electronic rather than mechanical means are usually used to make ripple-free DC.

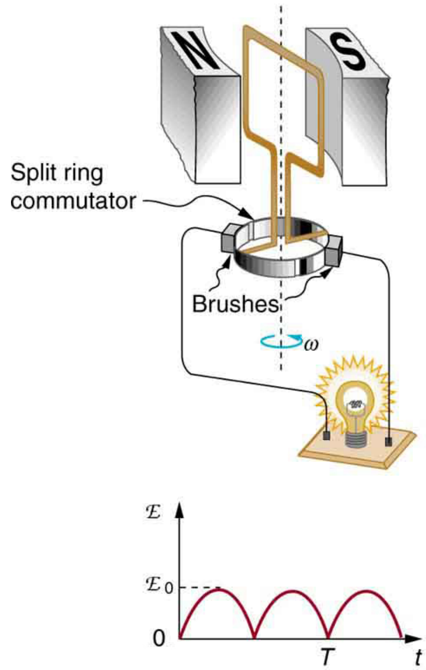


Figure 23.23 Split rings, called commutators, produce a pulsed DC emf output in this configuration.

Example 23.4 Calculating the Maximum Emf of a Generator

Calculate the maximum emf, emf_0, of the generator that was the subject of **Example 23.3**.

Strategy

Once ω, the angular velocity, is determined, $\text{emf}_0 = NAB\omega$ can be used to find emf_0. All other quantities are known.

Solution

Angular velocity is defined to be the change in angle per unit time:

$$\omega = \frac{\Delta\theta}{\Delta t}. \tag{23.21}$$

One-fourth of a revolution is $\pi/2$ radians, and the time is 0.0150 s; thus,

$$\begin{aligned} \omega &= \frac{\pi/2\text{ rad}}{0.0150\text{ s}} \\ &= 104.7\text{ rad/s}. \end{aligned} \tag{23.22}$$

104.7 rad/s is exactly 1000 rpm. We substitute this value for ω and the information from the previous example into $\text{emf}_0 = NAB\omega$, yielding

$$\begin{aligned} \text{emf}_0 &= NAB\omega \\ &= 200(7.85\times10^{-3}\text{ m}^2)(1.25\text{ T})(104.7\text{ rad/s})\cdot \\ &= 206\text{ V} \end{aligned} \tag{23.23}$$

Discussion

The maximum emf is greater than the average emf of 131 V found in the previous example, as it should be.

In real life, electric generators look a lot different than the figures in this section, but the principles are the same. The source of mechanical energy that turns the coil can be falling water (hydropower), steam produced by the burning of fossil fuels, or the kinetic energy of wind. **Figure 23.24** shows a cutaway view of a steam turbine; steam moves over the blades connected to the shaft, which rotates the coil within the generator.

Figure 23.24 Steam turbine/generator. The steam produced by burning coal impacts the turbine blades, turning the shaft which is connected to the generator. (credit: Nabonaco, Wikimedia Commons)

Generators illustrated in this section look very much like the motors illustrated previously. This is not coincidental. In fact, a motor becomes a generator when its shaft rotates. Certain early automobiles used their starter motor as a generator. In **Back Emf**, we shall further explore the action of a motor as a generator.

23.6 Back Emf

It has been noted that motors and generators are very similar. Generators convert mechanical energy into electrical energy, whereas motors convert electrical energy into mechanical energy. Furthermore, motors and generators have the same construction. When the coil of a motor is turned, magnetic flux changes, and an emf (consistent with Faraday's law of induction) is induced. The motor thus acts as a generator whenever its coil rotates. This will happen whether the shaft is turned by an external input, like a belt drive, or by the action of the motor itself. That is, when a motor is doing work and its shaft is turning, an emf is generated. Lenz's law tells us the emf opposes any change, so that the input emf that powers the motor will be opposed by the motor's self-generated emf, called the **back emf** of the motor. (See **Figure 23.25**.)

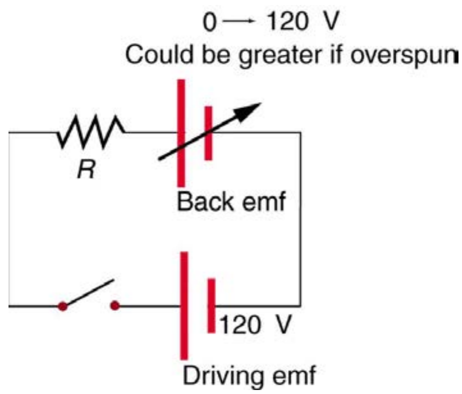


Figure 23.25 The coil of a DC motor is represented as a resistor in this schematic. The back emf is represented as a variable emf that opposes the one driving the motor. Back emf is zero when the motor is not turning, and it increases proportionally to the motor's angular velocity.

Back emf is the generator output of a motor, and so it is proportional to the motor's angular velocity ω. It is zero when the motor is first turned on, meaning that the coil receives the full driving voltage and the motor draws maximum current when it is on but not turning. As the motor turns faster and faster, the back emf grows, always opposing the driving emf, and reduces the voltage across the coil and the amount of current it draws. This effect is noticeable in a number of situations. When a vacuum cleaner, refrigerator, or washing machine is first turned on, lights in the same circuit dim briefly due to the IR drop produced in feeder lines by the large current drawn by the motor. When a motor first comes on, it draws more current than when it runs at its normal operating speed. When a mechanical load is placed on the motor, like an electric wheelchair going up a hill, the motor slows, the back emf drops, more current flows, and more work can be done. If the motor runs at too low a speed, the larger current can overheat it (via resistive power in the coil, $P = I^2R$), perhaps even burning it out. On the other hand, if there is no mechanical load on the motor, it will increase its angular velocity ω until the back emf is nearly equal to the driving emf. Then the motor uses only enough energy to overcome friction.

Consider, for example, the motor coils represented in **Figure 23.25**. The coils have a $0.400\ \Omega$ equivalent resistance and are driven by a 48.0 V emf. Shortly after being turned on, they draw a current $I = V/R = (48.0\text{ V})/(0.400\ \Omega) = 120\text{ A}$ and, thus, dissipate $P = I^2R = 5.76\text{ kW}$ of energy as heat transfer. Under normal operating conditions for this motor, suppose the back emf is 40.0 V. Then at operating speed, the total voltage across the coils is 8.0 V (48.0 V minus the 40.0 V back emf), and the current drawn is $I = V/R = (8.0\text{ V})/(0.400\ \Omega) = 20\text{ A}$. Under normal load, then, the power dissipated is $P = IV = (20\text{ A})/(8.0\text{ V}) = 160\text{ W}$. The latter will not cause a problem for this motor, whereas the former 5.76 kW would burn out the coils if sustained.

23.7 Transformers

Transformers do what their name implies—they transform voltages from one value to another (The term voltage is used rather than emf, because transformers have internal resistance). For example, many cell phones, laptops, video games, and power tools and small appliances have a transformer built into their plug-in unit (like that in **Figure 23.26**) that changes 120 V or 240 V AC into whatever voltage the device uses.

Transformers are also used at several points in the power distribution systems, such as illustrated in **Figure 23.27**. Power is sent long distances at high voltages, because less current is required for a given amount of power, and this means less line loss, as was discussed previously. But high voltages pose greater hazards, so that transformers are employed to produce lower voltage at the user's location.

Figure 23.26 The plug-in transformer has become increasingly familiar with the proliferation of electronic devices that operate on voltages other than common 120 V AC. Most are in the 3 to 12 V range. (credit: Shop Xtreme)

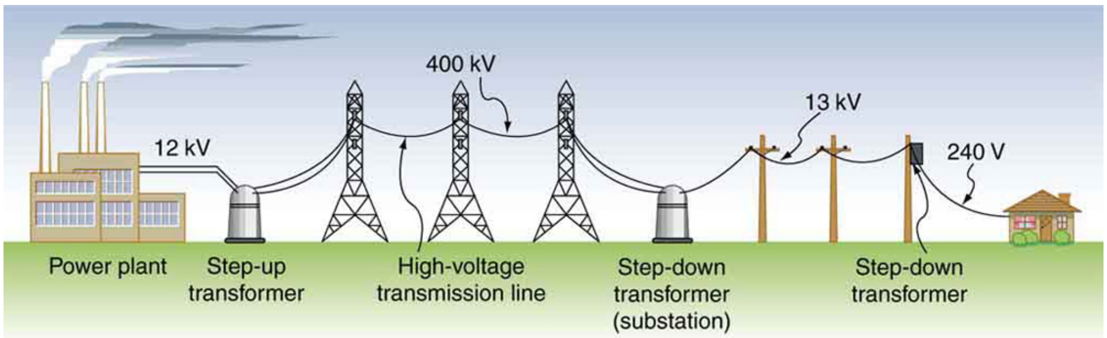


Figure 23.27 Transformers change voltages at several points in a power distribution system. Electric power is usually generated at greater than 10 kV, and transmitted long distances at voltages over 200 kV—sometimes as great as 700 kV—to limit energy losses. Local power distribution to neighborhoods or industries goes through a substation and is sent short distances at voltages ranging from 5 to 13 kV. This is reduced to 120, 240, or 480 V for safety at the individual user site.

The type of transformer considered in this text—see **Figure 23.28**—is based on Faraday's law of induction and is very similar in construction to the apparatus Faraday used to demonstrate magnetic fields could cause currents. The two coils are called the *primary* and *secondary coils*. In normal use, the input voltage is placed on the primary, and the secondary produces the transformed output voltage. Not only does the iron core trap the magnetic field created by the primary coil, its magnetization increases the field strength. Since the input voltage is AC, a time-varying magnetic flux is sent to the secondary, inducing its AC output voltage.

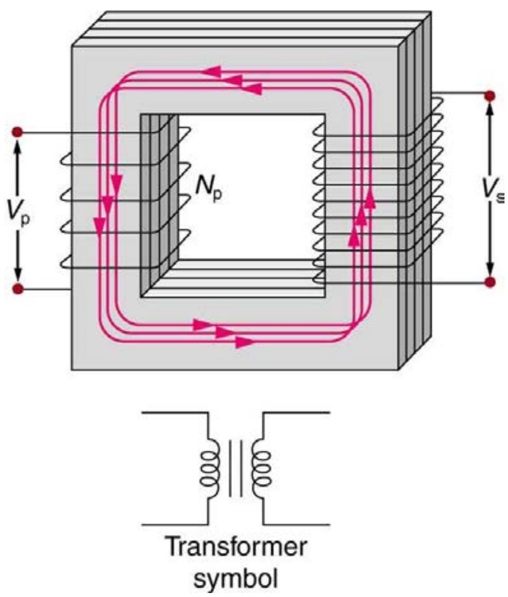


Figure 23.28 A typical construction of a simple transformer has two coils wound on a ferromagnetic core that is laminated to minimize eddy currents. The magnetic field created by the primary is mostly confined to and increased by the core, which transmits it to the secondary coil. Any change in current in the primary induces a current in the secondary.

For the simple transformer shown in **Figure 23.28**, the output voltage V_s depends almost entirely on the input voltage V_p and the ratio of the number of loops in the primary and secondary coils. Faraday's law of induction for the secondary coil gives its induced output voltage V_s to be

$$V_{\mathrm{s}} = -N_{\mathrm{s}}\frac{\Delta\Phi}{\Delta t}, \tag{23.24}$$

where N_{s} is the number of loops in the secondary coil and $\Delta\Phi/\Delta t$ is the rate of change of magnetic flux. Note that the output voltage equals the induced emf ($V_{\mathrm{s}} = \mathrm{emf}_{\mathrm{s}}$), provided coil resistance is small (a reasonable assumption for transformers). The cross-sectional area of the coils is the same on either side, as is the magnetic field strength, and so $\Delta\Phi/\Delta t$ is the same on either side. The input primary voltage V_{p} is also related to changing flux by

$$V_p = -N_{\mathrm{p}}\frac{\Delta\Phi}{\Delta t}. \tag{23.25}$$

The reason for this is a little more subtle. Lenz's law tells us that the primary coil opposes the change in flux caused by the input voltage V_{p} , hence the minus sign (This is an example of *self-inductance*, a topic to be explored in some detail in later sections). Assuming negligible coil resistance, Kirchhoff's loop rule tells us that the induced emf exactly equals the input voltage. Taking the ratio of these last two equations yields a useful relationship:

$$\frac{V_{\mathrm{s}}}{V_{\mathrm{p}}} = \frac{N_{\mathrm{s}}}{N_{\mathrm{p}}}. \tag{23.26}$$

This is known as the **transformer equation**, and it simply states that the ratio of the secondary to primary voltages in a transformer equals the ratio of the number of loops in their coils.

The output voltage of a transformer can be less than, greater than, or equal to the input voltage, depending on the ratio of the number of loops in their coils. Some transformers even provide a variable output by allowing connection to be made at different points on the secondary coil. A **step-up transformer** is one that increases voltage, whereas a **step-down transformer** decreases voltage. Assuming, as we have, that resistance is negligible, the electrical power output of a transformer equals its input. This is nearly true in practice—transformer efficiency often exceeds 99%. Equating the power input and output,

$$P_{\mathrm{p}} = I_{\mathrm{p}}V_{\mathrm{p}} = I_{\mathrm{s}}V_{\mathrm{s}} = P_{\mathrm{s}}. \tag{23.27}$$

Rearranging terms gives

$$\frac{V_{\mathrm{s}}}{V_{\mathrm{p}}} = \frac{I_{\mathrm{p}}}{I_{\mathrm{s}}}. \tag{23.28}$$

Combining this with $\frac{V_{\mathrm{s}}}{V_{\mathrm{p}}} = \frac{N_{\mathrm{s}}}{N_{\mathrm{p}}}$, we find that

$$\frac{I_{\mathrm{s}}}{I_{\mathrm{p}}} = \frac{N_{\mathrm{p}}}{N_{\mathrm{s}}} \tag{23.29}$$

is the relationship between the output and input currents of a transformer. So if voltage increases, current decreases. Conversely, if voltage decreases, current increases.

Example 23.5 Calculating Characteristics of a Step-Up Transformer

A portable x-ray unit has a step-up transformer, the 120 V input of which is transformed to the 100 kV output needed by the x-ray tube. The primary has 50 loops and draws a current of 10.00 A when in use. (a) What is the number of loops in the secondary? (b) Find the current output of the secondary.

Strategy and Solution for (a)

We solve $\frac{V_{\mathrm{s}}}{V_{\mathrm{p}}} = \frac{N_{\mathrm{s}}}{N_{\mathrm{p}}}$ for N_{s} , the number of loops in the secondary, and enter the known values. This gives

$$\begin{aligned} N_{\mathrm{s}} &= N_{\mathrm{p}}\frac{V_{\mathrm{s}}}{V_{\mathrm{p}}} \\ &= (50)\frac{100{,}000\ \mathrm{V}}{120\ \mathrm{V}} = 4.17\times10^4 . \end{aligned} \tag{23.30}$$

Discussion for (a)

A large number of loops in the secondary (compared with the primary) is required to produce such a large voltage. This would be true for neon sign transformers and those supplying high voltage inside TVs and CRTs.

Strategy and Solution for (b)

We can similarly find the output current of the secondary by solving $\frac{I_{\mathrm{s}}}{I_{\mathrm{p}}} = \frac{N_{\mathrm{p}}}{N_{\mathrm{s}}}$ for I_{s} and entering known values. This gives

$$\begin{aligned} I_s &= I_p \frac{N_p}{N_s} \\ &= (10.00\text{ A})\frac{50}{4.17\times 10^4} = 12.0\text{ mA}. \end{aligned} \tag{23.31}$$

Discussion for (b)

As expected, the current output is significantly less than the input. In certain spectacular demonstrations, very large voltages are used to produce long arcs, but they are relatively safe because the transformer output does not supply a large current. Note that the power input here is $P_p = I_p V_p = (10.00\text{ A})(120\text{ V}) = 1.20\text{ kW}$. This equals the power output $P_p = I_s V_s = (12.0\text{ mA})(100\text{ kV}) = 1.20\text{ kW}$, as we assumed in the derivation of the equations used.

The fact that transformers are based on Faraday's law of induction makes it clear why we cannot use transformers to change DC voltages. If there is no change in primary voltage, there is no voltage induced in the secondary. One possibility is to connect DC to the primary coil through a switch. As the switch is opened and closed, the secondary produces a voltage like that in Figure 23.29. This is not really a practical alternative, and AC is in common use wherever it is necessary to increase or decrease voltages.

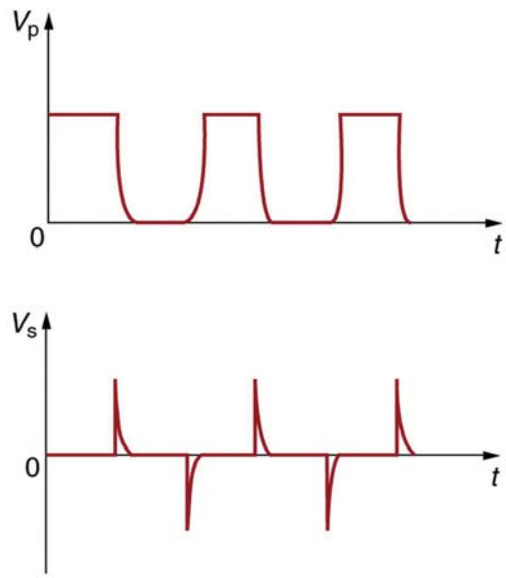


Figure 23.29 Transformers do not work for pure DC voltage input, but if it is switched on and off as on the top graph, the output will look something like that on the bottom graph. This is not the sinusoidal AC most AC appliances need.

Example 23.6 Calculating Characteristics of a Step-Down Transformer

A battery charger meant for a series connection of ten nickel-cadmium batteries (total emf of 12.5 V DC) needs to have a 15.0 V output to charge the batteries. It uses a step-down transformer with a 200-loop primary and a 120 V input. (a) How many loops should there be in the secondary coil? (b) If the charging current is 16.0 A, what is the input current?

Strategy and Solution for (a)

You would expect the secondary to have a small number of loops. Solving $\frac{V_s}{V_p} = \frac{N_s}{N_p}$ for N_s and entering known values gives

$$\begin{aligned} N_s &= N_p \frac{V_s}{V_p} \\ &= (200)\frac{15.0\text{ V}}{120\text{ V}} = 25. \end{aligned} \tag{23.32}$$

Strategy and Solution for (b)

The current input can be obtained by solving $\frac{I_s}{I_p} = \frac{N_p}{N_s}$ for I_p and entering known values. This gives

$$\begin{aligned} I_p &= I_s \frac{N_s}{N_p} \\ &= (16.0\text{ A})\frac{25}{200} = 2.00\text{ A}. \end{aligned} \tag{23.33}$$

Discussion

The number of loops in the secondary is small, as expected for a step-down transformer. We also see that a small input current produces a larger output current in a step-down transformer. When transformers are used to operate large magnets, they sometimes have a small number of very heavy loops in the secondary. This allows the secondary to have low internal resistance and produce large currents. Note again that this solution is based on the assumption of 100% efficiency—or power out equals power in ($P_\text{p} = P_\text{s}$)—reasonable for good transformers. In this case the primary and secondary power is 240 W. (Verify this for yourself as a consistency check.) Note that the Ni-Cd batteries need to be charged from a DC power source (as would a 12 V battery). So the AC output of the secondary coil needs to be converted into DC. This is done using something called a rectifier, which uses devices called diodes that allow only a one-way flow of current.

Transformers have many applications in electrical safety systems, which are discussed in **Electrical Safety: Systems and Devices**.

PhET Explorations: Generator

Generate electricity with a bar magnet! Discover the physics behind the phenomena by exploring magnets and how you can use them to make a bulb light.

PhET Interactive Simulation

Figure 23.30 Generator (http://cnx.org/content/m42414/1.5/generator_en.jar)

23.8 Electrical Safety: Systems and Devices

Electricity has two hazards. A **thermal hazard** occurs when there is electrical overheating. A **shock hazard** occurs when electric current passes through a person. Both hazards have already been discussed. Here we will concentrate on systems and devices that prevent electrical hazards.

Figure 23.31 shows the schematic for a simple AC circuit with no safety features. This is not how power is distributed in practice. Modern household and industrial wiring requires the **three-wire system**, shown schematically in **Figure 23.32**, which has several safety features. First is the familiar *circuit breaker* (or *fuse*) to prevent thermal overload. Second, there is a protective *case* around the appliance, such as a toaster or refrigerator. The case's safety feature is that it prevents a person from touching exposed wires and coming into electrical contact with the circuit, helping prevent shocks.

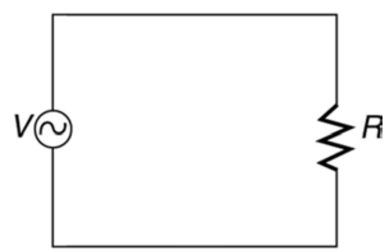


Figure 23.31 Schematic of a simple AC circuit with a voltage source and a single appliance represented by the resistance R . There are no safety features in this circuit.

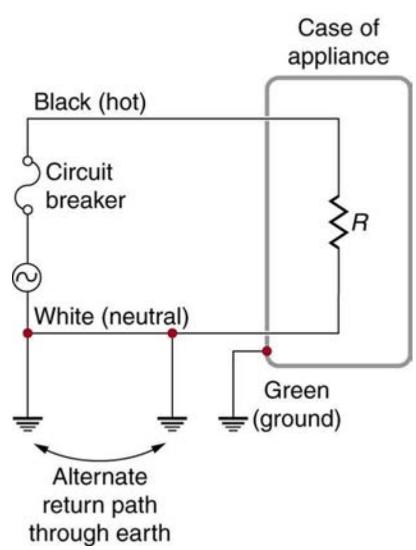


Figure 23.32 The three-wire system connects the neutral wire to the earth at the voltage source and user location, forcing it to be at zero volts and supplying an alternative return path for the current through the earth. Also grounded to zero volts is the case of the appliance. A circuit breaker or fuse protects against thermal overload and is in series on the active (live/hot) wire. Note that wire insulation colors vary with region and it is essential to check locally to determine which color codes are in use (and even if they were followed in the particular installation).

There are *three connections to earth or ground* (hereafter referred to as "earth/ground") shown in **Figure 23.32**. Recall that an earth/ground connection is a low-resistance path directly to the earth. The two earth/ground connections on the *neutral wire* force it to be at zero volts relative to the earth, giving the wire its name. This wire is therefore safe to touch even if its insulation, usually white, is missing. The neutral wire is the return path for the current to follow to complete the circuit. Furthermore, the two earth/ground connections supply an alternative path through the earth, a good conductor, to complete the circuit. The earth/ground connection closest to the power source could be at the generating plant, while the other is at the user's location. The third earth/ground is to the case of the appliance, through the green *earth/ground wire*, forcing the case, too, to be at zero volts. The *live* or *hot wire* (hereafter referred to as "live/hot") supplies voltage and current to operate the appliance. **Figure 23.33** shows a more pictorial version of how the three-wire system is connected through a three-prong plug to an appliance.

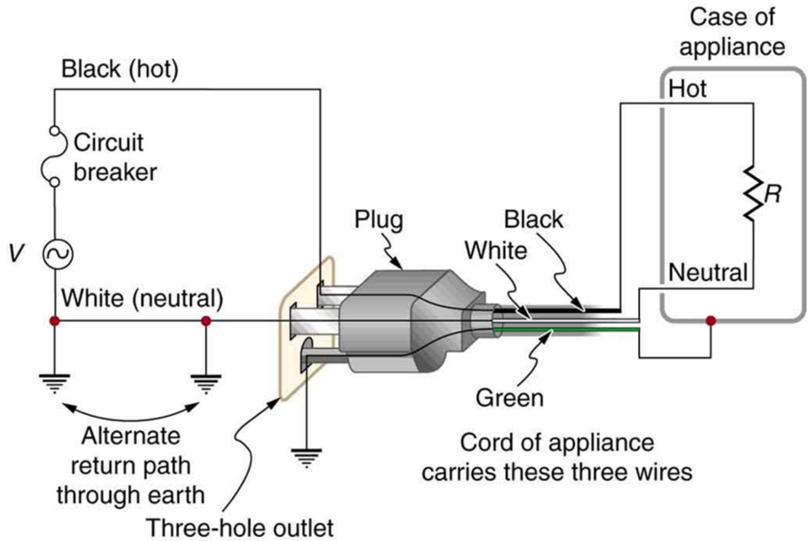


Figure 23.33 The standard three-prong plug can only be inserted in one way, to assure proper function of the three-wire system.

A note on insulation color-coding: Insulating plastic is color-coded to identify live/hot, neutral and ground wires but these codes vary around the world. Live/hot wires may be brown, red, black, blue or grey. Neutral wire may be blue, black or white. Since the same color may be used for live/hot or neutral in different parts of the world, it is essential to determine the color code in your region. The only exception is the earth/ground wire which is often green but may be yellow or just bare wire. Striped coatings are sometimes used for the benefit of those who are colorblind.

The three-wire system replaced the older two-wire system, which lacks an earth/ground wire. Under ordinary circumstances, insulation on the live/hot and neutral wires prevents the case from being directly in the circuit, so that the earth/ground wire may seem like double protection. Grounding the case solves more than one problem, however. The simplest problem is worn insulation on the live/hot wire that allows it to contact the case, as shown in **Figure 23.34**. Lacking an earth/ground connection (some people cut the third prong off the plug because they only have outdated two hole receptacles), a severe shock is possible. This is particularly dangerous in the kitchen, where a good connection to earth/ground is available through water on the floor or a water faucet. With the earth/ground connection intact, the circuit breaker will trip, forcing repair of the appliance. Why are some appliances still sold with two-prong plugs? These have nonconducting cases, such as power tools with impact resistant plastic cases, and are called *doubly insulated*. Modern two-prong plugs can be inserted into the asymmetric standard outlet in only one way, to ensure proper connection of live/hot and neutral wires.

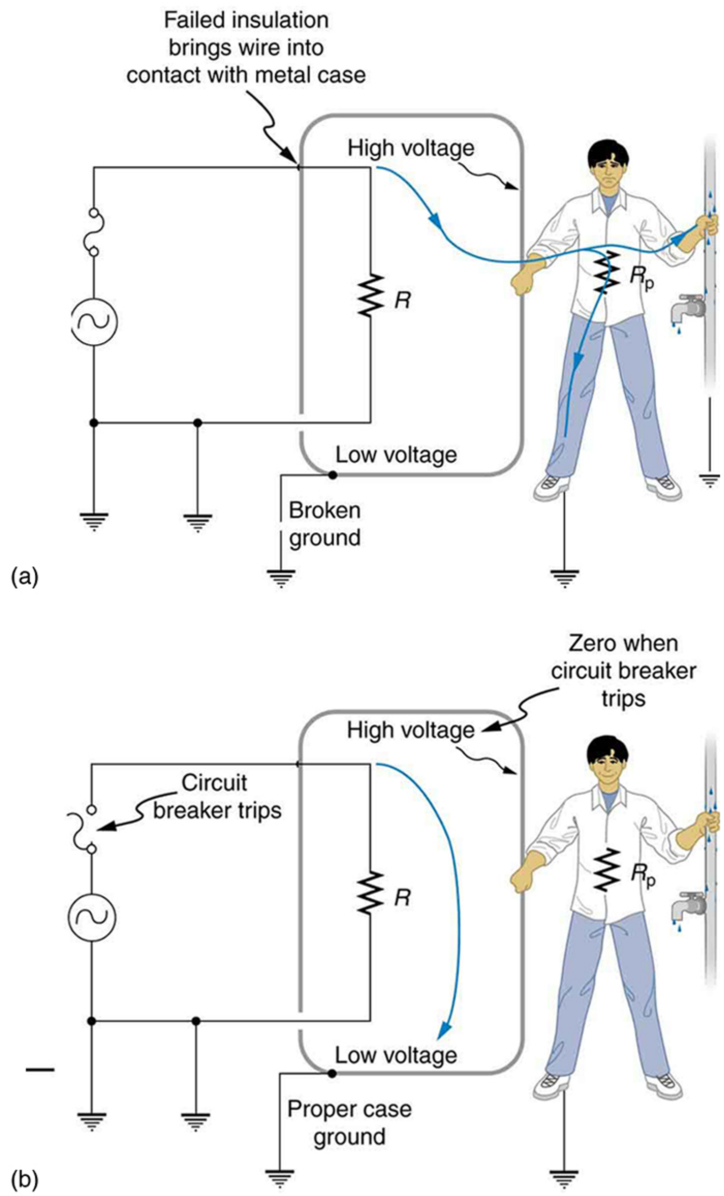


Figure 23.34 Worn insulation allows the live/hot wire to come into direct contact with the metal case of this appliance. (a) The earth/ground connection being broken, the person is severely shocked. The appliance may operate normally in this situation. (b) With a proper earth/ground, the circuit breaker trips, forcing repair of the appliance.

Electromagnetic induction causes a more subtle problem that is solved by grounding the case. The AC current in appliances can induce an emf on the case. If grounded, the case voltage is kept near zero, but if the case is not grounded, a shock can occur as pictured in **Figure 23.35**. Current driven by the induced case emf is called a *leakage current*, although current does not necessarily pass from the resistor to the case.

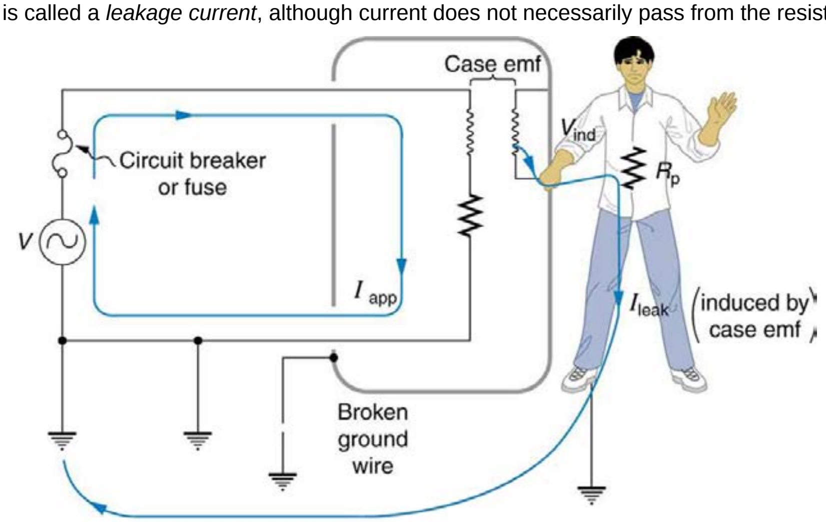


Figure 23.35 AC currents can induce an emf on the case of an appliance. The voltage can be large enough to cause a shock. If the case is grounded, the induced emf is kept near zero.

A *ground fault interrupter* (GFI) is a safety device found in updated kitchen and bathroom wiring that works based on electromagnetic induction. GFIs compare the currents in the live/hot and neutral wires. When live/hot and neutral currents are not equal, it is almost always because current in the neutral is less than in the live/hot wire. Then some of the current, again called a leakage current, is returning to the voltage source by a path other than through the neutral wire. It is assumed that this path presents a hazard, such as shown in **Figure 23.36**. GFIs are usually set to interrupt the circuit if the leakage current is greater than 5 mA, the accepted maximum harmless shock. Even if the leakage current goes safely to earth/ground through an intact earth/ground wire, the GFI will trip, forcing repair of the leakage.

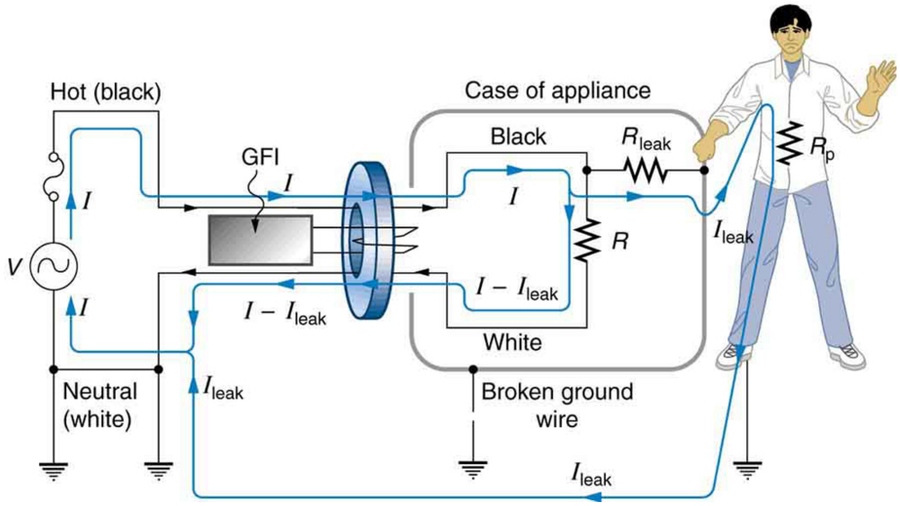


Figure 23.36 A ground fault interrupter (GFI) compares the currents in the live/hot and neutral wires and will trip if their difference exceeds a safe value. The leakage current here follows a hazardous path that could have been prevented by an intact earth/ground wire.

Figure 23.37 shows how a GFI works. If the currents in the live/hot and neutral wires are equal, then they induce equal and opposite emfs in the coil. If not, then the circuit breaker will trip.

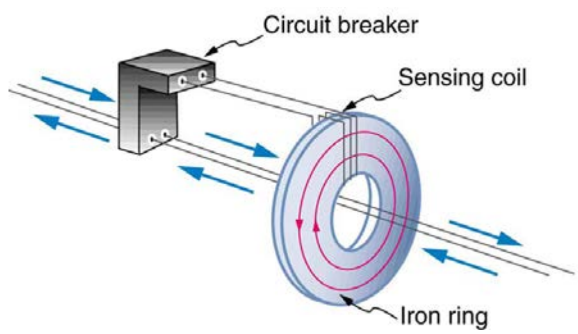


Figure 23.37 A GFI compares currents by using both to induce an emf in the same coil. If the currents are equal, they will induce equal but opposite emfs.

Another induction-based safety device is the *isolation transformer*, shown in **Figure 23.38**. Most isolation transformers have equal input and output voltages. Their function is to put a large resistance between the original voltage source and the device being operated. This prevents a complete circuit between them, even in the circumstance shown. There is a complete circuit through the appliance. But there is not a complete circuit for current to flow through the person in the figure, who is touching only one of the transformer's output wires, and neither output wire is grounded. The appliance is isolated from the original voltage source by the high resistance of the material between the transformer coils, hence the name isolation transformer. For current to flow through the person, it must pass through the high-resistance material between the coils, through the wire, the person, and back through the earth—a path with such a large resistance that the current is negligible.

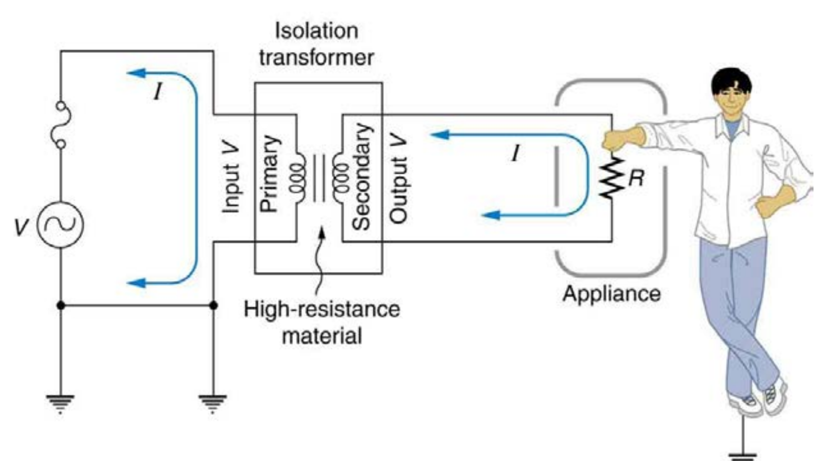


Figure 23.38 An isolation transformer puts a large resistance between the original voltage source and the device, preventing a complete circuit between them.

The basics of electrical safety presented here help prevent many electrical hazards. Electrical safety can be pursued to greater depths. There are, for example, problems related to different earth/ground connections for appliances in close proximity. Many other examples are found in hospitals. Microshock-sensitive patients, for instance, require special protection. For these people, currents as low as 0.1 mA may cause ventricular fibrillation. The interested reader can use the material presented here as a basis for further study.

23.9 Inductance

Inductors

Induction is the process in which an emf is induced by changing magnetic flux. Many examples have been discussed so far, some more effective than others. Transformers, for example, are designed to be particularly effective at inducing a desired voltage and current with very little loss of energy to other forms. Is there a useful physical quantity related to how “effective” a given device is? The answer is yes, and that physical quantity is called **inductance**.

Mutual inductance is the effect of Faraday’s law of induction for one device upon another, such as the primary coil in transmitting energy to the secondary in a transformer. See **Figure 23.39**, where simple coils induce emfs in one another.

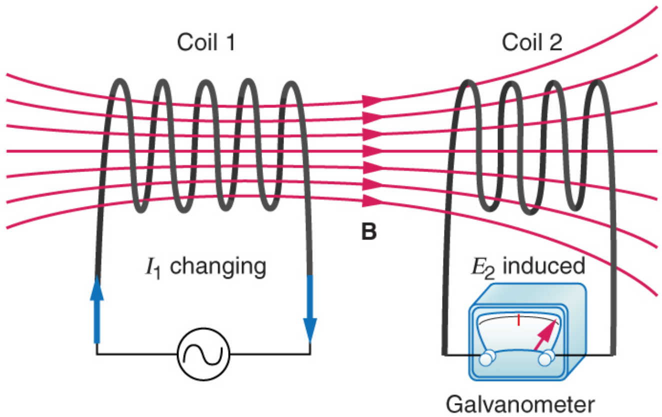


Figure 23.39 These coils can induce emfs in one another like an inefficient transformer. Their mutual inductance M indicates the effectiveness of the coupling between them. Here a change in current in coil 1 is seen to induce an emf in coil 2. (Note that " E_2 induced" represents the induced emf in coil 2.)

In the many cases where the geometry of the devices is fixed, flux is changed by varying current. We therefore concentrate on the rate of change of current, $\Delta I/\Delta t$, as the cause of induction. A change in the current I_1 in one device, coil 1 in the figure, induces an emf_2 in the other. We express this in equation form as

$$\text{emf}_2 = -M\frac{\Delta I_1}{\Delta t}, \tag{23.34}$$

where M is defined to be the mutual inductance between the two devices. The minus sign is an expression of Lenz’s law. The larger the mutual inductance M , the more effective the coupling. For example, the coils in **Figure 23.39** have a small M compared with the transformer coils in **Figure 23.28**. Units for M are $(\text{V}\cdot\text{s})/\text{A} = \Omega\cdot\text{s}$, which is named a **henry** (H), after Joseph Henry. That is, $1\text{ H} = 1\ \Omega\cdot\text{s}$.

Nature is symmetric here. If we change the current I_2 in coil 2, we induce an emf_1 in coil 1, which is given by

$$\text{emf}_1 = -M\frac{\Delta I_2}{\Delta t}, \tag{23.35}$$

where M is the same as for the reverse process. Transformers run backward with the same effectiveness, or mutual inductance M .

A large mutual inductance M may or may not be desirable. We want a transformer to have a large mutual inductance. But an appliance, such as an electric clothes dryer, can induce a dangerous emf on its case if the mutual inductance between its coils and the case is large. One way to reduce mutual inductance M is to counterwind coils to cancel the magnetic field produced. (See **Figure 23.40**.)

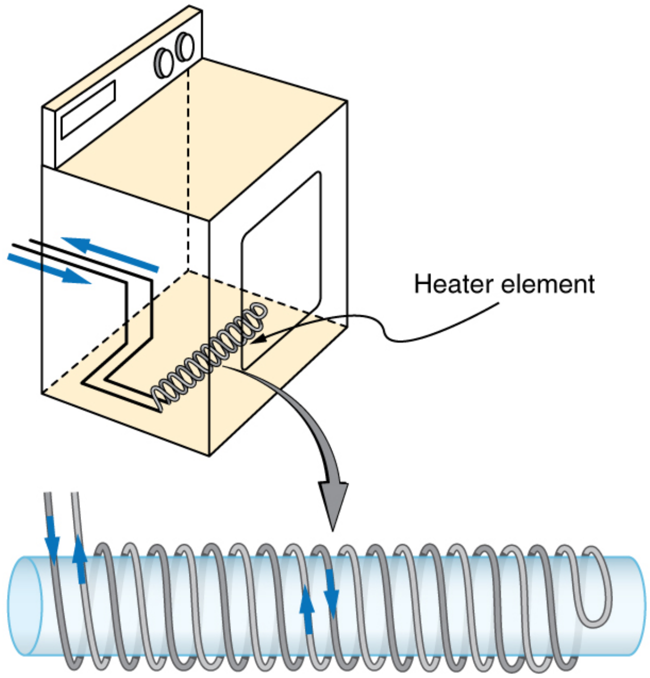


Figure 23.40 The heating coils of an electric clothes dryer can be counter-wound so that their magnetic fields cancel one another, greatly reducing the mutual inductance with the case of the dryer.

Self-inductance, the effect of Faraday's law of induction of a device on itself, also exists. When, for example, current through a coil is increased, the magnetic field and flux also increase, inducing a counter emf, as required by Lenz's law. Conversely, if the current is decreased, an emf is induced that opposes the decrease. Most devices have a fixed geometry, and so the change in flux is due entirely to the change in current ΔI through the device. The induced emf is related to the physical geometry of the device and the rate of change of current. It is given by

$$\text{emf} = -L\frac{\Delta I}{\Delta t}, \tag{23.36}$$

where L is the self-inductance of the device. A device that exhibits significant self-inductance is called an **inductor**, and given the symbol in **Figure 23.41**.

Figure 23.41

The minus sign is an expression of Lenz's law, indicating that emf opposes the change in current. Units of self-inductance are henries (H) just as for mutual inductance. The larger the self-inductance L of a device, the greater its opposition to any change in current through it. For example, a large coil with many turns and an iron core has a large L and will not allow current to change quickly. To avoid this effect, a small L must be achieved, such as by counterwinding coils as in **Figure 23.40**.

A 1 H inductor is a large inductor. To illustrate this, consider a device with $L = 1.0\text{ H}$ that has a 10 A current flowing through it. What happens if we try to shut off the current rapidly, perhaps in only 1.0 ms? An emf, given by $\text{emf} = -L(\Delta I / \Delta t)$, will oppose the change. Thus an emf will be induced given by $\text{emf} = -L(\Delta I / \Delta t) = (1.0\text{ H})[(10\text{ A}) / (1.0\text{ ms})] = 10{,}000\text{ V}$. The positive sign means this large voltage is in the same direction as the current, opposing its decrease. Such large emfs can cause arcs, damaging switching equipment, and so it may be necessary to change current more slowly.

There are uses for such a large induced voltage. Camera flashes use a battery, two inductors that function as a transformer, and a switching system or oscillator to induce large voltages. (Remember that we need a changing magnetic field, brought about by a changing current, to induce a voltage in another coil.) The oscillator system will do this many times as the battery voltage is boosted to over one thousand volts. (You may hear the high pitched whine from the transformer as the capacitor is being charged.) A capacitor stores the high voltage for later use in powering the flash. (See **Figure 23.42**.)

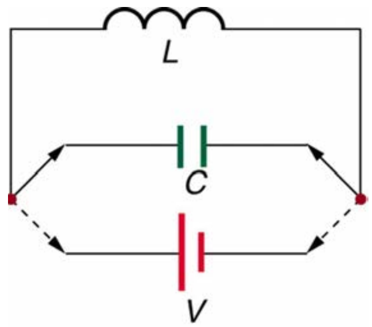


Figure 23.42 Through rapid switching of an inductor, 1.5 V batteries can be used to induce emfs of several thousand volts. This voltage can be used to store charge in a capacitor for later use, such as in a camera flash attachment.

It is possible to calculate L for an inductor given its geometry (size and shape) and knowing the magnetic field that it produces. This is difficult in most cases, because of the complexity of the field created. So in this text the inductance L is usually a given quantity. One exception is the solenoid, because it has a very uniform field inside, a nearly zero field outside, and a simple shape. It is instructive to derive an equation for its inductance. We start by noting that the induced emf is given by Faraday's law of induction as $\text{emf} = -N(\Delta\Phi / \Delta t)$ and, by the definition of self-inductance, as $\text{emf} = -L(\Delta I / \Delta t)$. Equating these yields

$$\text{emf} = -N\frac{\Delta\Phi}{\Delta t} = -L\frac{\Delta I}{\Delta t}. \tag{23.37}$$

Solving for L gives

$$L = N\frac{\Delta\Phi}{\Delta I}. \tag{23.38}$$

This equation for the self-inductance L of a device is always valid. It means that self-inductance L depends on how effective the current is in creating flux; the more effective, the greater $\Delta\Phi / \Delta I$ is.

Let us use this last equation to find an expression for the inductance of a solenoid. Since the area A of a solenoid is fixed, the change in flux is $\Delta\Phi = \Delta(BA) = A\Delta B$. To find ΔB , we note that the magnetic field of a solenoid is given by $B = \mu_0 nI = \mu_0\frac{NI}{\ell}$. (Here $n = N / \ell$, where N is the number of coils and ℓ is the solenoid's length.) Only the current changes, so that $\Delta\Phi = A\Delta B = \mu_0 NA\frac{\Delta I}{\ell}$. Substituting $\Delta\Phi$ into $L = N\frac{\Delta\Phi}{\Delta I}$ gives

$$L = N\frac{\Delta\Phi}{\Delta I} = N\frac{\mu_0 NA\frac{\Delta I}{\ell}}{\Delta I}. \tag{23.39}$$

This simplifies to

$$L = \frac{\mu_0 N^2 A}{\ell}(\text{solenoid}). \tag{23.40}$$

This is the self-inductance of a solenoid of cross-sectional area A and length ℓ . Note that the inductance depends only on the physical characteristics of the solenoid, consistent with its definition.

Example 23.7 Calculating the Self-inductance of a Moderate Size Solenoid

Calculate the self-inductance of a 10.0 cm long, 4.00 cm diameter solenoid that has 200 coils.

Strategy

This is a straightforward application of $L = \frac{\mu_0 N^2 A}{\ell}$, since all quantities in the equation except L are known.

Solution

Use the following expression for the self-inductance of a solenoid:

$$L = \frac{\mu_0 N^2 A}{\ell}. \tag{23.41}$$

The cross-sectional area in this example is $A = \pi r^2 = (3.14...)(0.0200\text{ m})^2 = 1.26\times10^{-3}\text{ m}^2$, N is given to be 200, and the length ℓ is 0.100 m. We know the permeability of free space is $\mu_0 = 4\pi\times10^{-7}\text{ T}\cdot\text{m/A}$. Substituting these into the expression for L gives

$$\begin{aligned} L &= \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(200)^2(1.26\times10^{-3}\ \text{m}^2)}{0.100\ \text{m}} \\ &= 0.632\ \text{mH}. \end{aligned} \tag{23.42}$$

Discussion

This solenoid is moderate in size. Its inductance of nearly a millihenry is also considered moderate.

One common application of inductance is used in traffic lights that can tell when vehicles are waiting at the intersection. An electrical circuit with an inductor is placed in the road under the place a waiting car will stop over. The body of the car increases the inductance and the circuit changes sending a signal to the traffic lights to change colors. Similarly, metal detectors used for airport security employ the same technique. A coil or inductor in the metal detector frame acts as both a transmitter and a receiver. The pulsed signal in the transmitter coil induces a signal in the receiver. The self-inductance of the circuit is affected by any metal object in the path. Such detectors can be adjusted for sensitivity and also can indicate the approximate location of metal found on a person. (But they will not be able to detect any plastic explosive such as that found on the “underwear bomber.”) See **Figure 23.43**.

Figure 23.43 The familiar security gate at an airport can not only detect metals but also indicate their approximate height above the floor. (credit: Alexbuirds, Wikimedia Commons)

Energy Stored in an Inductor

We know from Lenz’s law that inductances oppose changes in current. There is an alternative way to look at this opposition that is based on energy. Energy is stored in a magnetic field. It takes time to build up energy, and it also takes time to deplete energy; hence, there is an opposition to rapid change. In an inductor, the magnetic field is directly proportional to current and to the inductance of the device. It can be shown that the **energy stored in an inductor** E_{ind} is given by

$$E_{\text{ind}} = \frac{1}{2}LI^2. \tag{23.43}$$

This expression is similar to that for the energy stored in a capacitor.

Example 23.8 Calculating the Energy Stored in the Field of a Solenoid

How much energy is stored in the 0.632 mH inductor of the preceding example when a 30.0 A current flows through it?

Strategy

The energy is given by the equation $E_{\text{ind}} = \frac{1}{2}LI^2$, and all quantities except E_{ind} are known.

Solution

Substituting the value for L found in the previous example and the given current into $E_{\text{ind}} = \frac{1}{2}LI^2$ gives

$$\begin{aligned} E_{\text{ind}} &= \frac{1}{2}LI^2 \\ &= 0.5(0.632\times10^{-3}\ \text{H})(30.0\ \text{A})^2 = 0.284\ \text{J}. \end{aligned} \tag{23.44}$$

Discussion

This amount of energy is certainly enough to cause a spark if the current is suddenly switched off. It cannot be built up instantaneously unless the power input is infinite.

23.10 RL Circuits

We know that the current through an inductor L cannot be turned on or off instantaneously. The change in current changes flux, inducing an emf opposing the change (Lenz’s law). How long does the opposition last? Current *will* flow and *can* be turned off, but how long does it take? **Figure 23.44** shows a switching circuit that can be used to examine current through an inductor as a function of time.

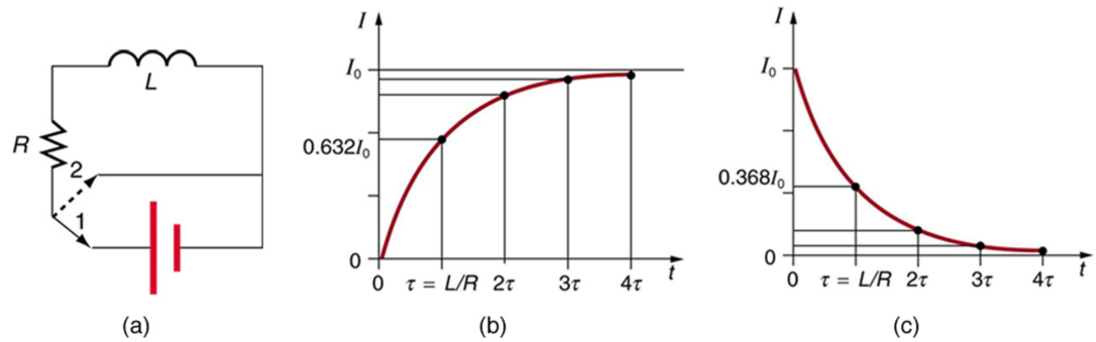


Figure 23.44 (a) An *RL* circuit with a switch to turn current on and off. When in position 1, the battery, resistor, and inductor are in series and a current is established. In position 2, the battery is removed and the current eventually stops because of energy loss in the resistor. (b) A graph of current growth versus time when the switch is moved to position 1. (c) A graph of current decay when the switch is moved to position 2.

When the switch is first moved to position 1 (at $t = 0$), the current is zero and it eventually rises to $I_0 = V/R$, where R is the total resistance of the circuit. The opposition of the inductor L is greatest at the beginning, because the amount of change is greatest. The opposition it poses is in the form of an induced emf, which decreases to zero as the current approaches its final value. The opposing emf is proportional to the amount of change left. This is the hallmark of an exponential behavior, and it can be shown with calculus that

$$I = I_0(1 - e^{-t/\tau}) \quad \text{(turning on)}, \tag{23.45}$$

is the current in an *RL* circuit when switched on (Note the similarity to the exponential behavior of the voltage on a charging capacitor). The initial current is zero and approaches $I_0 = V/R$ with a **characteristic time constant** τ for an *RL* circuit, given by

$$\tau = \frac{L}{R}, \tag{23.46}$$

where τ has units of seconds, since $1\ \text{H}=1\ \Omega{\cdot}\text{s}$. In the first period of time τ, the current rises from zero to $0.632I_0$, since $I = I_0(1 - e^{-1}) = I_0(1 - 0.368) = 0.632I_0$. The current will go 0.632 of the remainder in the next time τ. A well-known property of the exponential is that the final value is never exactly reached, but 0.632 of the remainder to that value is achieved in every characteristic time τ. In just a few multiples of the time τ, the final value is very nearly achieved, as the graph in **Figure 23.44**(b) illustrates.

The characteristic time τ depends on only two factors, the inductance L and the resistance R. The greater the inductance L, the greater τ is, which makes sense since a large inductance is very effective in opposing change. The smaller the resistance R, the greater τ is. Again this makes sense, since a small resistance means a large final current and a greater change to get there. In both cases—large L and small R —more energy is stored in the inductor and more time is required to get it in and out.

When the switch in **Figure 23.44**(a) is moved to position 2 and cuts the battery out of the circuit, the current drops because of energy dissipation by the resistor. But this is also not instantaneous, since the inductor opposes the decrease in current by inducing an emf in the same direction as the battery that drove the current. Furthermore, there is a certain amount of energy, $(1/2)LI_0^2$, stored in the inductor, and it is dissipated at a finite rate. As the current approaches zero, the rate of decrease slows, since the energy dissipation rate is I^2R. Once again the behavior is exponential, and I is found to be

$$I = I_0 e^{-t/\tau} \quad \text{(turning off)}. \tag{23.47}$$

(See **Figure 23.44**(c).) In the first period of time $\tau = L/R$ after the switch is closed, the current falls to 0.368 of its initial value, since $I = I_0 e^{-1} = 0.368I_0$. In each successive time τ, the current falls to 0.368 of the preceding value, and in a few multiples of τ, the current becomes very close to zero, as seen in the graph in **Figure 23.44**(c).

Example 23.9 Calculating Characteristic Time and Current in an *RL* Circuit

(a) What is the characteristic time constant for a 7.50 mH inductor in series with a $3.00\ \Omega$ resistor? (b) Find the current 5.00 ms after the switch is moved to position 2 to disconnect the battery, if it is initially 10.0 A.

Strategy for (a)

The time constant for an *RL* circuit is defined by $\tau = L/R$.

Solution for (a)

Entering known values into the expression for τ given in $\tau = L/R$ yields

$$\tau = \frac{L}{R} = \frac{7.50\ \text{mH}}{3.00\ \ \Omega} = 2.50\ \text{ms}. \tag{23.48}$$

Discussion for (a)

This is a small but definitely finite time. The coil will be very close to its full current in about ten time constants, or about 25 ms.

Strategy for (b)

We can find the current by using $I = I_0 e^{-t/\tau}$, or by considering the decline in steps. Since the time is twice the characteristic time, we consider the process in steps.

Solution for (b)

In the first 2.50 ms, the current declines to 0.368 of its initial value, which is

$$\begin{aligned} I &= 0.368 I_0 = (0.368)(10.0\text{ A}) \\ &= 3.68\text{ A at } t = 2.50\text{ ms.} \end{aligned} \tag{23.49}$$

After another 2.50 ms, or a total of 5.00 ms, the current declines to 0.368 of the value just found. That is,

$$\begin{aligned} I' &= 0.368 I = (0.368)(3.68\text{ A}) \\ &= 1.35\text{ A at } t = 5.00\text{ ms.} \end{aligned} \tag{23.50}$$

Discussion for (b)

After another 5.00 ms has passed, the current will be 0.183 A (see **Exercise 23.99**); so, although it does die out, the current certainly does not go to zero instantaneously.

In summary, when the voltage applied to an inductor is changed, the current also changes, *but the change in current lags the change in voltage in an RL circuit*. In **Reactance, Inductive and Capacitive**, we explore how an *RL* circuit behaves when a sinusoidal AC voltage is applied.

23.11 Reactance, Inductive and Capacitive

Many circuits also contain capacitors and inductors, in addition to resistors and an AC voltage source. We have seen how capacitors and inductors respond to DC voltage when it is switched on and off. We will now explore how inductors and capacitors react to sinusoidal AC voltage.

Inductors and Inductive Reactance

Suppose an inductor is connected directly to an AC voltage source, as shown in **Figure 23.45**. It is reasonable to assume negligible resistance, since in practice we can make the resistance of an inductor so small that it has a negligible effect on the circuit. Also shown is a graph of voltage and current as functions of time.

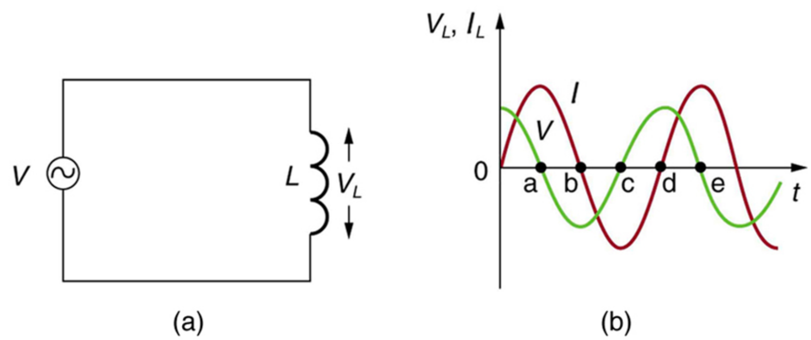


Figure 23.45 (a) An AC voltage source in series with an inductor having negligible resistance. (b) Graph of current and voltage across the inductor as functions of time.

The graph in **Figure 23.45**(b) starts with voltage at a maximum. Note that the current starts at zero and rises to its peak *after* the voltage that drives it, just as was the case when DC voltage was switched on in the preceding section. When the voltage becomes negative at point a, the current begins to decrease; it becomes zero at point b, where voltage is its most negative. The current then becomes negative, again following the voltage. The voltage becomes positive at point c and begins to make the current less negative. At point d, the current goes through zero just as the voltage reaches its positive peak to start another cycle. This behavior is summarized as follows:

AC Voltage in an Inductor

When a sinusoidal voltage is applied to an inductor, the voltage leads the current by one-fourth of a cycle, or by a 90^{o} phase angle.

Current lags behind voltage, since inductors oppose change in current. Changing current induces a back emf $V = -L(\Delta I / \Delta t)$. This is considered to be an effective resistance of the inductor to AC. The rms current I through an inductor L is given by a version of Ohm's law:

$$I = \frac{V}{X_L}, \tag{23.51}$$

where V is the rms voltage across the inductor and X_L is defined to be

$$X_L = 2\pi f L, \tag{23.52}$$

with f the frequency of the AC voltage source in hertz (An analysis of the circuit using Kirchhoff's loop rule and calculus actually produces this expression). X_L is called the **inductive reactance**, because the inductor reacts to impede the current. X_L has units of ohms ($1\text{ H} = 1\ \Omega \cdot \text{s}$, so that frequency times inductance has units of $(\text{cycles/s})(\Omega \cdot \text{s}) = \Omega$), consistent with its role as an effective resistance. It makes sense that X_L is proportional to L, since the greater the induction the greater its resistance to change. It is also reasonable that X_L is proportional to frequency f, since greater frequency means greater change in current. That is, $\Delta I/\Delta t$ is large for large frequencies (large f, small Δt). The greater the change, the greater the opposition of an inductor.

Example 23.10 Calculating Inductive Reactance and then Current

(a) Calculate the inductive reactance of a 3.00 mH inductor when 60.0 Hz and 10.0 kHz AC voltages are applied. (b) What is the rms current at each frequency if the applied rms voltage is 120 V?

Strategy

The inductive reactance is found directly from the expression $X_L = 2\pi fL$. Once X_L has been found at each frequency, Ohm's law as stated in the Equation $I = V / X_L$ can be used to find the current at each frequency.

Solution for (a)

Entering the frequency and inductance into Equation $X_L = 2\pi fL$ gives

$$X_L = 2\pi fL = 6.28(60.0 / \text{s})(3.00\text{ mH}) = 1.13\ \Omega \text{ at } 60\text{ Hz}. \tag{23.53}$$

Similarly, at 10 kHz,

$$X_L = 2\pi fL = 6.28(1.00{\times}10^4/\text{s})(3.00\text{ mH}) = 188\ \Omega \text{ at } 10\text{ kHz}. \tag{23.54}$$

Solution for (b)

The rms current is now found using the version of Ohm's law in Equation $I = V / X_L$, given the applied rms voltage is 120 V. For the first frequency, this yields

$$I = \frac{V}{X_L} = \frac{120\text{ V}}{1.13\ \Omega} = 106\text{ A at } 60\text{ Hz}. \tag{23.55}$$

Similarly, at 10 kHz,

$$I = \frac{V}{X_L} = \frac{120\text{ V}}{188\ \Omega} = 0.637\text{ A at } 10\text{ kHz}. \tag{23.56}$$

Discussion

The inductor reacts very differently at the two different frequencies. At the higher frequency, its reactance is large and the current is small, consistent with how an inductor impedes rapid change. Thus high frequencies are impeded the most. Inductors can be used to filter out high frequencies; for example, a large inductor can be put in series with a sound reproduction system or in series with your home computer to reduce high-frequency sound output from your speakers or high-frequency power spikes into your computer.

Note that although the resistance in the circuit considered is negligible, the AC current is not extremely large because inductive reactance impedes its flow. With AC, there is no time for the current to become extremely large.

Capacitors and Capacitive Reactance

Consider the capacitor connected directly to an AC voltage source as shown in **Figure 23.46**. The resistance of a circuit like this can be made so small that it has a negligible effect compared with the capacitor, and so we can assume negligible resistance. Voltage across the capacitor and current are graphed as functions of time in the figure.

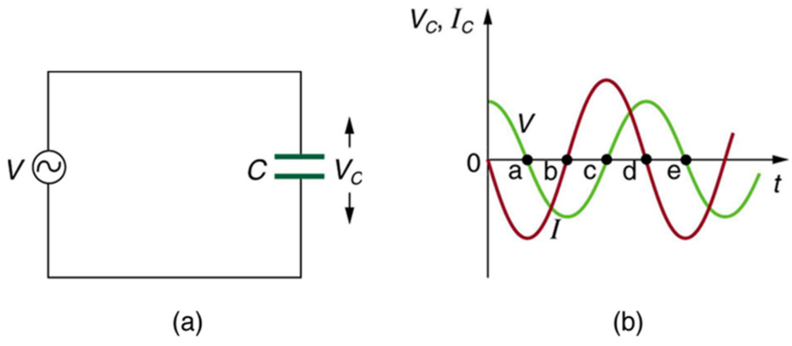


Figure 23.46 (a) An AC voltage source in series with a capacitor *C* having negligible resistance. (b) Graph of current and voltage across the capacitor as functions of time.

The graph in **Figure 23.46** starts with voltage across the capacitor at a maximum. The current is zero at this point, because the capacitor is fully charged and halts the flow. Then voltage drops and the current becomes negative as the capacitor discharges. At point a, the capacitor has fully discharged ($Q = 0$ on it) and the voltage across it is zero. The current remains negative between points a and b, causing the voltage on the capacitor to reverse. This is complete at point b, where the current is zero and the voltage has its most negative value. The current becomes positive after point b, neutralizing the charge on the capacitor and bringing the voltage to zero at point c, which allows the current to reach its maximum. Between points c and d, the current drops to zero as the voltage rises to its peak, and the process starts to repeat. Throughout the cycle, the voltage follows what the current is doing by one-fourth of a cycle:

AC Voltage in a Capacitor

When a sinusoidal voltage is applied to a capacitor, the voltage follows the current by one-fourth of a cycle, or by a 90° phase angle.

The capacitor is affecting the current, having the ability to stop it altogether when fully charged. Since an AC voltage is applied, there is an rms current, but it is limited by the capacitor. This is considered to be an effective resistance of the capacitor to AC, and so the rms current I in the circuit containing only a capacitor C is given by another version of Ohm's law to be

$$I = \frac{V}{X_C}, \tag{23.57}$$

where V is the rms voltage and X_C is defined (As with X_L, this expression for X_C results from an analysis of the circuit using Kirchhoff's rules and calculus) to be

$$X_C = \frac{1}{2\pi fC}, \tag{23.58}$$

where X_C is called the **capacitive reactance**, because the capacitor reacts to impede the current. X_C has units of ohms (verification left as an exercise for the reader). X_C is inversely proportional to the capacitance C; the larger the capacitor, the greater the charge it can store and the greater the current that can flow. It is also inversely proportional to the frequency f; the greater the frequency, the less time there is to fully charge the capacitor, and so it impedes current less.

Example 23.11 Calculating Capacitive Reactance and then Current

(a) Calculate the capacitive reactance of a 5.00 mF capacitor when 60.0 Hz and 10.0 kHz AC voltages are applied. (b) What is the rms current if the applied rms voltage is 120 V?

Strategy

The capacitive reactance is found directly from the expression in $X_C = \frac{1}{2\pi fC}$. Once X_C has been found at each frequency, Ohm's law stated as $I = V / X_C$ can be used to find the current at each frequency.

Solution for (a)

Entering the frequency and capacitance into $X_C = \frac{1}{2\pi fC}$ gives

$$\begin{aligned} X_C &= \frac{1}{2\pi fC} \\ &= \frac{1}{6.28(60.0/\text{s})(5.00\ \mu\text{F})} = 531\ \Omega \text{ at } 60 \text{ Hz}. \end{aligned} \tag{23.59}$$

Similarly, at 10 kHz,

$$\begin{aligned} X_C &= \frac{1}{2\pi fC} = \frac{1}{6.28(1.00\times 10^4/\text{s})(5.00\ \mu\text{F})}. \\ &= 3.18\ \Omega \text{ at } 10 \text{ kHz} \end{aligned} \tag{23.60}$$

Solution for (b)

The rms current is now found using the version of Ohm's law in $I = V / X_C$, given the applied rms voltage is 120 V. For the first frequency, this yields

$$I = \frac{V}{X_C} = \frac{120\text{ V}}{531\ \Omega} = 0.226 \text{ A at } 60 \text{ Hz}. \tag{23.61}$$

Similarly, at 10 kHz,

$$I = \frac{V}{X_C} = \frac{120\text{ V}}{3.18\ \Omega} = 37.7 \text{ A at } 10 \text{ kHz}. \tag{23.62}$$

Discussion

The capacitor reacts very differently at the two different frequencies, and in exactly the opposite way an inductor reacts. At the higher frequency, its reactance is small and the current is large. Capacitors favor change, whereas inductors oppose change. Capacitors impede low frequencies the most, since low frequency allows them time to become charged and stop the current. Capacitors can be used to filter out low frequencies. For example, a capacitor in series with a sound reproduction system rids it of the 60 Hz hum.

Although a capacitor is basically an open circuit, there is an rms current in a circuit with an AC voltage applied to a capacitor. This is because the voltage is continually reversing, charging and discharging the capacitor. If the frequency goes to zero (DC), X_C tends to infinity, and the current is zero once the capacitor is charged. At very high frequencies, the capacitor's reactance tends to zero—it has a negligible reactance and does not impede the current (it acts like a simple wire). *Capacitors have the opposite effect on AC circuits that inductors have*.

Resistors in an AC Circuit

Just as a reminder, consider **Figure 23.47**, which shows an AC voltage applied to a resistor and a graph of voltage and current versus time. The voltage and current are exactly *in phase* in a resistor. There is no frequency dependence to the behavior of plain resistance in a circuit:

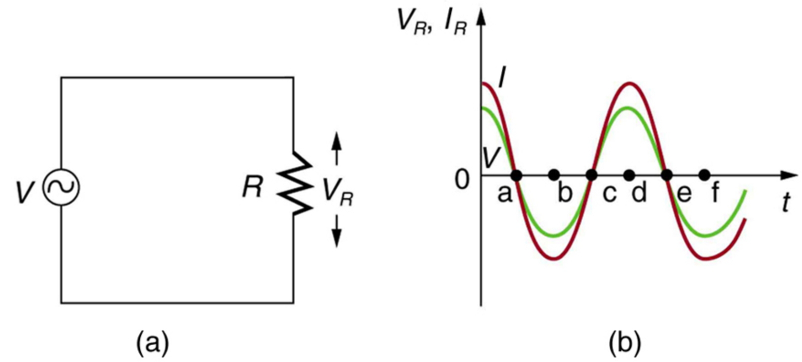


Figure 23.47 (a) An AC voltage source in series with a resistor. (b) Graph of current and voltage across the resistor as functions of time, showing them to be exactly in phase.

AC Voltage in a Resistor

When a sinusoidal voltage is applied to a resistor, the voltage is exactly in phase with the current—they have a 0° phase angle.

23.12 RLC Series AC Circuits

Impedance

When alone in an AC circuit, inductors, capacitors, and resistors all impede current. How do they behave when all three occur together? Interestingly, their individual resistances in ohms do not simply add. Because inductors and capacitors behave in opposite ways, they partially to totally cancel each other's effect. **Figure 23.48** shows an *RLC* series circuit with an AC voltage source, the behavior of which is the subject of this section. The crux of the analysis of an *RLC* circuit is the frequency dependence of X_L and X_C, and the effect they have on the phase of voltage versus current (established in the preceding section). These give rise to the frequency dependence of the circuit, with important "resonance" features that are the basis of many applications, such as radio tuners.

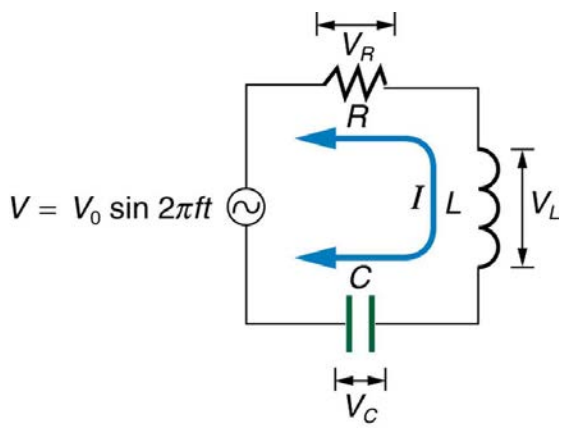


Figure 23.48 An *RLC* series circuit with an AC voltage source.

The combined effect of resistance R, inductive reactance X_L, and capacitive reactance X_C is defined to be **impedance**, an AC analogue to resistance in a DC circuit. Current, voltage, and impedance in an *RLC* circuit are related by an AC version of Ohm's law:

$$I_0 = \frac{V_0}{Z} \text{ or } I_{\text{rms}} = \frac{V_{\text{rms}}}{Z}. \tag{23.63}$$

Here I_0 is the peak current, V_0 the peak source voltage, and Z is the impedance of the circuit. The units of impedance are ohms, and its effect on the circuit is as you might expect: the greater the impedance, the smaller the current. To get an expression for Z in terms of R, X_L, and X_C, we will now examine how the voltages across the various components are related to the source voltage. Those voltages are labeled V_R, V_L, and V_C in **Figure 23.48**.

Conservation of charge requires current to be the same in each part of the circuit at all times, so that we can say the currents in R, L, and C are equal and in phase. But we know from the preceding section that the voltage across the inductor V_L leads the current by one-fourth of a cycle, the voltage across the capacitor V_C follows the current by one-fourth of a cycle, and the voltage across the resistor V_R is exactly in phase with the current. **Figure 23.49** shows these relationships in one graph, as well as showing the total voltage around the circuit $V = V_R + V_L + V_C$, where all four voltages are the instantaneous values. According to Kirchhoff's loop rule, the total voltage around the circuit V is also the voltage of the source. You can see from **Figure 23.49** that while V_R is in phase with the current, V_L leads by 90°, and V_C follows by 90°. Thus V_L and V_C are 180° out of phase (crest to trough) and tend to cancel, although not completely unless they have the same magnitude. Since the peak voltages are not aligned (not in phase), the peak voltage V_0 of the source does *not* equal the sum of the peak voltages across R, L, and C. The actual relationship is

$$V_0 = \sqrt{{V_{0R}}^2 + (V_{0L} - V_{0C})^2}, \tag{23.64}$$

where V_{0R}, V_{0L}, and V_{0C} are the peak voltages across R, L, and C, respectively. Now, using Ohm's law and definitions from **Reactance, Inductive and Capacitive**, we substitute $V_0 = I_0 Z$ into the above, as well as $V_{0R} = I_0 R$, $V_{0L} = I_0 X_L$, and $V_{0C} = I_0 X_C$, yielding

$$I_0 Z = \sqrt{{I_0}^2 R^2 + (I_0 X_L - I_0 X_C)^2} = I_0 \sqrt{R^2 + (X_L - X_C)^2}. \tag{23.65}$$

I_0 cancels to yield an expression for Z:

$$Z = \sqrt{R^2 + (X_L - X_C)^2}, \tag{23.66}$$

which is the impedance of an *RLC* series AC circuit. For circuits without a resistor, take $R = 0$; for those without an inductor, take $X_L = 0$; and for those without a capacitor, take $X_C = 0$.

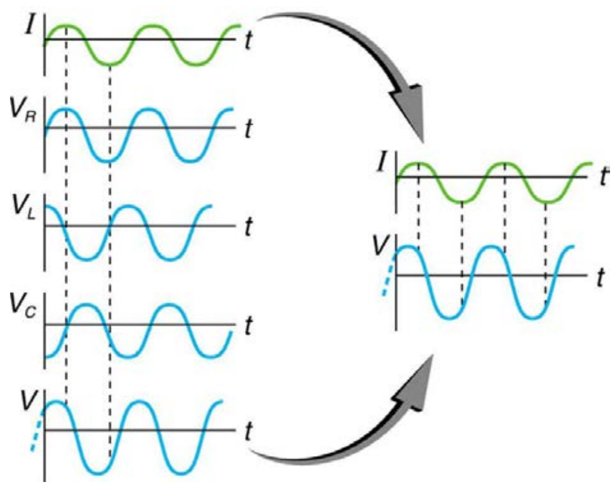


Figure 23.49 This graph shows the relationships of the voltages in an *RLC* circuit to the current. The voltages across the circuit elements add to equal the voltage of the source, which is seen to be out of phase with the current.

Example 23.12 Calculating Impedance and Current

An *RLC* series circuit has a $40.0\ \Omega$ resistor, a 3.00 mH inductor, and a $5.00\ \mu\text{F}$ capacitor. (a) Find the circuit's impedance at 60.0 Hz and 10.0 kHz, noting that these frequencies and the values for L and C are the same as in **Example 23.10** and **Example 23.11**. (b) If the voltage source has $V_{\text{rms}} = 120\ \text{V}$, what is I_{rms} at each frequency?

Strategy

For each frequency, we use $Z = \sqrt{R^2 + (X_L - X_C)^2}$ to find the impedance and then Ohm's law to find current. We can take advantage of the results of the previous two examples rather than calculate the reactances again.

Solution for (a)

At 60.0 Hz, the values of the reactances were found in **Example 23.10** to be $X_L = 1.13\ \ \Omega$ and in **Example 23.11** to be $X_C = 531\ \ \Omega$.

Entering these and the given $40.0\ \Omega$ for resistance into $Z = \sqrt{R^2 + (X_L - X_C)^2}$ yields

$$\begin{aligned} Z &= \sqrt{R^2 + (X_L - X_C)^2} \\ &= \sqrt{(40.0\ \ \Omega\ )^2 + (1.13\ \ \Omega\ - 531\ \ \Omega\ )^2} \\ &= 531\ \ \Omega\ \text{ at } 60.0\ \text{Hz.} \end{aligned} \tag{23.67}$$

Similarly, at 10.0 kHz, $X_L = 188\ \ \Omega$ and $X_C = 3.18\ \ \Omega$, so that

$$\begin{aligned} Z &= \sqrt{(40.0\ \ \Omega\ )^2 + (188\ \ \Omega\ - 3.18\ \ \Omega\ )^2} \\ &= 190\ \ \Omega\ \text{ at } 10.0\ \text{kHz.} \end{aligned} \tag{23.68}$$

Discussion for (a)

In both cases, the result is nearly the same as the largest value, and the impedance is definitely not the sum of the individual values. It is clear that X_L dominates at high frequency and X_C dominates at low frequency.

Solution for (b)

The current I_{rms} can be found using the AC version of Ohm's law in Equation $I_{\text{rms}} = V_{\text{rms}}/Z$:

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{120\ \text{V}}{531\ \ \Omega} = 0.226\ \text{A} \text{ at 60.0 Hz}$$

Finally, at 10.0 kHz, we find

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{120\ \text{V}}{190\ \ \Omega} = 0.633\ \text{A} \text{ at 10.0 kHz}$$

Discussion for (a)

The current at 60.0 Hz is the same (to three digits) as found for the capacitor alone in **Example 23.11**. The capacitor dominates at low frequency. The current at 10.0 kHz is only slightly different from that found for the inductor alone in **Example 23.10**. The inductor dominates at high frequency.

Resonance in *RLC* Series AC Circuits

How does an *RLC* circuit behave as a function of the frequency of the driving voltage source? Combining Ohm's law, $I_{\text{rms}} = V_{\text{rms}}/Z$, and the expression for impedance Z from $Z = \sqrt{R^2 + (X_L - X_C)^2}$ gives

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{\sqrt{R^2 + (X_L - X_C)^2}}. \tag{23.69}$$

The reactances vary with frequency, with X_L large at high frequencies and X_C large at low frequencies, as we have seen in three previous examples. At some intermediate frequency f_0 , the reactances will be equal and cancel, giving $Z = R$ —this is a minimum value for impedance, and a maximum value for I_{rms} results. We can get an expression for f_0 by taking

$$X_L = X_C. \tag{23.70}$$

Substituting the definitions of X_L and X_C ,

$$2\pi f_0 L = \frac{1}{2\pi f_0 C}. \tag{23.71}$$

Solving this expression for f_0 yields

$$f_0 = \frac{1}{2\pi\sqrt{LC}}, \tag{23.72}$$

where f_0 is the **resonant frequency** of an *RLC* series circuit. This is also the *natural frequency* at which the circuit would oscillate if not driven by the voltage source. At f_0 , the effects of the inductor and capacitor cancel, so that $Z = R$, and I_{rms} is a maximum.

Resonance in AC circuits is analogous to mechanical resonance, where resonance is defined to be a forced oscillation—in this case, forced by the voltage source—at the natural frequency of the system. The receiver in a radio is an *RLC* circuit that oscillates best at its f_0 . A variable capacitor is often used to adjust f_0 to receive a desired frequency and to reject others. **Figure 23.50** is a graph of current as a function of frequency, illustrating a resonant peak in I_{rms} at f_0 . The two curves are for two different circuits, which differ only in the amount of resistance in them. The peak is lower and broader for the higher-resistance circuit. Thus the higher-resistance circuit does not resonate as strongly and would not be as selective in a radio receiver, for example.

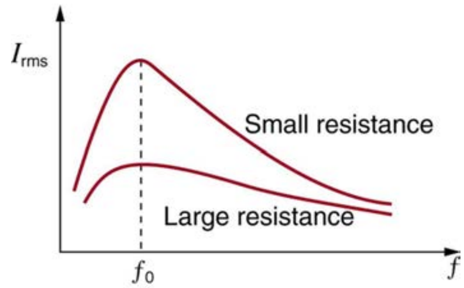


Figure 23.50 A graph of current versus frequency for two *RLC* series circuits differing only in the amount of resistance. Both have a resonance at f_0, but that for the higher resistance is lower and broader. The driving AC voltage source has a fixed amplitude V_0.

Example 23.13 Calculating Resonant Frequency and Current

For the same *RLC* series circuit having a $40.0\ \Omega$ resistor, a 3.00 mH inductor, and a $5.00\ \mu\text{F}$ capacitor: (a) Find the resonant frequency. (b) Calculate I_{rms} at resonance if V_{rms} is 120 V.

Strategy

The resonant frequency is found by using the expression in $f_0 = \frac{1}{2\pi\sqrt{LC}}$. The current at that frequency is the same as if the resistor alone were in the circuit.

Solution for (a)

Entering the given values for L and C into the expression given for f_0 in $f_0 = \frac{1}{2\pi\sqrt{LC}}$ yields

$$\begin{aligned} f_0 &= \frac{1}{2\pi\sqrt{LC}} \\ &= \frac{1}{2\pi\sqrt{(3.00\times10^{-3}\text{ H})(5.00\times10^{-6}\text{ F})}} = 1.30\text{ kHz}. \end{aligned} \tag{23.73}$$

Discussion for (a)

We see that the resonant frequency is between 60.0 Hz and 10.0 kHz, the two frequencies chosen in earlier examples. This was to be expected, since the capacitor dominated at the low frequency and the inductor dominated at the high frequency. Their effects are the same at this intermediate frequency.

Solution for (b)

The current is given by Ohm's law. At resonance, the two reactances are equal and cancel, so that the impedance equals the resistance alone. Thus,

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{120\text{ V}}{40.0\ \ \Omega} = 3.00\text{ A}. \tag{23.74}$$

Discussion for (b)

At resonance, the current is greater than at the higher and lower frequencies considered for the same circuit in the preceding example.

Power in *RLC* Series AC Circuits

If current varies with frequency in an *RLC* circuit, then the power delivered to it also varies with frequency. But the average power is not simply current times voltage, as it is in purely resistive circuits. As was seen in **Figure 23.49**, voltage and current are out of phase in an *RLC* circuit. There is a **phase angle** ϕ between the source voltage V and the current I, which can be found from

$$\cos\phi = \frac{R}{Z}. \tag{23.75}$$

For example, at the resonant frequency or in a purely resistive circuit $Z = R$, so that $\cos\phi = 1$. This implies that $\phi = 0\ ^\text{o}$ and that voltage and current are in phase, as expected for resistors. At other frequencies, average power is less than at resonance. This is both because voltage and current are out of phase and because I_{rms} is lower. The fact that source voltage and current are out of phase affects the power delivered to the circuit. It can be shown that the *average power* is

$$P_{\text{ave}} = I_{\text{rms}}V_{\text{rms}}\cos\phi, \tag{23.76}$$

Thus $\cos\phi$ is called the **power factor**, which can range from 0 to 1. Power factors near 1 are desirable when designing an efficient motor, for example. At the resonant frequency, $\cos\phi = 1$.

Example 23.14 Calculating the Power Factor and Power

For the same *RLC* series circuit having a $40.0\ \Omega$ resistor, a 3.00 mH inductor, a $5.00\ \mu\text{F}$ capacitor, and a voltage source with a V_{rms} of 120 V: (a) Calculate the power factor and phase angle for $f = 60.0\text{Hz}$. (b) What is the average power at 50.0 Hz? (c) Find the average power at the circuit's resonant frequency.

Strategy and Solution for (a)

The power factor at 60.0 Hz is found from

$$\cos\phi = \frac{R}{Z}. \tag{23.77}$$

We know $Z = 531\ \Omega$ from **Example 23.12**, so that

$$\cos\phi = \frac{40.0\ \ \Omega}{531\ \ \Omega} = 0.0753 \text{ at } 60.0 \text{ Hz}. \tag{23.78}$$

This small value indicates the voltage and current are significantly out of phase. In fact, the phase angle is

$$\phi = \cos^{-1} 0.0753 = 85.7^{\text{o}} \text{ at } 60.0 \text{ Hz}. \tag{23.79}$$

Discussion for (a)

The phase angle is close to 90^{o}, consistent with the fact that the capacitor dominates the circuit at this low frequency (a pure *RC* circuit has its voltage and current 90^{o} out of phase).

Strategy and Solution for (b)

The average power at 60.0 Hz is

$$P_{\text{ave}} = I_{\text{rms}}V_{\text{rms}}\cos\phi. \tag{23.80}$$

I_{rms} was found to be 0.226 A in **Example 23.12**. Entering the known values gives

$$P_{\text{ave}} = (0.226 \text{ A})(120 \text{ V})(0.0753) = 2.04 \text{ W at } 60.0 \text{ Hz}. \tag{23.81}$$

Strategy and Solution for (c)

At the resonant frequency, we know $\cos\phi = 1$, and I_{rms} was found to be 6.00 A in **Example 23.13**. Thus,

$P_{\text{ave}} = (3.00 \text{ A})(120 \text{ V})(1) = 360 \text{ W}$ at resonance (1.30 kHz)

Discussion

Both the current and the power factor are greater at resonance, producing significantly greater power than at higher and lower frequencies.

Power delivered to an *RLC* series AC circuit is dissipated by the resistance alone. The inductor and capacitor have energy input and output but do not dissipate it out of the circuit. Rather they transfer energy back and forth to one another, with the resistor dissipating exactly what the voltage source puts into the circuit. This assumes no significant electromagnetic radiation from the inductor and capacitor, such as radio waves. Such radiation can happen and may even be desired, as we will see in the next chapter on electromagnetic radiation, but it can also be suppressed as is the case in this chapter. The circuit is analogous to the wheel of a car driven over a corrugated road as shown in **Figure 23.51**. The regularly spaced bumps in the road are analogous to the voltage source, driving the wheel up and down. The shock absorber is analogous to the resistance damping and limiting the amplitude of the oscillation. Energy within the system goes back and forth between kinetic (analogous to maximum current, and energy stored in an inductor) and potential energy stored in the car spring (analogous to no current, and energy stored in the electric field of a capacitor). The amplitude of the wheels' motion is a maximum if the bumps in the road are hit at the resonant frequency.

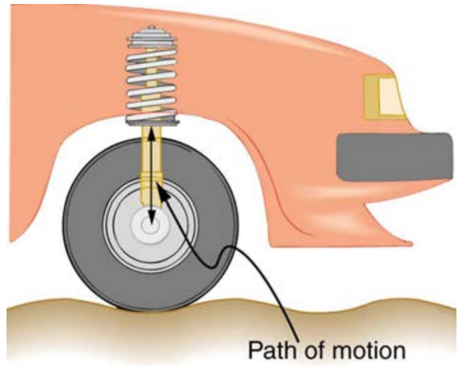


Figure 23.51 The forced but damped motion of the wheel on the car spring is analogous to an *RLC* series AC circuit. The shock absorber damps the motion and dissipates energy, analogous to the resistance in an *RLC* circuit. The mass and spring determine the resonant frequency.

A pure *LC* circuit with negligible resistance oscillates at f_0, the same resonant frequency as an *RLC* circuit. It can serve as a frequency standard or clock circuit—for example, in a digital wristwatch. With a very small resistance, only a very small energy input is necessary to maintain the oscillations. The circuit is analogous to a car with no shock absorbers. Once it starts oscillating, it continues at its natural frequency for some time. **Figure 23.52** shows the analogy between an *LC* circuit and a mass on a spring.

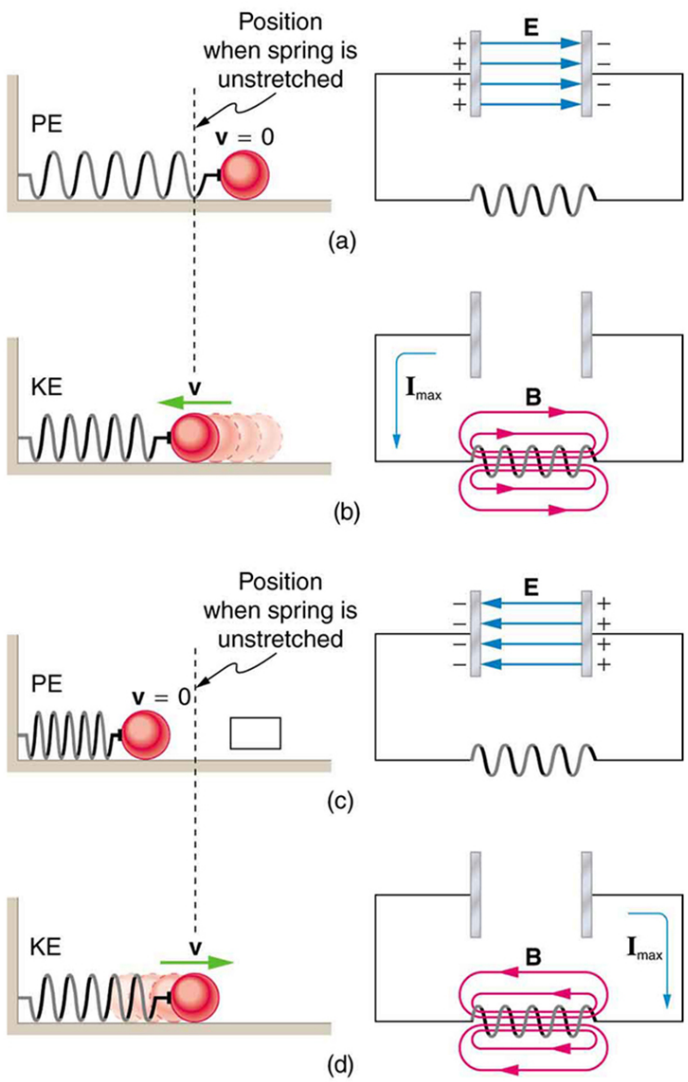


Figure 23.52 An *LC* circuit is analogous to a mass oscillating on a spring with no friction and no driving force. Energy moves back and forth between the inductor and capacitor, just as it moves from kinetic to potential in the mass-spring system.

PhET Explorations: Circuit Construction Kit (AC+DC), Virtual Lab

Build circuits with capacitors, inductors, resistors and AC or DC voltage sources, and inspect them using lab instruments such as voltmeters and ammeters.

PhET Interactive Simulation

Figure 23.53 Circuit Construction Kit (AC+DC), Virtual Lab (http://cnx.org/content/m42431/1.5/circuit-construction-kit-ac-virtual-lab_en.jar)

Glossary

back emf: the emf generated by a running motor, because it consists of a coil turning in a magnetic field; it opposes the voltage powering the motor

capacitive reactance: the opposition of a capacitor to a change in current; calculated by $X_C = \frac{1}{2\pi fC}$

characteristic time constant: denoted by τ, of a particular series *RL* circuit is calculated by $\tau = \frac{L}{R}$, where L is the inductance and R is the resistance

eddy current: a current loop in a conductor caused by motional emf

electric generator: a device for converting mechanical work into electric energy; it induces an emf by rotating a coil in a magnetic field

electromagnetic induction: the process of inducing an emf (voltage) with a change in magnetic flux

emf induced in a generator coil: $\text{emf} = NAB\omega \sin \omega t$, where A is the area of an N-turn coil rotated at a constant angular velocity ω in a uniform magnetic field B, over a period of time t

energy stored in an inductor: self-explanatory; calculated by $E_{\text{ind}} = \frac{1}{2}LI^2$

Faraday's law of induction: the means of calculating the emf in a coil due to changing magnetic flux, given by $\text{emf} = -N\frac{\Delta\Phi}{\Delta t}$

henry: the unit of inductance; $1 \text{ H} = 1 \ \Omega \cdot \text{s}$

impedance: the AC analogue to resistance in a DC circuit; it is the combined effect of resistance, inductive reactance, and capacitive reactance in the form $Z = \sqrt{R^2 + (X_L - X_C)^2}$

inductance: a property of a device describing how efficient it is at inducing emf in another device

induction: (magnetic induction) the creation of emfs and hence currents by magnetic fields

inductive reactance: the opposition of an inductor to a change in current; calculated by $X_L = 2\pi fL$

inductor: a device that exhibits significant self-inductance

Lenz's law: the minus sign in Faraday's law, signifying that the emf induced in a coil opposes the change in magnetic flux

magnetic damping: the drag produced by eddy currents

magnetic flux: the amount of magnetic field going through a particular area, calculated with $\Phi = BA \cos \theta$ where B is the magnetic field strength over an area A at an angle θ with the perpendicular to the area

mutual inductance: how effective a pair of devices are at inducing emfs in each other

peak emf: $\text{emf}_0 = NAB\omega$

phase angle: denoted by ϕ, the amount by which the voltage and current are out of phase with each other in a circuit

power factor: the amount by which the power delivered in the circuit is less than the theoretical maximum of the circuit due to voltage and current being out of phase; calculated by $\cos \phi$

resonant frequency: the frequency at which the impedance in a circuit is at a minimum, and also the frequency at which the circuit would oscillate if not driven by a voltage source; calculated by $f_0 = \frac{1}{2\pi\sqrt{LC}}$

self-inductance: how effective a device is at inducing emf in itself

shock hazard: the term for electrical hazards due to current passing through a human

step-down transformer: a transformer that decreases voltage

step-up transformer: a transformer that increases voltage

thermal hazard: the term for electrical hazards due to overheating

three-wire system: the wiring system used at present for safety reasons, with live, neutral, and ground wires

transformer equation: the equation showing that the ratio of the secondary to primary voltages in a transformer equals the ratio of the number of loops in their coils; $\frac{V_\text{s}}{V_\text{p}} = \frac{N_\text{s}}{N_\text{p}}$

transformer: a device that transforms voltages from one value to another using induction

Section Summary

23.1 Induced Emf and Magnetic Flux

- The crucial quantity in induction is magnetic flux Φ, defined to be $\Phi = BA\cos\theta$, where B is the magnetic field strength over an area A at an angle θ with the perpendicular to the area.
- Units of magnetic flux Φ are $\text{T}\cdot\text{m}^2$.
- Any change in magnetic flux Φ induces an emf—the process is defined to be electromagnetic induction.

23.2 Faraday's Law of Induction: Lenz's Law

- Faraday's law of induction states that the emfinduced by a change in magnetic flux is
$$\text{emf} = -N\frac{\Delta\Phi}{\Delta t}$$
when flux changes by $\Delta\Phi$ in a time Δt.
- If emf is induced in a coil, N is its number of turns.
- The minus sign means that the emf creates a current I and magnetic field B that *oppose the change in flux* $\Delta\Phi$ —this opposition is known as Lenz's law.

23.3 Motional Emf

- An emf induced by motion relative to a magnetic field B is called a *motional emf* and is given by
$$\text{emf} = B\ell v \qquad (B, \ell, \text{and } v \text{ perpendicular}),$$
where ℓ is the length of the object moving at speed v relative to the field.

23.4 Eddy Currents and Magnetic Damping

- Current loops induced in moving conductors are called eddy currents.
- They can create significant drag, called magnetic damping.

23.5 Electric Generators

- An electric generator rotates a coil in a magnetic field, inducing an emfgiven as a function of time by
$$\text{emf} = NAB\omega \sin \omega t,$$
where A is the area of an N-turn coil rotated at a constant angular velocity ω in a uniform magnetic field B.
- The peak emf emf_0 of a generator is
$$\text{emf}_0 = NAB\omega.$$

23.6 Back Emf

- Any rotating coil will have an induced emf—in motors, this is called back emf, since it opposes the emf input to the motor.

23.7 Transformers

- Transformers use induction to transform voltages from one value to another.
- For a transformer, the voltages across the primary and secondary coils are related by
$$\frac{V_\text{s}}{V_\text{p}} = \frac{N_\text{s}}{N_\text{p}},$$
where V_p and V_s are the voltages across primary and secondary coils having N_p and N_s turns.
- The currents I_p and I_s in the primary and secondary coils are related by $\frac{I_\text{s}}{I_\text{p}} = \frac{N_\text{p}}{N_\text{s}}$.
- A step-up transformer increases voltage and decreases current, whereas a step-down transformer decreases voltage and increases current.

23.8 Electrical Safety: Systems and Devices

- Electrical safety systems and devices are employed to prevent thermal and shock hazards.

- Circuit breakers and fuses interrupt excessive currents to prevent thermal hazards.
- The three-wire system guards against thermal and shock hazards, utilizing live/hot, neutral, and earth/ground wires, and grounding the neutral wire and case of the appliance.
- A ground fault interrupter (GFI) prevents shock by detecting the loss of current to unintentional paths.
- An isolation transformer insulates the device being powered from the original source, also to prevent shock.
- Many of these devices use induction to perform their basic function.

23.9 Inductance

- Inductance is the property of a device that tells how effectively it induces an emf in another device.
- Mutual inductance is the effect of two devices in inducing emfs in each other.
- A change in current $\Delta I_1 / \Delta t$ in one induces an emf emf_2 in the second:
$$\text{emf}_2 = -M\frac{\Delta I_1}{\Delta t},$$
where M is defined to be the mutual inductance between the two devices, and the minus sign is due to Lenz's law.
- Symmetrically, a change in current $\Delta I_2 / \Delta t$ through the second device induces an emf emf_1 in the first:
$$\text{emf}_1 = -M\frac{\Delta I_2}{\Delta t},$$
where M is the same mutual inductance as in the reverse process.
- Current changes in a device induce an emf in the device itself.
- Self-inductance is the effect of the device inducing emf in itself.
- The device is called an inductor, and the emf induced in it by a change in current through it is
$$\text{emf} = -L\frac{\Delta I}{\Delta t},$$
where L is the self-inductance of the inductor, and $\Delta I / \Delta t$ is the rate of change of current through it. The minus sign indicates that emf opposes the change in current, as required by Lenz's law.
- The unit of self- and mutual inductance is the henry (H), where $1\text{ H} = 1\ \Omega \cdot \text{s}$.
- The self-inductance L of an inductor is proportional to how much flux changes with current. For an N-turn inductor,
$$L = N\frac{\Delta\Phi}{\Delta I}.$$
- The self-inductance of a solenoid is
$$L = \frac{\mu_0 N^2 A}{\ell}\text{(solenoid)},$$
where N is its number of turns in the solenoid, A is its cross-sectional area, ℓ is its length, and $\mu_0 = 4\pi\times10^{-7}\text{ T}\cdot\text{m/A}$ is the permeability of free space.
- The energy stored in an inductor E_{ind} is
$$E_{\text{ind}} = \frac{1}{2}LI^2.$$

23.10 RL Circuits

- When a series connection of a resistor and an inductor—an *RL* circuit—is connected to a voltage source, the time variation of the current is
$$I = I_0(1 - e^{-t/\tau}) \quad \text{(turning on)}.$$
where $I_0 = V/R$ is the final current.
- The characteristic time constant τ is $\tau = \frac{L}{R}$, where L is the inductance and R is the resistance.
- In the first time constant τ, the current rises from zero to $0.632I_0$, and 0.632 of the remainder in every subsequent time interval τ.
- When the inductor is shorted through a resistor, current decreases as
$$I = I_0 e^{-t/\tau} \quad \text{(turning off)}.$$
Here I_0 is the initial current.
- Current falls to $0.368I_0$ in the first time interval τ, and 0.368 of the remainder toward zero in each subsequent time τ.

23.11 Reactance, Inductive and Capacitive

- For inductors in AC circuits, we find that when a sinusoidal voltage is applied to an inductor, the voltage leads the current by one-fourth of a cycle, or by a 90° phase angle.
- The opposition of an inductor to a change in current is expressed as a type of AC resistance.
- Ohm's law for an inductor is
$$I = \frac{V}{X_L},$$
where V is the rms voltage across the inductor.
- X_L is defined to be the inductive reactance, given by

$$X_L = 2\pi fL,$$

with f the frequency of the AC voltage source in hertz.

- Inductive reactance X_L has units of ohms and is greatest at high frequencies.
- For capacitors, we find that when a sinusoidal voltage is applied to a capacitor, the voltage follows the current by one-fourth of a cycle, or by a 90° phase angle.
- Since a capacitor can stop current when fully charged, it limits current and offers another form of AC resistance; Ohm's law for a capacitor is

$$I = \frac{V}{X_C},$$

where V is the rms voltage across the capacitor.

- X_C is defined to be the capacitive reactance, given by

$$X_C = \frac{1}{2\pi fC}.$$

- X_C has units of ohms and is greatest at low frequencies.

23.12 RLC Series AC Circuits

- The AC analogy to resistance is impedance Z, the combined effect of resistors, inductors, and capacitors, defined by the AC version of Ohm's law:

$$I_0 = \frac{V_0}{Z} \text{ or } I_{\text{rms}} = \frac{V_{\text{rms}}}{Z},$$

where I_0 is the peak current and V_0 is the peak source voltage.

- Impedance has units of ohms and is given by $Z = \sqrt{R^2 + (X_L - X_C)^2}$.
- The resonant frequency f_0, at which $X_L = X_C$, is

$$f_0 = \frac{1}{2\pi\sqrt{LC}}.$$

- In an AC circuit, there is a phase angle ϕ between source voltage V and the current I, which can be found from

$$\cos\phi = \frac{R}{Z},$$

- $\phi = 0^\circ$ for a purely resistive circuit or an *RLC* circuit at resonance.
- The average power delivered to an *RLC* circuit is affected by the phase angle and is given by

$$P_{\text{ave}} = I_{\text{rms}}V_{\text{rms}} \cos\phi,$$

$\cos\phi$ is called the power factor, which ranges from 0 to 1.

Conceptual Questions

23.1 Induced Emf and Magnetic Flux

1. How do the multiple-loop coils and iron ring in the version of Faraday's apparatus shown in **Figure 23.3** enhance the observation of induced emf?

2. When a magnet is thrust into a coil as in **Figure 23.4**(a), what is the direction of the force exerted by the coil on the magnet? Draw a diagram showing the direction of the current induced in the coil and the magnetic field it produces, to justify your response. How does the magnitude of the force depend on the resistance of the galvanometer?

3. Explain how magnetic flux can be zero when the magnetic field is not zero.

4. Is an emf induced in the coil in **Figure 23.54** when it is stretched? If so, state why and give the direction of the induced current.

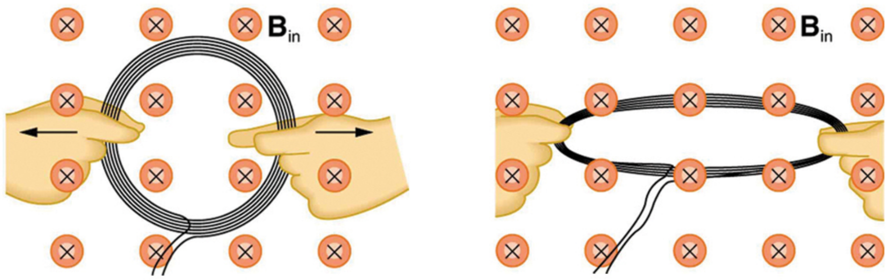


Figure 23.54 A circular coil of wire is stretched in a magnetic field.

23.2 Faraday's Law of Induction: Lenz's Law

5. A person who works with large magnets sometimes places her head inside a strong field. She reports feeling dizzy as she quickly turns her head. How might this be associated with induction?

6. A particle accelerator sends high-velocity charged particles down an evacuated pipe. Explain how a coil of wire wrapped around the pipe could detect the passage of individual particles. Sketch a graph of the voltage output of the coil as a single particle passes through it.

23.3 Motional Emf

7. Why must part of the circuit be moving relative to other parts, to have usable motional emf? Consider, for example, that the rails in **Figure 23.11** are stationary relative to the magnetic field, while the rod moves.

8. A powerful induction cannon can be made by placing a metal cylinder inside a solenoid coil. The cylinder is forcefully expelled when solenoid current is turned on rapidly. Use Faraday's and Lenz's laws to explain how this works. Why might the cylinder get live/hot when the cannon is fired?

9. An induction stove heats a pot with a coil carrying an alternating current located beneath the pot (and without a hot surface). Can the stove surface be a conductor? Why won't a coil carrying a direct current work?

10. Explain how you could thaw out a frozen water pipe by wrapping a coil carrying an alternating current around it. Does it matter whether or not the pipe is a conductor? Explain.

23.4 Eddy Currents and Magnetic Damping

11. Explain why magnetic damping might not be effective on an object made of several thin conducting layers separated by insulation.

12. Explain how electromagnetic induction can be used to detect metals? This technique is particularly important in detecting buried landmines for disposal, geophysical prospecting and at airports.

23.5 Electric Generators

13. Using RHR-1, show that the emfs in the sides of the generator loop in **Figure 23.23** are in the same sense and thus add.

14. The source of a generator's electrical energy output is the work done to turn its coils. How is the work needed to turn the generator related to Lenz's law?

23.6 Back Emf

15. Suppose you find that the belt drive connecting a powerful motor to an air conditioning unit is broken and the motor is running freely. Should you be worried that the motor is consuming a great deal of energy for no useful purpose? Explain why or why not.

23.7 Transformers

16. Explain what causes physical vibrations in transformers at twice the frequency of the AC power involved.

23.8 Electrical Safety: Systems and Devices

17. Does plastic insulation on live/hot wires prevent shock hazards, thermal hazards, or both?

18. Why are ordinary circuit breakers and fuses ineffective in preventing shocks?

19. A GFI may trip just because the live/hot and neutral wires connected to it are significantly different in length. Explain why.

23.9 Inductance

20. How would you place two identical flat coils in contact so that they had the greatest mutual inductance? The least?

21. How would you shape a given length of wire to give it the greatest self-inductance? The least?

22. Verify, as was concluded without proof in **Example 23.7**, that units of $\mathrm{T \cdot m^2} / A = \Omega \cdot \mathrm{s} = \mathrm{H}$.

23.11 Reactance, Inductive and Capacitive

23. Presbycusis is a hearing loss due to age that progressively affects higher frequencies. A hearing aid amplifier is designed to amplify all frequencies equally. To adjust its output for presbycusis, would you put a capacitor in series or parallel with the hearing aid's speaker? Explain.

24. Would you use a large inductance or a large capacitance in series with a system to filter out low frequencies, such as the 100 Hz hum in a sound system? Explain.

25. High-frequency noise in AC power can damage computers. Does the plug-in unit designed to prevent this damage use a large inductance or a large capacitance (in series with the computer) to filter out such high frequencies? Explain.

26. Does inductance depend on current, frequency, or both? What about inductive reactance?

27. Explain why the capacitor in **Figure 23.55**(a) acts as a low-frequency filter between the two circuits, whereas that in **Figure 23.55**(b) acts as a high-frequency filter.

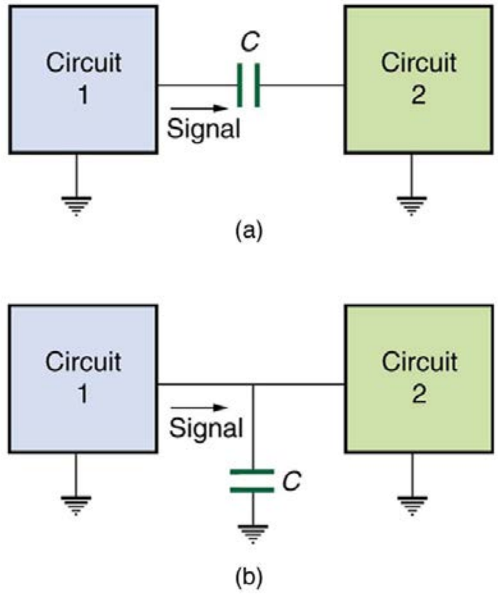


Figure 23.55 Capacitors and inductors. Capacitor with high frequency and low frequency.

28. If the capacitors in **Figure 23.55** are replaced by inductors, which acts as a low-frequency filter and which as a high-frequency filter?

23.12 RLC Series AC Circuits

29. Does the resonant frequency of an AC circuit depend on the peak voltage of the AC source? Explain why or why not.

30. Suppose you have a motor with a power factor significantly less than 1. Explain why it would be better to improve the power factor as a method of improving the motor's output, rather than to increase the voltage input.

Problems & Exercises

23.1 Induced Emf and Magnetic Flux

31. What is the value of the magnetic flux at coil 2 in **Figure 23.56** due to coil 1?

Figure 23.56 (a) The planes of the two coils are perpendicular. (b) The wire is perpendicular to the plane of the coil.

32. What is the value of the magnetic flux through the coil in **Figure 23.56**(b) due to the wire?

23.2 Faraday's Law of Induction: Lenz's Law

33. Referring to **Figure 23.57**(a), what is the direction of the current induced in coil 2: (a) If the current in coil 1 increases? (b) If the current in coil 1 decreases? (c) If the current in coil 1 is constant? Explicitly show how you follow the steps in the **Problem-Solving Strategy for Lenz's Law**.

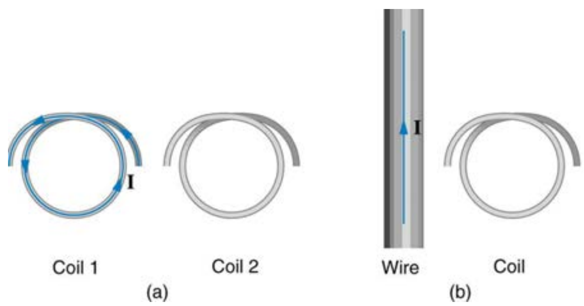


Figure 23.57 (a) The coils lie in the same plane. (b) The wire is in the plane of the coil

34. Referring to **Figure 23.57**(b), what is the direction of the current induced in the coil: (a) If the current in the wire increases? (b) If the current in the wire decreases? (c) If the current in the wire suddenly changes direction? Explicitly show how you follow the steps in the **Problem-Solving Strategy for Lenz's Law**.

35. Referring to **Figure 23.58**, what are the directions of the currents in coils 1, 2, and 3 (assume that the coils are lying in the plane of the circuit): (a) When the switch is first closed? (b) When the switch has been closed for a long time? (c) Just after the switch is opened?

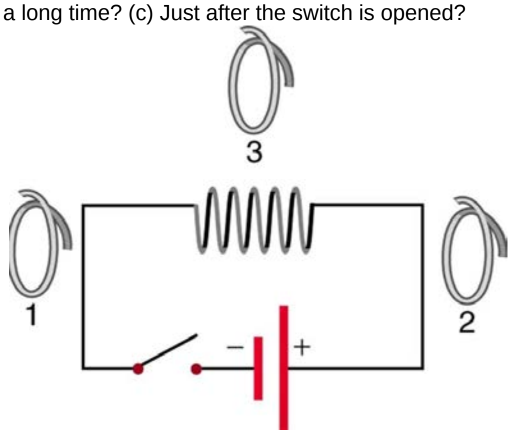


Figure 23.58

36. Repeat the previous problem with the battery reversed.

37. Verify that the units of $\Delta\Phi / \Delta t$ are volts. That is, show that $1\ \text{T}\cdot\text{m}^2/\text{s} = 1\ \text{V}$.

38. Suppose a 50-turn coil lies in the plane of the page in a uniform magnetic field that is directed into the page. The coil originally has an area of $0.250\ \text{m}^2$. It is stretched to have no area in 0.100 s. What is the direction and magnitude of the induced emf if the uniform magnetic field has a strength of 1.50 T?

39. (a) An MRI technician moves his hand from a region of very low magnetic field strength into an MRI scanner's 2.00 T field with his fingers pointing in the direction of the field. Find the average emf induced in his wedding ring, given its diameter is 2.20 cm and assuming it takes 0.250 s to move it into the field. (b) Discuss whether this current would significantly change the temperature of the ring.

40. Integrated Concepts

Referring to the situation in the previous problem: (a) What current is induced in the ring if its resistance is 0.0100 Ω? (b) What average power is dissipated? (c) What magnetic field is induced at the center of the ring? (d) What is the direction of the induced magnetic field relative to the MRI's field?

41. An emf is induced by rotating a 1000-turn, 20.0 cm diameter coil in the Earth's $5.00\times10^{-5}\ \text{T}$ magnetic field. What average emf is induced, given the plane of the coil is originally perpendicular to the Earth's field and is rotated to be parallel to the field in 10.0 ms?

42. A 0.250 m radius, 500-turn coil is rotated one-fourth of a revolution in 4.17 ms, originally having its plane perpendicular to a uniform magnetic field. (This is 60 rev/s.) Find the magnetic field strength needed to induce an average emf of 10,000 V.

43. Integrated Concepts

Approximately how does the emf induced in the loop in **Figure 23.57**(b) depend on the distance of the center of the loop from the wire?

44. Integrated Concepts

(a) A lightning bolt produces a rapidly varying magnetic field. If the bolt strikes the earth vertically and acts like a current in a long straight wire, it will induce a voltage in a loop aligned like that in **Figure 23.57**(b). What voltage is induced in a 1.00 m diameter loop 50.0 m from a $2.00\times10^{6}\ \text{A}$ lightning strike, if the current falls to zero in $25.0\ \mu\text{s}$? (b) Discuss circumstances under which such a voltage would produce noticeable consequences.

23.3 Motional Emf

45. Use Faraday's law, Lenz's law, and RHR-1 to show that the magnetic force on the current in the moving rod in **Figure 23.11** is in the opposite direction of its velocity.

46. If a current flows in the Satellite Tether shown in **Figure 23.12**, use Faraday's law, Lenz's law, and RHR-1 to show that there is a magnetic force on the tether in the direction opposite to its velocity.

47. (a) A jet airplane with a 75.0 m wingspan is flying at 280 m/s. What emf is induced between wing tips if the vertical component of the Earth's field is $3.00\times10^{-5}\ \text{T}$? (b) Is an emf of this magnitude likely to have any consequences? Explain.

48. (a) A nonferrous screwdriver is being used in a 2.00 T magnetic field. What maximum emf can be induced along its 12.0 cm length when it moves at 6.00 m/s? (b) Is it likely that this emf will have any consequences or even be noticed?

49. At what speed must the sliding rod in **Figure 23.11** move to produce an emf of 1.00 V in a 1.50 T field, given the rod's length is 30.0 cm?

50. The 12.0 cm long rod in **Figure 23.11** moves at 4.00 m/s. What is the strength of the magnetic field if a 95.0 V emf is induced?

51. Prove that when B, ℓ, and v are not mutually perpendicular, motional emf is given by $\text{emf} = B\ell v\sin\theta$. If v is perpendicular to B, then θ is the angle between ℓ and B. If ℓ is perpendicular to B, then θ is the angle between v and B.

52. In the August 1992 space shuttle flight, only 250 m of the conducting tether considered in **Example 23.2** could be let out. A 40.0 V motional emf was generated in the Earth's 5.00×10^{-5} T field, while moving at 7.80×10^{3} m/s. What was the angle between the shuttle's velocity and the Earth's field, assuming the conductor was perpendicular to the field?

53. Integrated Concepts

Derive an expression for the current in a system like that in **Figure 23.11**, under the following conditions. The resistance between the rails is R, the rails and the moving rod are identical in cross section A and have the same resistivity ρ. The distance between the rails is l, and the rod moves at constant speed v perpendicular to the uniform field B. At time zero, the moving rod is next to the resistance R.

54. Integrated Concepts

The Tethered Satellite in **Figure 23.12** has a mass of 525 kg and is at the end of a 20.0 km long, 2.50 mm diameter cable with the tensile strength of steel. (a) How much does the cable stretch if a 100 N force is exerted to pull the satellite in? (Assume the satellite and shuttle are at the same altitude above the Earth.) (b) What is the effective force constant of the cable? (c) How much energy is stored in it when stretched by the 100 N force?

55. Integrated Concepts

The Tethered Satellite discussed in this module is producing 5.00 kV, and a current of 10.0 A flows. (a) What magnetic drag force does this produce if the system is moving at 7.80 km/s? (b) How much kinetic energy is removed from the system in 1.00 h, neglecting any change in altitude or velocity during that time? (c) What is the change in velocity if the mass of the system is 100,000 kg? (d) Discuss the long term consequences (say, a week-long mission) on the space shuttle's orbit, noting what effect a decrease in velocity has and assessing the magnitude of the effect.

23.4 Eddy Currents and Magnetic Damping

56. Make a drawing similar to **Figure 23.14**, but with the pendulum moving in the opposite direction. Then use Faraday's law, Lenz's law, and RHR-1 to show that magnetic force opposes motion.

57.

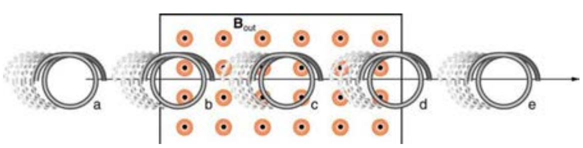


Figure 23.59 A coil is moved into and out of a region of uniform magnetic field. A coil is moved through a magnetic field as shown in **Figure 23.59**. The field is uniform inside the rectangle and zero outside. What is the direction of the induced current and what is the direction of the magnetic force on the coil at each position shown?

23.5 Electric Generators

58. Calculate the peak voltage of a generator that rotates its 200-turn, 0.100 m diameter coil at 3600 rpm in a 0.800 T field.

59. At what angular velocity in rpm will the peak voltage of a generator be 480 V, if its 500-turn, 8.00 cm diameter coil rotates in a 0.250 T field?

60. What is the peak emf generated by rotating a 1000-turn, 20.0 cm diameter coil in the Earth's 5.00×10^{-5} T magnetic field, given the plane of the coil is originally perpendicular to the Earth's field and is rotated to be parallel to the field in 10.0 ms?

61. What is the peak emf generated by a 0.250 m radius, 500-turn coil is rotated one-fourth of a revolution in 4.17 ms, originally having its plane perpendicular to a uniform magnetic field. (This is 60 rev/s.)

62. (a) A bicycle generator rotates at 1875 rad/s, producing an 18.0 V peak emf. It has a 1.00 by 3.00 cm rectangular coil in a 0.640 T field. How many turns are in the coil? (b) Is this number of turns of wire practical for a 1.00 by 3.00 cm coil?

63. Integrated Concepts

This problem refers to the bicycle generator considered in the previous problem. It is driven by a 1.60 cm diameter wheel that rolls on the outside rim of the bicycle tire. (a) What is the velocity of the bicycle if the generator's angular velocity is 1875 rad/s? (b) What is the maximum emf of the generator when the bicycle moves at 10.0 m/s, noting that it was 18.0 V under the original conditions? (c) If the sophisticated generator can vary its own magnetic field, what field strength will it need at 5.00 m/s to produce a 9.00 V maximum emf?

64. (a) A car generator turns at 400 rpm when the engine is idling. Its 300-turn, 5.00 by 8.00 cm rectangular coil rotates in an adjustable magnetic field so that it can produce sufficient voltage even at low rpms. What is the field strength needed to produce a 24.0 V peak emf? (b) Discuss how this required field strength compares to those available in permanent and electromagnets.

65. Show that if a coil rotates at an angular velocity ω, the period of its AC output is $2\pi/\omega$.

66. A 75-turn, 10.0 cm diameter coil rotates at an angular velocity of 8.00 rad/s in a 1.25 T field, starting with the plane of the coil parallel to the field. (a) What is the peak emf? (b) At what time is the peak emf first reached? (c) At what time is the emf first at its most negative? (d) What is the period of the AC voltage output?

67. (a) If the emf of a coil rotating in a magnetic field is zero at $t = 0$, and increases to its first peak at $t = 0.100$ ms, what is the angular velocity of the coil? (b) At what time will its next maximum occur? (c) What is the period of the output? (d) When is the output first one-fourth of its maximum? (e) When is it next one-fourth of its maximum?

68. Unreasonable Results

A 500-turn coil with a 0.250 m^2 area is spun in the Earth's 5.00×10^{-5} T field, producing a 12.0 kV maximum emf. (a) At what angular velocity must the coil be spun? (b) What is unreasonable about this result? (c) Which assumption or premise is responsible?

23.6 Back Emf

69. Suppose a motor connected to a 120 V source draws 10.0 A when it first starts. (a) What is its resistance? (b) What current does it draw at its normal operating speed when it develops a 100 V back emf?

70. A motor operating on 240 V electricity has a 180 V back emf at operating speed and draws a 12.0 A current. (a) What is its resistance? (b) What current does it draw when it is first started?

71. What is the back emf of a 120 V motor that draws 8.00 A at its normal speed and 20.0 A when first starting?

72. The motor in a toy car operates on 6.00 V, developing a 4.50 V back emf at normal speed. If it draws 3.00 A at normal speed, what current does it draw when starting?

73. Integrated Concepts

The motor in a toy car is powered by four batteries in series, which produce a total emf of 6.00 V. The motor draws 3.00 A and develops a 4.50 V back emf at normal speed. Each battery has a $0.100\ \Omega$ internal resistance. What is the resistance of the motor?

23.7 Transformers

74. A plug-in transformer, like that in **Figure 23.29**, supplies 9.00 V to a video game system. (a) How many turns are in its secondary coil, if its input voltage is 120 V and the primary coil has 400 turns? (b) What is its input current when its output is 1.30 A?

75. An American traveler in New Zealand carries a transformer to convert New Zealand's standard 240 V to 120 V so that she can use some small appliances on her trip. (a) What is the ratio of turns in the primary and secondary coils of her transformer? (b) What is the ratio of input to output current? (c) How could a New Zealander traveling in the United States use this same transformer to power her 240 V appliances from 120 V?

76. A cassette recorder uses a plug-in transformer to convert 120 V to 12.0 V, with a maximum current output of 200 mA. (a) What is the current input? (b) What is the power input? (c) Is this amount of power reasonable for a small appliance?

77. (a) What is the voltage output of a transformer used for rechargeable flashlight batteries, if its primary has 500 turns, its secondary 4 turns, and the input voltage is 120 V? (b) What input current is required to produce a 4.00 A output? (c) What is the power input?

78. (a) The plug-in transformer for a laptop computer puts out 7.50 V and can supply a maximum current of 2.00 A. What is the maximum input current if the input voltage is 240 V? Assume 100% efficiency. (b) If the actual efficiency is less than 100%, would the input current need to be greater or smaller? Explain.

79. A multipurpose transformer has a secondary coil with several points at which a voltage can be extracted, giving outputs of 5.60, 12.0, and 480 V. (a) The input voltage is 240 V to a primary coil of 280 turns. What are the numbers of turns in the parts of the secondary used to produce the output voltages? (b) If the maximum input current is 5.00 A, what are the maximum output currents (each used alone)?

80. A large power plant generates electricity at 12.0 kV. Its old transformer once converted the voltage to 335 kV. The secondary of this transformer is being replaced so that its output can be 750 kV for more efficient cross-country transmission on upgraded transmission lines. (a) What is the ratio of turns in the new secondary compared with the old secondary? (b) What is the ratio of new current output to old output (at 335 kV) for the same power? (c) If the upgraded transmission lines have the same resistance, what is the ratio of new line power loss to old?

81. If the power output in the previous problem is 1000 MW and line resistance is $2.00\ \Omega$, what were the old and new line losses?

82. Unreasonable Results

The 335 kV AC electricity from a power transmission line is fed into the primary coil of a transformer. The ratio of the number of turns in the secondary to the number in the primary is $N_\text{s}/N_\text{p} = 1000$. (a) What voltage is induced in the secondary? (b) What is unreasonable about this result? (c) Which assumption or premise is responsible?

83. Construct Your Own Problem

Consider a double transformer to be used to create very large voltages. The device consists of two stages. The first is a transformer that produces a much larger output voltage than its input. The output of the first transformer is used as input to a second transformer that further increases the voltage. Construct a problem in which you calculate the output voltage of the final stage based on the input voltage of the first stage and the number of turns or loops in both parts of both transformers (four coils in all). Also calculate the maximum output current of the final stage based on the input current. Discuss the possibility of power losses in the devices and the effect on the output current and power.

23.8 Electrical Safety: Systems and Devices

84. Integrated Concepts

A short circuit to the grounded metal case of an appliance occurs as shown in Figure 23.60. The person touching the case is wet and only has a $3.00\ \text{k}\Omega$ resistance to earth/ground. (a) What is the voltage on the case if 5.00 mA flows through the person? (b) What is the current in the short circuit if the resistance of the earth/ground wire is $0.200\ \Omega$? (c) Will this trigger the 20.0 A circuit breaker supplying the appliance?

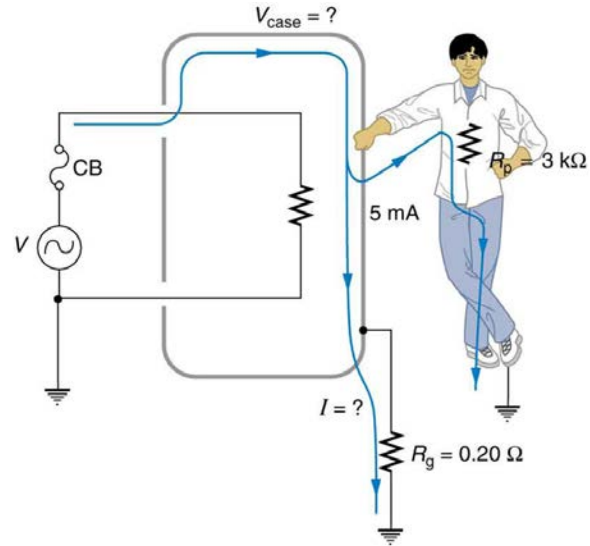


Figure 23.60 A person can be shocked even when the case of an appliance is grounded. The large short circuit current produces a voltage on the case of the appliance, since the resistance of the earth/ground wire is not zero.

23.9 Inductance

85. Two coils are placed close together in a physics lab to demonstrate Faraday's law of induction. A current of 5.00 A in one is switched off in 1.00 ms, inducing a 9.00 V emf in the other. What is their mutual inductance?

86. If two coils placed next to one another have a mutual inductance of 5.00 mH, what voltage is induced in one when the 2.00 A current in the other is switched off in 30.0 ms?

87. The 4.00 A current through a 7.50 mH inductor is switched off in 8.33 ms. What is the emf induced opposing this?

88. A device is turned on and 3.00 A flows through it 0.100 ms later. What is the self-inductance of the device if an induced 150 V emf opposes this?

89. Starting with $\text{emf}_2 = -M\dfrac{\Delta I_1}{\Delta t}$, show that the units of inductance are $(\text{V}\cdot\text{s})/\text{A} = \Omega\cdot\text{s}$.

90. Camera flashes charge a capacitor to high voltage by switching the current through an inductor on and off rapidly. In what time must the 0.100 A current through a 2.00 mH inductor be switched on or off to induce a 500 V emf?

91. A large research solenoid has a self-inductance of 25.0 H. (a) What induced emf opposes shutting it off when 100 A of current through it is switched off in 80.0 ms? (b) How much energy is stored in the inductor at full current? (c) At what rate in watts must energy be dissipated to switch the current off in 80.0 ms? (d) In view of the answer to the last part, is it surprising that shutting it down this quickly is difficult?

92. (a) Calculate the self-inductance of a 50.0 cm long, 10.0 cm diameter solenoid having 1000 loops. (b) How much energy is stored in this inductor when 20.0 A of current flows through it? (c) How fast can it be turned off if the induced emf cannot exceed 3.00 V?

93. A precision laboratory resistor is made of a coil of wire 1.50 cm in diameter and 4.00 cm long, and it has 500 turns. (a) What is its self-inductance? (b) What average emf is induced if the 12.0 A current through it is turned on in 5.00 ms (one-fourth of a cycle for 50 Hz AC)? (c) What is its inductance if it is shortened to half its length and counter-wound (two layers of 250 turns in opposite directions)?

94. The heating coils in a hair dryer are 0.800 cm in diameter, have a combined length of 1.00 m, and a total of 400 turns. (a) What is their total self-inductance assuming they act like a single solenoid? (b) How much energy is stored in them when 6.00 A flows? (c) What average emf opposes shutting them off if this is done in 5.00 ms (one-fourth of a cycle for 50 Hz AC)?

95. When the 20.0 A current through an inductor is turned off in 1.50 ms, an 800 V emf is induced, opposing the change. What is the value of the self-inductance?

96. How fast can the 150 A current through a 0.250 H inductor be shut off if the induced emf cannot exceed 75.0 V?

97. Integrated Concepts

A very large, superconducting solenoid such as one used in MRI scans, stores 1.00 MJ of energy in its magnetic field when 100 A flows. (a) Find its self-inductance. (b) If the coils "go normal," they gain resistance and start to dissipate thermal energy. What temperature increase is produced if all the stored energy goes into heating the 1000 kg magnet, given its average specific heat is 200 J/kg·ºC ?

98. Unreasonable Results

A 25.0 H inductor has 100 A of current turned off in 1.00 ms. (a) What voltage is induced to oppose this? (b) What is unreasonable about this result? (c) Which assumption or premise is responsible?

23.10 RL Circuits

99. If you want a characteristic *RL* time constant of 1.00 s, and you have a $500\ \Omega$ resistor, what value of self-inductance is needed?

100. Your *RL* circuit has a characteristic time constant of 20.0 ns, and a resistance of $5.00\text{ M}\Omega$. (a) What is the inductance of the circuit? (b) What resistance would give you a 1.00 ns time constant, perhaps needed for quick response in an oscilloscope?

101. A large superconducting magnet, used for magnetic resonance imaging, has a 50.0 H inductance. If you want current through it to be adjustable with a 1.00 s characteristic time constant, what is the minimum resistance of system?

102. Verify that after a time of 10.0 ms, the current for the situation considered in **Example 23.9** will be 0.183 A as stated.

103. Suppose you have a supply of inductors ranging from 1.00 nH to 10.0 H, and resistors ranging from $0.100\ \Omega$ to $1.00\text{ M}\Omega$. What is the range of characteristic *RL* time constants you can produce by connecting a single resistor to a single inductor?

104. (a) What is the characteristic time constant of a 25.0 mH inductor that has a resistance of $4.00\ \Omega$? (b) If it is connected to a 12.0 V battery, what is the current after 12.5 ms?

105. What percentage of the final current I_0 flows through an inductor L in series with a resistor R , three time constants after the circuit is completed?

106. The 5.00 A current through a 1.50 H inductor is dissipated by a $2.00\ \Omega$ resistor in a circuit like that in **Figure 23.44** with the switch in position 2. (a) What is the initial energy in the inductor? (b) How long will it take the current to decline to 5.00% of its initial value? (c) Calculate the average power dissipated, and compare it with the initial power dissipated by the resistor.

107. (a) Use the exact exponential treatment to find how much time is required to bring the current through an 80.0 mH inductor in series with a $15.0\ \Omega$ resistor to 99.0% of its final value, starting from zero. (b) Compare your answer to the approximate treatment using integral numbers of τ . (c) Discuss how significant the difference is.

108. (a) Using the exact exponential treatment, find the time required for the current through a 2.00 H inductor in series with a $0.500\ \Omega$ resistor to be reduced to 0.100% of its original value. (b) Compare your answer to the approximate treatment using integral numbers of τ . (c) Discuss how significant the difference is.

23.11 Reactance, Inductive and Capacitive

109. At what frequency will a 30.0 mH inductor have a reactance of $100\ \Omega$?

110. What value of inductance should be used if a $20.0\text{ k}\Omega$ reactance is needed at a frequency of 500 Hz?

111. What capacitance should be used to produce a $2.00\text{ M}\Omega$ reactance at 60.0 Hz?

112. At what frequency will an 80.0 mF capacitor have a reactance of $0.250\ \Omega$?

113. (a) Find the current through a 0.500 H inductor connected to a 60.0 Hz, 480 V AC source. (b) What would the current be at 100 kHz?

114. (a) What current flows when a 60.0 Hz, 480 V AC source is connected to a $0.250\ \mu\text{F}$ capacitor? (b) What would the current be at 25.0 kHz?

115. A 20.0 kHz, 16.0 V source connected to an inductor produces a 2.00 A current. What is the inductance?

116. A 20.0 Hz, 16.0 V source produces a 2.00 mA current when connected to a capacitor. What is the capacitance?

117. (a) An inductor designed to filter high-frequency noise from power supplied to a personal computer is placed in series with the computer. What minimum inductance should it have to produce a $2.00\text{ k}\Omega$ reactance for 15.0 kHz noise? (b) What is its reactance at 60.0 Hz?

118. The capacitor in **Figure 23.55**(a) is designed to filter low-frequency signals, impeding their transmission between circuits. (a) What capacitance is needed to produce a $100\text{ k}\Omega$ reactance at a frequency of 120 Hz? (b) What would its reactance be at 1.00 MHz? (c) Discuss the implications of your answers to (a) and (b).

119. The capacitor in **Figure 23.55**(b) will filter high-frequency signals by shorting them to earth/ground. (a) What capacitance is needed to produce a reactance of $10.0\text{ m}\Omega$ for a 5.00 kHz signal? (b) What would its reactance be at 3.00 Hz? (c) Discuss the implications of your answers to (a) and (b).

120. Unreasonable Results

In a recording of voltages due to brain activity (an EEG), a 10.0 mV signal with a 0.500 Hz frequency is applied to a capacitor, producing a current of 100 mA. Resistance is negligible. (a) What is the capacitance? (b) What is unreasonable about this result? (c) Which assumption or premise is responsible?

121. Construct Your Own Problem

Consider the use of an inductor in series with a computer operating on 60 Hz electricity. Construct a problem in which you calculate the relative reduction in voltage of incoming high frequency noise compared to 60 Hz voltage. Among the things to consider are the acceptable series reactance of the inductor for 60 Hz power and the likely frequencies of noise coming through the power lines.

23.12 RLC Series AC Circuits

122. An *RL* circuit consists of a $40.0\ \Omega$ resistor and a 3.00 mH inductor. (a) Find its impedance Z at 60.0 Hz and 10.0 kHz. (b) Compare these values of Z with those found in **Example 23.12** in which there was also a capacitor.

123. An *RC* circuit consists of a $40.0\ \Omega$ resistor and a $5.00\ \mu\text{F}$ capacitor. (a) Find its impedance at 60.0 Hz and 10.0 kHz. (b) Compare these values of Z with those found in **Example 23.12**, in which there was also an inductor.

124. An *LC* circuit consists of a 3.00 mH inductor and a $5.00\ \mu F$ capacitor. (a) Find its impedance at 60.0 Hz and 10.0 kHz. (b) Compare these values of Z with those found in **Example 23.12** in which there was also a resistor.

125. What is the resonant frequency of a 0.500 mH inductor connected to a $40.0\ \mu\text{F}$ capacitor?

126. To receive AM radio, you want an *RLC* circuit that can be made to resonate at any frequency between 500 and 1650 kHz. This is accomplished with a fixed $1.00\ \mu\text{H}$ inductor connected to a variable capacitor. What range of capacitance is needed?

127. Suppose you have a supply of inductors ranging from 1.00 nH to 10.0 H, and capacitors ranging from 1.00 pF to 0.100 F. What is the range of resonant frequencies that can be achieved from combinations of a single inductor and a single capacitor?

128. What capacitance do you need to produce a resonant frequency of 1.00 GHz, when using an 8.00 nH inductor?

129. What inductance do you need to produce a resonant frequency of 60.0 Hz, when using a $2.00\ \mu\text{F}$ capacitor?

130. The lowest frequency in the FM radio band is 88.0 MHz. (a) What inductance is needed to produce this resonant frequency if it is connected to a 2.50 pF capacitor? (b) The capacitor is variable, to allow the resonant frequency to be adjusted to as high as 108 MHz. What must the capacitance be at this frequency?

131. An *RLC* series circuit has a $2.50\ \Omega$ resistor, a $100\ \mu\text{H}$ inductor, and an $80.0\ \mu\text{F}$ capacitor.(a) Find the circuit's impedance at 120 Hz. (b) Find the circuit's impedance at 5.00 kHz. (c) If the voltage source has $V_{\text{rms}} = 5.60\ \text{V}$, what is I_{rms} at each frequency? (d) What is the resonant frequency of the circuit? (e) What is I_{rms} at resonance?

132. An *RLC* series circuit has a $1.00\ \text{k}\Omega$ resistor, a $150\ \mu\text{H}$ inductor, and a 25.0 nF capacitor. (a) Find the circuit's impedance at 500 Hz. (b) Find the circuit's impedance at 7.50 kHz. (c) If the voltage source has $V_{\text{rms}} = 408\ \text{V}$, what is I_{rms} at each frequency? (d) What is the resonant frequency of the circuit? (e) What is I_{rms} at resonance?

133. An *RLC* series circuit has a $2.50\ \Omega$ resistor, a $100\ \mu\text{H}$ inductor, and an $80.0\ \mu\text{F}$ capacitor. (a) Find the power factor at $f = 120\ \text{Hz}$. (b) What is the phase angle at 120 Hz? (c) What is the average power at 120 Hz? (d) Find the average power at the circuit's resonant frequency.

134. An *RLC* series circuit has a $1.00\ \text{k}\Omega$ resistor, a $150\ \mu\text{H}$ inductor, and a 25.0 nF capacitor. (a) Find the power factor at $f = 7.50\ \text{Hz}$. (b) What is the phase angle at this frequency? (c) What is the average power at this frequency? (d) Find the average power at the circuit's resonant frequency.

135. An *RLC* series circuit has a $200\ \Omega$ resistor and a 25.0 mH inductor. At 8000 Hz, the phase angle is 45.0^{o}. (a) What is the impedance? (b) Find the circuit's capacitance. (c) If $V_{\text{rms}} = 408\ \text{V}$ is applied, what is the average power supplied?

136. Referring to **Example 23.14**, find the average power at 10.0 kHz.

24 ELECTROMAGNETIC WAVES

Figure 24.1 Human eyes detect these orange “sea goldie” fish swimming over a coral reef in the blue waters of the Gulf of Eilat (Red Sea) using visible light. (credit: Daviddarom, Wikimedia Commons)

Learning Objectives

24.1. Maxwell’s Equations: Electromagnetic Waves Predicted and Observed
24.2. Production of Electromagnetic Waves
24.3. The Electromagnetic Spectrum
24.4. Energy in Electromagnetic Waves

Introduction to Electromagnetic Waves

The beauty of a coral reef, the warm radiance of sunshine, the sting of sunburn, the X-ray revealing a broken bone, even microwave popcorn—all are brought to us by **electromagnetic waves**. The list of the various types of electromagnetic waves, ranging from radio transmission waves to nuclear gamma-ray (γ -ray) emissions, is interesting in itself.

Even more intriguing is that all of these widely varied phenomena are different manifestations of the same thing—electromagnetic waves. (See **Figure 24.2**.) What are electromagnetic waves? How are they created, and how do they travel? How can we understand and organize their widely varying properties? What is their relationship to electric and magnetic effects? These and other questions will be explored.

Misconception Alert: Sound Waves vs. Radio Waves

Many people confuse sound waves with **radio waves**, one type of electromagnetic (EM) wave. However, sound and radio waves are completely different phenomena. Sound creates pressure variations (waves) in matter, such as air or water, or your eardrum. Conversely, radio waves are *electromagnetic waves*, like visible light, infrared, ultraviolet, X-rays, and gamma rays. EM waves don’t need a medium in which to propagate; they can travel through a vacuum, such as outer space.

A radio works because sound waves played by the D.J. at the radio station are converted into electromagnetic waves, then encoded and transmitted in the radio-frequency range. The radio in your car receives the radio waves, decodes the information, and uses a speaker to change it back into a sound wave, bringing sweet music to your ears.

Discovering a New Phenomenon

It is worth noting at the outset that the general phenomenon of electromagnetic waves was predicted by theory before it was realized that light is a form of electromagnetic wave. The prediction was made by James Clerk Maxwell in the mid-19th century when he formulated a single theory combining all the electric and magnetic effects known by scientists at that time. “Electromagnetic waves” was the name he gave to the phenomena his theory predicted.

Such a theoretical prediction followed by experimental verification is an indication of the power of science in general, and physics in particular. The underlying connections and unity of physics allow certain great minds to solve puzzles without having all the pieces. The prediction of electromagnetic waves is one of the most spectacular examples of this power. Certain others, such as the prediction of antimatter, will be discussed in later modules.

Figure 24.2 The electromagnetic waves sent and received by this 50-foot radar dish antenna at Kennedy Space Center in Florida are not visible, but help track expendable launch vehicles with high-definition imagery. The first use of this C-band radar dish was for the launch of the Atlas V rocket sending the New Horizons probe toward Pluto. (credit: NASA)

24.1 Maxwell’s Equations: Electromagnetic Waves Predicted and Observed

The Scotsman James Clerk Maxwell (1831–1879) is regarded as the greatest theoretical physicist of the 19th century. (See **Figure 24.3**.) Although he died young, Maxwell not only formulated a complete electromagnetic theory, represented by **Maxwell’s equations**, he also developed the kinetic theory of gases and made significant contributions to the understanding of color vision and the nature of Saturn’s rings.

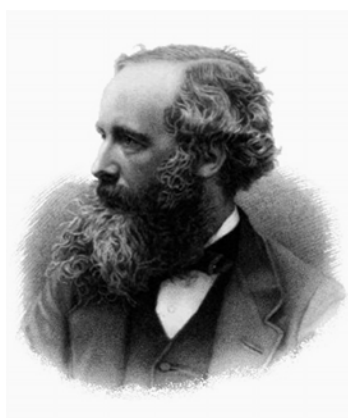


Figure 24.3 James Clerk Maxwell, a 19th-century physicist, developed a theory that explained the relationship between electricity and magnetism and correctly predicted that visible light is caused by electromagnetic waves. (credit: G. J. Stodart)

Maxwell brought together all the work that had been done by brilliant physicists such as Oersted, Coulomb, Gauss, and Faraday, and added his own insights to develop the overarching theory of electromagnetism. Maxwell’s equations are paraphrased here in words because their mathematical statement is beyond the level of this text. However, the equations illustrate how apparently simple mathematical statements can elegantly unite and express a multitude of concepts—why mathematics is the language of science.

Maxwell’s Equations

1. **Electric field lines** originate on positive charges and terminate on negative charges. The electric field is defined as the force per unit charge on a test charge, and the strength of the force is related to the electric constant ε_0, also known as the permittivity of free space. From Maxwell’s first equation we obtain a special form of Coulomb’s law known as Gauss’s law for electricity.
2. **Magnetic field lines** are continuous, having no beginning or end. No magnetic monopoles are known to exist. The strength of the magnetic force is related to the magnetic constant μ_0, also known as the permeability of free space. This second of Maxwell’s equations is known as Gauss’s law for magnetism.
3. A changing magnetic field induces an electromotive force (emf) and, hence, an electric field. The direction of the emf opposes the change. This third of Maxwell’s equations is Faraday’s law of induction, and includes Lenz’s law.

4. Magnetic fields are generated by moving charges or by changing electric fields. This fourth of Maxwell's equations encompasses Ampere's law and adds another source of magnetism—changing electric fields.

Maxwell's equations encompass the major laws of electricity and magnetism. What is not so apparent is the symmetry that Maxwell introduced in his mathematical framework. Especially important is his addition of the hypothesis that changing electric fields create magnetic fields. This is exactly analogous (and symmetric) to Faraday's law of induction and had been suspected for some time, but fits beautifully into Maxwell's equations.

Symmetry is apparent in nature in a wide range of situations. In contemporary research, symmetry plays a major part in the search for sub-atomic particles using massive multinational particle accelerators such as the new Large Hadron Collider at CERN.

Making Connections: Unification of Forces

Maxwell's complete and symmetric theory showed that electric and magnetic forces are not separate, but different manifestations of the same thing—the electromagnetic force. This classical unification of forces is one motivation for current attempts to unify the four basic forces in nature—the gravitational, electrical, strong, and weak nuclear forces.

Since changing electric fields create relatively weak magnetic fields, they could not be easily detected at the time of Maxwell's hypothesis. Maxwell realized, however, that oscillating charges, like those in AC circuits, produce changing electric fields. He predicted that these changing fields would propagate from the source like waves generated on a lake by a jumping fish.

The waves predicted by Maxwell would consist of oscillating electric and magnetic fields—defined to be an electromagnetic wave (EM wave). Electromagnetic waves would be capable of exerting forces on charges great distances from their source, and they might thus be detectable. Maxwell calculated that electromagnetic waves would propagate at a speed given by the equation

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}. \tag{24.1}$$

When the values for μ_0 and ε_0 are entered into the equation for c, we find that

$$c = \frac{1}{\sqrt{(8.85\times 10^{-12}\ \frac{\text{C}^2}{\text{N}\cdot\text{m}^2})(4\pi\times 10^{-7}\ \frac{\text{T}\cdot\text{m}}{\text{A}})}} = 3.00\times 10^8\ \text{m/s}, \tag{24.2}$$

which is the speed of light. In fact, Maxwell concluded that light is an electromagnetic wave having such wavelengths that it can be detected by the eye.

Other wavelengths should exist—it remained to be seen if they did. If so, Maxwell's theory and remarkable predictions would be verified, the greatest triumph of physics since Newton. Experimental verification came within a few years, but not before Maxwell's death.

Hertz's Observations

The German physicist Heinrich Hertz (1857–1894) was the first to generate and detect certain types of electromagnetic waves in the laboratory. Starting in 1887, he performed a series of experiments that not only confirmed the existence of electromagnetic waves, but also verified that they travel at the speed of light.

Hertz used an AC RLC (resistor-inductor-capacitor) circuit that resonates at a known frequency $f_0 = \frac{1}{2\pi\sqrt{LC}}$ and connected it to a loop of wire as shown in **Figure 24.4**. High voltages induced across the gap in the loop produced sparks that were visible evidence of the current in the circuit and that helped generate electromagnetic waves.

Across the laboratory, Hertz had another loop attached to another RLC circuit, which could be tuned (as the dial on a radio) to the same resonant frequency as the first and could, thus, be made to receive electromagnetic waves. This loop also had a gap across which sparks were generated, giving solid evidence that electromagnetic waves had been received.

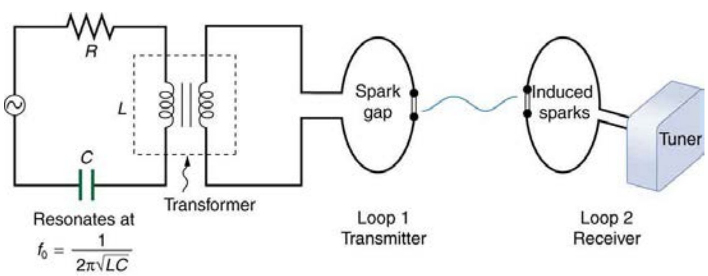


Figure 24.4 The apparatus used by Hertz in 1887 to generate and detect electromagnetic waves. An RLC circuit connected to the first loop caused sparks across a gap in the wire loop and generated electromagnetic waves. Sparks across a gap in the second loop located across the laboratory gave evidence that the waves had been received.

Hertz also studied the reflection, refraction, and interference patterns of the electromagnetic waves he generated, verifying their wave character. He was able to determine wavelength from the interference patterns, and knowing their frequency, he could calculate the propagation speed using the equation $v = f\lambda$ (velocity—or speed—equals frequency times wavelength). Hertz was thus able to prove that electromagnetic waves travel at the speed of light. The SI unit for frequency, the hertz ($1\ \text{Hz} = 1\ \text{cycle/sec}$), is named in his honor.

24.2 Production of Electromagnetic Waves

We can get a good understanding of **electromagnetic waves** (EM) by considering how they are produced. Whenever a current varies, associated electric and magnetic fields vary, moving out from the source like waves. Perhaps the easiest situation to visualize is a varying current in a long straight wire, produced by an AC generator at its center, as illustrated in **Figure 24.5**.

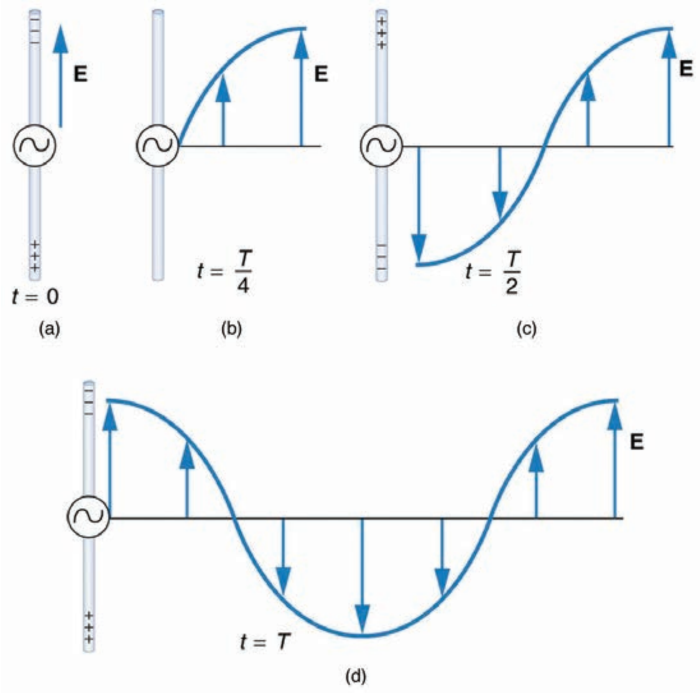


Figure 24.5 This long straight gray wire with an AC generator at its center becomes a broadcast antenna for electromagnetic waves. Shown here are the charge distributions at four different times. The electric field ($\mathbf{E}$) propagates away from the antenna at the speed of light, forming part of an electromagnetic wave.

The **electric field** ($\mathbf{E}$) shown surrounding the wire is produced by the charge distribution on the wire. Both the $\mathbf{E}$ and the charge distribution vary as the current changes. The changing field propagates outward at the speed of light.

There is an associated **magnetic field** ($\mathbf{B}$) which propagates outward as well (see **Figure 24.6**). The electric and magnetic fields are closely related and propagate as an electromagnetic wave. This is what happens in broadcast antennae such as those in radio and TV stations.

Closer examination of the one complete cycle shown in **Figure 24.5** reveals the periodic nature of the generator-driven charges oscillating up and down in the antenna and the electric field produced. At time $t = 0$, there is the maximum separation of charge, with negative charges at the top and positive charges at the bottom, producing the maximum magnitude of the electric field (or E -field) in the upward direction. One-fourth of a cycle later, there is no charge separation and the field next to the antenna is zero, while the maximum E -field has moved away at speed c .

As the process continues, the charge separation reverses and the field reaches its maximum downward value, returns to zero, and rises to its maximum upward value at the end of one complete cycle. The outgoing wave has an **amplitude** proportional to the maximum separation of charge. Its **wavelength** (λ) is proportional to the period of the oscillation and, hence, is smaller for short periods or high frequencies. (As usual, wavelength and **frequency** (f) are inversely proportional.)

Electric and Magnetic Waves: Moving Together

Following Ampere’s law, current in the antenna produces a magnetic field, as shown in **Figure 24.6**. The relationship between $\mathbf{E}$ and $\mathbf{B}$ is shown at one instant in **Figure 24.6** (a). As the current varies, the magnetic field varies in magnitude and direction.

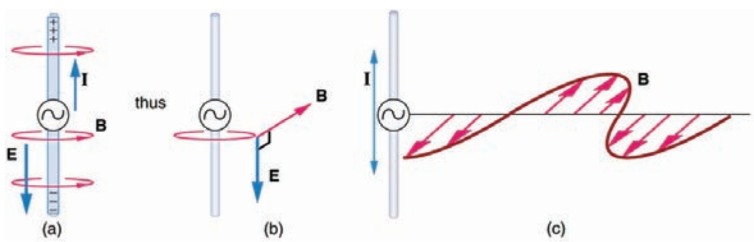


Figure 24.6 (a) The current in the antenna produces the circular magnetic field lines. The current ($\mathbf{I}$) produces the separation of charge along the wire, which in turn creates the electric field as shown. (b) The electric and magnetic fields ($\mathbf{E}$ and $\mathbf{B}$) near the wire are perpendicular; they are shown here for one point in space. (c) The magnetic field varies with current and propagates away from the antenna at the speed of light.

The magnetic field lines also propagate away from the antenna at the speed of light, forming the other part of the electromagnetic wave, as seen in **Figure 24.6** (b). The magnetic part of the wave has the same period and wavelength as the electric part, since they are both produced by the same movement and separation of charges in the antenna.

The electric and magnetic waves are shown together at one instant in time in **Figure 24.7**. The electric and magnetic fields produced by a long straight wire antenna are exactly in phase. Note that they are perpendicular to one another and to the direction of propagation, making this a **transverse wave**.

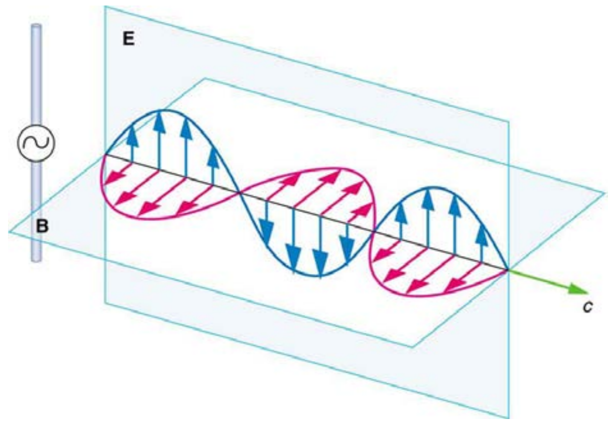


Figure 24.7 A part of the electromagnetic wave sent out from the antenna at one instant in time. The electric and magnetic fields ($\mathbf{E}$ and $\mathbf{B}$) are in phase, and they are perpendicular to one another and the direction of propagation. For clarity, the waves are shown only along one direction, but they propagate out in other directions too.

Electromagnetic waves generally propagate out from a source in all directions, sometimes forming a complex radiation pattern. A linear antenna like this one will not radiate parallel to its length, for example. The wave is shown in one direction from the antenna in **Figure 24.7** to illustrate its basic characteristics.

Instead of the AC generator, the antenna can also be driven by an AC circuit. In fact, charges radiate whenever they are accelerated. But while a current in a circuit needs a complete path, an antenna has a varying charge distribution forming a **standing wave**, driven by the AC. The dimensions of the antenna are critical for determining the frequency of the radiated electromagnetic waves. This is a **resonant** phenomenon and when we tune radios or TV, we vary electrical properties to achieve appropriate resonant conditions in the antenna.

Receiving Electromagnetic Waves

Electromagnetic waves carry energy away from their source, similar to a sound wave carrying energy away from a standing wave on a guitar string. An antenna for receiving EM signals works in reverse. And like antennas that produce EM waves, receiver antennas are specially designed to resonate at particular frequencies.

An incoming electromagnetic wave accelerates electrons in the antenna, setting up a standing wave. If the radio or TV is switched on, electrical components pick up and amplify the signal formed by the accelerating electrons. The signal is then converted to audio and/or video format. Sometimes big receiver dishes are used to focus the signal onto an antenna.

In fact, charges radiate whenever they are accelerated. When designing circuits, we often assume that energy does not quickly escape AC circuits, and mostly this is true. A broadcast antenna is specially designed to enhance the rate of electromagnetic radiation, and shielding is necessary to keep the radiation close to zero. Some familiar phenomena are based on the production of electromagnetic waves by varying currents. Your microwave oven, for example, sends electromagnetic waves, called microwaves, from a concealed antenna that has an oscillating current imposed on it.

Relating E-Field and B-Field Strengths

There is a relationship between the E- and B-field strengths in an electromagnetic wave. This can be understood by again considering the antenna just described. The stronger the E-field created by a separation of charge, the greater the current and, hence, the greater the B-field created.

Since current is directly proportional to voltage (Ohm's law) and voltage is directly proportional to E-field strength, the two should be directly proportional. It can be shown that the magnitudes of the fields do have a constant ratio, equal to the speed of light. That is,

$$\frac{E}{B} = c \tag{24.3}$$

is the ratio of E-field strength to B-field strength in any electromagnetic wave. This is true at all times and at all locations in space. A simple and elegant result.

Example 24.1 Calculating B-Field Strength in an Electromagnetic Wave

What is the maximum strength of the B-field in an electromagnetic wave that has a maximum E-field strength of $1000\ \text{V/m}$?

Strategy

To find the B-field strength, we rearrange the above equation to solve for B, yielding

$$B = \frac{E}{c}. \tag{24.4}$$

Solution

We are given E, and c is the speed of light. Entering these into the expression for B yields

$$B = \frac{1000\text{ V/m}}{3.00\times10^{8}\text{ m/s}} = 3.33\times10^{-6}\text{ T,} \tag{24.5}$$

Where T stands for Tesla, a measure of magnetic field strength.

Discussion

The B -field strength is less than a tenth of the Earth's admittedly weak magnetic field. This means that a relatively strong electric field of 1000 V/m is accompanied by a relatively weak magnetic field. Note that as this wave spreads out, say with distance from an antenna, its field strengths become progressively weaker.

The result of this example is consistent with the statement made in the module **Maxwell's Equations: Electromagnetic Waves Predicted and Observed** that changing electric fields create relatively weak magnetic fields. They can be detected in electromagnetic waves, however, by taking advantage of the phenomenon of resonance, as Hertz did. A system with the same natural frequency as the electromagnetic wave can be made to oscillate. All radio and TV receivers use this principle to pick up and then amplify weak electromagnetic waves, while rejecting all others not at their resonant frequency.

Take-Home Experiment: Antennas

For your TV or radio at home, identify the antenna, and sketch its shape. If you don't have cable, you might have an outdoor or indoor TV antenna. Estimate its size. If the TV signal is between 60 and 216 MHz for basic channels, then what is the wavelength of those EM waves?

Try tuning the radio and note the small range of frequencies at which a reasonable signal for that station is received. (This is easier with digital readout.) If you have a car with a radio and extendable antenna, note the quality of reception as the length of the antenna is changed.

PhET Explorations: Radio Waves and Electromagnetic Fields

Broadcast radio waves from KPhET. Wiggle the transmitter electron manually or have it oscillate automatically. Display the field as a curve or vectors. The strip chart shows the electron positions at the transmitter and at the receiver.

PhET Interactive Simulation

Figure 24.8 Radio Waves and Electromagnetic Fields (http://cnx.org/content/m42440/1.5/radio-waves_en.jar)

24.3 The Electromagnetic Spectrum

In this module we examine how electromagnetic waves are classified into categories such as radio, infrared, ultraviolet, and so on, so that we can understand some of their similarities as well as some of their differences. We will also find that there are many connections with previously discussed topics, such as wavelength and resonance. A brief overview of the production and utilization of electromagnetic waves is found in **Table 24.1**.

Table 24.1 Electromagnetic Waves

Type of EM wave	Production	Applications	Life sciences aspect	Issues
Radio & TV	Accelerating charges	Communications Remote controls	MRI	Requires controls for band use
Microwaves	Accelerating charges & thermal agitation	Communications Ovens Radar	Deep heating	Cell phone use
Infrared	Thermal agitations & electronic transitions	Thermal imaging Heating	Absorbed by atmosphere	Greenhouse effect
Visible light	Thermal agitations & electronic transitions	All pervasive	Photosynthesis Human vision	
Ultraviolet	Thermal agitations & electronic transitions	Sterilization Cancer control	Vitamin D production	Ozone depletion Cancer causing
X-rays	Inner electronic transitions and fast collisions	Medical Security	Medical diagnosis Cancer therapy	Cancer causing
Gamma rays	Nuclear decay	Nuclear medicineSecurity	Medical diagnosis Cancer therapy	Cancer causing Radiation damage

Connections: Waves

There are many types of waves, such as water waves and even earthquakes. Among the many shared attributes of waves are propagation speed, frequency, and wavelength. These are always related by the expression $v_{\text{W}} = f\lambda$. This module concentrates on EM waves, but other modules contain examples of all of these characteristics for sound waves and submicroscopic particles.

As noted before, an electromagnetic wave has a frequency and a wavelength associated with it and travels at the speed of light, or c . The relationship among these wave characteristics can be described by $v_{\text{W}} = f\lambda$, where v_{W} is the propagation speed of the wave, f is the frequency, and λ is the wavelength. Here $v_{\text{W}} = c$, so that for all electromagnetic waves,

$$c = f\lambda. \tag{24.6}$$

Thus, for all electromagnetic waves, the greater the frequency, the smaller the wavelength.

Figure 24.9 shows how the various types of electromagnetic waves are categorized according to their wavelengths and frequencies—that is, it shows the electromagnetic spectrum. Many of the characteristics of the various types of electromagnetic waves are related to their frequencies and wavelengths, as we shall see.

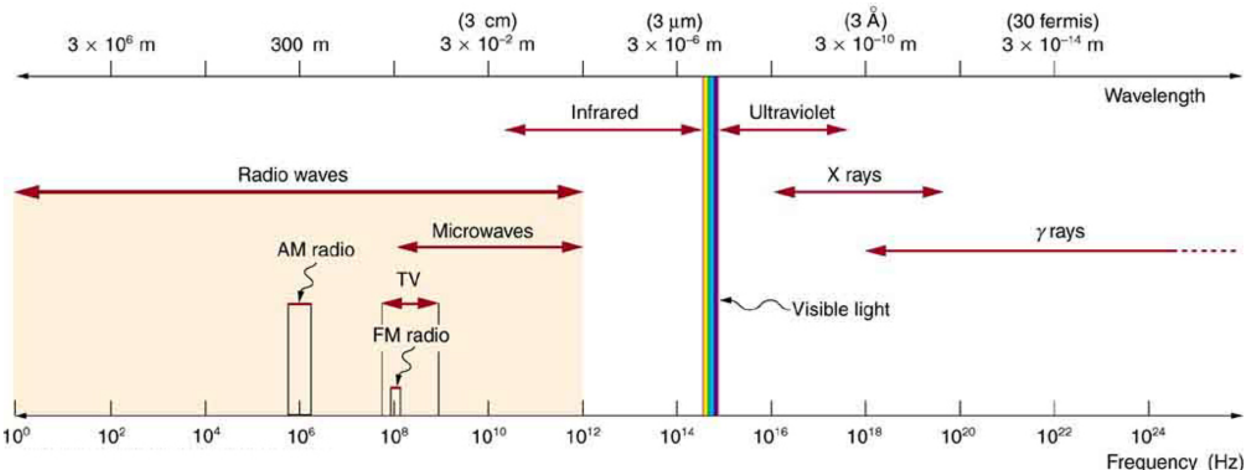


Figure 24.9 The electromagnetic spectrum, showing the major categories of electromagnetic waves. The range of frequencies and wavelengths is remarkable. The dividing line between some categories is distinct, whereas other categories overlap.

Electromagnetic Spectrum: Rules of Thumb

Three rules that apply to electromagnetic waves in general are as follows:

- High-frequency electromagnetic waves are more energetic and are more able to penetrate than low-frequency waves.
- High-frequency electromagnetic waves can carry more information per unit time than low-frequency waves.
- The shorter the wavelength of any electromagnetic wave probing a material, the smaller the detail it is possible to resolve.

Note that there are exceptions to these rules of thumb.

Transmission, Reflection, and Absorption

What happens when an electromagnetic wave impinges on a material? If the material is transparent to the particular frequency, then the wave can largely be transmitted. If the material is opaque to the frequency, then the wave can be totally reflected. The wave can also be absorbed by the material, indicating that there is some interaction between the wave and the material, such as the thermal agitation of molecules.

Of course it is possible to have partial transmission, reflection, and absorption. We normally associate these properties with visible light, but they do apply to all electromagnetic waves. What is not obvious is that something that is transparent to light may be opaque at other frequencies. For example, ordinary glass is transparent to visible light but largely opaque to ultraviolet radiation. Human skin is opaque to visible light—we cannot see through people—but transparent to X-rays.

Radio and TV Waves

The broad category of **radio waves** is defined to contain any electromagnetic wave produced by currents in wires and circuits. Its name derives from their most common use as a carrier of audio information (i.e., radio). The name is applied to electromagnetic waves of similar frequencies regardless of source. Radio waves from outer space, for example, do not come from alien radio stations. They are created by many astronomical phenomena, and their study has revealed much about nature on the largest scales.

There are many uses for radio waves, and so the category is divided into many subcategories, including microwaves and those electromagnetic waves used for AM and FM radio, cellular telephones, and TV.

The lowest commonly encountered radio frequencies are produced by high-voltage AC power transmission lines at frequencies of 50 or 60 Hz. (See **Figure 24.10**.) These extremely long wavelength electromagnetic waves (about 6000 km!) are one means of energy loss in long-distance power transmission.

Figure 24.10 This high-voltage traction power line running to Eutingen Railway Substation in Germany radiates electromagnetic waves with very long wavelengths. (credit: Zonk43, Wikimedia Commons)

There is an ongoing controversy regarding potential health hazards associated with exposure to these electromagnetic fields (E -fields). Some people suspect that living near such transmission lines may cause a variety of illnesses, including cancer. But demographic data are either inconclusive or simply do not support the hazard theory. Recent reports that have looked at many European and American epidemiological studies have found no increase in risk for cancer due to exposure to E -fields.

Extremely low frequency (ELF) radio waves of about 1 kHz are used to communicate with submerged submarines. The ability of radio waves to penetrate salt water is related to their wavelength (much like ultrasound penetrating tissue)—the longer the wavelength, the farther they penetrate. Since salt water is a good conductor, radio waves are strongly absorbed by it, and very long wavelengths are needed to reach a submarine under the surface. (See **Figure 24.11**.)

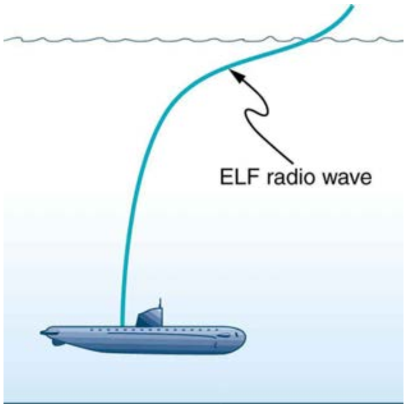


Figure 24.11 Very long wavelength radio waves are needed to reach this submarine, requiring extremely low frequency signals (ELF). Shorter wavelengths do not penetrate to any significant depth.

AM radio waves are used to carry commercial radio signals in the frequency range from 540 to 1600 kHz. The abbreviation AM stands for **amplitude modulation**, which is the method for placing information on these waves. (See **Figure 24.12**.) A **carrier wave** having the basic frequency of the radio station, say 1530 kHz, is varied or modulated in amplitude by an audio signal. The resulting wave has a constant frequency, but a varying amplitude.

A radio receiver tuned to have the same resonant frequency as the carrier wave can pick up the signal, while rejecting the many other frequencies impinging on its antenna. The receiver's circuitry is designed to respond to variations in amplitude of the carrier wave to replicate the original audio signal. That audio signal is amplified to drive a speaker or perhaps to be recorded.

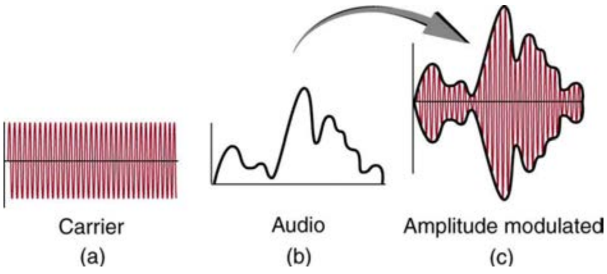


Figure 24.12 Amplitude modulation for AM radio. (a) A carrier wave at the station's basic frequency. (b) An audio signal at much lower audible frequencies. (c) The amplitude of the carrier is modulated by the audio signal without changing its basic frequency.

FM Radio Waves

FM radio waves are also used for commercial radio transmission, but in the frequency range of 88 to 108 MHz. FM stands for **frequency modulation**, another method of carrying information. (See **Figure 24.13**.) Here a carrier wave having the basic frequency of the radio station, perhaps 105.1 MHz, is modulated in frequency by the audio signal, producing a wave of constant amplitude but varying frequency.

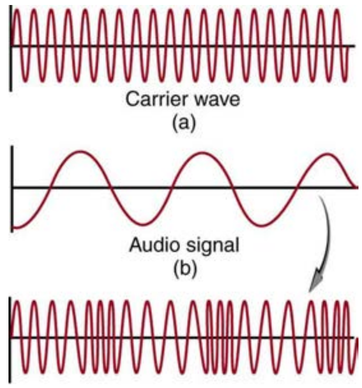


Figure 24.13 Frequency modulation for FM radio. (a) A carrier wave at the station's basic frequency. (b) An audio signal at much lower audible frequencies. (c) The frequency of the carrier is modulated by the audio signal without changing its amplitude.

Since audible frequencies range up to 20 kHz (or 0.020 MHz) at most, the frequency of the FM radio wave can vary from the carrier by as much as 0.020 MHz. Thus the carrier frequencies of two different radio stations cannot be closer than 0.020 MHz. An FM receiver is tuned to resonate at the carrier frequency and has circuitry that responds to variations in frequency, reproducing the audio information.

FM radio is inherently less subject to noise from stray radio sources than AM radio. The reason is that amplitudes of waves add. So an AM receiver would interpret noise added onto the amplitude of its carrier wave as part of the information. An FM receiver can be made to reject amplitudes other than that of the basic carrier wave and only look for variations in frequency. It is thus easier to reject noise from FM, since noise produces a variation in amplitude.

Television is also broadcast on electromagnetic waves. Since the waves must carry a great deal of visual as well as audio information, each channel requires a larger range of frequencies than simple radio transmission. TV channels utilize frequencies in the range of 54 to 88 MHz and 174 to 222 MHz. (The entire FM radio band lies between channels 88 MHz and 174 MHz.) These TV channels are called VHF (for **very high frequency**). Other channels called UHF (for **ultra high frequency**) utilize an even higher frequency range of 470 to 1000 MHz.

The TV video signal is AM, while the TV audio is FM. Note that these frequencies are those of free transmission with the user utilizing an old-fashioned roof antenna. Satellite dishes and cable transmission of TV occurs at significantly higher frequencies and is rapidly evolving with the use of the high-definition or HD format.

Example 24.2 Calculating Wavelengths of Radio Waves

Calculate the wavelengths of a 1530-kHz AM radio signal, a 105.1-MHz FM radio signal, and a 1.90-GHz cell phone signal.

Strategy

The relationship between wavelength and frequency is $c = f\lambda$, where $c = 3.00\times10^8\ \text{m/s}$ is the speed of light (the speed of light is only very slightly smaller in air than it is in a vacuum). We can rearrange this equation to find the wavelength for all three frequencies.

Solution

Rearranging gives

$$\lambda = \frac{c}{f}. \tag{24.7}$$

(a) For the $f = 1530\ \text{kHz}$ AM radio signal, then,

$$\begin{aligned}\lambda &= \frac{3.00\times10^8\ \text{m/s}}{1530\times10^3\ \text{cycles/s}}\\ &= 196\ \text{m.}\end{aligned} \tag{24.8}$$

(b) For the $f = 105.1\ \text{MHz}$ FM radio signal,

$$\begin{aligned}\lambda &= \frac{3.00\times10^8\ \text{m/s}}{105.1\times10^6\ \text{cycles/s}}\\ &= 2.85\ \text{m.}\end{aligned} \tag{24.9}$$

(c) And for the $f = 1.90\ \text{GHz}$ cell phone,

$$\begin{aligned}\lambda &= \frac{3.00\times10^8\ \text{m/s}}{1.90\times10^9\ \text{cycles/s}}\\ &= 0.158\ \text{m.}\end{aligned} \tag{24.10}$$

Discussion

These wavelengths are consistent with the spectrum in **Figure 24.9**. The wavelengths are also related to other properties of these electromagnetic waves, as we shall see.

The wavelengths found in the preceding example are representative of AM, FM, and cell phones, and account for some of the differences in how they are broadcast and how well they travel. The most efficient length for a linear antenna, such as discussed in **Production of Electromagnetic Waves**, is $\lambda / 2$, half the wavelength of the electromagnetic wave. Thus a very large antenna is needed to efficiently broadcast typical AM radio with its carrier wavelengths on the order of hundreds of meters.

One benefit to these long AM wavelengths is that they can go over and around rather large obstacles (like buildings and hills), just as ocean waves can go around large rocks. FM and TV are best received when there is a line of sight between the broadcast antenna and receiver, and they are often sent from very tall structures. FM, TV, and mobile phone antennas themselves are much smaller than those used for AM, but they are elevated to achieve an unobstructed line of sight. (See **Figure 24.14**.)

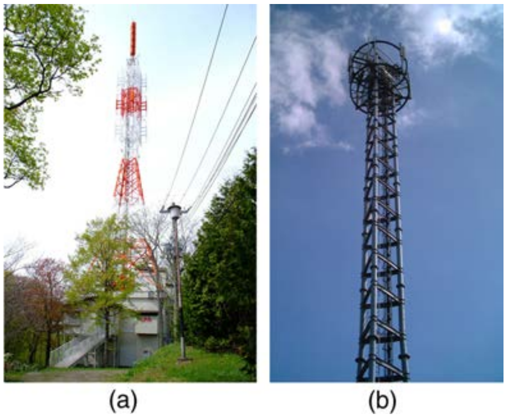


Figure 24.14 (a) A large tower is used to broadcast TV signals. The actual antennas are small structures on top of the tower—they are placed at great heights to have a clear line of sight over a large broadcast area. (credit: Ozizo, Wikimedia Commons) (b) The NTT Dokomo mobile phone tower at Tokorozawa City, Japan. (credit: tokoroten, Wikimedia Commons)

Radio Wave Interference

Astronomers and astrophysicists collect signals from outer space using electromagnetic waves. A common problem for astrophysicists is the “pollution” from electromagnetic radiation pervading our surroundings from communication systems in general. Even everyday gadgets like our car keys having the facility to lock car doors remotely and being able to turn TVs on and off using remotes involve radio-wave frequencies. In order to prevent interference between all these electromagnetic signals, strict regulations are drawn up for different organizations to utilize different radio frequency bands.

One reason why we are sometimes asked to switch off our mobile phones (operating in the range of 1.9 GHz) on airplanes and in hospitals is that important communications or medical equipment often uses similar radio frequencies and their operation can be affected by frequencies used in the communication devices.

For example, radio waves used in magnetic resonance imaging (MRI) have frequencies on the order of 100 MHz, although this varies significantly depending on the strength of the magnetic field used and the nuclear type being scanned. MRI is an important medical imaging and research tool, producing highly detailed two- and three-dimensional images. Radio waves are broadcast, absorbed, and reemitted in a resonance process that is sensitive to the density of nuclei (usually protons or hydrogen nuclei).

The wavelength of 100-MHz radio waves is 3 m, yet using the sensitivity of the resonant frequency to the magnetic field strength, details smaller than a millimeter can be imaged. This is a good example of an exception to a rule of thumb (in this case, the rubric that details much smaller than the probe’s wavelength cannot be detected). The intensity of the radio waves used in MRI presents little or no hazard to human health.

Microwaves

Microwaves are the highest-frequency electromagnetic waves that can be produced by currents in macroscopic circuits and devices. Microwave frequencies range from about $10^9\ \text{Hz}$ to the highest practical LC resonance at nearly $10^{12}\ \text{Hz}$. Since they have high frequencies, their wavelengths are short compared with those of other radio waves—hence the name “microwave.”

Microwaves can also be produced by atoms and molecules. They are, for example, a component of electromagnetic radiation generated by **thermal agitation**. The thermal motion of atoms and molecules in any object at a temperature above absolute zero causes them to emit and absorb radiation.

Since it is possible to carry more information per unit time on high frequencies, microwaves are quite suitable for communications. Most satellite-transmitted information is carried on microwaves, as are land-based long-distance transmissions. A clear line of sight between transmitter and receiver is needed because of the short wavelengths involved.

Radar is a common application of microwaves that was first developed in World War II. By detecting and timing microwave echoes, radar systems can determine the distance to objects as diverse as clouds and aircraft. A Doppler shift in the radar echo can be used to determine the speed of a car or the intensity of a rainstorm. Sophisticated radar systems are used to map the Earth and other planets, with a resolution limited by wavelength. (See **Figure 24.15**.) The shorter the wavelength of any probe, the smaller the detail it is possible to observe.

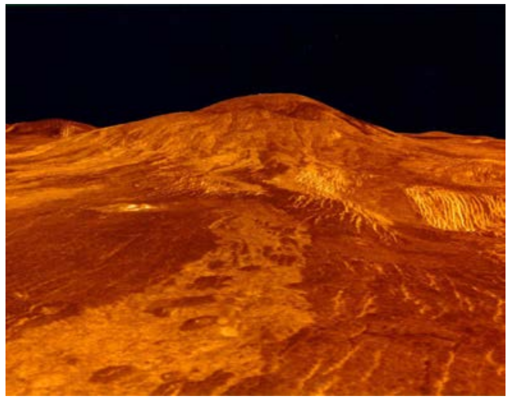


Figure 24.15 An image of Sif Mons with lava flows on Venus, based on Magellan synthetic aperture radar data combined with radar altimetry to produce a three-dimensional map of the surface. The Venusian atmosphere is opaque to visible light, but not to the microwaves that were used to create this image. (credit: NSSDC, NASA/JPL)

Heating with Microwaves

How does the ubiquitous microwave oven produce microwaves electronically, and why does food absorb them preferentially? Microwaves at a frequency of 2.45 GHz are produced by accelerating electrons. The microwaves are then used to induce an alternating electric field in the oven.

Water and some other constituents of food have a slightly negative charge at one end and a slightly positive charge at one end (called polar molecules). The range of microwave frequencies is specially selected so that the polar molecules, in trying to keep orienting themselves with the electric field, absorb these energies and increase their temperatures—called dielectric heating.

The energy thereby absorbed results in thermal agitation heating food and not the plate, which does not contain water. Hot spots in the food are related to constructive and destructive interference patterns. Rotating antennas and food turntables help spread out the hot spots.

Another use of microwaves for heating is within the human body. Microwaves will penetrate more than shorter wavelengths into tissue and so can accomplish “deep heating” (called microwave diathermy). This is used for treating muscular pains, spasms, tendonitis, and rheumatoid arthritis.

Making Connections: Take-Home Experiment—Microwave Ovens

1. Look at the door of a microwave oven. Describe the structure of the door. Why is there a metal grid on the door? How does the size of the holes in the grid compare with the wavelengths of microwaves used in microwave ovens? What is this wavelength?
2. Place a glass of water (about 250 ml) in the microwave and heat it for 30 seconds. Measure the temperature gain (the $\Delta\mathrm{T}$). Assuming that the power output of the oven is 1000 W, calculate the efficiency of the heat-transfer process.
3. Remove the rotating turntable or moving plate and place a cup of water in several places along a line parallel with the opening. Heat for 30 seconds and measure the $\Delta\mathrm{T}$ for each position. Do you see cases of destructive interference?

Microwaves generated by atoms and molecules far away in time and space can be received and detected by electronic circuits. Deep space acts like a blackbody with a 2.7 K temperature, radiating most of its energy in the microwave frequency range. In 1964, Penzias and Wilson detected this radiation and eventually recognized that it was the radiation of the Big Bang’s cooled remnants.

Infrared Radiation

The microwave and infrared regions of the electromagnetic spectrum overlap (see Figure 24.9). **Infrared radiation** is generally produced by thermal motion and the vibration and rotation of atoms and molecules. Electronic transitions in atoms and molecules can also produce infrared radiation.

The range of infrared frequencies extends up to the lower limit of visible light, just below red. In fact, infrared means “below red.” Frequencies at its upper limit are too high to be produced by accelerating electrons in circuits, but small systems, such as atoms and molecules, can vibrate fast enough to produce these waves.

Water molecules rotate and vibrate particularly well at infrared frequencies, emitting and absorbing them so efficiently that the emissivity for skin is $e=0.97$ in the infrared. Night-vision scopes can detect the infrared emitted by various warm objects, including humans, and convert it to visible light.

We can examine radiant heat transfer from a house by using a camera capable of detecting infrared radiation. Reconnaissance satellites can detect buildings, vehicles, and even individual humans by their infrared emissions, whose power radiation is proportional to the fourth power of the absolute temperature. More mundanely, we use infrared lamps, some of which are called quartz heaters, to preferentially warm us because we absorb infrared better than our surroundings.

The Sun radiates like a nearly perfect blackbody (that is, it has $e=1$), with a 6000 K surface temperature. About half of the solar energy arriving at the Earth is in the infrared region, with most of the rest in the visible part of the spectrum, and a relatively small amount in the ultraviolet. On average, 50 percent of the incident solar energy is absorbed by the Earth.

The relatively constant temperature of the Earth is a result of the energy balance between the incoming solar radiation and the energy radiated from the Earth. Most of the infrared radiation emitted from the Earth is absorbed by $\mathrm{CO_2}$ and $\mathrm{H_2O}$ in the atmosphere and then radiated back to Earth or into outer space. This radiation back to Earth is known as the greenhouse effect, and it maintains the surface temperature of the Earth about $40^\circ\mathrm{C}$ higher than it would be if there is no absorption. Some scientists think that the increased concentration of $\mathrm{CO_2}$ and other greenhouse gases in the atmosphere, resulting from increases in fossil fuel burning, has increased global average temperatures.

Visible Light

Visible light is the narrow segment of the electromagnetic spectrum to which the normal human eye responds. Visible light is produced by vibrations and rotations of atoms and molecules, as well as by electronic transitions within atoms and molecules. The receivers or detectors of light largely utilize electronic transitions. We say the atoms and molecules are excited when they absorb and relax when they emit through electronic transitions.

Figure 24.16 shows this part of the spectrum, together with the colors associated with particular pure wavelengths. We usually refer to visible light as having wavelengths of between 400 nm and 750 nm. (The retina of the eye actually responds to the lowest ultraviolet frequencies, but these do not normally reach the retina because they are absorbed by the cornea and lens of the eye.)

Red light has the lowest frequencies and longest wavelengths, while violet has the highest frequencies and shortest wavelengths. Blackbody radiation from the Sun peaks in the visible part of the spectrum but is more intense in the red than in the violet, making the Sun yellowish in appearance.

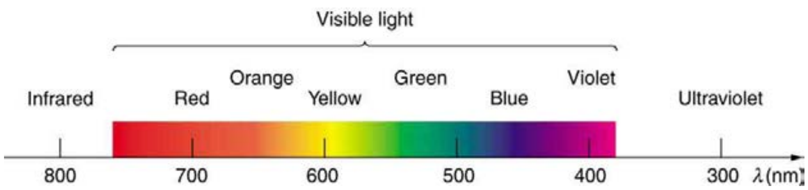


Figure 24.16 A small part of the electromagnetic spectrum that includes its visible components. The divisions between infrared, visible, and ultraviolet are not perfectly distinct, nor are those between the seven rainbow colors.

Living things—plants and animals—have evolved to utilize and respond to parts of the electromagnetic spectrum they are embedded in. Visible light is the most predominant and we enjoy the beauty of nature through visible light. Plants are more selective. Photosynthesis makes use of parts of the visible spectrum to make sugars.

Example 24.3 Integrated Concept Problem: Correcting Vision with Lasers

During laser vision correction, a brief burst of 193-nm ultraviolet light is projected onto the cornea of a patient. It makes a spot 0.80 mm in diameter and evaporates a layer of cornea $0.30\ \mu\text{m}$ thick. Calculate the energy absorbed, assuming the corneal tissue has the same properties as water; it is initially at 34°C. Assume the evaporated tissue leaves at a temperature of 100°C.

Strategy

The energy from the laser light goes toward raising the temperature of the tissue and also toward evaporating it. Thus we have two amounts of heat to add together. Also, we need to find the mass of corneal tissue involved.

Solution

To figure out the heat required to raise the temperature of the tissue to 100°C, we can apply concepts of thermal energy. We know that

$$Q = mc\Delta T, \tag{24.11}$$

where Q is the heat required to raise the temperature, ΔT is the desired change in temperature, m is the mass of tissue to be heated, and c is the specific heat of water equal to 4186 J/kg/K.

Without knowing the mass m at this point, we have

$$Q = m(4186\ \text{J/kg/K})(100^\circ\text{C} - 34^\circ\text{C}) = m(276{,}276\ \text{J/kg}) = m(276\ \text{kJ/kg}). \tag{24.12}$$

The latent heat of vaporization of water is 2256 kJ/kg, so that the energy needed to evaporate mass m is

$$Q_\text{v} = mL_\text{v} = m(2256\ \text{kJ/kg}). \tag{24.13}$$

To find the mass m, we use the equation $\rho = m/\text{V}$, where ρ is the density of the tissue and V is its volume. For this case,

$$\begin{aligned} m &= \rho\text{V} \\ &= (1000\ \text{kg/m}^3)(\text{area}\times\text{thickness}(\text{m}^3)) \\ &= (1000\ \text{kg/m}^3)(\pi(0.80\times10^{-3}\ \text{m})^2/4)(0.30\times10^{-6}\ \text{m}) \\ &= 0.151\times10^{-9}\ \text{kg}. \end{aligned} \tag{24.14}$$

Therefore, the total energy absorbed by the tissue in the eye is the sum of Q and Q_v:

$$\text{Q}_\text{tot} = m(c\Delta\text{T} + \text{L}_\text{v}) = (0.151\times10^{-9}\ \text{kg})(276\ \text{kJ/kg} + 2256\ \text{kJ/kg}) = 382\times10^{-9}\ \text{kJ}. \tag{24.15}$$

Discussion

The lasers used for this eye surgery are excimer lasers, whose light is well absorbed by biological tissue. They evaporate rather than burn the tissue, and can be used for precision work. Most lasers used for this type of eye surgery have an average power rating of about one watt. For our example, if we assume that each laser burst from this pulsed laser lasts for 10 ns, and there are 400 bursts per second, then the average power is $\text{Q}_\text{tot}\times400 = 150\ \text{mW}$.

Optics is the study of the behavior of visible light and other forms of electromagnetic waves. Optics falls into two distinct categories. When electromagnetic radiation, such as visible light, interacts with objects that are large compared with its wavelength, its motion can be represented by straight lines like rays. Ray optics is the study of such situations and includes lenses and mirrors.

When electromagnetic radiation interacts with objects about the same size as the wavelength or smaller, its wave nature becomes apparent. For example, observable detail is limited by the wavelength, and so visible light can never detect individual atoms, because they are so much smaller than its wavelength. Physical or wave optics is the study of such situations and includes all wave characteristics.

Take-Home Experiment: Colors That Match

When you light a match you see largely orange light; when you light a gas stove you see blue light. Why are the colors different? What other colors are present in these?

Ultraviolet Radiation

Ultraviolet means "above violet." The electromagnetic frequencies of **ultraviolet radiation (UV)** extend upward from violet, the highest-frequency visible light. Ultraviolet is also produced by atomic and molecular motions and electronic transitions. The wavelengths of ultraviolet extend from 400 nm down to about 10 nm at its highest frequencies, which overlap with the lowest X-ray frequencies. It was recognized as early as 1801 by Johann Ritter that the solar spectrum had an invisible component beyond the violet range.

Solar UV radiation is broadly subdivided into three regions: UV-A (320–400 nm), UV-B (290–320 nm), and UV-C (220–290 nm), ranked from long to shorter wavelengths (from smaller to larger energies). Most UV-B and all UV-C is absorbed by ozone ($\mathrm{O_3}$) molecules in the upper atmosphere.

Consequently, 99% of the solar UV radiation reaching the Earth's surface is UV-A.

Human Exposure to UV Radiation

It is largely exposure to UV-B that causes skin cancer. It is estimated that as many as 20% of adults will develop skin cancer over the course of their lifetime. Again, treatment is often successful if caught early. Despite very little UV-B reaching the Earth's surface, there are substantial increases in skin-cancer rates in countries such as Australia, indicating how important it is that UV-B and UV-C continue to be absorbed by the upper atmosphere.

All UV radiation can damage collagen fibers, resulting in an acceleration of the aging process of skin and the formation of wrinkles. Because there is so little UV-B and UV-C reaching the Earth's surface, sunburn is caused by large exposures, and skin cancer from repeated exposure. Some studies indicate a link between overexposure to the Sun when young and melanoma later in life.

The tanning response is a defense mechanism in which the body produces pigments to absorb future exposures in inert skin layers above living cells. Basically UV-B radiation excites DNA molecules, distorting the DNA helix, leading to mutations and the possible formation of cancerous cells.

Repeated exposure to UV-B may also lead to the formation of cataracts in the eyes—a cause of blindness among people living in the equatorial belt where medical treatment is limited. Cataracts, clouding in the eye's lens and a loss of vision, are age related; 60% of those between the ages of 65 and 74 will develop cataracts. However, treatment is easy and successful, as one replaces the lens of the eye with a plastic lens. Prevention is important. Eye protection from UV is more effective with plastic sunglasses than those made of glass.

A major acute effect of extreme UV exposure is the suppression of the immune system, both locally and throughout the body.

Low-intensity ultraviolet is used to sterilize haircutting implements, implying that the energy associated with ultraviolet is deposited in a manner different from lower-frequency electromagnetic waves. (Actually this is true for all electromagnetic waves with frequencies greater than visible light.)

Flash photography is generally not allowed of precious artworks and colored prints because the UV radiation from the flash can cause photo-degradation in the artworks. Often artworks will have an extra-thick layer of glass in front of them, which is especially designed to absorb UV radiation.

UV Light and the Ozone Layer

If all of the Sun's ultraviolet radiation reached the Earth's surface, there would be extremely grave effects on the biosphere from the severe cell damage it causes. However, the layer of ozone ($\mathrm{O_3}$) in our upper atmosphere (10 to 50 km above the Earth) protects life by absorbing most of the dangerous UV radiation.

Unfortunately, today we are observing a depletion in ozone concentrations in the upper atmosphere. This depletion has led to the formation of an "ozone hole" in the upper atmosphere. The hole is more centered over the southern hemisphere, and changes with the seasons, being largest in the spring. This depletion is attributed to the breakdown of ozone molecules by refrigerant gases called chlorofluorocarbons (CFCs).

The UV radiation helps dissociate the CFC's, releasing highly reactive chlorine (Cl) atoms, which catalyze the destruction of the ozone layer. For example, the reaction of $\mathrm{CFCl_3}$ with a photon of light (hv) can be written as:

$$\mathrm{CFCl_3 + h}v \rightarrow \mathrm{CFCl_2 + Cl.} \tag{24.16}$$

The Cl atom then catalyzes the breakdown of ozone as follows:

$$\mathrm{Cl + O_3 \rightarrow ClO + O_2 \text{ and } ClO + O_3 \rightarrow Cl + 2O_2.} \tag{24.17}$$

A single chlorine atom could destroy ozone molecules for up to two years before being transported down to the surface. The CFCs are relatively stable and will contribute to ozone depletion for years to come. CFCs are found in refrigerants, air conditioning systems, foams, and aerosols.

International concern over this problem led to the establishment of the "Montreal Protocol" agreement (1987) to phase out CFC production in most countries. However, developing-country participation is needed if worldwide production and elimination of CFCs is to be achieved. Probably the largest contributor to CFC emissions today is India. But the protocol seems to be working, as there are signs of an ozone recovery. (See **Figure 24.17**.)

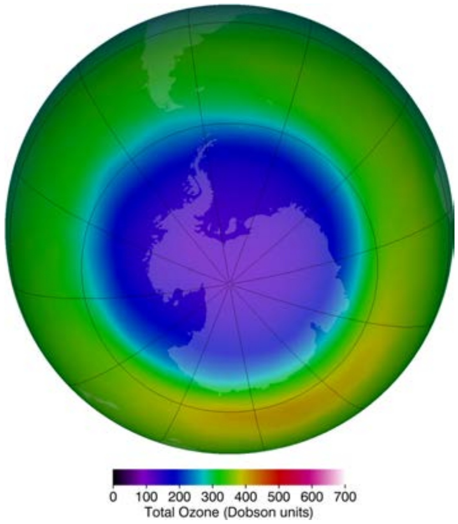


Figure 24.17 This map of ozone concentration over Antarctica in October 2011 shows severe depletion suspected to be caused by CFCs. Less dramatic but more general depletion has been observed over northern latitudes, suggesting the effect is global. With less ozone, more ultraviolet radiation from the Sun reaches the surface, causing more damage. (credit: NASA Ozone Watch)

Benefits of UV Light

Besides the adverse effects of ultraviolet radiation, there are also benefits of exposure in nature and uses in technology. Vitamin D production in the skin (epidermis) results from exposure to UVB radiation, generally from sunlight. A number of studies indicate lack of vitamin D can result in the development of a range of cancers (prostate, breast, colon), so a certain amount of UV exposure is helpful. Lack of vitamin D is also linked to osteoporosis. Exposures (with no sunscreen) of 10 minutes a day to arms, face, and legs might be sufficient to provide the accepted dietary level. However, in the winter time north of about 37° latitude, most UVB gets blocked by the atmosphere.

UV radiation is used in the treatment of infantile jaundice and in some skin conditions. It is also used in sterilizing workspaces and tools, and killing germs in a wide range of applications. It is also used as an analytical tool to identify substances.

When exposed to ultraviolet, some substances, such as minerals, glow in characteristic visible wavelengths, a process called fluorescence. So-called black lights emit ultraviolet to cause posters and clothing to fluoresce in the visible. Ultraviolet is also used in special microscopes to detect details smaller than those observable with longer-wavelength visible-light microscopes.

Things Great and Small: A Submicroscopic View of X-Ray Production

X-rays can be created in a high-voltage discharge. They are emitted in the material struck by electrons in the discharge current. There are two mechanisms by which the electrons create X-rays.

The first method is illustrated in **Figure 24.18**. An electron is accelerated in an evacuated tube by a high positive voltage. The electron strikes a metal plate (e.g., copper) and produces X-rays. Since this is a high-voltage discharge, the electron gains sufficient energy to ionize the atom.

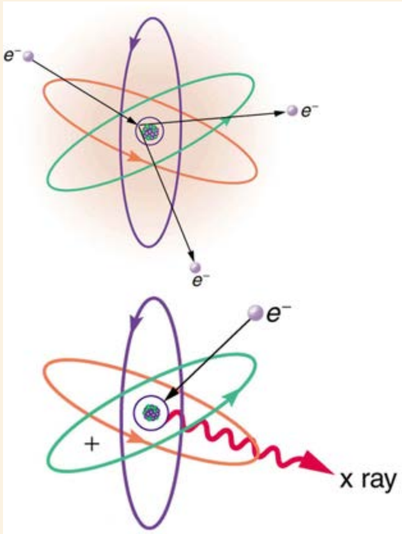


Figure 24.18 Artist's conception of an electron ionizing an atom followed by the recapture of an electron and emission of an X-ray. An energetic electron strikes an atom and knocks an electron out of one of the orbits closest to the nucleus. Later, the atom captures another electron, and the energy released by its fall into a low orbit generates a high-energy EM wave called an X-ray.

In the case shown, an inner-shell electron (one in an orbit relatively close to and tightly bound to the nucleus) is ejected. A short time later, another electron is captured and falls into the orbit in a single great plunge. The energy released by this fall is given to an EM wave known as an X-ray. Since the orbits of the atom are unique to the type of atom, the energy of the X-ray is characteristic of the atom, hence the name characteristic X-ray.

The second method by which an energetic electron creates an X-ray when it strikes a material is illustrated in **Figure 24.19**. The electron interacts with charges in the material as it penetrates. These collisions transfer kinetic energy from the electron to the electrons and atoms in the material.

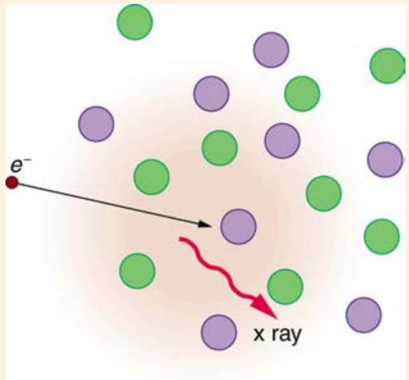


Figure 24.19 Artist's conception of an electron being slowed by collisions in a material and emitting X-ray radiation. This energetic electron makes numerous collisions with electrons and atoms in a material it penetrates. An accelerated charge radiates EM waves, a second method by which X-rays are created.

A loss of kinetic energy implies an acceleration, in this case decreasing the electron's velocity. Whenever a charge is accelerated, it radiates EM waves. Given the high energy of the electron, these EM waves can have high energy. We call them X-rays. Since the process is random, a broad spectrum of X-ray energy is emitted that is more characteristic of the electron energy than the type of material the electron encounters. Such EM radiation is called "bremsstrahlung" (German for "braking radiation").

X-Rays

In the 1850s, scientists (such as Faraday) began experimenting with high-voltage electrical discharges in tubes filled with rarefied gases. It was later found that these discharges created an invisible, penetrating form of very high frequency electromagnetic radiation. This radiation was called an **X-ray**, because its identity and nature were unknown.

As described in **Things Great and Small**, there are two methods by which X-rays are created—both are submicroscopic processes and can be caused by high-voltage discharges. While the low-frequency end of the X-ray range overlaps with the ultraviolet, X-rays extend to much higher frequencies (and energies).

X-rays have adverse effects on living cells similar to those of ultraviolet radiation, and they have the additional liability of being more penetrating, affecting more than the surface layers of cells. Cancer and genetic defects can be induced by exposure to X-rays. Because of their effect on rapidly dividing cells, X-rays can also be used to treat and even cure cancer.

The widest use of X-rays is for imaging objects that are opaque to visible light, such as the human body or aircraft parts. In humans, the risk of cell damage is weighed carefully against the benefit of the diagnostic information obtained. However, questions have risen in recent years as to accidental overexposure of some people during CT scans—a mistake at least in part due to poor monitoring of radiation dose.

The ability of X-rays to penetrate matter depends on density, and so an X-ray image can reveal very detailed density information. **Figure 24.20** shows an example of the simplest type of X-ray image, an X-ray shadow on film. The amount of information in a simple X-ray image is impressive, but more sophisticated techniques, such as CT scans, can reveal three-dimensional information with details smaller than a millimeter.

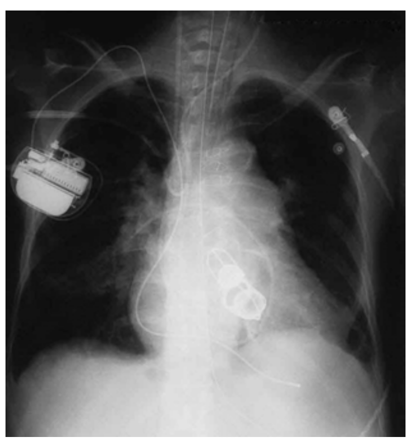


Figure 24.20 This shadow X-ray image shows many interesting features, such as artificial heart valves, a pacemaker, and the wires used to close the sternum. (credit: P. P. Urone)

The use of X-ray technology in medicine is called radiology—an established and relatively cheap tool in comparison to more sophisticated technologies. Consequently, X-rays are widely available and used extensively in medical diagnostics. During World War I, mobile X-ray units, advocated by Madame Marie Curie, were used to diagnose soldiers.

Because they can have wavelengths less than 0.01 nm, X-rays can be scattered (a process called X-ray diffraction) to detect the shape of molecules and the structure of crystals. X-ray diffraction was crucial to Crick, Watson, and Wilkins in the determination of the shape of the double-helix DNA molecule.

X-rays are also used as a precise tool for trace-metal analysis in X-ray induced fluorescence, in which the energy of the X-ray emissions are related to the specific types of elements and amounts of materials present.

Gamma Rays

Soon after nuclear radioactivity was first detected in 1896, it was found that at least three distinct types of radiation were being emitted. The most penetrating nuclear radiation was called a **gamma ray (γ ray)** (again a name given because its identity and character were unknown), and it was later found to be an extremely high frequency electromagnetic wave.

In fact, γ rays are any electromagnetic radiation emitted by a nucleus. This can be from natural nuclear decay or induced nuclear processes in nuclear reactors and weapons. The lower end of the $\gamma\text{-ray}$ frequency range overlaps the upper end of the X-ray range, but γ rays can have the highest frequency of any electromagnetic radiation.

Gamma rays have characteristics identical to X-rays of the same frequency—they differ only in source. At higher frequencies, γ rays are more penetrating and more damaging to living tissue. They have many of the same uses as X-rays, including cancer therapy. Gamma radiation from radioactive materials is used in nuclear medicine.

Figure 24.21 shows a medical image based on γ rays. Food spoilage can be greatly inhibited by exposing it to large doses of γ radiation, thereby obliterating responsible microorganisms. Damage to food cells through irradiation occurs as well, and the long-term hazards of consuming radiation-preserved food are unknown and controversial for some groups. Both X-ray and $\gamma\text{-ray}$ technologies are also used in scanning luggage at airports.

Figure 24.21 This is an image of the γ rays emitted by nuclei in a compound that is concentrated in the bones and eliminated through the kidneys. Bone cancer is evidenced by nonuniform concentration in similar structures. For example, some ribs are darker than others. (credit: P. P. Urone)

Detecting Electromagnetic Waves from Space

A final note on star gazing. The entire electromagnetic spectrum is used by researchers for investigating stars, space, and time. As noted earlier, Penzias and Wilson detected microwaves to identify the background radiation originating from the Big Bang. Radio telescopes such as the Arecibo Radio Telescope in Puerto Rico and Parkes Observatory in Australia were designed to detect radio waves.

Infrared telescopes need to have their detectors cooled by liquid nitrogen to be able to gather useful signals. Since infrared radiation is predominantly from thermal agitation, if the detectors were not cooled, the vibrations of the molecules in the antenna would be stronger than the signal being collected.

The most famous of these infrared sensitive telescopes is the James Clerk Maxwell Telescope in Hawaii. The earliest telescopes, developed in the seventeenth century, were optical telescopes, collecting visible light. Telescopes in the ultraviolet, X-ray, and γ -ray regions are placed outside the atmosphere on satellites orbiting the Earth.

The Hubble Space Telescope (launched in 1990) gathers ultraviolet radiation as well as visible light. In the X-ray region, there is the Chandra X-ray Observatory (launched in 1999), and in the γ -ray region, there is the new Fermi Gamma-ray Space Telescope (launched in 2008—taking the place of the Compton Gamma Ray Observatory, 1991–2000.).

PhET Explorations: Color Vision

Make a whole rainbow by mixing red, green, and blue light. Change the wavelength of a monochromatic beam or filter white light. View the light as a solid beam, or see the individual photons.

PhET Interactive Simulation

Figure 24.22 Color Vision (http://cnx.org/content/m42444/1.6/color-vision_en.jar)

24.4 Energy in Electromagnetic Waves

Anyone who has used a microwave oven knows there is energy in **electromagnetic waves**. Sometimes this energy is obvious, such as in the warmth of the summer sun. Other times it is subtle, such as the unfelt energy of gamma rays, which can destroy living cells.

Electromagnetic waves can bring energy into a system by virtue of their **electric and magnetic fields**. These fields can exert forces and move charges in the system and, thus, do work on them. If the frequency of the electromagnetic wave is the same as the natural frequencies of the system (such as microwaves at the resonant frequency of water molecules), the transfer of energy is much more efficient.

Connections: Waves and Particles

The behavior of electromagnetic radiation clearly exhibits wave characteristics. But we shall find in later modules that at high frequencies, electromagnetic radiation also exhibits particle characteristics. These particle characteristics will be used to explain more of the properties of the electromagnetic spectrum and to introduce the formal study of modern physics.

Another startling discovery of modern physics is that particles, such as electrons and protons, exhibit wave characteristics. This simultaneous sharing of wave and particle properties for all submicroscopic entities is one of the great symmetries in nature.

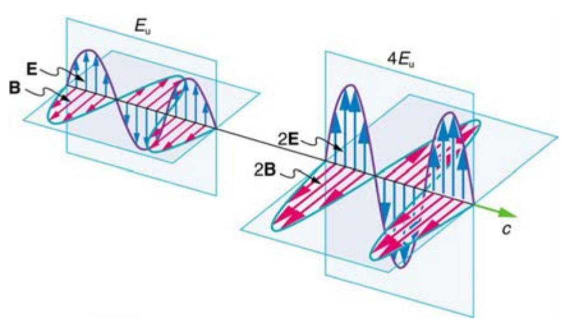


Figure 24.23 Energy carried by a wave is proportional to its amplitude squared. With electromagnetic waves, larger E -fields and B -fields exert larger forces and can do more work.

But there is energy in an electromagnetic wave, whether it is absorbed or not. Once created, the fields carry energy away from a source. If absorbed, the field strengths are diminished and anything left travels on. Clearly, the larger the strength of the electric and magnetic fields, the more work they can do and the greater the energy the electromagnetic wave carries.

A wave's energy is proportional to its **amplitude** squared (E^2 or B^2). This is true for waves on guitar strings, for water waves, and for sound waves, where amplitude is proportional to pressure. In electromagnetic waves, the amplitude is the **maximum field strength** of the electric and magnetic fields. (See **Figure 24.23**.)

Thus the energy carried and the **intensity** I of an electromagnetic wave is proportional to E^2 and B^2 . In fact, for a continuous sinusoidal electromagnetic wave, the average intensity I_{ave} is given by

$$I_{\text{ave}} = \frac{c\varepsilon_0 E_0^2}{2}, \tag{24.18}$$

where c is the speed of light, ε_0 is the permittivity of free space, and E_0 is the maximum electric field strength; intensity, as always, is power per unit area (here in W/m^2).

The average intensity of an electromagnetic wave I_{ave} can also be expressed in terms of the magnetic field strength by using the relationship $B = E / c$, and the fact that $\varepsilon_0 = 1 / \mu_0 c^2$, where μ_0 is the permeability of free space. Algebraic manipulation produces the relationship

$$I_{\text{ave}} = \frac{cB_0^2}{2\mu_0}, \tag{24.19}$$

where B_0 is the maximum magnetic field strength.

One more expression for I_{ave} in terms of both electric and magnetic field strengths is useful. Substituting the fact that $c \cdot B_0 = E_0$, the previous expression becomes

$$I_{\text{ave}} = \frac{E_0 B_0}{2\mu_0}. \tag{24.20}$$

Whichever of the three preceding equations is most convenient can be used, since they are really just different versions of the same principle: Energy in a wave is related to amplitude squared. Furthermore, since these equations are based on the assumption that the electromagnetic waves are sinusoidal, peak intensity is twice the average; that is, $I_0 = 2I_{\text{ave}}$.

Example 24.4 Calculate Microwave Intensities and Fields

On its highest power setting, a certain microwave oven projects 1.00 kW of microwaves onto a 30.0 by 40.0 cm area. (a) What is the intensity in W/m^2? (b) Calculate the peak electric field strength E_0 in these waves. (c) What is the peak magnetic field strength B_0?

Strategy

In part (a), we can find intensity from its definition as power per unit area. Once the intensity is known, we can use the equations below to find the field strengths asked for in parts (b) and (c).

Solution for (a)

Entering the given power into the definition of intensity, and noting the area is 0.300 by 0.400 m, yields

$$I = \frac{P}{A} = \frac{1.00\ \text{kW}}{0.300\ \text{m} \times 0.400\ \text{m}}. \tag{24.21}$$

Here $I = I_{\text{ave}}$, so that

$$I_{\text{ave}} = \frac{1000\ \text{W}}{0.120\ \text{m}^2} = 8.33\times10^3\ \text{W/m}^2. \tag{24.22}$$

Note that the peak intensity is twice the average:

$$I_0 = 2I_{\text{ave}} = 1.67\times10^4\ \text{W} / \text{m}^2. \tag{24.23}$$

Solution for (b)

To find E_0, we can rearrange the first equation given above for I_{ave} to give

$$E_0 = \left(\frac{2I_{\text{ave}}}{c\varepsilon_0}\right)^{1/2}. \tag{24.24}$$

Entering known values gives

$$\begin{aligned} E_0 &= \sqrt{\frac{2(8.33\times10^3\ \text{W/m}^2)}{(3.00\times10^8\ \text{m/s})(8.85\times10^{-12}\ \text{C}^2 / \text{N}\cdot\text{m}^2)}} \\ &= 2.51\times10^3\ \text{V/m}. \end{aligned} \tag{24.25}$$

Solution for (c)

Perhaps the easiest way to find magnetic field strength, now that the electric field strength is known, is to use the relationship given by

$$B_0 = \frac{E_0}{c}. \tag{24.26}$$

Entering known values gives

$$\begin{aligned} B_0 &= \frac{2.51\times10^3\ \text{V/m}}{3.0\times10^8\ \text{m/s}} \\ &= 8.35\times10^{-6}\ \text{T}. \end{aligned} \tag{24.27}$$

Discussion

As before, a relatively strong electric field is accompanied by a relatively weak magnetic field in an electromagnetic wave, since $B = E / c$, and c is a large number.

Glossary

amplitude modulation (AM): a method for placing information on electromagnetic waves by modulating the amplitude of a carrier wave with an audio signal, resulting in a wave with constant frequency but varying amplitude

amplitude: the height, or magnitude, of an electromagnetic wave

carrier wave: an electromagnetic wave that carries a signal by modulation of its amplitude or frequency

electric field lines: a pattern of imaginary lines that extend between an electric source and charged objects in the surrounding area, with arrows pointed away from positively charged objects and toward negatively charged objects. The more lines in the pattern, the stronger the electric field in that region

electric field strength: the magnitude of the electric field, denoted *E*-field

electric field: a vector quantity (**E**); the lines of electric force per unit charge, moving radially outward from a positive charge and in toward a negative charge

electromagnetic spectrum: the full range of wavelengths or frequencies of electromagnetic radiation

electromagnetic waves: radiation in the form of waves of electric and magnetic energy

electromotive force (emf): energy produced per unit charge, drawn from a source that produces an electrical current

extremely low frequency (ELF): electromagnetic radiation with wavelengths usually in the range of 0 to 300 Hz, but also about 1kHz

frequency modulation (FM): a method of placing information on electromagnetic waves by modulating the frequency of a carrier wave with an audio signal, producing a wave of constant amplitude but varying frequency

frequency: the number of complete wave cycles (up-down-up) passing a given point within one second (cycles/second)

gamma ray: (γ ray); extremely high frequency electromagnetic radiation emitted by the nucleus of an atom, either from natural nuclear decay or induced nuclear processes in nuclear reactors and weapons. The lower end of the γ-ray frequency range overlaps the upper end of the X-ray range, but γ rays can have the highest frequency of any electromagnetic radiation

hertz: an SI unit denoting the frequency of an electromagnetic wave, in cycles per second

infrared radiation (IR): a region of the electromagnetic spectrum with a frequency range that extends from just below the red region of the visible light spectrum up to the microwave region, or from $0.74\ \mu m$ to $300\ \mu m$

intensity: the power of an electric or magnetic field per unit area, for example, Watts per square meter

Maxwell's equations: a set of four equations that comprise a complete, overarching theory of electromagnetism

magnetic field lines: a pattern of continuous, imaginary lines that emerge from and enter into opposite magnetic poles. The density of the lines indicates the magnitude of the magnetic field

magnetic field strength: the magnitude of the magnetic field, denoted *B*-field

magnetic field: a vector quantity (**B**); can be used to determine the magnetic force on a moving charged particle

maximum field strength: the maximum amplitude an electromagnetic wave can reach, representing the maximum amount of electric force and/or magnetic flux that the wave can exert

microwaves: electromagnetic waves with wavelengths in the range from 1 mm to 1 m; they can be produced by currents in macroscopic circuits and devices

oscillate: to fluctuate back and forth in a steady beat

RLC circuit: an electric circuit that includes a resistor, capacitor and inductor

radar: a common application of microwaves. Radar can determine the distance to objects as diverse as clouds and aircraft, as well as determine the speed of a car or the intensity of a rainstorm

radio waves: electromagnetic waves with wavelengths in the range from 1 mm to 100 km; they are produced by currents in wires and circuits and by astronomical phenomena

resonant: a system that displays enhanced oscillation when subjected to a periodic disturbance of the same frequency as its natural frequency

speed of light: in a vacuum, such as space, the speed of light is a constant 3×10^8 m/s

standing wave: a wave that oscillates in place, with nodes where no motion happens

TV: video and audio signals broadcast on electromagnetic waves

thermal agitation: the thermal motion of atoms and molecules in any object at a temperature above absolute zero, which causes them to emit and absorb radiation

transverse wave: a wave, such as an electromagnetic wave, which oscillates perpendicular to the axis along the line of travel

ultra-high frequency (UHF): TV channels in an even higher frequency range than VHF, of 470 to 1000 MHz

ultraviolet radiation (UV): electromagnetic radiation in the range extending upward in frequency from violet light and overlapping with the lowest X-ray frequencies, with wavelengths from 400 nm down to about 10 nm

very high frequency (VHF): TV channels utilizing frequencies in the two ranges of 54 to 88 MHz and 174 to 222 MHz

visible light: the narrow segment of the electromagnetic spectrum to which the normal human eye responds

wavelength: the distance from one peak to the next in a wave

X-ray: invisible, penetrating form of very high frequency electromagnetic radiation, overlapping both the ultraviolet range and the γ -ray range

Section Summary

24.1 Maxwell's Equations: Electromagnetic Waves Predicted and Observed

- Electromagnetic waves consist of oscillating electric and magnetic fields and propagate at the speed of light c . They were predicted by Maxwell, who also showed that

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}},$$

  where μ_0 is the permeability of free space and ε_0 is the permittivity of free space.
- Maxwell's prediction of electromagnetic waves resulted from his formulation of a complete and symmetric theory of electricity and magnetism, known as Maxwell's equations.
- These four equations are paraphrased in this text, rather than presented numerically, and encompass the major laws of electricity and magnetism. First is Gauss's law for electricity, second is Gauss's law for magnetism, third is Faraday's law of induction, including Lenz's law, and fourth is Ampere's law in a symmetric formulation that adds another source of magnetism—changing electric fields.

24.2 Production of Electromagnetic Waves

- Electromagnetic waves are created by oscillating charges (which radiate whenever accelerated) and have the same frequency as the oscillation.
- Since the electric and magnetic fields in most electromagnetic waves are perpendicular to the direction in which the wave moves, it is ordinarily a transverse wave.
- The strengths of the electric and magnetic parts of the wave are related by

$$\frac{E}{B} = c,$$

  which implies that the magnetic field B is very weak relative to the electric field E .

24.3 The Electromagnetic Spectrum

- The relationship among the speed of propagation, wavelength, and frequency for any wave is given by $v_\mathrm{W} = f\lambda$, so that for electromagnetic waves,

$$c = f\lambda,$$

  where f is the frequency, λ is the wavelength, and c is the speed of light.
- The electromagnetic spectrum is separated into many categories and subcategories, based on the frequency and wavelength, source, and uses of the electromagnetic waves.
- Any electromagnetic wave produced by currents in wires is classified as a radio wave, the lowest frequency electromagnetic waves. Radio waves are divided into many types, depending on their applications, ranging up to microwaves at their highest frequencies.
- Infrared radiation lies below visible light in frequency and is produced by thermal motion and the vibration and rotation of atoms and molecules. Infrared's lower frequencies overlap with the highest-frequency microwaves.
- Visible light is largely produced by electronic transitions in atoms and molecules, and is defined as being detectable by the human eye. Its colors vary with frequency, from red at the lowest to violet at the highest.
- Ultraviolet radiation starts with frequencies just above violet in the visible range and is produced primarily by electronic transitions in atoms and molecules.
- X-rays are created in high-voltage discharges and by electron bombardment of metal targets. Their lowest frequencies overlap the ultraviolet range but extend to much higher values, overlapping at the high end with gamma rays.
- Gamma rays are nuclear in origin and are defined to include the highest-frequency electromagnetic radiation of any type.

24.4 Energy in Electromagnetic Waves

- The energy carried by any wave is proportional to its amplitude squared. For electromagnetic waves, this means intensity can be expressed as

$$I_\text{ave} = \frac{c\varepsilon_0 E_0^2}{2},$$

where I_{ave} is the average intensity in W/m^2, and E_0 is the maximum electric field strength of a continuous sinusoidal wave.

- This can also be expressed in terms of the maximum magnetic field strength B_0 as

$$I_{\text{ave}} = \frac{cB_0^2}{2\mu_0}$$

and in terms of both electric and magnetic fields as

$$I_{\text{ave}} = \frac{E_0 B_0}{2\mu_0}.$$

- The three expressions for I_{ave} are all equivalent.

Conceptual Questions

24.2 Production of Electromagnetic Waves

1. The direction of the electric field shown in each part of **Figure 24.5** is that produced by the charge distribution in the wire. Justify the direction shown in each part, using the Coulomb force law and the definition of $\mathbf{E} = \mathbf{F} / q$, where q is a positive test charge.

2. Is the direction of the magnetic field shown in **Figure 24.6** (a) consistent with the right-hand rule for current (RHR-2) in the direction shown in the figure?

3. Why is the direction of the current shown in each part of **Figure 24.6** opposite to the electric field produced by the wire's charge separation?

4. In which situation shown in **Figure 24.24** will the electromagnetic wave be more successful in inducing a current in the wire? Explain.

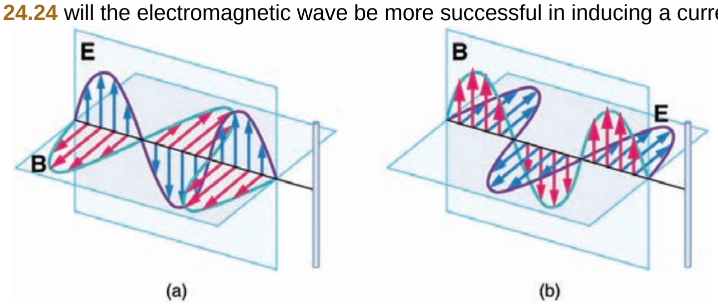


Figure 24.24 Electromagnetic waves approaching long straight wires.

5. In which situation shown in **Figure 24.25** will the electromagnetic wave be more successful in inducing a current in the loop? Explain.

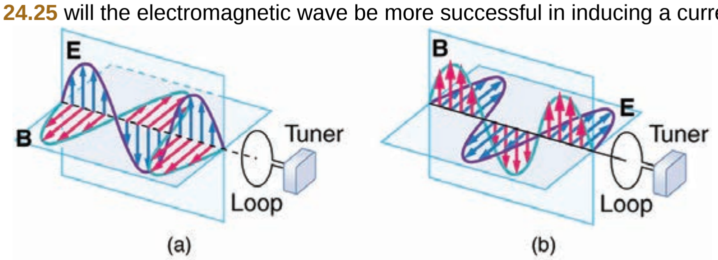


Figure 24.25 Electromagnetic waves approaching a wire loop.

6. Should the straight wire antenna of a radio be vertical or horizontal to best receive radio waves broadcast by a vertical transmitter antenna? How should a loop antenna be aligned to best receive the signals? (Note that the direction of the loop that produces the best reception can be used to determine the location of the source. It is used for that purpose in tracking tagged animals in nature studies, for example.)

7. Under what conditions might wires in a DC circuit emit electromagnetic waves?

8. Give an example of interference of electromagnetic waves.

9. **Figure 24.26** shows the interference pattern of two radio antennas broadcasting the same signal. Explain how this is analogous to the interference pattern for sound produced by two speakers. Could this be used to make a directional antenna system that broadcasts preferentially in certain directions? Explain.

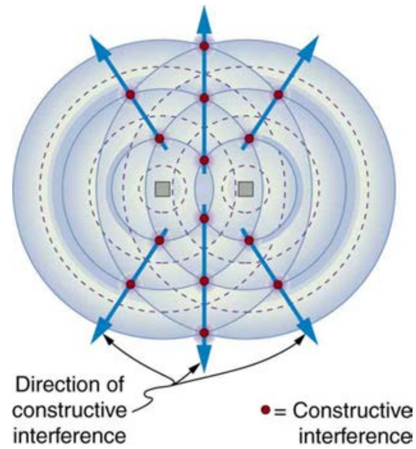


Figure 24.26 An overhead view of two radio broadcast antennas sending the same signal, and the interference pattern they produce.

10. Can an antenna be any length? Explain your answer.

24.3 The Electromagnetic Spectrum

11. If you live in a region that has a particular TV station, you can sometimes pick up some of its audio portion on your FM radio receiver. Explain how this is possible. Does it imply that TV audio is broadcast as FM?

12. Explain why people who have the lens of their eye removed because of cataracts are able to see low-frequency ultraviolet.

13. How do fluorescent soap residues make clothing look “brighter and whiter” in outdoor light? Would this be effective in candlelight?

14. Give an example of resonance in the reception of electromagnetic waves.

15. Illustrate that the size of details of an object that can be detected with electromagnetic waves is related to their wavelength, by comparing details observable with two different types (for example, radar and visible light or infrared and X-rays).

16. Why don’t buildings block radio waves as completely as they do visible light?

17. Make a list of some everyday objects and decide whether they are transparent or opaque to each of the types of electromagnetic waves.

18. Your friend says that more patterns and colors can be seen on the wings of birds if viewed in ultraviolet light. Would you agree with your friend? Explain your answer.

19. The rate at which information can be transmitted on an electromagnetic wave is proportional to the frequency of the wave. Is this consistent with the fact that laser telephone transmission at visible frequencies carries far more conversations per optical fiber than conventional electronic transmission in a wire? What is the implication for ELF radio communication with submarines?

20. Give an example of energy carried by an electromagnetic wave.

21. In an MRI scan, a higher magnetic field requires higher frequency radio waves to resonate with the nuclear type whose density and location is being imaged. What effect does going to a larger magnetic field have on the most efficient antenna to broadcast those radio waves? Does it favor a smaller or larger antenna?

22. Laser vision correction often uses an excimer laser that produces 193-nm electromagnetic radiation. This wavelength is extremely strongly absorbed by the cornea and ablates it in a manner that reshapes the cornea to correct vision defects. Explain how the strong absorption helps concentrate the energy in a thin layer and thus give greater accuracy in shaping the cornea. Also explain how this strong absorption limits damage to the lens and retina of the eye.

Problems & Exercises

24.1 Maxwell's Equations: Electromagnetic Waves Predicted and Observed

23. Verify that the correct value for the speed of light c is obtained when numerical values for the permeability and permittivity of free space (μ_0 and ε_0) are entered into the equation $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$.

24. Show that, when SI units for μ_0 and ε_0 are entered, the units given by the right-hand side of the equation in the problem above are m/s.

24.2 Production of Electromagnetic Waves

25. What is the maximum electric field strength in an electromagnetic wave that has a maximum magnetic field strength of $5.00\times10^{-4}\ \text{T}$ (about 10 times the Earth's)?

26. The maximum magnetic field strength of an electromagnetic field is $5\times10^{-6}\ \text{T}$. Calculate the maximum electric field strength if the wave is traveling in a medium in which the speed of the wave is $0.75c$.

27. Verify the units obtained for magnetic field strength B in **Example 24.1** (using the equation $B = \frac{E}{c}$) are in fact teslas (T).

24.3 The Electromagnetic Spectrum

28. (a) Two microwave frequencies are authorized for use in microwave ovens: 900 and 2560 MHz. Calculate the wavelength of each. (b) Which frequency would produce smaller hot spots in foods due to interference effects?

29. (a) Calculate the range of wavelengths for AM radio given its frequency range is 540 to 1600 kHz. (b) Do the same for the FM frequency range of 88.0 to 108 MHz.

30. A radio station utilizes frequencies between commercial AM and FM. What is the frequency of a 11.12-m-wavelength channel?

31. Find the frequency range of visible light, given that it encompasses wavelengths from 380 to 760 nm.

32. Combing your hair leads to excess electrons on the comb. How fast would you have to move the comb up and down to produce red light?

33. Electromagnetic radiation having a $15.0 - \mu\text{m}$ wavelength is classified as infrared radiation. What is its frequency?

34. Approximately what is the smallest detail observable with a microscope that uses ultraviolet light of frequency $1.20\times10^{15}\ \text{Hz}$?

35. A radar used to detect the presence of aircraft receives a pulse that has reflected off an object $6\times10^{-5}\ \text{s}$ after it was transmitted. What is the distance from the radar station to the reflecting object?

36. Some radar systems detect the size and shape of objects such as aircraft and geological terrain. Approximately what is the smallest observable detail utilizing 500-MHz radar?

37. Determine the amount of time it takes for X-rays of frequency $3\times10^{18}\ \text{Hz}$ to travel (a) 1 mm and (b) 1 cm.

38. If you wish to detect details of the size of atoms (about $1\times10^{-10}\ \text{m}$) with electromagnetic radiation, it must have a wavelength of about this size. (a) What is its frequency? (b) What type of electromagnetic radiation might this be?

39. If the Sun suddenly turned off, we would not know it until its light stopped coming. How long would that be, given that the Sun is $1.50\times10^{11}\ \text{m}$ away?

40. Distances in space are often quoted in units of light years, the distance light travels in one year. (a) How many meters is a light year? (b) How many meters is it to Andromeda, the nearest large galaxy, given that it is 2.00×10^{6} light years away? (c) The most distant galaxy yet discovered is 12.0×10^{9} light years away. How far is this in meters?

41. A certain 50.0-Hz AC power line radiates an electromagnetic wave having a maximum electric field strength of 13.0 kV/m. (a) What is the wavelength of this very low frequency electromagnetic wave? (b) What is its maximum magnetic field strength?

42. During normal beating, the heart creates a maximum 4.00-mV potential across 0.300 m of a person's chest, creating a 1.00-Hz electromagnetic wave. (a) What is the maximum electric field strength created? (b) What is the corresponding maximum magnetic field strength in the electromagnetic wave? (c) What is the wavelength of the electromagnetic wave?

43. (a) The ideal size (most efficient) for a broadcast antenna with one end on the ground is one-fourth the wavelength ($\lambda/4$) of the electromagnetic radiation being sent out. If a new radio station has such an antenna that is 50.0 m high, what frequency does it broadcast most efficiently? Is this in the AM or FM band? (b) Discuss the analogy of the fundamental resonant mode of an air column closed at one end to the resonance of currents on an antenna that is one-fourth their wavelength.

44. (a) What is the wavelength of 100-MHz radio waves used in an MRI unit? (b) If the frequencies are swept over a ±1.00 range centered on 100 MHz, what is the range of wavelengths broadcast?

45. (a) What is the frequency of the 193-nm ultraviolet radiation used in laser eye surgery? (b) Assuming the accuracy with which this EM radiation can ablate the cornea is directly proportional to wavelength, how much more accurate can this UV be than the shortest visible wavelength of light?

46. TV-reception antennas for VHF are constructed with cross wires supported at their centers, as shown in **Figure 24.27**. The ideal length for the cross wires is one-half the wavelength to be received, with the more expensive antennas having one for each channel. Suppose you measure the lengths of the wires for particular channels and find them to be 1.94 and 0.753 m long, respectively. What are the frequencies for these channels?

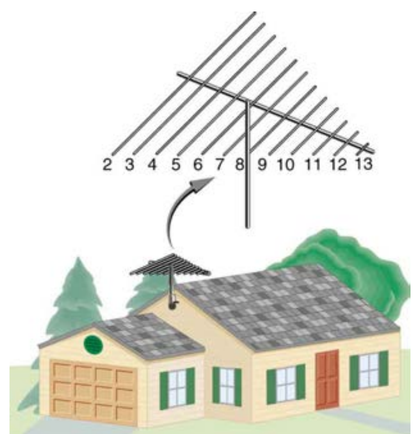


Figure 24.27 A television reception antenna has cross wires of various lengths to most efficiently receive different wavelengths.

47. Conversations with astronauts on lunar walks had an echo that was used to estimate the distance to the Moon. The sound spoken by the person on Earth was transformed into a radio signal sent to the Moon, and transformed back into sound on a speaker inside the astronaut's space suit. This sound was picked up by the microphone in the space suit (intended for the astronaut's voice) and sent back to Earth as a radio echo of sorts. If the round-trip time was 2.60 s, what was the approximate distance to the Moon, neglecting any delays in the electronics?

48. Lunar astronauts placed a reflector on the Moon's surface, off which a laser beam is periodically reflected. The distance to the Moon is calculated from the round-trip time. (a) To what accuracy in meters can the distance to the Moon be determined, if this time can be measured to

0.100 ns? (b) What percent accuracy is this, given the average distance to the Moon is $3.84\times10^{8}\ \text{m}$?

49. Radar is used to determine distances to various objects by measuring the round-trip time for an echo from the object. (a) How far away is the planet Venus if the echo time is 1000 s? (b) What is the echo time for a car 75.0 m from a Highway Police radar unit? (c) How accurately (in nanoseconds) must you be able to measure the echo time to an airplane 12.0 km away to determine its distance within 10.0 m?

50. Integrated Concepts

(a) Calculate the ratio of the highest to lowest frequencies of electromagnetic waves the eye can see, given the wavelength range of visible light is from 380 to 760 nm. (b) Compare this with the ratio of highest to lowest frequencies the ear can hear.

51. Integrated Concepts

(a) Calculate the rate in watts at which heat transfer through radiation occurs (almost entirely in the infrared) from $1.0\ \text{m}^2$ of the Earth's surface at night. Assume the emissivity is 0.90, the temperature of the Earth is 15°C, and that of outer space is 2.7 K. (b) Compare the intensity of this radiation with that coming to the Earth from the Sun during the day, which averages about $800\ \text{W/m}^2$, only half of which is absorbed. (c) What is the maximum magnetic field strength in the outgoing radiation, assuming it is a continuous wave?

24.4 Energy in Electromagnetic Waves

52. What is the intensity of an electromagnetic wave with a peak electric field strength of 125 V/m?

53. Find the intensity of an electromagnetic wave having a peak magnetic field strength of $4.00\times10^{-9}\ \text{T}$.

54. Assume the helium-neon lasers commonly used in student physics laboratories have power outputs of 0.250 mW. (a) If such a laser beam is projected onto a circular spot 1.00 mm in diameter, what is its intensity? (b) Find the peak magnetic field strength. (c) Find the peak electric field strength.

55. An AM radio transmitter broadcasts 50.0 kW of power uniformly in all directions. (a) Assuming all of the radio waves that strike the ground are completely absorbed, and that there is no absorption by the atmosphere or other objects, what is the intensity 30.0 km away? (Hint: Half the power will be spread over the area of a hemisphere.) (b) What is the maximum electric field strength at this distance?

56. Suppose the maximum safe intensity of microwaves for human exposure is taken to be $1.00\ \text{W/m}^2$. (a) If a radar unit leaks 10.0 W of microwaves (other than those sent by its antenna) uniformly in all directions, how far away must you be to be exposed to an intensity considered to be safe? Assume that the power spreads uniformly over the area of a sphere with no complications from absorption or reflection. (b) What is the maximum electric field strength at the safe intensity? (Note that early radar units leaked more than modern ones do. This caused identifiable health problems, such as cataracts, for people who worked near them.)

57. A 2.50-m-diameter university communications satellite dish receives TV signals that have a maximum electric field strength (for one channel) of $7.50\ \mu\text{V/m}$. (See **Figure 24.28**.) (a) What is the intensity of this wave? (b) What is the power received by the antenna? (c) If the orbiting satellite broadcasts uniformly over an area of $1.50\times10^{13}\ \text{m}^2$ (a large fraction of North America), how much power does it radiate?

Figure 24.28 Satellite dishes receive TV signals sent from orbit. Although the signals are quite weak, the receiver can detect them by being tuned to resonate at their frequency.

58. Lasers can be constructed that produce an extremely high intensity electromagnetic wave for a brief time—called pulsed lasers. They are used to ignite nuclear fusion, for example. Such a laser may produce an electromagnetic wave with a maximum electric field strength of $1.00\times10^{11}\ \text{V/m}$ for a time of 1.00 ns. (a) What is the maximum magnetic field strength in the wave? (b) What is the intensity of the beam? (c) What energy does it deliver on a 1.00-mm^2 area?

59. Show that for a continuous sinusoidal electromagnetic wave, the peak intensity is twice the average intensity ($I_0 = 2I_{\text{ave}}$), using either the fact that $E_0 = \sqrt{2}E_{\text{rms}}$, or $B_0 = \sqrt{2}B_{\text{rms}}$, where rms means average (actually root mean square, a type of average).

60. Suppose a source of electromagnetic waves radiates uniformly in all directions in empty space where there are no absorption or interference effects. (a) Show that the intensity is inversely proportional to r^2, the distance from the source squared. (b) Show that the magnitudes of the electric and magnetic fields are inversely proportional to r.

61. Integrated Concepts

An LC circuit with a 5.00-pF capacitor oscillates in such a manner as to radiate at a wavelength of 3.30 m. (a) What is the resonant frequency? (b) What inductance is in series with the capacitor?

62. Integrated Concepts

What capacitance is needed in series with an $800-\mu\text{H}$ inductor to form a circuit that radiates a wavelength of 196 m?

63. Integrated Concepts

Police radar determines the speed of motor vehicles using the same Doppler-shift technique employed for ultrasound in medical diagnostics. Beats are produced by mixing the double Doppler-shifted echo with the original frequency. If $1.50\times10^{9}\text{-Hz}$ microwaves are used and a beat frequency of 150 Hz is produced, what is the speed of the vehicle? (Assume the same Doppler-shift formulas are valid with the speed of sound replaced by the speed of light.)

64. Integrated Concepts

Assume the mostly infrared radiation from a heat lamp acts like a continuous wave with wavelength $1.50\ \mu\text{m}$. (a) If the lamp's 200-W output is focused on a person's shoulder, over a circular area 25.0 cm in diameter, what is the intensity in W/m^2? (b) What is the peak electric field strength? (c) Find the peak magnetic field strength. (d) How long will it take to increase the temperature of the 4.00-kg shoulder by $2.00^\circ\ \text{C}$, assuming no other heat transfer and given that its specific heat is $3.47\times10^{3}\ \text{J/kg}\cdot\text{ºC}$?

65. Integrated Concepts

On its highest power setting, a microwave oven increases the temperature of 0.400 kg of spaghetti by 45.0ºC in 120 s. (a) What was the rate of power absorption by the spaghetti, given that its specific heat is $3.76\times10^{3}\ \text{J/kg}\cdot\text{ºC}$? (b) Find the average intensity of the microwaves, given that they are absorbed over a circular area 20.0 cm in diameter. (c) What is the peak electric field strength of the microwave? (d) What is its peak magnetic field strength?

66. Integrated Concepts

Electromagnetic radiation from a 5.00-mW laser is concentrated on a 1.00-mm^2 area. (a) What is the intensity in W/m^2? (b) Suppose a 2.00-nC static charge is in the beam. What is the maximum electric force it experiences? (c) If the static charge moves at 400 m/s, what maximum magnetic force can it feel?

67. Integrated Concepts

A 200-turn flat coil of wire 30.0 cm in diameter acts as an antenna for FM radio at a frequency of 100 MHz. The magnetic field of the incoming electromagnetic wave is perpendicular to the coil and has a maximum strength of $1.00\times10^{-12}\ \text{T}$. (a) What power is incident on the coil? (b) What average emf is induced in the coil over one-fourth of a cycle? (c) If the radio receiver has an inductance of $2.50\ \mu\text{H}$, what capacitance must it have to resonate at 100 MHz?

68. Integrated Concepts

If electric and magnetic field strengths vary sinusoidally in time, being zero at $t = 0$, then $E = E_0 \sin 2\pi ft$ and $B = B_0 \sin 2\pi ft$. Let $f = 1.00\ \text{GHz}$ here. (a) When are the field strengths first zero? (b) When do they reach their most negative value? (c) How much time is needed for them to complete one cycle?

69. Unreasonable Results

A researcher measures the wavelength of a 1.20-GHz electromagnetic wave to be 0.500 m. (a) Calculate the speed at which this wave propagates. (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

70. Unreasonable Results

The peak magnetic field strength in a residential microwave oven is $9.20\times10^{-5}\ \text{T}$. (a) What is the intensity of the microwave? (b) What is unreasonable about this result? (c) What is wrong about the premise?

71. Unreasonable Results

An LC circuit containing a 2.00-H inductor oscillates at such a frequency that it radiates at a 1.00-m wavelength. (a) What is the capacitance of the circuit? (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

72. Unreasonable Results

An LC circuit containing a 1.00-pF capacitor oscillates at such a frequency that it radiates at a 300-nm wavelength. (a) What is the inductance of the circuit? (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

73. Create Your Own Problem

Consider electromagnetic fields produced by high voltage power lines. Construct a problem in which you calculate the intensity of this electromagnetic radiation in W/m^2 based on the measured magnetic field strength of the radiation in a home near the power lines. Assume these magnetic field strengths are known to average less than a μT.

The intensity is small enough that it is difficult to imagine mechanisms for biological damage due to it. Discuss how much energy may be radiating from a section of power line several hundred meters long and compare this to the power likely to be carried by the lines. An idea of how much power this is can be obtained by calculating the approximate current responsible for μT fields at distances of tens of meters.

74. Create Your Own Problem

Consider the most recent generation of residential satellite dishes that are a little less than half a meter in diameter. Construct a problem in which you calculate the power received by the dish and the maximum electric field strength of the microwave signals for a single channel received by the dish. Among the things to be considered are the power broadcast by the satellite and the area over which the power is spread, as well as the area of the receiving dish.

A ATOMIC MASSES

Table A1 Atomic Masses

Atomic Number, Z	Name	Atomic Mass Number, A	Symbol	Atomic Mass (u)	Percent Abundance or Decay Mode	Half-life, $t_{1/2}$
0	neutron	1	n	1.008 665	β^-	10.37 min
1	Hydrogen	1	^1H	1.007 825	99.985%	
	Deuterium	2	^2H or D	2.014 102	0.015%	
	Tritium	3	^3H or T	3.016 050	β^-	12.33 y
2	Helium	3	^3He	3.016 030	$1.38\times10^{-4}\%$	
		4	^4He	4.002 603	$\approx 100\%$	
3	Lithium	6	^6Li	6.015 121	7.5%	
		7	^7Li	7.016 003	92.5%	
4	Beryllium	7	^7Be	7.016 928	EC	53.29 d
		9	^9Be	9.012 182	100%	
5	Boron	10	^{10}B	10.012 937	19.9%	
		11	^{11}B	11.009 305	80.1%	
6	Carbon	11	^{11}C	11.011 432	EC, β^+	
		12	^{12}C	12.000 000	98.90%	
		13	^{13}C	13.003 355	1.10%	
		14	^{14}C	14.003 241	β^-	5730 y
7	Nitrogen	13	^{13}N	13.005 738	β^+	9.96 min
		14	^{14}N	14.003 074	99.63%	
		15	^{15}N	15.000 108	0.37%	
8	Oxygen	15	^{15}O	15.003 065	EC, β^+	122 s
		16	^{16}O	15.994 915	99.76%	
		18	^{18}O	17.999 160	0.200%	
9	Fluorine	18	^{18}F	18.000 937	EC, β^+	1.83 h
		19	^{19}F	18.998 403	100%	
10	Neon	20	^{20}Ne	19.992 435	90.51%	
		22	^{22}Ne	21.991 383	9.22%	
11	Sodium	22	^{22}Na	21.994 434	β^+	2.602 y
		23	^{23}Na	22.989 767	100%	
		24	^{24}Na	23.990 961	β^-	14.96 h
12	Magnesium	24	^{24}Mg	23.985 042	78.99%	
13	Aluminum	27	^{27}Al	26.981 539	100%	
14	Silicon	28	^{28}Si	27.976 927	92.23%	2.62h

Atomic Number, *Z*	Name	Atomic Mass Number, *A*	Symbol	Atomic Mass (u)	Percent Abundance or Decay Mode	Half-life, $t_{1/2}$
		31	^{31}Si	30.975 362	β^-	
15	Phosphorus	31	^{31}P	30.973 762	100%	
		32	^{32}P	31.973 907	β^-	14.28 d
16	Sulfur	32	^{32}S	31.972 070	95.02%	
		35	^{35}S	34.969 031	β^-	87.4 d
17	Chlorine	35	^{35}Cl	34.968 852	75.77%	
		37	^{37}Cl	36.965 903	24.23%	
18	Argon	40	^{40}Ar	39.962 384	99.60%	
19	Potassium	39	^{39}K	38.963 707	93.26%	
		40	^{40}K	39.963 999	0.0117%, EC, β^-	1.28×10^9 y
20	Calcium	40	^{40}Ca	39.962 591	96.94%	
21	Scandium	45	^{45}Sc	44.955 910	100%	
22	Titanium	48	^{48}Ti	47.947 947	73.8%	
23	Vanadium	51	^{51}V	50.943 962	99.75%	
24	Chromium	52	^{52}Cr	51.940 509	83.79%	
25	Manganese	55	^{55}Mn	54.938 047	100%	
26	Iron	56	^{56}Fe	55.934 939	91.72%	
27	Cobalt	59	^{59}Co	58.933 198	100%	
		60	^{60}Co	59.933 819	β^-	5.271 y
28	Nickel	58	^{58}Ni	57.935 346	68.27%	
		60	^{60}Ni	59.930 788	26.10%	
29	Copper	63	^{63}Cu	62.939 598	69.17%	
		65	^{65}Cu	64.927 793	30.83%	
30	Zinc	64	^{64}Zn	63.929 145	48.6%	
		66	^{66}Zn	65.926 034	27.9%	
31	Gallium	69	^{69}Ga	68.925 580	60.1%	
32	Germanium	72	^{72}Ge	71.922 079	27.4%	
		74	^{74}Ge	73.921 177	36.5%	
33	Arsenic	75	^{75}As	74.921 594	100%	
34	Selenium	80	^{80}Se	79.916 520	49.7%	
35	Bromine	79	^{79}Br	78.918 336	50.69%	
36	Krypton	84	^{84}Kr	83.911 507	57.0%	
37	Rubidium	85	^{85}Rb	84.911 794	72.17%	

Atomic Number, Z	Name	Atomic Mass Number, A	Symbol	Atomic Mass (u)	Percent Abundance or Decay Mode	Half-life, $t_{1/2}$
38	Strontium	86	^{86}Sr	85.909 267	9.86%	
		88	^{88}Sr	87.905 619	82.58%	
		90	^{90}Sr	89.907 738	β^-	28.8 y
39	Yttrium	89	^{89}Y	88.905 849	100%	
		90	^{90}Y	89.907 152	β^-	64.1 h
40	Zirconium	90	^{90}Zr	89.904 703	51.45%	
41	Niobium	93	^{93}Nb	92.906 377	100%	
42	Molybdenum	98	^{98}Mo	97.905 406	24.13%	
43	Technetium	98	^{98}Tc	97.907 215	β^-	4.2×10^{6}y
44	Ruthenium	102	^{102}Ru	101.904 348	31.6%	
45	Rhodium	103	^{103}Rh	102.905 500	100%	
46	Palladium	106	^{106}Pd	105.903 478	27.33%	
47	Silver	107	^{107}Ag	106.905 092	51.84%	
		109	^{109}Ag	108.904 757	48.16%	
48	Cadmium	114	^{114}Cd	113.903 357	28.73%	
49	Indium	115	^{115}In	114.903 880	95.7%, β^-	4.4×10^{14}y
50	Tin	120	^{120}Sn	119.902 200	32.59%	
51	Antimony	121	^{121}Sb	120.903 821	57.3%	
52	Tellurium	130	^{130}Te	129.906 229	33.8%, β^-	2.5×10^{21}y
53	Iodine	127	^{127}I	126.904 473	100%	
		131	^{131}I	130.906 114	β^-	8.040 d
54	Xenon	132	^{132}Xe	131.904 144	26.9%	
		136	^{136}Xe	135.907 214	8.9%	
55	Cesium	133	^{133}Cs	132.905 429	100%	
		134	^{134}Cs	133.906 696	EC, β^-	2.06 y
56	Barium	137	^{137}Ba	136.905 812	11.23%	
		138	^{138}Ba	137.905 232	71.70%	
57	Lanthanum	139	^{139}La	138.906 346	99.91%	
58	Cerium	140	^{140}Ce	139.905 433	88.48%	
59	Praseodymium	141	^{141}Pr	140.907 647	100%	
60	Neodymium	142	^{142}Nd	141.907 719	27.13%	
61	Promethium	145	^{145}Pm	144.912 743	EC, α	17.7 y
62	Samarium	152	^{152}Sm	151.919 729	26.7%	

Atomic Number, *Z*	Name	Atomic Mass Number, *A*	Symbol	Atomic Mass (u)	Percent Abundance or Decay Mode	Half-life, $t_{1/2}$
63	Europium	153	^{153}Eu	152.921 225	52.2%	
64	Gadolinium	158	^{158}Gd	157.924 099	24.84%	
65	Terbium	159	^{159}Tb	158.925 342	100%	
66	Dysprosium	164	^{164}Dy	163.929 171	28.2%	
67	Holmium	165	^{165}Ho	164.930 319	100%	
68	Erbium	166	^{166}Er	165.930 290	33.6%	
69	Thulium	169	^{169}Tm	168.934 212	100%	
70	Ytterbium	174	^{174}Yb	173.938 859	31.8%	
71	Lutecium	175	^{175}Lu	174.940 770	97.41%	
72	Hafnium	180	^{180}Hf	179.946 545	35.10%	
73	Tantalum	181	^{181}Ta	180.947 992	99.98%	
74	Tungsten	184	^{184}W	183.950 928	30.67%	
75	Rhenium	187	^{187}Re	186.955 744	62.6%, β^-	$4.6\times10^{10}\text{y}$
76	Osmium	191	^{191}Os	190.960 920	β^-	15.4 d
		192	^{192}Os	191.961 467	41.0%	
77	Iridium	191	^{191}Ir	190.960 584	37.3%	
		193	^{193}Ir	192.962 917	62.7%	
78	Platinum	195	^{195}Pt	194.964 766	33.8%	
79	Gold	197	^{197}Au	196.966 543	100%	
		198	^{198}Au	197.968 217	β^-	2.696 d
80	Mercury	199	^{199}Hg	198.968 253	16.87%	
		202	^{202}Hg	201.970 617	29.86%	
81	Thallium	205	^{205}Tl	204.974 401	70.48%	
82	Lead	206	^{206}Pb	205.974 440	24.1%	
		207	^{207}Pb	206.975 872	22.1%	
		208	^{208}Pb	207.976 627	52.4%	
		210	^{210}Pb	209.984 163	α, β^-	22.3 y
		211	^{211}Pb	210.988 735	β^-	36.1 min
		212	^{212}Pb	211.991 871	β^-	10.64 h
83	Bismuth	209	^{209}Bi	208.980 374	100%	
		211	^{211}Bi	210.987 255	α, β^-	2.14 min
84	Polonium	210	^{210}Po	209.982 848	α	138.38 d
85	Astatine	218	^{218}At	218.008 684	α, β^-	1.6 s

Atomic Number, Z	Name	Atomic Mass Number, A	Symbol	Atomic Mass (u)	Percent Abundance or Decay Mode	Half-life, $t_{1/2}$
86	Radon	222	^{222}Rn	222.017 570	α	3.82 d
87	Francium	223	^{223}Fr	223.019 733	α, β^-	21.8 min
88	Radium	226	^{226}Ra	226.025 402	α	$1.60\times10^3 y$
89	Actinium	227	^{227}Ac	227.027 750	α, β^-	21.8 y
90	Thorium	228	^{228}Th	228.028 715	α	1.91 y
		232	^{232}Th	232.038 054	100%, α	$1.41\times10^{10} y$
91	Protactinium	231	^{231}Pa	231.035 880	α	$3.28\times10^4 y$
92	Uranium	233	^{233}U	233.039 628	α	$1.59\times10^3 y$
		235	^{235}U	235.043 924	0.720%, α	$7.04\times10^8 y$
		236	^{236}U	236.045 562	α	$2.34\times10^7 y$
		238	^{238}U	238.050 784	99.2745%, α	$4.47\times10^9 y$
		239	^{239}U	239.054 289	β^-	23.5 min
93	Neptunium	239	^{239}Np	239.052 933	β^-	2.355 d
94	Plutonium	239	^{239}Pu	239.052 157	α	$2.41\times10^4 y$
95	Americium	243	^{243}Am	243.061 375	α, fission	$7.37\times10^3 y$
96	Curium	245	^{245}Cm	245.065 483	α	$8.50\times10^3 y$
97	Berkelium	247	^{247}Bk	247.070 300	α	$1.38\times10^3 y$
98	Californium	249	^{249}Cf	249.074 844	α	351 y
99	Einsteinium	254	^{254}Es	254.088 019	α, β^-	276 d
100	Fermium	253	^{253}Fm	253.085 173	EC, α	3.00 d
101	Mendelevium	255	^{255}Md	255.091 081	EC, α	27 min
102	Nobelium	255	^{255}No	255.093 260	EC, α	3.1 min
103	Lawrencium	257	^{257}Lr	257.099 480	EC, α	0.646 s
104	Rutherfordium	261	^{261}Rf	261.108 690	α	1.08 min
105	Dubnium	262	^{262}Db	262.113 760	α, fission	34 s
106	Seaborgium	263	^{263}Sg	263.11 86	α, fission	0.8 s
107	Bohrium	262	^{262}Bh	262.123 1	α	0.102 s
108	Hassium	264	^{264}Hs	264.128 5	α	0.08 ms
109	Meitnerium	266	^{266}Mt	266.137 8	α	3.4 ms

B SELECTED RADIOACTIVE ISOTOPES

Decay modes are α, β^-, β^+, electron capture (EC) and isomeric transition (IT). EC results in the same daughter nucleus as would β^+ decay. IT is a transition from a metastable excited state. Energies for $\beta^\pm$ decays are the maxima; average energies are roughly one-half the maxima.

Table B1 Selected Radioactive Isotopes

Isotope	$t_{1/2}$	DecayMode(s)	Energy(MeV)	Percent		γ -Ray Energy(MeV)	Percent
^{3}H	12.33 y	β^-	0.0186	100%			
^{14}C	5730 y	β^-	0.156	100%			
^{13}N	9.96 min	β^+	1.20	100%			
^{22}Na	2.602 y	β^+	0.55	90%	γ	1.27	100%
^{32}P	14.28 d	β^-	1.71	100%			
^{35}S	87.4 d	β^-	0.167	100%			
^{36}Cl	3.00×10^{5}y	β^-	0.710	100%			
^{40}K	1.28×10^{9}y	β^-	1.31	89%			
^{43}K	22.3 h	β^-	0.827	87%	γs	0.373	87%
						0.618	87%
^{45}Ca	165 d	β^-	0.257	100%			
^{51}Cr	27.70 d	EC			γ	0.320	10%
^{52}Mn	5.59d	β^+	3.69	28%	γs	1.33	28%
						1.43	28%
^{52}Fe	8.27 h	β^+	1.80	43%		0.169	43%
						0.378	43%
^{59}Fe	44.6 d	β^-s	0.273	45%	γs	1.10	57%
			0.466	55%		1.29	43%
^{60}Co	5.271 y	β^-	0.318	100%	γs	1.17	100%
						1.33	100%
^{65}Zn	244.1 d	EC			γ	1.12	51%
^{67}Ga	78.3 h	EC			γs	0.0933	70%
						0.185	35%
						0.300	19%
						others	
^{75}Se	118.5 d	EC			γs	0.121	20%
						0.136	65%
						0.265	68%
						0.280	20%
						others	
^{86}Rb	18.8 d	β^-s	0.69	9%	γ	1.08	9%
			1.77	91%			
^{85}Sr	64.8 d	EC			γ	0.514	100%
^{90}Sr	28.8 y	β^-	0.546	100%			
^{90}Y	64.1 h	β^-	2.28	100%			
^{99m}Tc	6.02 h	IT			γ	0.142	100%

Isotope	$t_{1/2}$	DecayMode(s)	Energy(MeV)	Percent		γ -Ray Energy(MeV)	Percent
^{113m}In	99.5 min	IT			γ	0.392	100%
^{123}I	13.0 h	EC			γ	0.159	$\approx 100\%$
^{131}I	8.040 d	β^- s	0.248	7%	γ s	0.364	85%
			0.607	93%		others	
			others				
^{129}Cs	32.3 h	EC			γ s	0.0400	35%
						0.372	32%
						0.411	25%
						others	
^{137}Cs	30.17 y	β^- s	0.511	95%	γ	0.662	95%
			1.17	5%			
^{140}Ba	12.79 d	β^-	1.035	$\approx 100\%$	γ s	0.030	25%
						0.044	65%
						0.537	24%
						others	
^{198}Au	2.696 d	β^-	1.161	$\approx 100\%$	γ	0.412	$\approx 100\%$
^{197}Hg	64.1 h	EC			γ	0.0733	100%
^{210}Po	138.38 d	α	5.41	100%			
^{226}Ra	$1.60 \times 10^3 y$	αs	4.68	5%	γ	0.186	100%
			4.87	95%			
^{235}U	$7.038 \times 10^8 y$	α	4.68	$\approx 100\%$	γ s	numerous	<0.400%
^{238}U	$4.468 \times 10^9 y$	αs	4.22	23%	γ	0.050	23%
			4.27	77%			
^{237}Np	$2.14 \times 10^6 y$	αs	numerous		γ s	numerous	<0.250%
			4.96 (max.)				
^{239}Pu	$2.41 \times 10^4 y$	αs	5.19	11%	γ s	7.5×10^{-5}	73%
			5.23	15%		0.013	15%
			5.24	73%		0.052	10%
						others	
^{243}Am	$7.37 \times 10^3 y$	αs	Max. 5.44		γ s	0.075	
			5.37	88%		others	
			5.32	11%			
			others				

C USEFUL INFORMATION

This appendix is broken into several tables.

- **Table C1**, Important Constants
- **Table C2**, Submicroscopic Masses
- **Table C3**, Solar System Data
- **Table C4**, Metric Prefixes for Powers of Ten and Their Symbols
- **Table C5**, The Greek Alphabet
- **Table C6**, SI units
- **Table C7**, Selected British Units
- **Table C8**, Other Units
- **Table C9**, Useful Formulae

Table C1 Important Constants[1]

Symbol	Meaning	Best Value	Approximate Value
c	Speed of light in vacuum	$2.99792458 \times 10^{8}\ \text{m/s}$	$3.00 \times 10^{8}\ \text{m/s}$
G	Gravitational constant	$6.67384(80) \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2$	$6.67 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2$
N_A	Avogadro's number	$6.02214129(27) \times 10^{23}$	6.02×10^{23}
k	Boltzmann's constant	$1.3806488(13) \times 10^{-23}\ \text{J/K}$	$1.38 \times 10^{-23}\ \text{J/K}$
R	Gas constant	$8.3144621(75)\ \text{J/mol} \cdot \text{K}$	$8.31\ \text{J/mol} \cdot \text{K} = 1.99\ \text{cal/mol} \cdot \text{K} = 0.0821\text{atm} \cdot \text{L/mol} \cdot \text{K}$
σ	Stefan-Boltzmann constant	$5.670373(21) \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}$	$5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}$
k	Coulomb force constant	$8.987551788... \times 10^{9}\ \text{N} \cdot \text{m}^2/\text{C}^2$	$8.99 \times 10^{9}\ \text{N} \cdot \text{m}^2/\text{C}^2$
q_e	Charge on electron	$-1.602176565(35) \times 10^{-19}\ \text{C}$	$-1.60 \times 10^{-19}\ \text{C}$
ε_0	Permittivity of free space	$8.854187817... \times 10^{-12}\ \text{C}^2/\text{N} \cdot \text{m}^2$	$8.85 \times 10^{-12}\ \text{C}^2/\text{N} \cdot \text{m}^2$
μ_0	Permeability of free space	$4\pi \times 10^{-7}\ \text{T} \cdot \text{m/A}$	$1.26 \times 10^{-6}\ \text{T} \cdot \text{m/A}$
h	Planck's constant	$6.62606957(29) \times 10^{-34}\ \text{J} \cdot \text{s}$	$6.63 \times 10^{-34}\ \text{J} \cdot \text{s}$

Table C2 Submicroscopic Masses[2]

Symbol	Meaning	Best Value	Approximate Value
m_e	Electron mass	$9.10938291(40) \times 10^{-31}\text{kg}$	$9.11 \times 10^{-31}\text{kg}$
m_p	Proton mass	$1.672621777(74) \times 10^{-27}\text{kg}$	$1.6726 \times 10^{-27}\text{kg}$
m_n	Neutron mass	$1.674927351(74) \times 10^{-27}\text{kg}$	$1.6749 \times 10^{-27}\text{kg}$
u	Atomic mass unit	$1.660538921(73) \times 10^{-27}\text{kg}$	$1.6605 \times 10^{-27}\text{kg}$

1. Stated values are according to the National Institute of Standards and Technology Reference on Constants, Units, and Uncertainty, **www.physics.nist.gov/cuu (http://www.physics.nist.gov/cuu)** (accessed May 18, 2012). Values in parentheses are the uncertainties in the last digits. Numbers without uncertainties are exact as defined.
2. Stated values are according to the National Institute of Standards and Technology Reference on Constants, Units, and Uncertainty, **www.physics.nist.gov/cuu (http://www.physics.nist.gov/cuu)** (accessed May 18, 2012). Values in parentheses are the uncertainties in the last digits. Numbers without uncertainties are exact as defined.

Table C3 Solar System Data

Sun	mass	$1.99 \times 10^{30} kg$
	average radius	$6.96 \times 10^{8} m$
	Earth-sun distance (average)	$1.496 \times 10^{11} m$
Earth	mass	$5.9736 \times 10^{24} kg$
	average radius	$6.376 \times 10^{6} m$
	orbital period	$3.16 \times 10^{7} s$
Moon	mass	$7.35 \times 10^{22} kg$
	average radius	$1.74 \times 10^{6} m$
	orbital period (average)	$2.36 \times 10^{6} s$
	Earth-moon distance (average)	$3.84 \times 10^{8} m$

Table C4 Metric Prefixes for Powers of Ten and Their Symbols

Prefix	Symbol	Value	Prefix	Symbol	Value
tera	T	10^{12}	deci	d	10^{-1}
giga	G	10^{9}	centi	c	10^{-2}
mega	M	10^{6}	milli	m	10^{-3}
kilo	k	10^{3}	micro	μ	10^{-6}
hecto	h	10^{2}	nano	n	10^{-9}
deka	da	10^{1}	pico	p	10^{-12}
—	—	$10^{0}(=1)$	femto	f	10^{-15}

Table C5 The Greek Alphabet

Alpha	A	α	Eta	H	η	Nu	N	ν	Tau	T	τ
Beta	B	β	Theta	Θ	θ	Xi	Ξ	ξ	Upsilon	Υ	υ
Gamma	Γ	γ	Iota	I	ι	Omicron	O	o	Phi	Φ	ϕ
Delta	Δ	δ	Kappa	K	κ	Pi	Π	π	Chi	X	χ
Epsilon	E	ε	Lambda	Λ	λ	Rho	P	ρ	Psi	Ψ	ψ
Zeta	Z	ζ	Mu	M	μ	Sigma	Σ	σ	Omega	Ω	ω

Table C6 SI Units

	Entity	Abbreviation	Name
Fundamental units	Length	m	meter
	Mass	kg	kilogram
	Time	s	second
	Current	A	ampere
Supplementary unit	Angle	rad	radian
Derived units	Force	$N = kg \cdot m / s^2$	newton
	Energy	$J = kg \cdot m^2 / s^2$	joule
	Power	$W = J / s$	watt
	Pressure	$Pa = N / m^2$	pascal
	Frequency	$Hz = 1 / s$	hertz
	Electronic potential	$V = J / C$	volt
	Capacitance	$F = C / V$	farad
	Charge	$C = s \cdot A$	coulomb
	Resistance	$\Omega = V / A$	ohm
	Magnetic field	$T = N / (A \cdot m)$	tesla
	Nuclear decay rate	$Bq = 1 / s$	becquerel

Table C7 Selected British Units

Length	$1 \text{ inch (in.)} = 2.54 \text{ cm (exactly)}$
	$1 \text{ foot (ft)} = 0.3048 \text{ m}$
	$1 \text{ mile (mi)} = 1.609 \text{ km}$
Force	$1 \text{ pound (lb)} = 4.448 \text{ N}$
Energy	$1 \text{ British thermal unit (Btu)} = 1.055 \times 10^3 \text{ J}$
Power	$1 \text{ horsepower (hp)} = 746 \text{ W}$
Pressure	$1 \text{ lb} / \text{in}^2 = 6.895 \times 10^3 \text{ Pa}$

Table C8 Other Units

Length	1 light year (ly) $= 9.46 \times 10^{15}$ m
	1 astronomical unit (au) $= 1.50 \times 10^{11}$ m
	1 nautical mile $= 1.852$ km
	1 angstrom(Å) $= 10^{-10}$ m
Area	1 acre (ac) $= 4.05 \times 10^{3}\ \text{m}^2$
	1 square foot (ft^2) $= 9.29 \times 10^{-2}\ \text{m}^2$
	1 barn (b) $= 10^{-28}\ \text{m}^2$
Volume	1 liter (L) $= 10^{-3}\ \text{m}^3$
	1 U.S. gallon (gal) $= 3.785 \times 10^{-3}\ \text{m}^3$
Mass	1 solar mass $= 1.99 \times 10^{30}$ kg
	1 metric ton $= 10^{3}$ kg
	1 atomic mass unit (u) $= 1.6605 \times 10^{-27}$ kg
Time	1 year (y) $= 3.16 \times 10^{7}$ s
	1 day (d) $= 86{,}400$ s
Speed	1 mile per hour (mph) $= 1.609$ km / h
	1 nautical mile per hour (naut) $= 1.852$ km / h
Angle	1 degree (°) $= 1.745 \times 10^{-2}$ rad
	1 minute of arc (′) $= 1 / 60$ degree
	1 second of arc (″) $= 1 / 60$ minute of arc
	1 grad $= 1.571 \times 10^{-2}$ rad
Energy	1 kiloton TNT (kT) $= 4.2 \times 10^{12}$ J
	1 kilowatt hour ($\text{kW} \cdot h$) $= 3.60 \times 10^{6}$ J
	1 food calorie (kcal) $= 4186$ J
	1 calorie (cal) $= 4.186$ J
	1 electron volt (eV) $= 1.60 \times 10^{-19}$ J
Pressure	1 atmosphere (atm) $= 1.013 \times 10^{5}$ Pa
	1 millimeter of mercury (mm Hg) $= 133.3$ Pa
	1 torricelli (torr) $= 1$ mm Hg $= 133.3$ Pa
Nuclear decay rate	1 curie (Ci) $= 3.70 \times 10^{10}$ Bq

Table C9 Useful Formulae

Circumference of a circle with radius r or diameter d	$C = 2\pi r = \pi d$
Area of a circle with radius r or diameter d	$A = \pi r^2 = \pi d^2 / 4$
Area of a sphere with radius r	$A = 4\pi r^2$
Volume of a sphere with radius r	$V = (4/3)\left(\pi r^3\right)$

3. Stated values are according to the National Institute of Standards and Technology Reference on Constants, Units, and Uncertainty, **www.physics.nist.gov/cuu (http://www.physics.nist.gov/cuu)** (accessed May 18, 2012). Values in parentheses are the uncertainties in the last digits. Numbers without uncertainties are exact as defined.
4. Stated values are according to the National Institute of Standards and Technology Reference on Constants, Units, and Uncertainty, **www.physics.nist.gov/cuu (http://www.physics.nist.gov/cuu)** (accessed May 18, 2012). Values in parentheses are the uncertainties in the last digits. Numbers without uncertainties are exact as defined.

D GLOSSARY OF KEY SYMBOLS AND NOTATION

In this glossary, key symbols and notation are briefly defined.

Table D1

Symbol	Definition
$\overline{\text{any symbol}}$	average (indicated by a bar over a symbol—e.g., $\bar{v}$ is average velocity)
°C	Celsius degree
°F	Fahrenheit degree
//	parallel
$\perp$	perpendicular
$\propto$	proportional to
$\pm$	plus or minus
$_0$	zero as a subscript denotes an initial value
α	alpha rays
α	angular acceleration
α	temperature coefficient(s) of resistivity
β	beta rays
β	sound level
β	volume coefficient of expansion
β^-	electron emitted in nuclear beta decay
β^+	positron decay
γ	gamma rays
γ	surface tension
$\gamma = 1/\sqrt{1 - v^2/c^2}$	a constant used in relativity
Δ	change in whatever quantity follows
δ	uncertainty in whatever quantity follows
ΔE	change in energy between the initial and final orbits of an electron in an atom
ΔE	uncertainty in energy
Δm	difference in mass between initial and final products
ΔN	number of decays that occur
Δp	change in momentum
Δp	uncertainty in momentum
$\Delta \text{PE}_\text{g}$	change in gravitational potential energy
$\Delta \theta$	rotation angle
Δs	distance traveled along a circular path
Δt	uncertainty in time
Δt_0	proper time as measured by an observer at rest relative to the process
ΔV	potential difference
Δx	uncertainty in position
ε_0	permittivity of free space
η	viscosity

Symbol	Definition
θ	angle between the force vector and the displacement vector
θ	angle between two lines
θ	contact angle
θ	direction of the resultant
θ_b	Brewster's angle
θ_c	critical angle
κ	dielectric constant
λ	decay constant of a nuclide
λ	wavelength
λ_n	wavelength in a medium
μ_0	permeability of free space
μ_{k}	coefficient of kinetic friction
μ_{s}	coefficient of static friction
v_e	electron neutrino
π^+	positive pion
π^-	negative pion
π^0	neutral pion
ρ	density
ρ_{c}	critical density, the density needed to just halt universal expansion
ρ_{fl}	fluid density
$\bar{\rho}_{\text{obj}}$	average density of an object
ρ / ρ_{w}	specific gravity
τ	characteristic time constant for a resistance and inductance (RL) or resistance and capacitance (RC) circuit
τ	characteristic time for a resistor and capacitor (RC) circuit
τ	torque
Υ	upsilon meson
Φ	magnetic flux
ϕ	phase angle
Ω	ohm (unit)
ω	angular velocity
A	ampere (current unit)
A	area
A	cross-sectional area
A	total number of nucleons
a	acceleration
a_{B}	Bohr radius
a_{c}	centripetal acceleration

Symbol	Definition
a_{t}	tangential acceleration
AC	alternating current
AM	amplitude modulation
atm	atmosphere
B	baryon number
B	blue quark color
$\bar{B}$	antiblue (yellow) antiquark color
b	quark flavor bottom or beauty
B	bulk modulus
B	magnetic field strength
B_{int}	electron's intrinsic magnetic field
B_{orb}	orbital magnetic field
BE	binding energy of a nucleus—it is the energy required to completely disassemble it into separate protons and neutrons
BE/A	binding energy per nucleon
Bq	becquerel—one decay per second
C	capacitance (amount of charge stored per volt)
C	coulomb (a fundamental SI unit of charge)
C_{p}	total capacitance in parallel
C_{s}	total capacitance in series
CG	center of gravity
CM	center of mass
c	quark flavor charm
c	specific heat
c	speed of light
Cal	kilocalorie
cal	calorie
COP_{hp}	heat pump's coefficient of performance
COP_{ref}	coefficient of performance for refrigerators and air conditioners
$\cos\theta$	cosine
$\cot\theta$	cotangent
$\csc\theta$	cosecant
D	diffusion constant
d	displacement
d	quark flavor down
dB	decibel
d_{i}	distance of an image from the center of a lens
d_{o}	distance of an object from the center of a lens
DC	direct current

Symbol	Definition
E	electric field strength
ε	emf (voltage) or Hall electromotive force
emf	electromotive force
E	energy of a single photon
E	nuclear reaction energy
E	relativistic total energy
E	total energy
E_0	ground state energy for hydrogen
E_0	rest energy
EC	electron capture
E_{cap}	energy stored in a capacitor
Eff	efficiency—the useful work output divided by the energy input
Eff_C	Carnot efficiency
E_{in}	energy consumed (food digested in humans)
E_{ind}	energy stored in an inductor
E_{out}	energy output
e	emissivity of an object
e^+	antielectron or positron
eV	electron volt
F	farad (unit of capacitance, a coulomb per volt)
F	focal point of a lens
$\mathbf{F}$	force
F	magnitude of a force
F	restoring force
F_{B}	buoyant force
F_{c}	centripetal force
F_{i}	force input
$\mathbf{F}_{\text{net}}$	net force
F_{o}	force output
FM	frequency modulation
f	focal length
f	frequency
f_0	resonant frequency of a resistance, inductance, and capacitance (RLC) series circuit
f_0	threshold frequency for a particular material (photoelectric effect)
f_1	fundamental
f_2	first overtone
f_3	second overtone

Symbol	Definition
f_{B}	beat frequency
f_{k}	magnitude of kinetic friction
f_{s}	magnitude of static friction
G	gravitational constant
G	green quark color
$\bar{G}$	antigreen (magenta) antiquark color
g	acceleration due to gravity
g	gluons (carrier particles for strong nuclear force)
h	change in vertical position
h	height above some reference point
h	maximum height of a projectile
h	Planck's constant
hf	photon energy
h_{i}	height of the image
h_{o}	height of the object
I	electric current
I	intensity
I	intensity of a transmitted wave
I	moment of inertia (also called rotational inertia)
I_0	intensity of a polarized wave before passing through a filter
I_{ave}	average intensity for a continuous sinusoidal electromagnetic wave
I_{rms}	average current
J	joule
J / Ψ	Joules/psi meson
K	kelvin
k	Boltzmann constant
k	force constant of a spring
K_{α}	x rays created when an electron falls into an $n=1$ shell vacancy from the $n=3$ shell
K_{β}	x rays created when an electron falls into an $n=2$ shell vacancy from the $n=3$ shell
kcal	kilocalorie
KE	translational kinetic energy
$\mathrm{KE}+\mathrm{PE}$	mechanical energy
KE_e	kinetic energy of an ejected electron
$\mathrm{KE}_{\mathrm{rel}}$	relativistic kinetic energy
$\mathrm{KE}_{\mathrm{rot}}$	rotational kinetic energy
$\overline{\mathrm{KE}}$	thermal energy

Symbol	Definition
kg	kilogram (a fundamental SI unit of mass)
L	angular momentum
L	liter
L	magnitude of angular momentum
L	self-inductance
ℓ	angular momentum quantum number
L_α	x rays created when an electron falls into an $n=2$ shell from the $n=3$ shell
L_e	electron total family number
L_μ	muon family total number
L_τ	tau family total number
L_f	heat of fusion
L_f and L_v	latent heat coefficients
L_orb	orbital angular momentum
L_s	heat of sublimation
L_v	heat of vaporization
L_z	*z* - component of the angular momentum
M	angular magnification
M	mutual inductance
m	indicates metastable state
m	magnification
m	mass
m	mass of an object as measured by a person at rest relative to the object
m	meter (a fundamental SI unit of length)
m	order of interference
m	overall magnification (product of the individual magnifications)
$m\left({}^A\mathrm{X}\right)$	atomic mass of a nuclide
MA	mechanical advantage
m_e	magnification of the eyepiece
m_e	mass of the electron
m_ℓ	angular momentum projection quantum number
m_n	mass of a neutron
m_o	magnification of the objective lens
mol	mole
m_p	mass of a proton
m_s	spin projection quantum number
N	magnitude of the normal force
N	newton
$\mathbf{N}$	normal force

Symbol	Definition
N	number of neutrons
n	index of refraction
n	number of free charges per unit volume
N_A	Avogadro's number
N_r	Reynolds number
$N \cdot m$	newton-meter (work-energy unit)
$N \cdot m$	newtons times meters (SI unit of torque)
OE	other energy
P	power
P	power of a lens
P	pressure
$\mathbf{p}$	momentum
p	momentum magnitude
p	relativistic momentum
$\mathbf{p}_{tot}$	total momentum
$\mathbf{p}'_{tot}$	total momentum some time later
P_{abs}	absolute pressure
P_{atm}	atmospheric pressure
P_{atm}	standard atmospheric pressure
PE	potential energy
PE_{el}	elastic potential energy
PE_{elec}	electric potential energy
PE_s	potential energy of a spring
P_g	gauge pressure
P_{in}	power consumption or input
P_{out}	useful power output going into useful work or a desired, form of energy
Q	latent heat
Q	net heat transferred into a system
Q	flow rate—volume per unit time flowing past a point
$+Q$	positive charge
$-Q$	negative charge
q	electron charge
q_p	charge of a proton
q	test charge
QF	quality factor
R	activity, the rate of decay
R	radius of curvature of a spherical mirror

Symbol	Definition
R	red quark color
$\bar{R}$	antired (cyan) quark color
R	resistance
R	resultant or total displacement
R	Rydberg constant
R	universal gas constant
r	distance from pivot point to the point where a force is applied
r	internal resistance
$r_{\perp}$	perpendicular lever arm
r	radius of a nucleus
r	radius of curvature
r	resistivity
r or rad	radiation dose unit
rem	roentgen equivalent man
rad	radian
RBE	relative biological effectiveness
RC	resistor and capacitor circuit
rms	root mean square
r_n	radius of the *n*th H-atom orbit
R_{p}	total resistance of a parallel connection
R_{s}	total resistance of a series connection
R_{s}	Schwarzschild radius
S	entropy
$\mathbf{S}$	intrinsic spin (intrinsic angular momentum)
S	magnitude of the intrinsic (internal) spin angular momentum
S	shear modulus
S	strangeness quantum number
s	quark flavor strange
s	second (fundamental SI unit of time)
s	spin quantum number
$\mathbf{s}$	total displacement
$\sec\theta$	secant
$\sin\theta$	sine
s_z	*z*-component of spin angular momentum
T	period—time to complete one oscillation
T	temperature
T_{c}	critical temperature—temperature below which a material becomes a superconductor
T	tension
T	tesla (magnetic field strength *B*)

Symbol	Definition
t	quark flavor top or truth
t	time
$t_{1/2}$	half-life—the time in which half of the original nuclei decay
$\tan\theta$	tangent
U	internal energy
u	quark flavor up
u	unified atomic mass unit
$\mathbf{u}$	velocity of an object relative to an observer
$\mathbf{u}'$	velocity relative to another observer
V	electric potential
V	terminal voltage
V	volt (unit)
V	volume
$\mathbf{v}$	relative velocity between two observers
v	speed of light in a material
$\mathbf{v}$	velocity
$\bar{\mathbf{v}}$	average fluid velocity
$V_\mathrm{B} - V_\mathrm{A}$	change in potential
$\mathbf{v}_\mathrm{d}$	drift velocity
V_p	transformer input voltage
V_rms	rms voltage
V_s	transformer output voltage
$\mathbf{v}_\mathrm{tot}$	total velocity
v_w	propagation speed of sound or other wave
$\mathbf{v}_\mathrm{w}$	wave velocity
W	work
W	net work done by a system
W	watt
w	weight
w_fl	weight of the fluid displaced by an object
W_c	total work done by all conservative forces
W_nc	total work done by all nonconservative forces
W_out	useful work output
X	amplitude
X	symbol for an element
${}^{Z}X_{N}$	notation for a particular nuclide
x	deformation or displacement from equilibrium
x	displacement of a spring from its undeformed position

Symbol	Definition
x	horizontal axis
X_{C}	capacitive reactance
X_{L}	inductive reactance
x_{rms}	root mean square diffusion distance
y	vertical axis
Y	elastic modulus or Young's modulus
Z	atomic number (number of protons in a nucleus)
Z	impedance

Index

V

W

X

Y

Z